Acoustic emission from one-dimensional vibrating porous panels in a single-sided flow

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The acoustic far-field pressure is determined for one-dimensional finite-chord panels with uniform porosity in a single-sided uniform flow. The unsteady, non-circulatory pressure on the panel is computed using a previously established analysis method. The acoustic field is computed using the Green's method. Results from this acoustic analysis identify the sensitivity of the far-field pressure magnitude and directivity to changes in flow Mach number, the reduced frequency of the panel vibration, and the panel porosity level characterized by a Darcy-type porosity boundary condition.

I. Introduction

Vibrating panels are common sound sources in many engineering devices such as passively tuned vibration absorbers (TVA)[1,2] and continue to be an active subject of research .[3,4] Acoustical data collected by Fahy and Gardonio[4] for vibrating porous panels demonstrate that high-porosity panels reduce their sound radiation efficiency by a factor of at least five relative to nonporous panels. However, few studies provide theoretical methods for determining the effect of panel porosity on the panel vibration and associated noise. In fact, the accompanying theoretical results found in the Fahy and Gardonio paper for non-perforated panels are not applicable to perforated ones.

Recent research in the field of aerodynamics indicates that porosity on airfoils and wings also leads to noise suppression .[5,6] In particular, a large body of research has recently emerged to predict the impact of a porous edge condition on the trailing edge turbulence scattering mechanism.[5–13] Howe[8] examined the scattering of turbulent noise sources from a semi-infinite rigid plane with porosity at the trailing edge section. Porosity and elasticity are combined to study the transmission of incident sound through an infinite poroelastic plate,[9] and this model has been employed by Jaworski and Peake[5] to investigate the scattering of turbulent noise sources from a poroelastic half-plane. Accordingly, trailing-edge porosity and elasticity can be tuned to eliminate in a scaling sense the predominant scattering mechanism of trailing edge noise.

Recent theoretical research into the aerodynamics of porous airfoils by Hajian and Jaworski motivates the current study, which develops a theoretical approach to investigate the use of porosity as a means of structural noise suppression for vibrating panels. To develop the theoretical model, a simple one-dimensional panel geometry is considered. The unsteady pressure on a vibrating one-dimensional panel has been studied theoretically as a step towards the development of a full unsteady aerodynamic analysis of porous airfoils.[14] The present work is an extension that focuses on the noise created by the vibrating one-dimensional porous panel exposed on one side to a uniform, low-speed flow.

The acoustic pressure field created by a vibrating panel in a baffle may be computed from the Rayleigh integral,[3] which is a convolution of the vibrational velocity and an appropriate Green's function. The analysis proceeds in the frequency domain by using the Green's function method to propagate the known surface pressure at a given frequency into the acoustic field. This method is used extensively in both structural acoustics[15,16] and aeroacoustics.[17,18] It is of interest to determine if the acoustic field directivity is affected by the vibrational frequency and the flow Mach number as is the case in aeroacoustic applications.[18] Of most significant interest, though, is the impact of a Darcy-type panel porosity on the noise generated by the vibrating panel.

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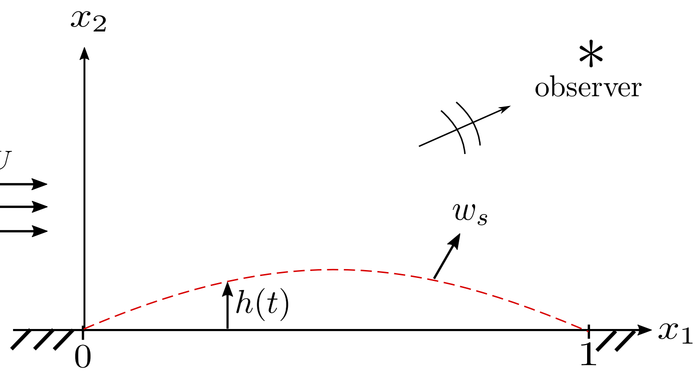

Figure 1: Schematic of a baffled, one-dimensional porous panel with seepage velocity $w_s(x,t)$ in a single-sided flow of speed U that is undergoing unsteady deformations $z_a(x,t)$.

