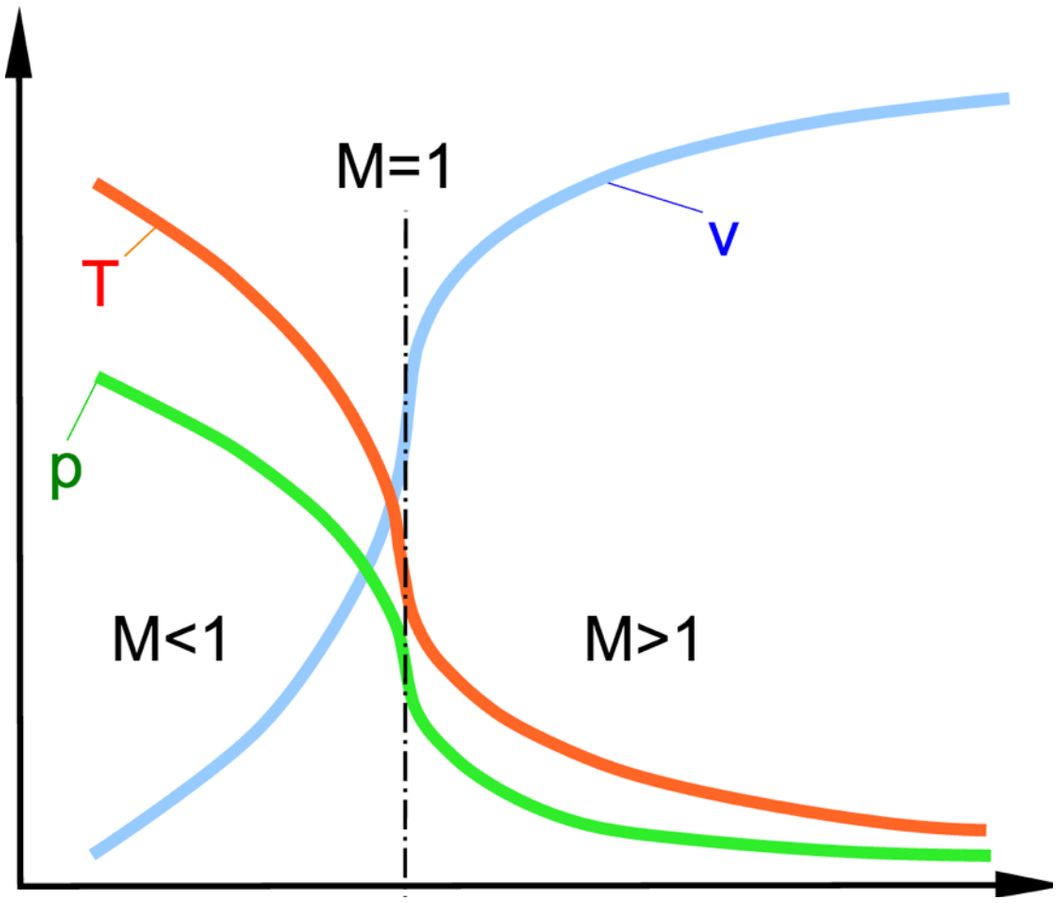
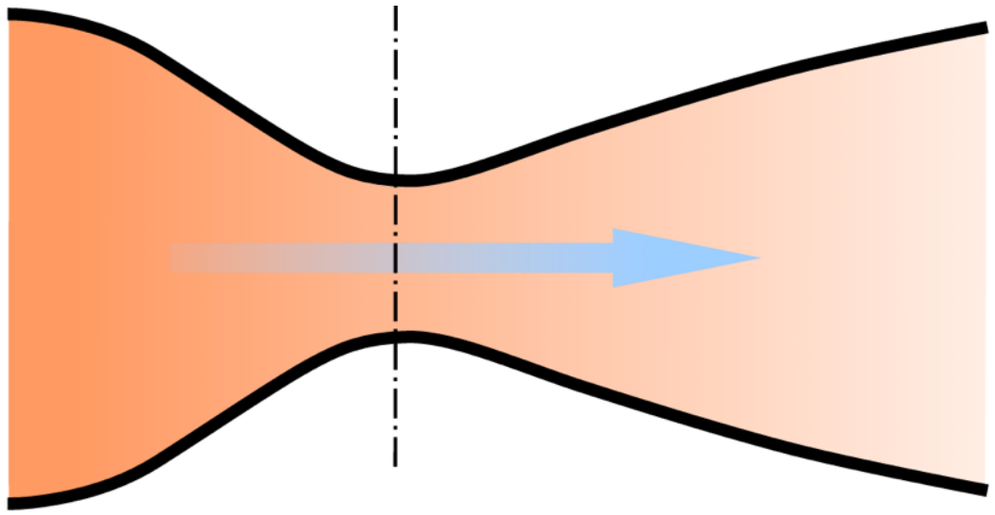
FLOW THROUGH A DE LAVAL NOZZLE

Connor Robinson
Computational Fluid Dynamics, ME 702
December 20th, 2016

Nozzles change flow speeds by changing the area through which the fluid flows





We can gain physical insight about the flow using the governing equations

$$\nabla \cdot (\rho u) = 0$$

$$u\frac{du}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx}$$

$$(1 - M^2)\frac{1}{v}\frac{du}{dx} = -\frac{1}{A}\frac{dA}{dx}$$

We can gain physical insight about the flow using the governing equations

$$\nabla \cdot (\rho u) = 0$$

$$u\frac{du}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx}$$

$$(1 - M^2)\frac{1}{v}\frac{du}{dx} = -\frac{1}{A}\frac{dA}{dx}$$

Nothing too interesting happens in the subsonic case

$$(1 - M^2)\frac{1}{v}\frac{du}{dx} = -\frac{1}{A}\frac{dA}{dx}$$

$M < 1 \rightarrow +$

Signs are opposite: As area increases, speed decreases

Thinking about a nozzle: Starts slow, speeds up in the throat, then slows down again.

In the supersonic case, get smooth acceleration throughout the nozzle

$$(1 - M^2)\frac{1}{v}\frac{du}{dx} = -\frac{1}{A}\frac{dA}{dx}$$

$M > 1 \rightarrow -$

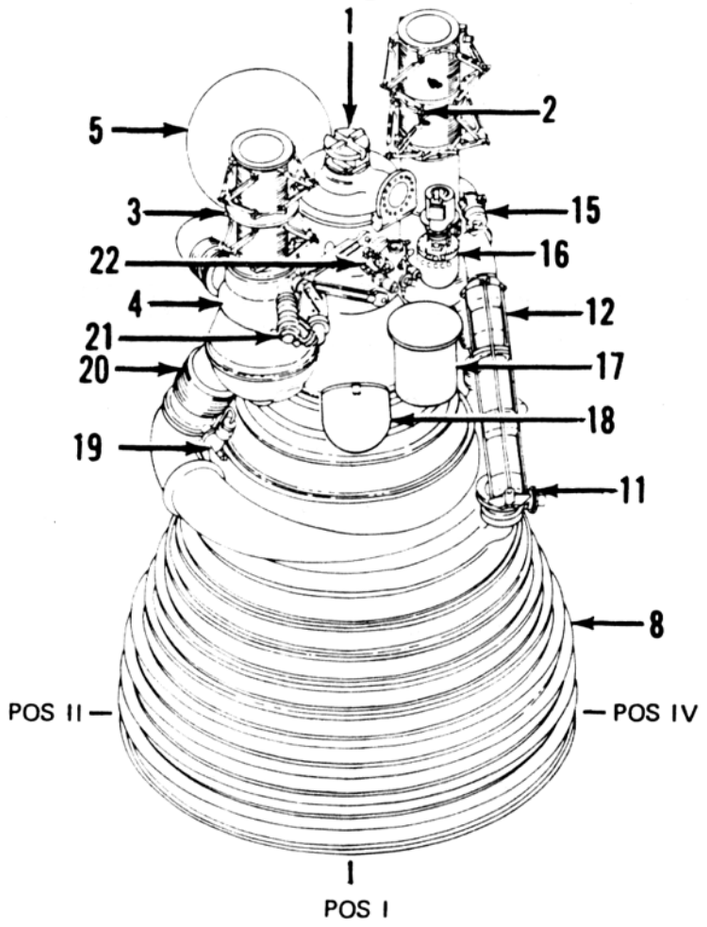
Signs are the same: As area increases, speed increases