II. Mathematical model

Consider a thin panel undergoing prescribed unsteady motions in a two-dimensional steady, single-sided incompressible flow. When the panel is in a baffle, there is no wake produced. Therefore the analysis does not require a wake vortex sheet nor the imposition of a Kutta condition. As such, one can consider only the non-circulatory force on the panel. The non-circulatory pressure distribution on such a porous panel has been determined in Ref. [14]. The method does not include the mass and stiffness of the panel, as would be the case in the structural acoustics literature. In this section, the method for determining the associated acoustics is described.

A. Acoustics of a porous panel

In the half-plane above the panel, the governing equation for the acoustic pressure is the 2D convective wave equation:[17]

$$\left[M_\infty^2\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x_1}\right)^2-\nabla^2\right]p(\mathbf{x},t)=0, \tag{1}$$

where c is the speed of sound and M_∞ is the Mach number. For a chord length l, mean flow speed U, and fluid density ρ, all terms in Eq. (1) have been nondimensionalized using l, l/U, and $\frac{1}{2}\rho U^2$ as the length, time, and pressure scales, respectively.

The wave equation is transformed using the Prandtl-Glauert transformation ($\tilde{x}_1 = x_1$, $\tilde{x}_2 = \beta_\infty x_2$), and the additional transformation $P = p(\mathbf{x},t)e^{-i(\omega t+M_\infty K x_1)}$ following Reissner[19] and Graham[20] to obtain

$$\begin{aligned}&\left(M_\infty^2-1\right)\tilde{\nabla}^2 P+2iM_\infty\left(\omega M_\infty+KM_\infty^2-K\right)\frac{\partial P}{\partial x_1}\\&+M_\infty^2\left(K^2-2M_\infty K\omega-\omega^2-M_\infty^2K^2\right)P=0,\end{aligned} \tag{2}$$

where ω is a dimensionless frequency $\omega=\omega_0 l/U$, for dimensional ω_0. By choosing

$$K=\frac{\omega M_\infty}{{\beta_\infty}^2}, \tag{3}$$

where $\beta_\infty^2 = 1-M_\infty^2$, the coefficient of $\partial P/\partial x_1$ vanishes, and the convective wave equation is reduced to a Helmholtz equation for P:

$$\left(\tilde{\nabla}^2+K^2\right)P=0. \tag{4}$$

Green's method is now employed in the fluid half space to evaluate the values of P in the field:

$$P(\tilde{\mathbf{x}})=\frac{1}{2\pi}\int_0^1\left[P(\tilde{\mathbf{y}})\frac{\partial G(\tilde{\mathbf{y}}|\tilde{\mathbf{x}})}{\partial y_2}-G(\tilde{\mathbf{y}}|\tilde{\mathbf{x}})\frac{\partial P(\tilde{\mathbf{y}})}{\partial y_2}\right]dy_1. \tag{5}$$

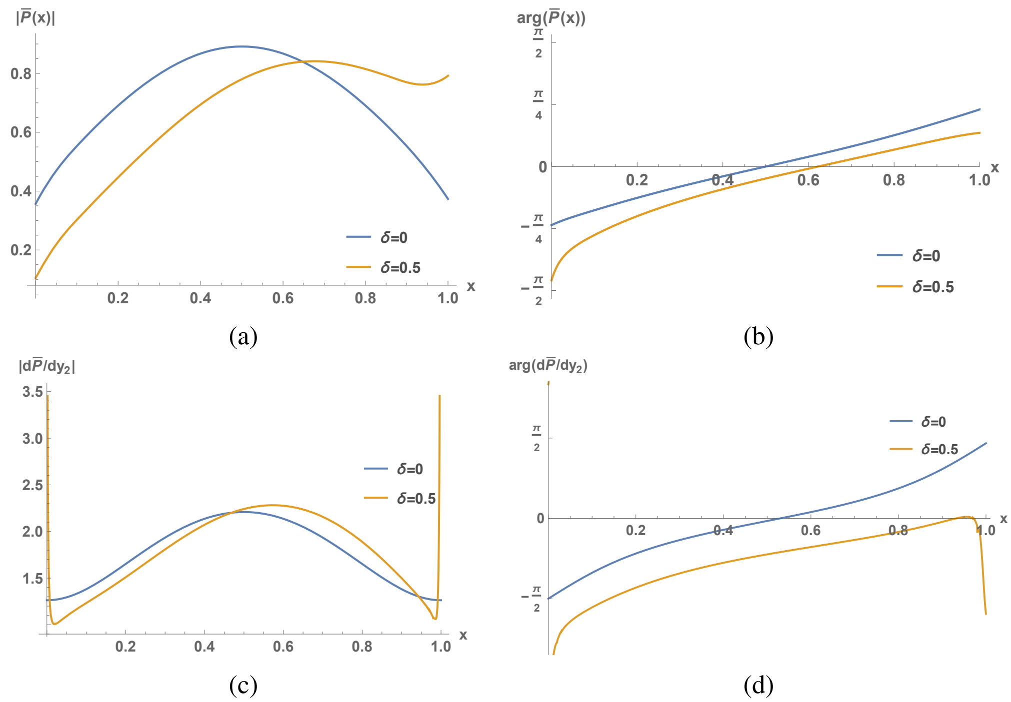

Figure 2: Magnitude and argument of $\bar{P}$ and $\partial\bar{P}/\partial y_2$ on non-porous ($\delta = 0$) and porous panel with $\delta = 0.2$ at $\omega = 10$: (a) magnitude of $\bar{P}$; (b) argument of $\bar{P}$; (c) magnitude of $\partial\bar{P}/\partial y_2$; (d) argument of $\partial\bar{P}/\partial y_2$.

Here $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ are the observation and source points, respectively, in the Prandtl-Glauert plane. In Eq. (5), G denotes the appropriate Green's function satisfying a Neumann boundary condition on the panel and baffle. As such, the Green's function must satisfy the two-dimensional Helmholtz equation:

$$\left(\tilde{\nabla}^2_{\tilde{\mathbf{y}}} + K^2\right) G(\tilde{\mathbf{y}}|\tilde{\mathbf{x}}) = -2\pi\boldsymbol{\delta}(\tilde{\mathbf{y}} - \tilde{\mathbf{x}}), \tag{6}$$

where $\boldsymbol{\delta}$ is the Dirac delta function. The appropriate Green's function is:

$$G(\tilde{\mathbf{y}}|\tilde{\mathbf{x}}) = -i\frac{\pi}{2} H_0^{(2)}\left(K|\tilde{\mathbf{y}} - \tilde{\mathbf{x}}|\right), \tag{7}$$

where $H_0^{(2)}$ is the Hankel function of the second kind. Now, $\partial G/\partial y_2$ and $\partial P/\partial y_2$ are needed to evaluate Eq. (5):

$$\frac{\partial G(\tilde{\mathbf{y}}|\tilde{\mathbf{x}})}{\partial \tilde{y}_2}\Big|_{\tilde{y}_2=0} = -\frac{i\pi K\beta_\infty x_2}{2}\frac{H_1^{(2)}\left(K|\tilde{\mathbf{y}} - \tilde{\mathbf{x}}|\right)}{|\tilde{\mathbf{y}} - \tilde{\mathbf{x}}|}, \tag{8}$$

and $\partial P/\partial y_2$ can be evaluated using the linearized Euler equation for incompressible flows:

$$\begin{aligned} \frac{\partial P(\tilde{\mathbf{y}})}{\partial \tilde{y}_2}\Big|_{\tilde{y}_2=0} &= \frac{\partial p(\tilde{\mathbf{y}}, t)}{\partial \tilde{y}_2}\Big|_{\tilde{y}_2=0} e^{-i(\omega t + M_\infty K\tilde{y}_1)} \\ &= \frac{-2}{\beta_\infty}\left(\frac{\partial^2\phi}{\partial y_2 \partial t} + \frac{\partial^2\phi}{\partial y_2\partial y_1}\right) e^{-i(\omega t + M_\infty K y_1)} \\ &= \frac{-2}{\beta_\infty}\Big(\frac{\partial w(y_1, t)}{\partial t} + \frac{\partial w(y_1, t)}{\partial y_1}\Big) e^{-i(\omega t + M_\infty K y_1)}, \end{aligned} \tag{9}$$

where $w(y_1, t) = \partial\phi/\partial y_2|_{y_2=0}$ is the perturbation flow velocity on the panel surface and is given by[21]

$$w(y_1, t) = w_s + \frac{\partial z_a}{\partial y_1} + \frac{\partial z_a}{\partial t}. \tag{10}$$