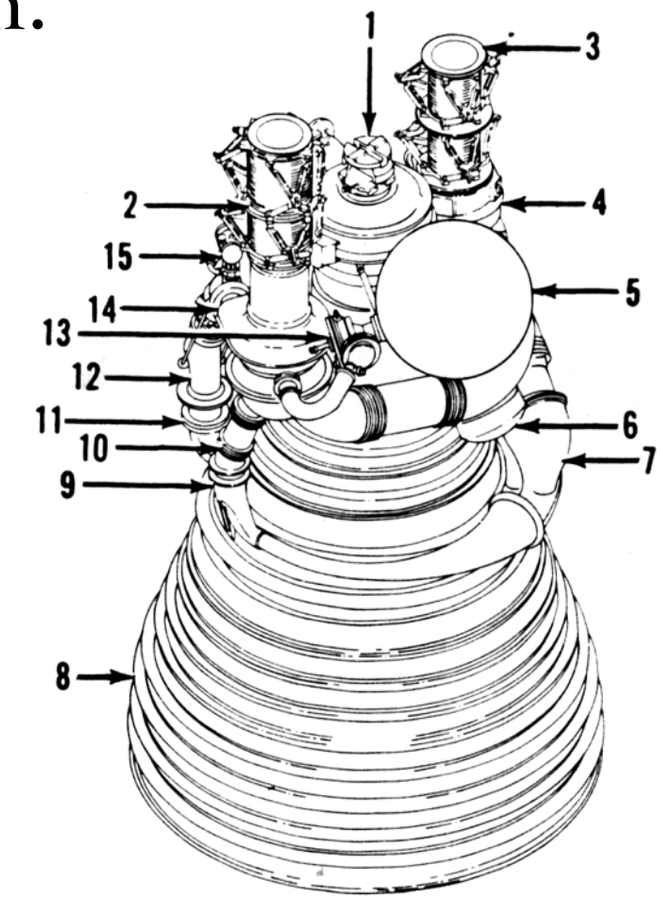
Thinking about a nozzle: Starts slow, speeds up in the throat, then continues to speed up!

This is known as a de Laval nozzle.

Transforming subsonic flow into supersonic flow has many applications

For example: Going to the moon.





VIEW ROTATED 180°

NASA Apollo 11 Flight Journal

LEGEND

1. GIMBAL
2. FUEL INLET DUCT
3. OXIDIZER INLET DUCT
4. OXIDIZER TURBOPUMP
5. START TANK
6. AUXILIARY FLIGHT INSTRUMENTATION PACKAGE
7. EXHAUST MANIFOLD
8. THRUST CHAMBER
9. OXIDIZER TURBINE BYPASS VALVE
10. TURBINE BYPASS DUCT
11. MAIN FUEL VALVE
12. HIGH PRESSURE FUEL DUCT
13. START TANK DISCHARGE VALVE
14. FUEL TURBOPUMP
15. FUEL BLEED VALVE
16. GAS GENERATOR
17. ELECTRICAL CONTROL PACKAGE
18. PRIMARY FLIGHT INSTR. PACKAGE
19. ANTI-FLOOD CHECK VALVE
20. HEAT EXCHANGER
21. MIXTURE RATIO CONTROL VALVE
22. PNEUMATIC CONTROL PACKAGE

There are also astrophysical applications: Solar wind

Parker (1963)

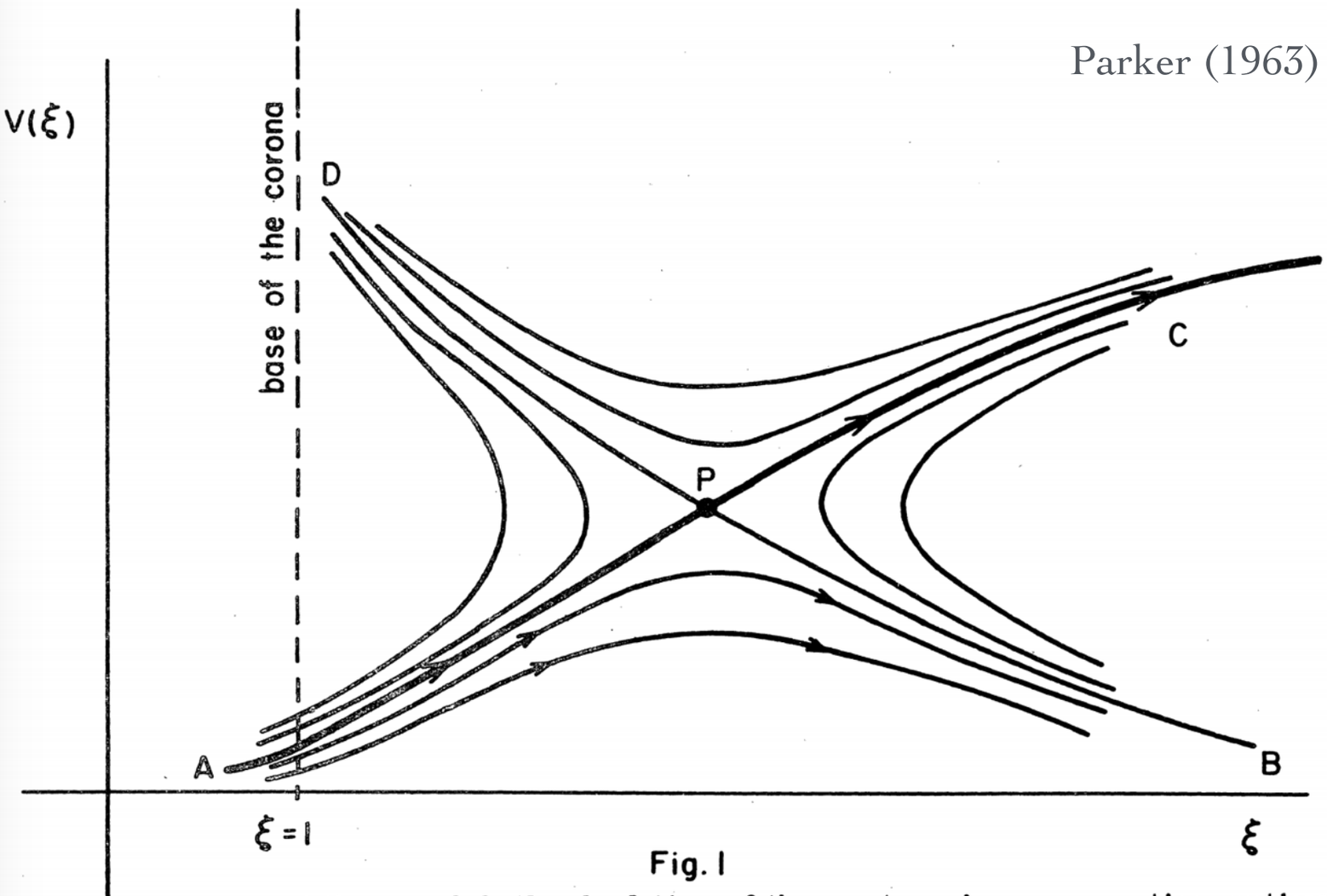
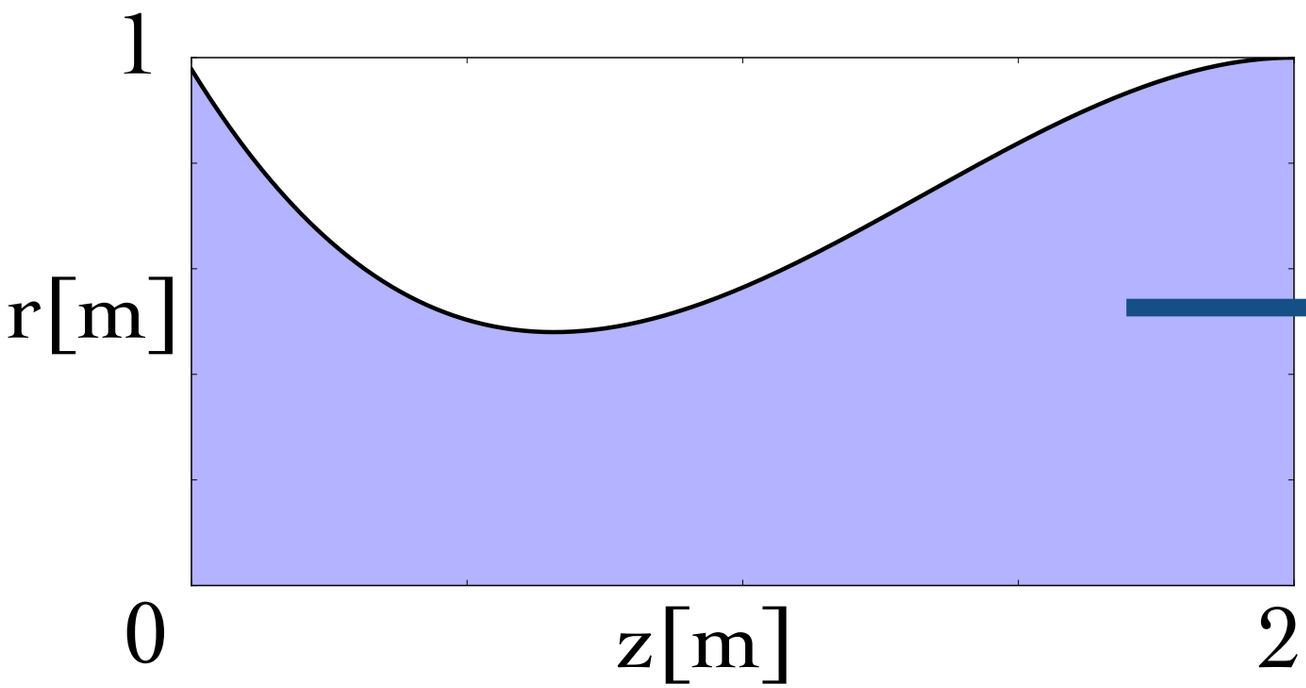
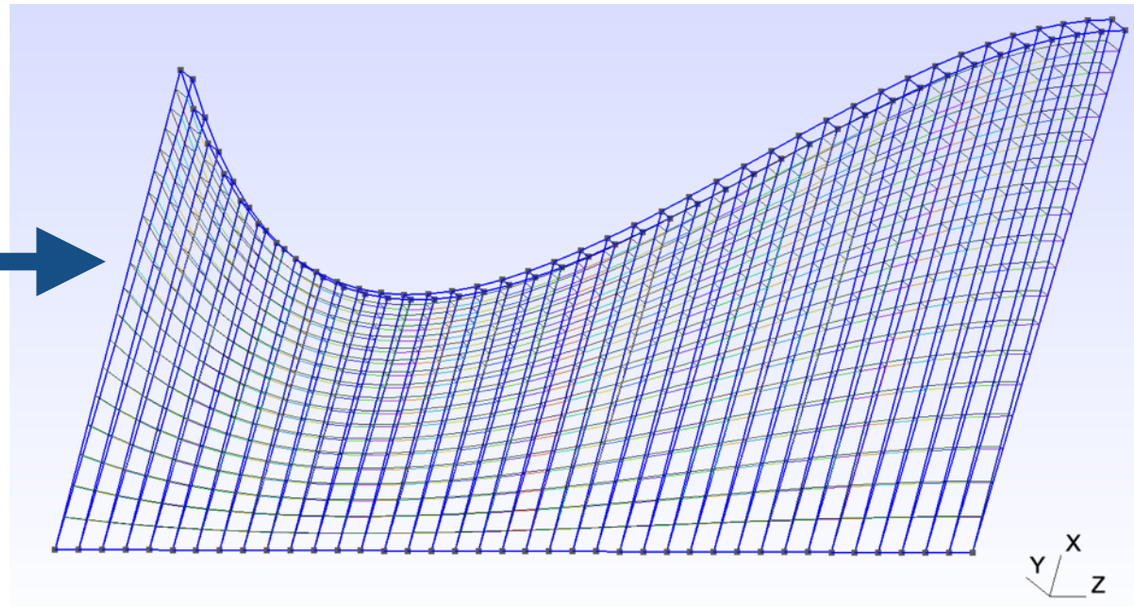


Fig. 1

A sketch of the general family of solutions of the momentum and mass conservation equations when the critical point P exists. Here $\xi = r/a$ represents distance from the sun in units of the solar radius and $V(\xi)$ is the velocity of expansion of the corona.

Constructing the mesh for the nozzle

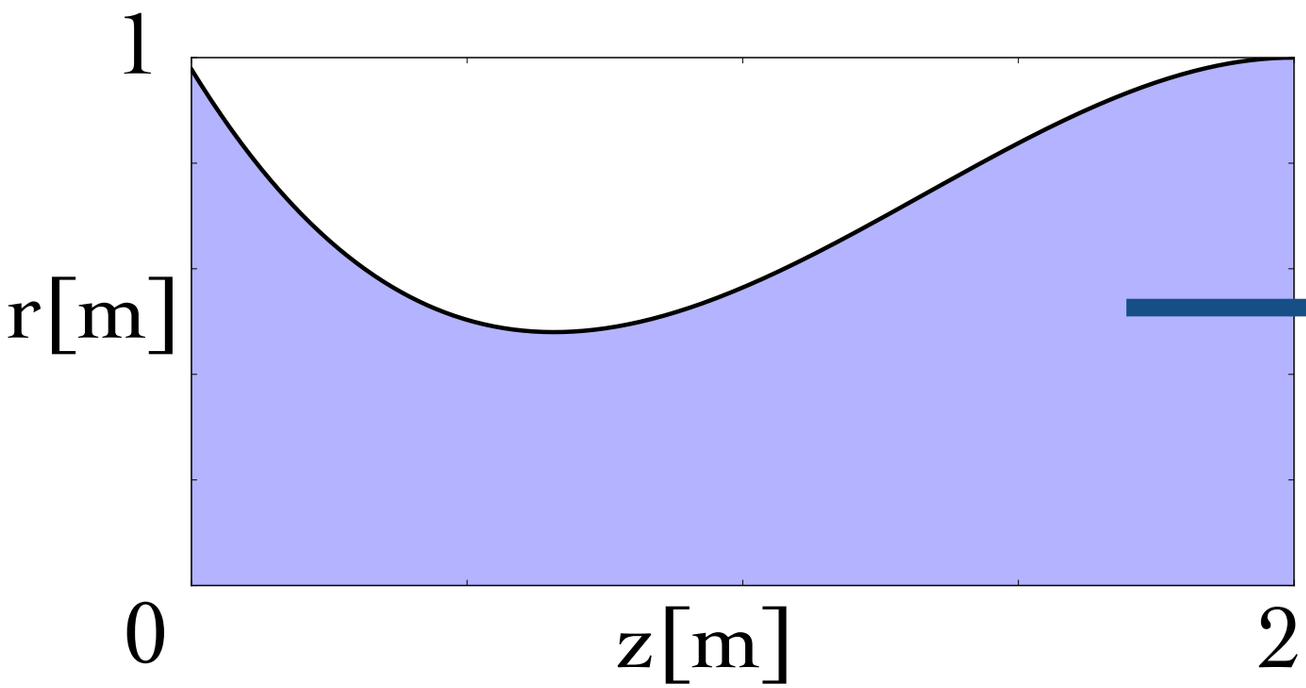


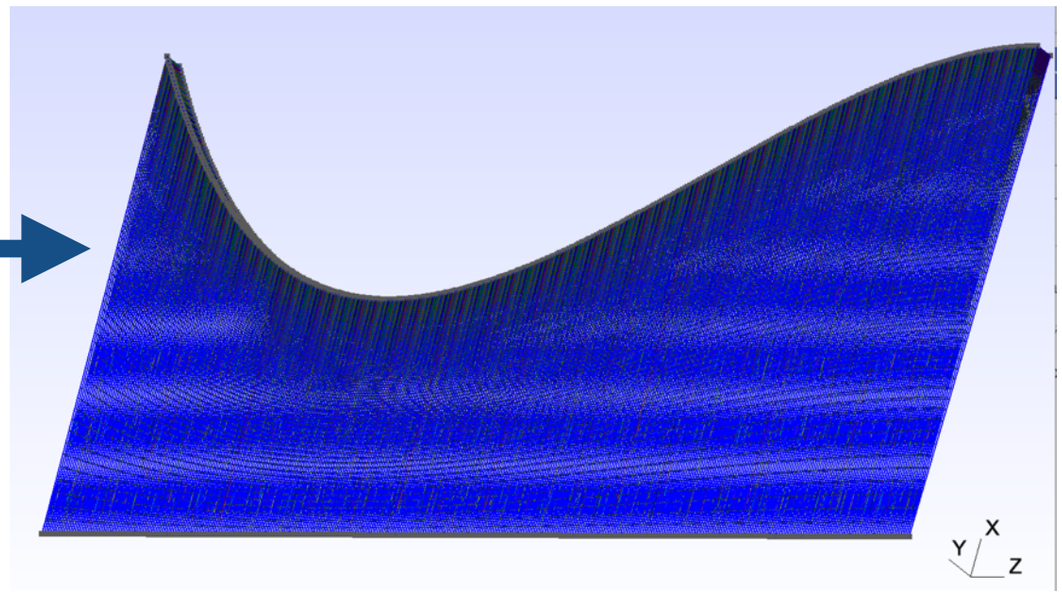


Cells: 400 x 200

Transfinite scaling: 0.95

Constructing the mesh for the nozzle

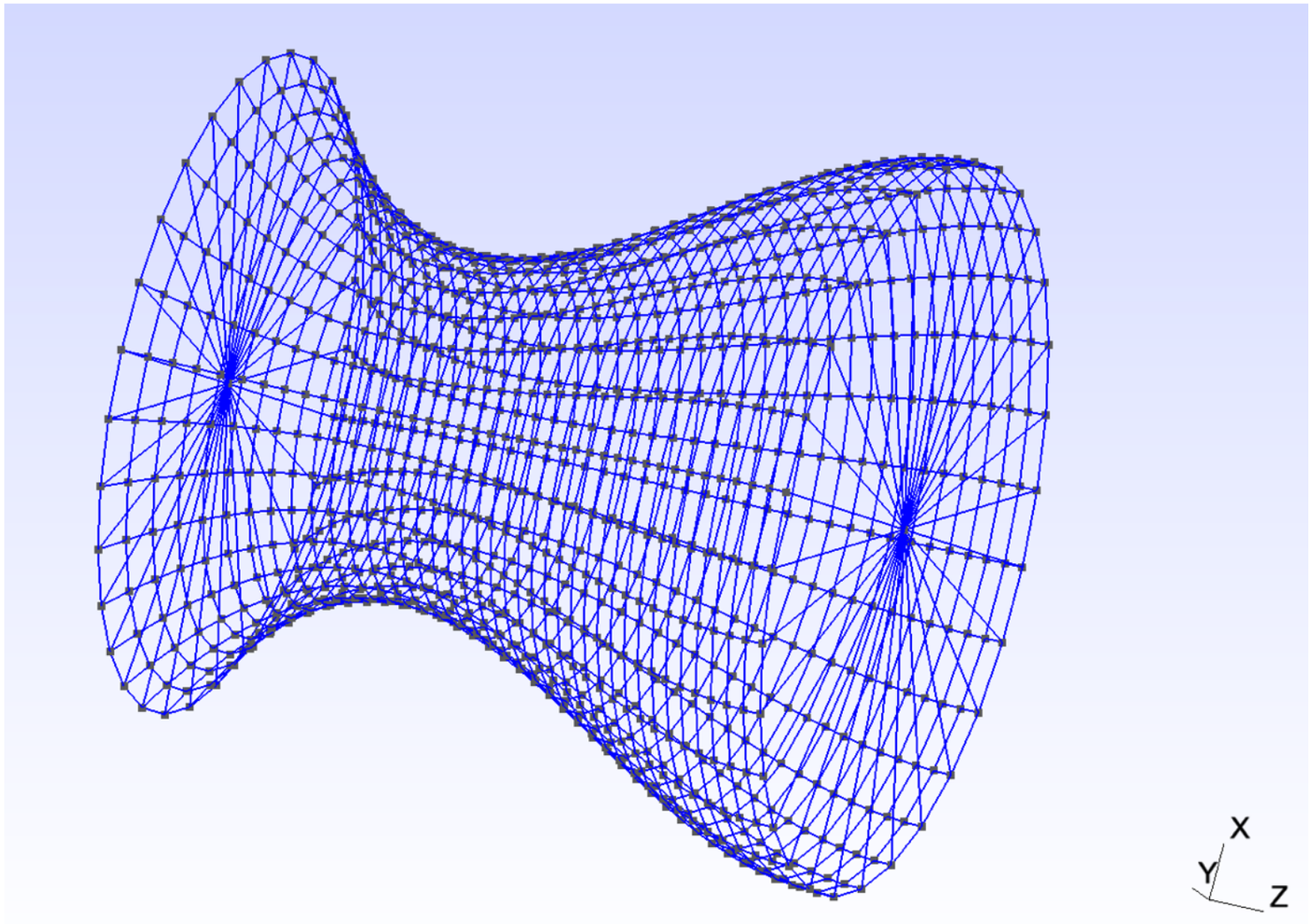




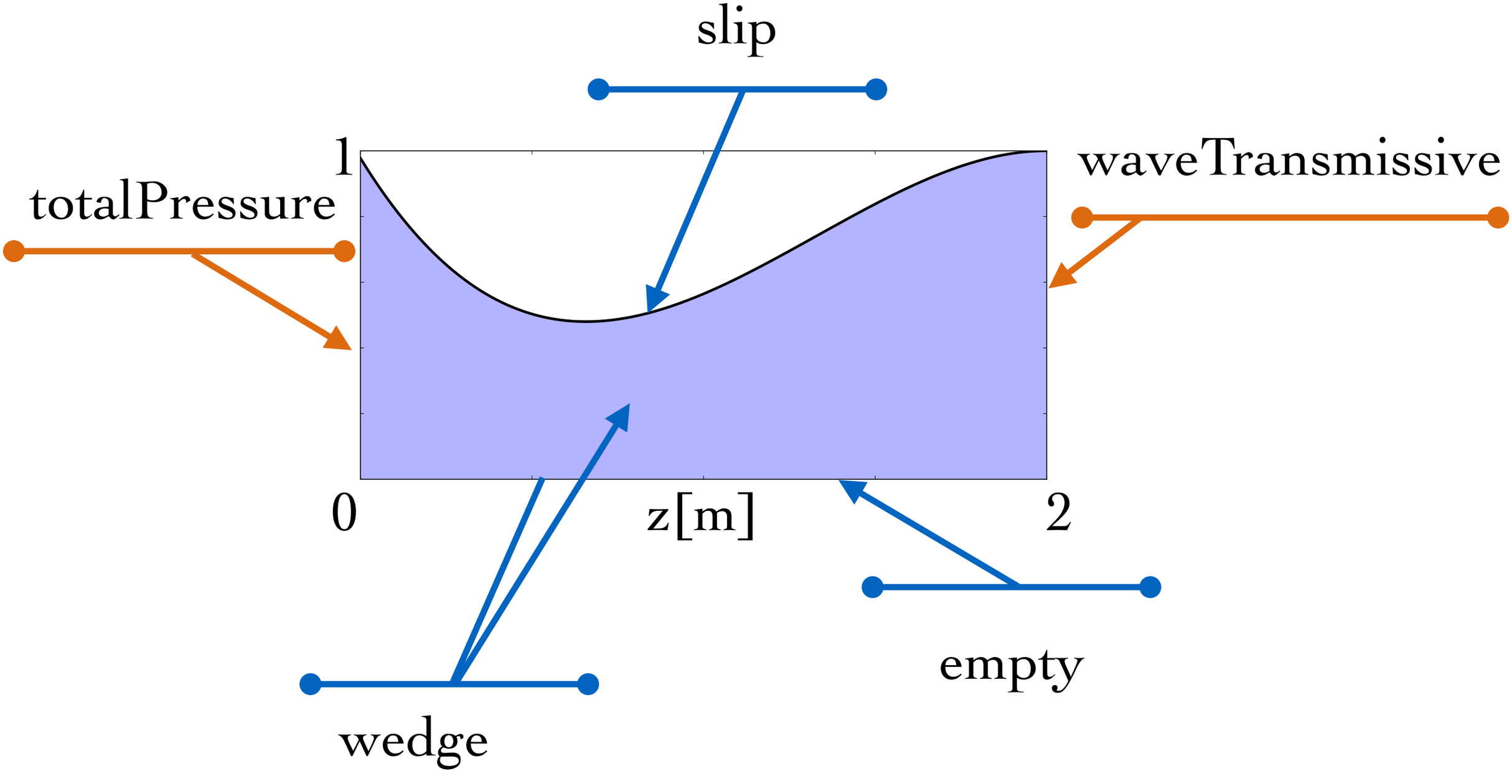