Here w_s denotes the seepage velocity. For a panel with a Darcy-type porosity distribution, the local flow rate is linearly proportional to the porosity and the dimensionless pressure distribution:[21,22]

$$w_s = -\frac{1}{2}\rho U C R(y_1) p(y_1, t). \tag{11}$$

The integration in Eq. (5) is formally performed from $y_1 = -\infty$ to $y_1 = +\infty$ and then around a semicircle at $|\mathbf{y}| \to \infty$. Both G and $\partial G/\partial y_2$ tend to zero on the semicircle, leaving only the integration over the y_1 axis. $\partial P/\partial y_2$ is zero on the baffle, but P is not. The contribution of baffle pressure is not considered in the current work. As such, the integration is then only over the panel itself.

III. Uniformly-porous panels with simply-supported ends

The acoustic emission from the special case of uniformly-porous panels, $R(x) = 1$, with simply-supported ends is presented here. For uniformly-porous panels with harmonic motions, such that $z_a(x, t) = X(x)e^{i\omega t}$ and $p(y_1, t) = \bar{P}(y_1)e^{i\omega t}$, where $\bar{P}(y_1)$ is a complex-valued function, the non-circulatory fluid pressure on the panel is:[14]

$$\bar{P}(y_1) = \frac{O(y_1)}{1+\delta^2} - \frac{\delta}{\pi(1+\delta^2)}\left(\frac{y_1}{1-y_1}\right)^{\frac{1}{\pi}\tan^{-1}\delta} \fint_0^1 \frac{O(\xi)}{\xi - y_1}\left(\frac{1-\xi}{\xi}\right)^{\frac{1}{\pi}\tan^{-1}\delta} d\xi, \tag{12}$$

where

$$O(y_1) = \frac{2}{\pi}\fint_0^1 \frac{X'(\xi) + i\omega X(\xi)}{\xi - y_1} d\xi - \frac{2i\omega}{\pi}\int_0^1 [X'(\xi) + i\omega X(\xi)] \ln|y_1 - \xi| d\xi. \tag{13}$$

Considering the simply-supported boundary condition $X(y_1) = \epsilon \sin(\pi y_1)$, $\epsilon = 0.01$, Eq. (9) can be recast in the following form:

$$\frac{\partial P(\tilde{y}_1, \tilde{y}_2)}{\partial \tilde{y}_2}\Big|_{\tilde{y}_2=0} = \frac{1}{\beta_\infty}\Big[2\epsilon(\pi^2 + \omega^2)\sin(\pi y_1) - 4i\epsilon\pi\omega\cos(\pi y_1) + \delta\left(\frac{\partial \bar{P}(y_1)}{\partial y_1} + i\omega \bar{P}(y_1)\right)\Big] e^{-iM_\infty K y_1}, \tag{14}$$

where $\bar{P}$ defines the transformed pressure distribution on the panel given by Eq. (12). The magnitude and argument of $\bar{P}$ and $\partial \bar{P}/\partial y_2$ on the surface are shown in Fig. 2 for non-porous and porous panels at $\omega = 10$.

Substituting Eqs. (7,8,12,14) into Eq. (5) and performing the integration lead to the pressure evaluated at any point $(\tilde{x}_1, \tilde{x}_2)$ in the field. Note that the theory presented in Ref. [14] is derived for incompressible flows, where the results for the panel surface pressures are strictly valid in the zero Mach number limit. Therefore, in the present study, acoustic pressures are evaluated for background mean flows with $M \lesssim 0.3$ for consistency.