Cells: 400 x 200

Transfinite scaling: 0.95

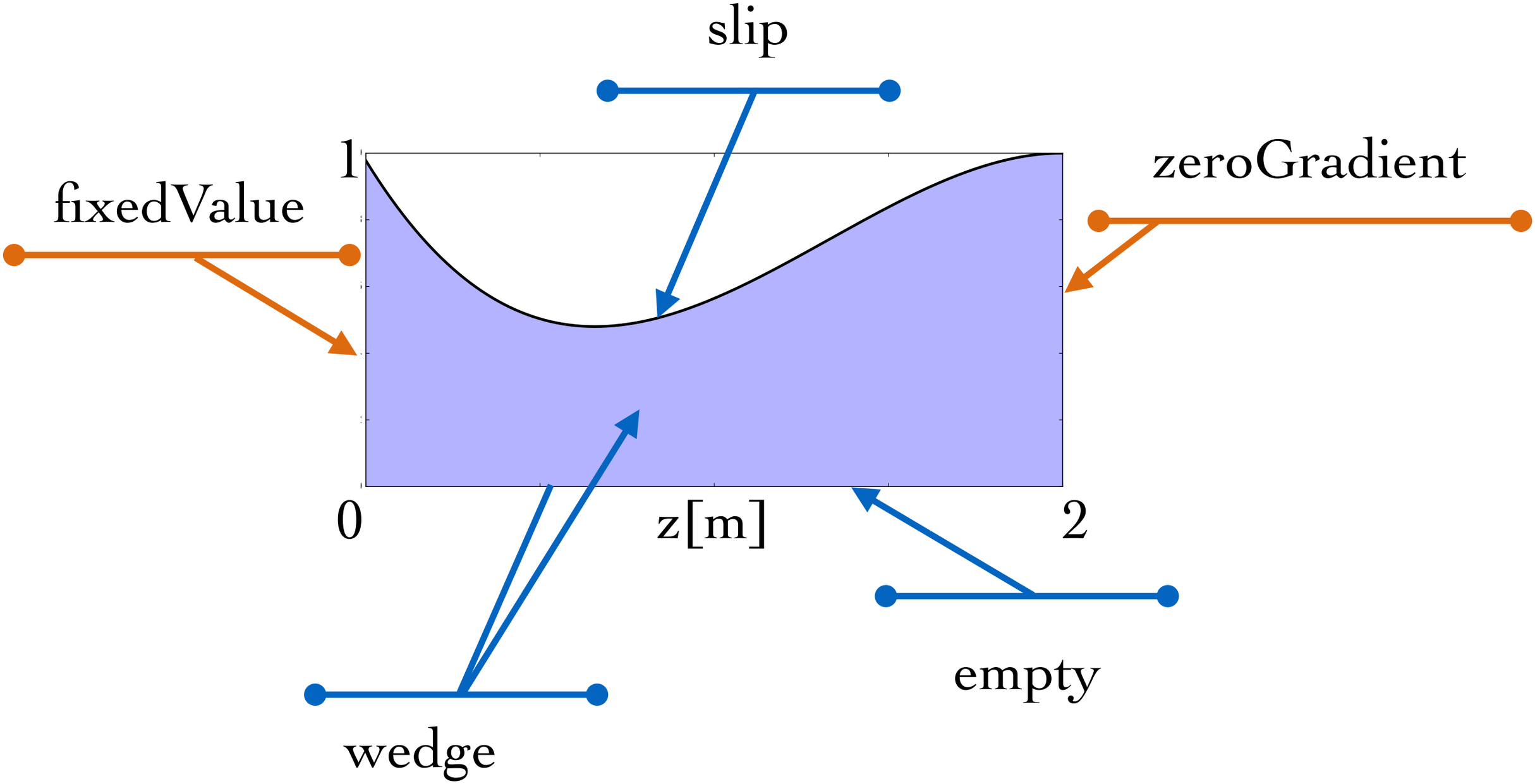
Use wedge boundary conditions to take advantage of this symmetry.



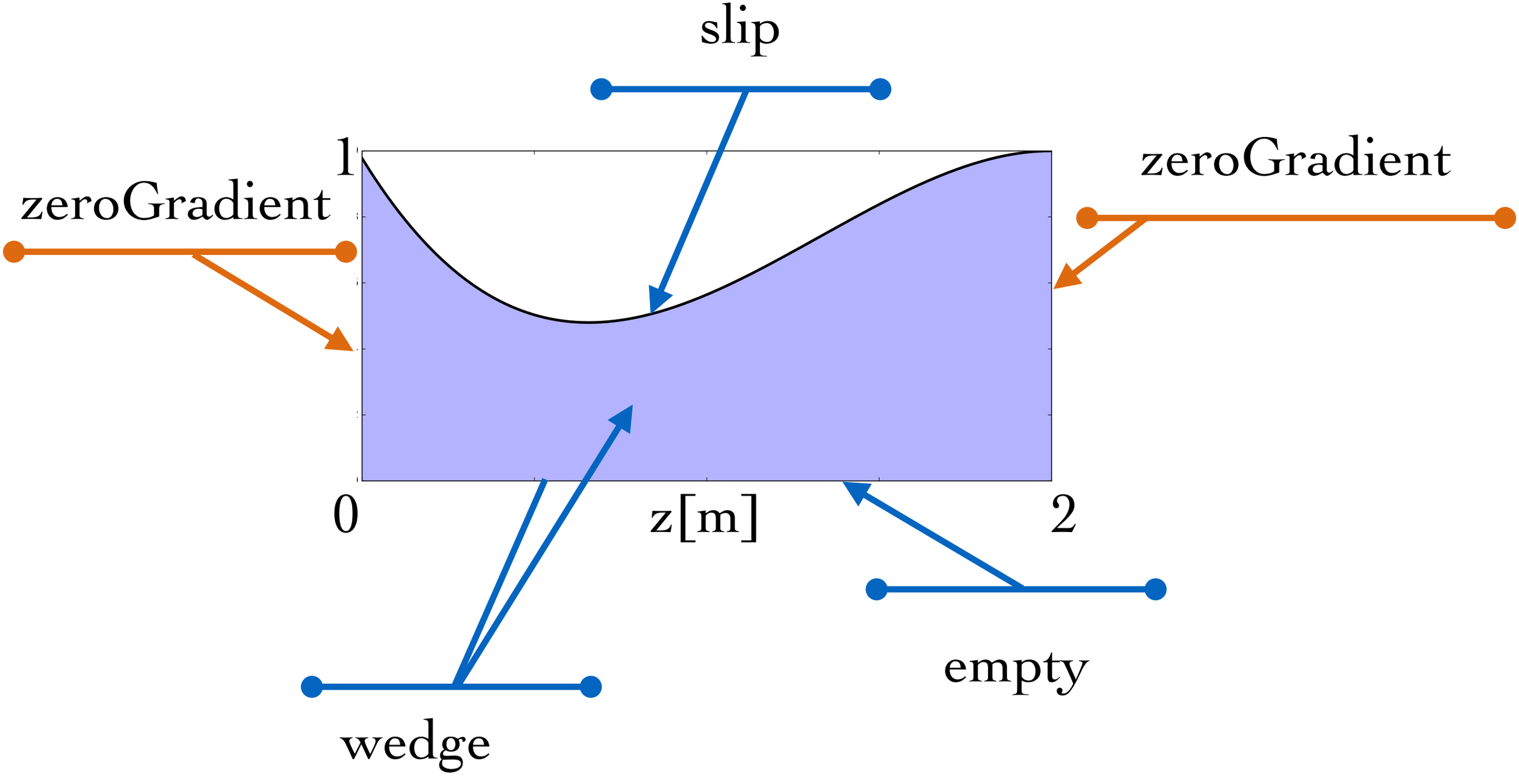
Conditions used in the construction of the nozzle: Pressure



Conditions used in the construction of the nozzle: Temperature



Conditions used in the construction of the nozzle: Velocity

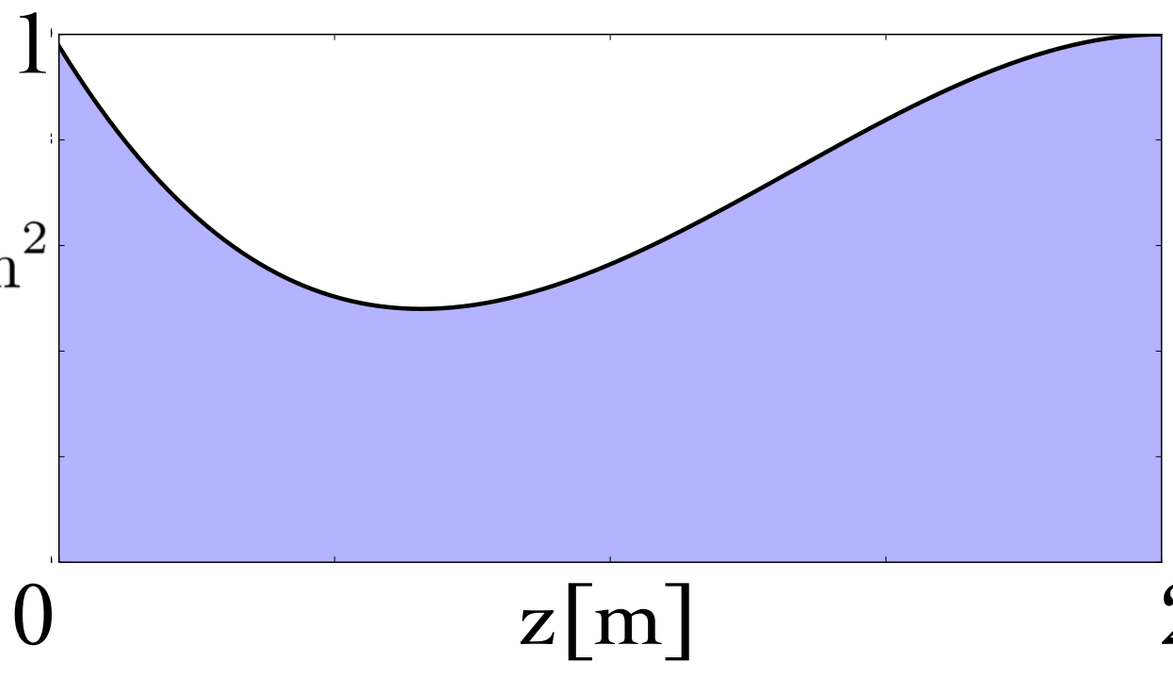


The inlet and outlet boundaries are set with a pressure gradient

Inlet:

$p = 1.01 \times 10^5\,\mathrm{N/m^2}$

$T = 298\mathrm{K}$



Outlet:

$p = 2.5 \times 10^3\,\mathrm{N/m^2}$

U_{in}, U_{out}, and T_{out} are set by the simulation

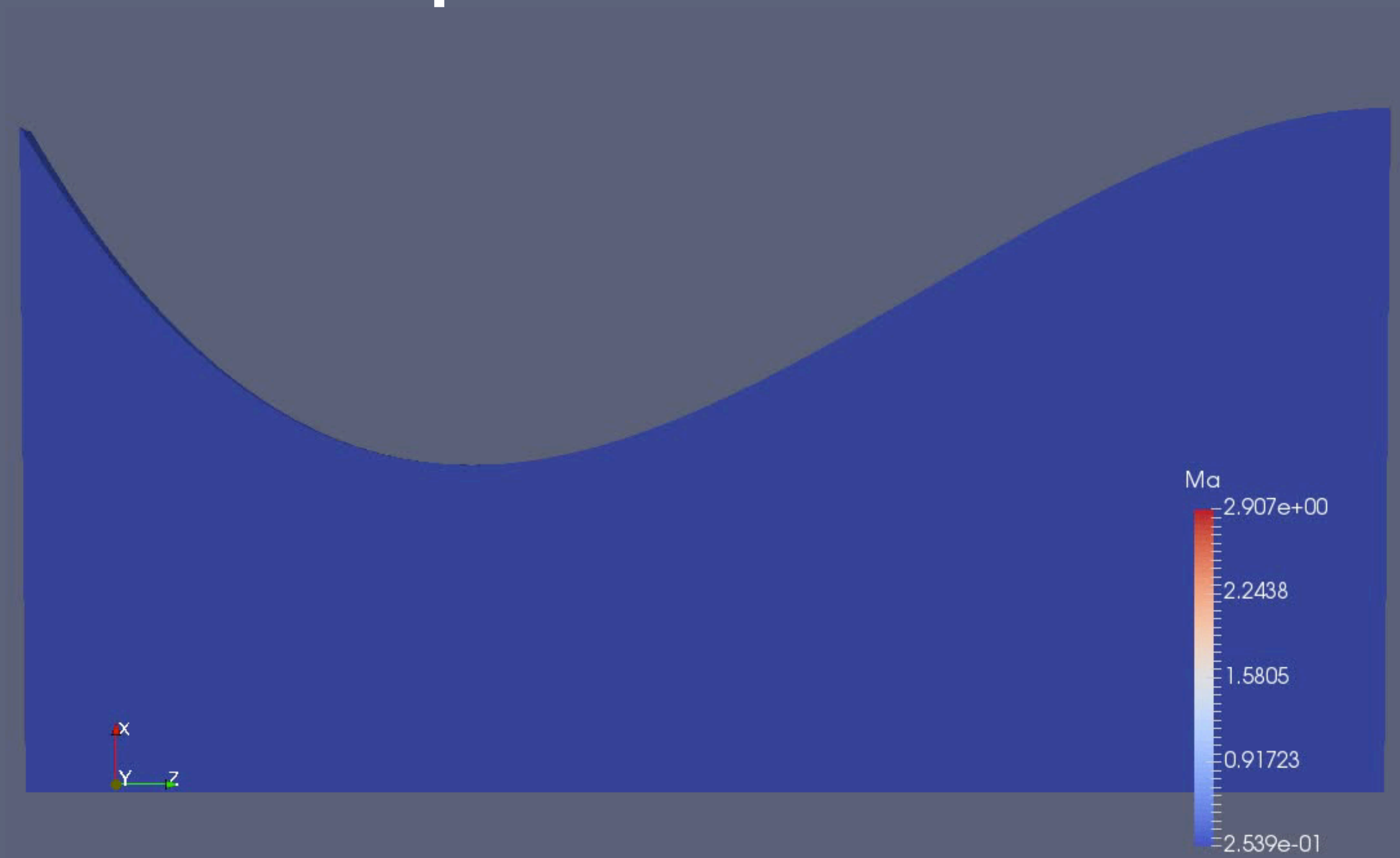
Simulation uses the rhoCentralFoam solver