Figure 3 shows the amplitude of acoustic pressure produced by a non-porous and a porous panel for $M = 0.1$ and $\delta = 0.5$ at different values of reduced frequency. The panel considered in this problem is exposed to a single-sided flow and therefore propagates the acoustic pressure into the field similarly to a volumetric monopole sound source. Note that Green's theorem produces both monopole and dipole terms, *i.e.* the first term $P\partial G/\partial y_2$ in Eq. (5) produces dipole contribution and the term $G\partial P/\partial y_2$ produces monopole contribution. However, for the parameters considered here, the monopole contribution is dominant.

Figure 3(a) indicates that the far-field pressure produced by a non-porous panel is symmetric for a fixed Mach number. Moreover, at low frequencies, the amplitude of the produced sound decreases by increasing ω. However, for frequencies in Fig. 3(b) that are larger than a critical value, say, ω^*, the sound produced by structural vibration increases for larger values of frequencies. Similar behavior is observed in Fig. 3(c) and 3(d) for vibrating porous panels with $\delta = 0.5$; however, porosity breaks the left-right symmetry of the directivity pattern for large ω and increases the value of ω^*.

A comparison is made between the acoustic emission from porous and non-porous panels in Fig. 4. As illustrated in Fig. 4(b), for a fixed Mach number $M = 0.1$, the acoustic pressure emission from a non-porous panel decreases by introducing porosity at frequency $\omega = 5$. However, Fig. 4(a) indicates that a porous panel produces a larger sound pressure at the lower reduced frequency $\omega = 0.1$. Previous results in the literature[6,13,23,24] predict the attenuation of far-field sound at low frequencies by introducing porosity, which is different from the result obtained in the present work. However, the papers mentioned above consider the acoustic field from a sound wave hitting a porous panel or edge, whereas in this study the sound is produced by forced panel motion.

Figure 5 investigates the effect of Mach number in the far-field acoustic emission for non-porous and porous panels. At a constant reduced frequency $\omega = 5$, increasing the Mach number reduces the sound pressure level and it

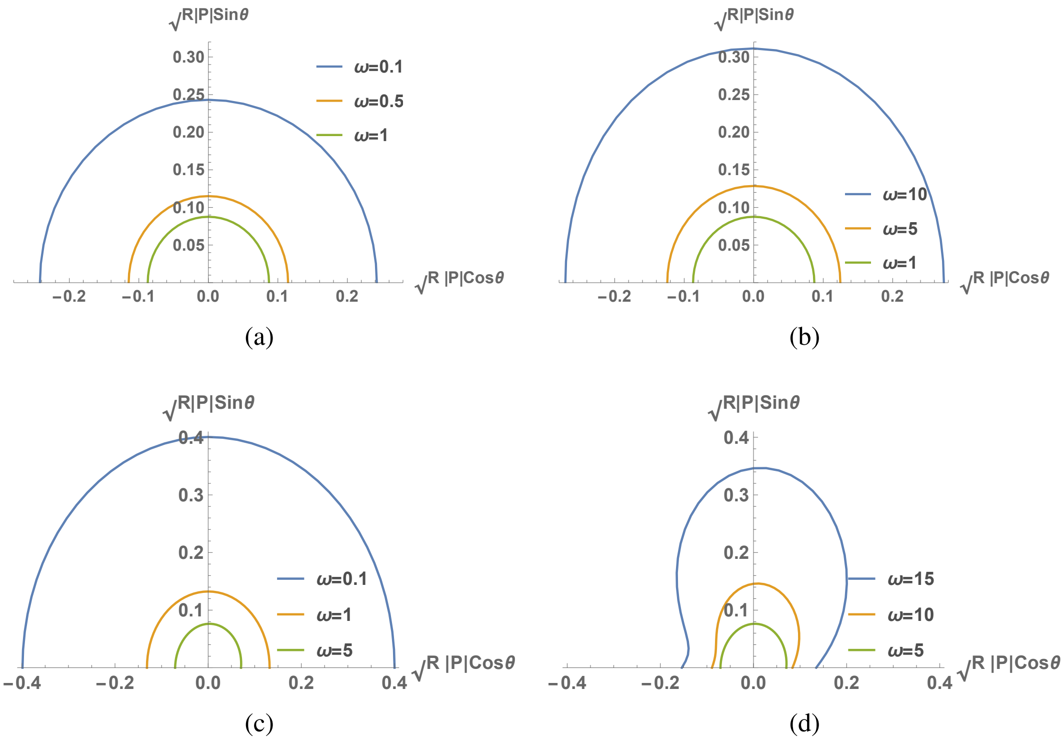

Figure 3: Acoustic emission at $M = 0.1$ for different values of frequency: (a)-(b) from a non-porous vibrating panel, (c)-(d) from a porous vibrating panel with $\delta = 0.5$.