Density-based compressible flow solver based on central-upwind schemes

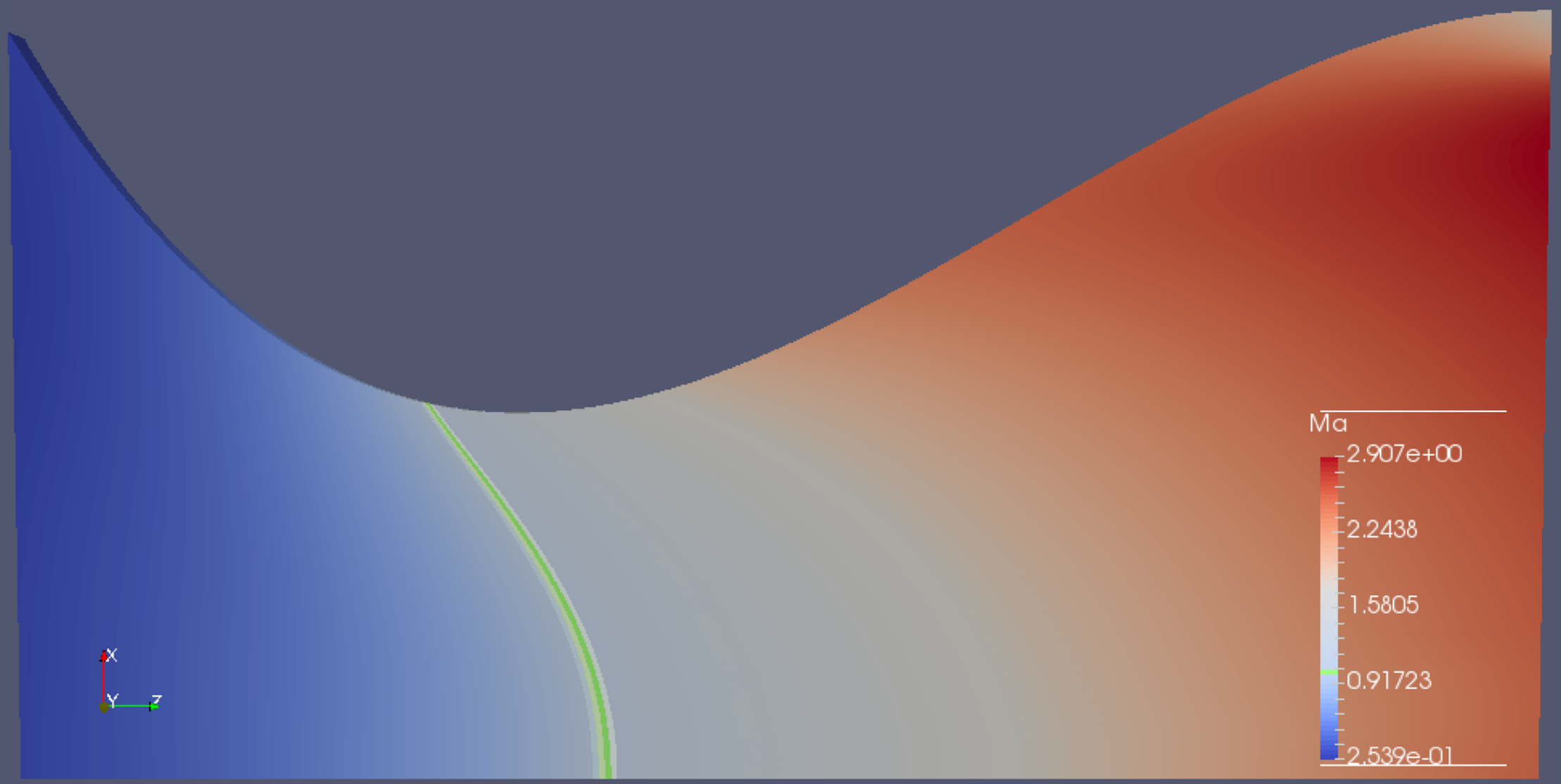
Uses Sutherland's law to calculate the dynamic viscosity

Assumed the flow was laminar (no turbulence)

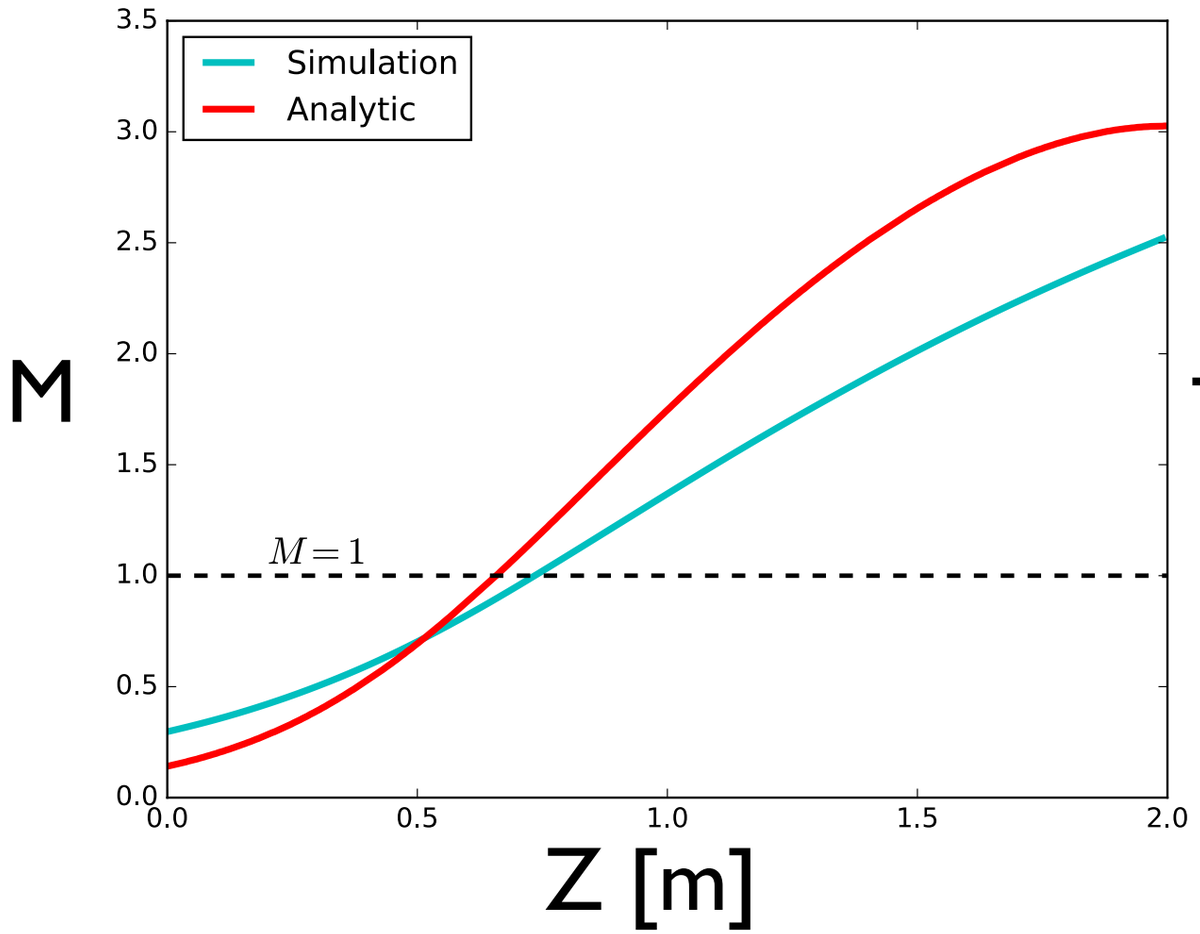
Initially get waves and shocks that dissipate over time