rotates the directivity clockwise, *i.e.* in the downstream direction, this rotation can be interpreted from Eq. (14). For high porosity case, $\delta = 0.5$, an increase in the pressure level is observed in the plane of the panel along the upstream direction. Therefore, larger values of porosity parameter δ are needed to reduce the sound generated from vibrating panels in all directions. The result of this study indicates that even at high frequencies, the introduction of porosity does not always reduce the sound pressure.

IV. Conclusion

The present study determines the acoustic far-field pressure for finite-chord porous panels with simply-supported end conditions and no wake effect. The free space Green's function for the two-dimensional Helmholtz equation propagates into the acoustic field the unsteady non-circulatory forces on the panel, which are known in closed form from the established analysis. The amplitude of the sound produced by panels with different porosity is compared for different values of the dimensionless porosity parameter δ and reduced frequency ω. Results from this study indicate that, at low Mach numbers, increasing the magnitude of a Darcy-type porosity parameter leads to a reduction in the acoustic emission from a vibrating panel at high frequencies, while the introduction of porosity does not necessarily reduce the sound pressure for lower frequencies and larger Mach numbers in the low-subsonic range.

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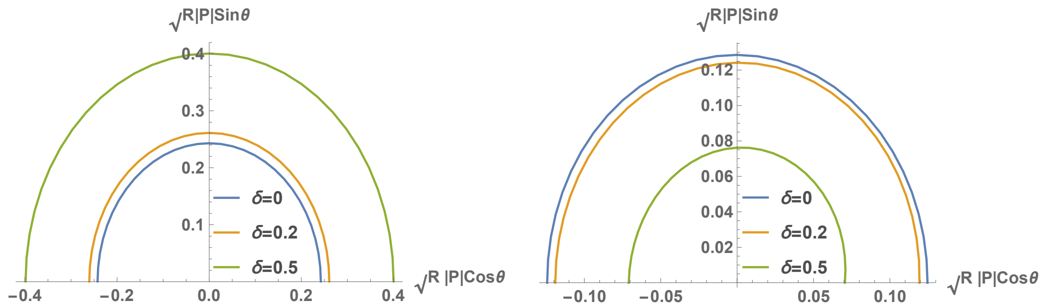
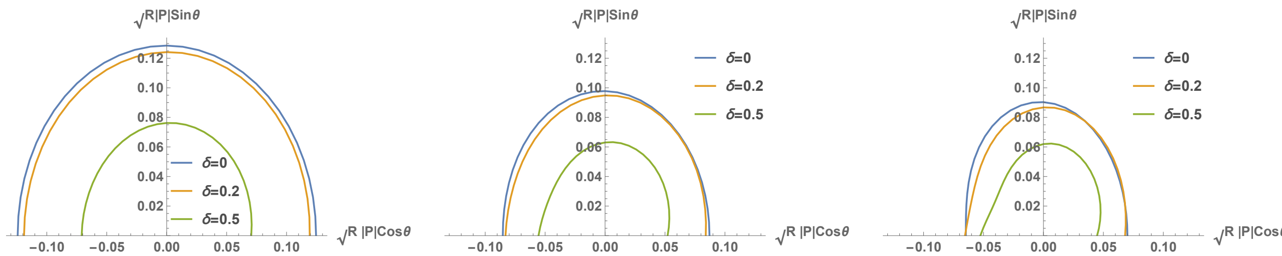


(a) (b)

Figure 4: Frequency dependence of the acoustic emission from porous and non-porous panels at $M = 0.1$: (a) $\omega = 0.1$; (b) $\omega = 5$.



(a) (b) (c)

Figure 5: Mach number dependence of the acoustic emission from non-porous and porous panels with $\omega = 5$: (a) $M = 0.1$; (b) $M = 0.2$; (c) $M = 0.3$.

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