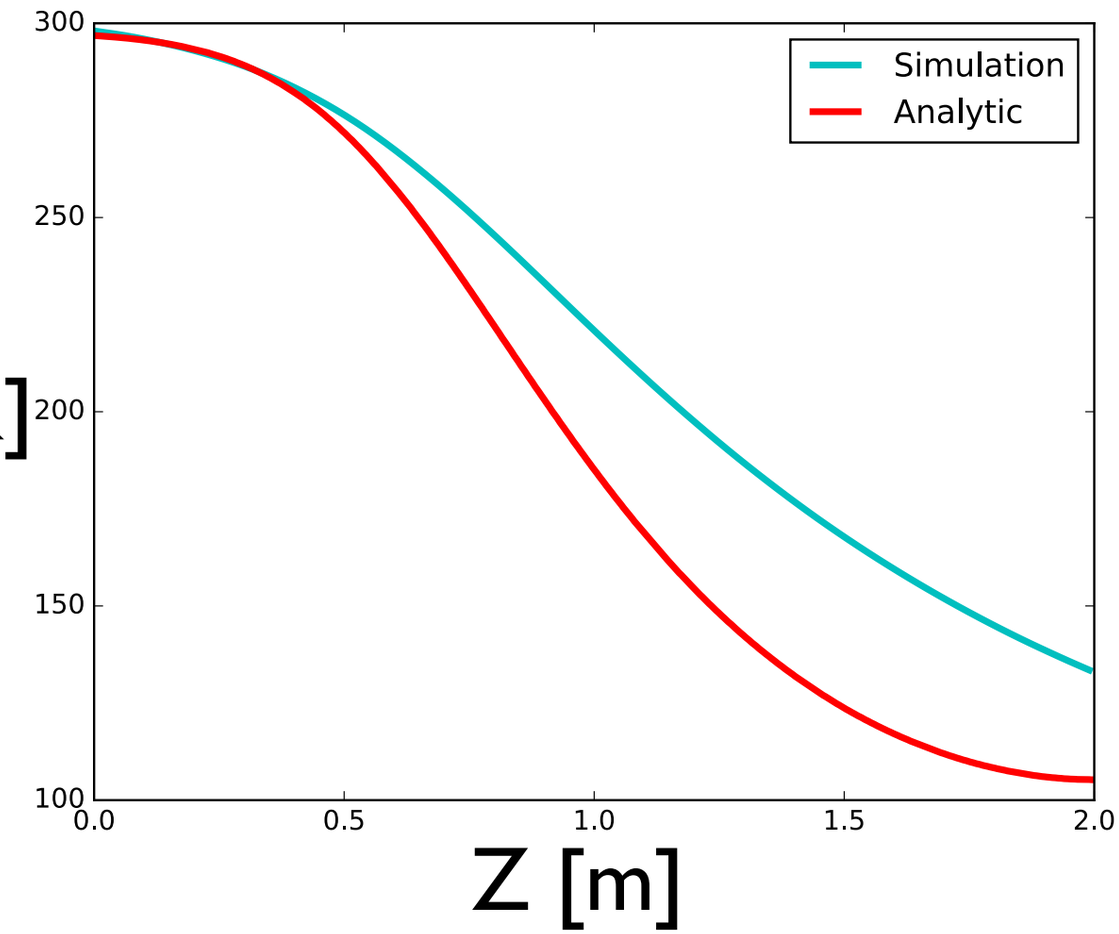


Finally settle into a supersonic steady state flow

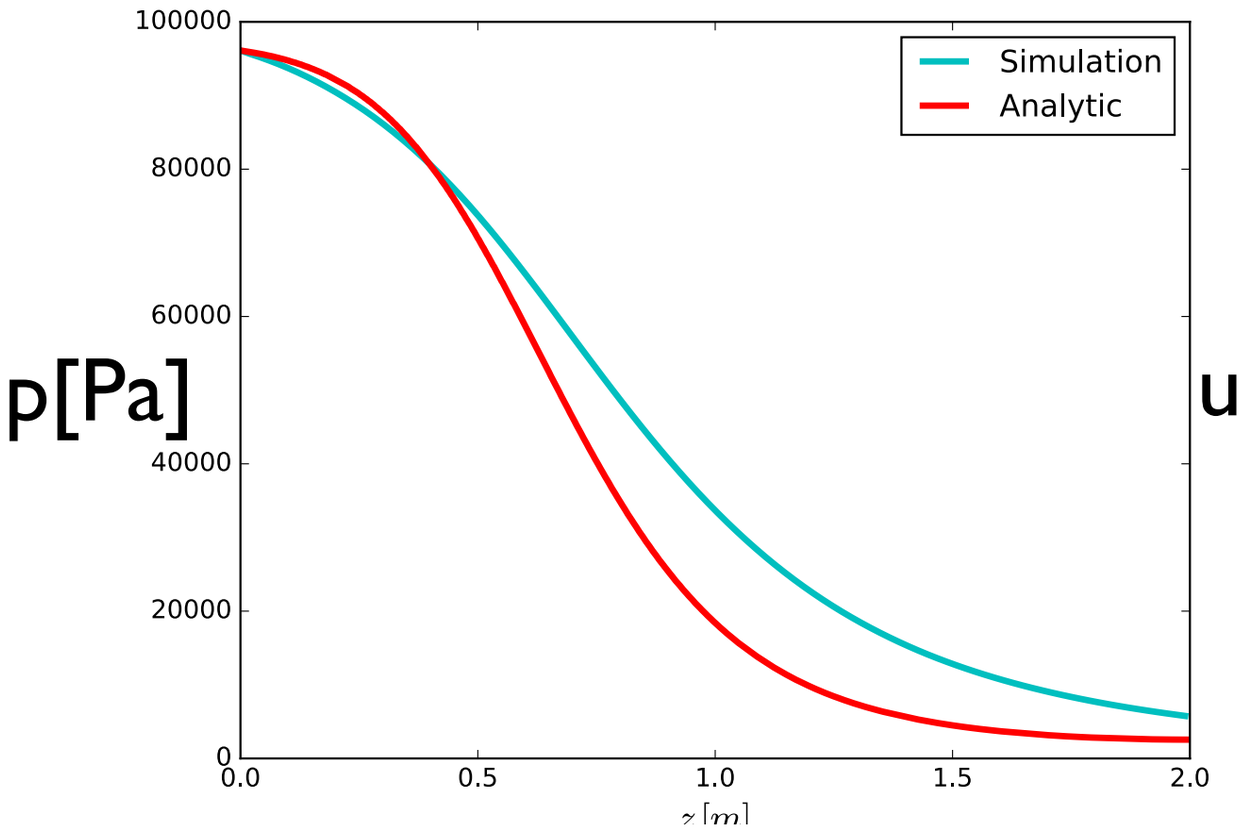


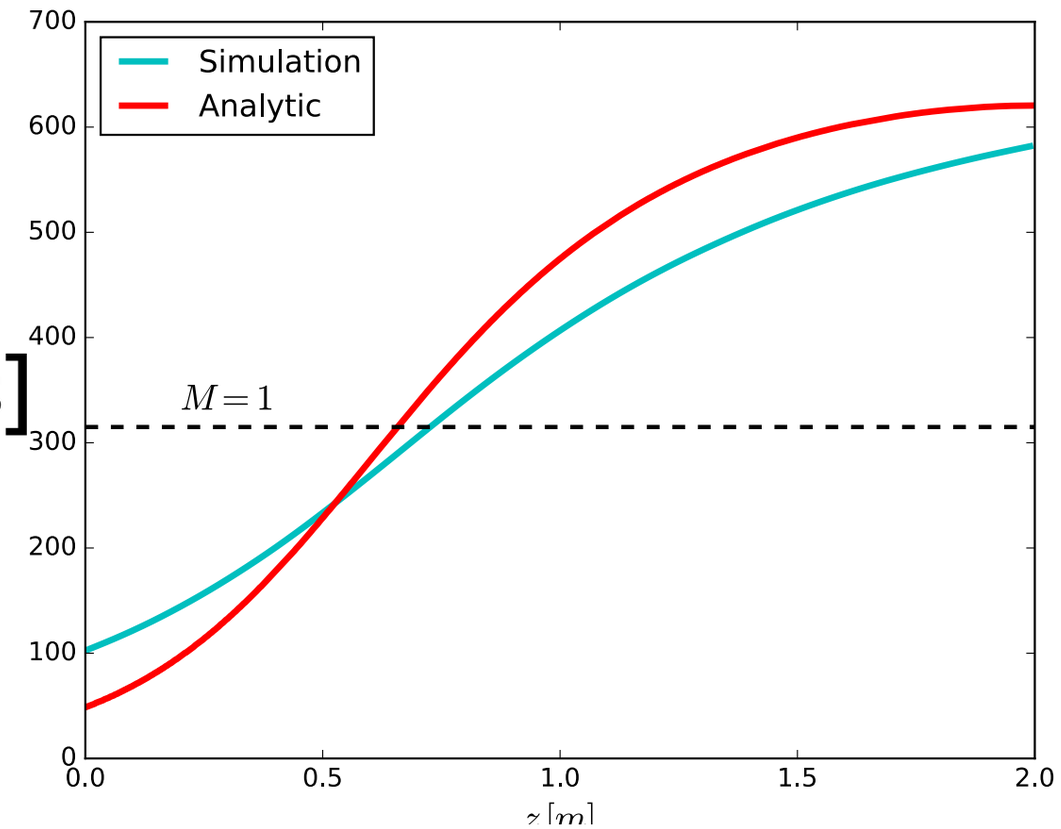
We can compare the simulation results to analytic inviscid solutions



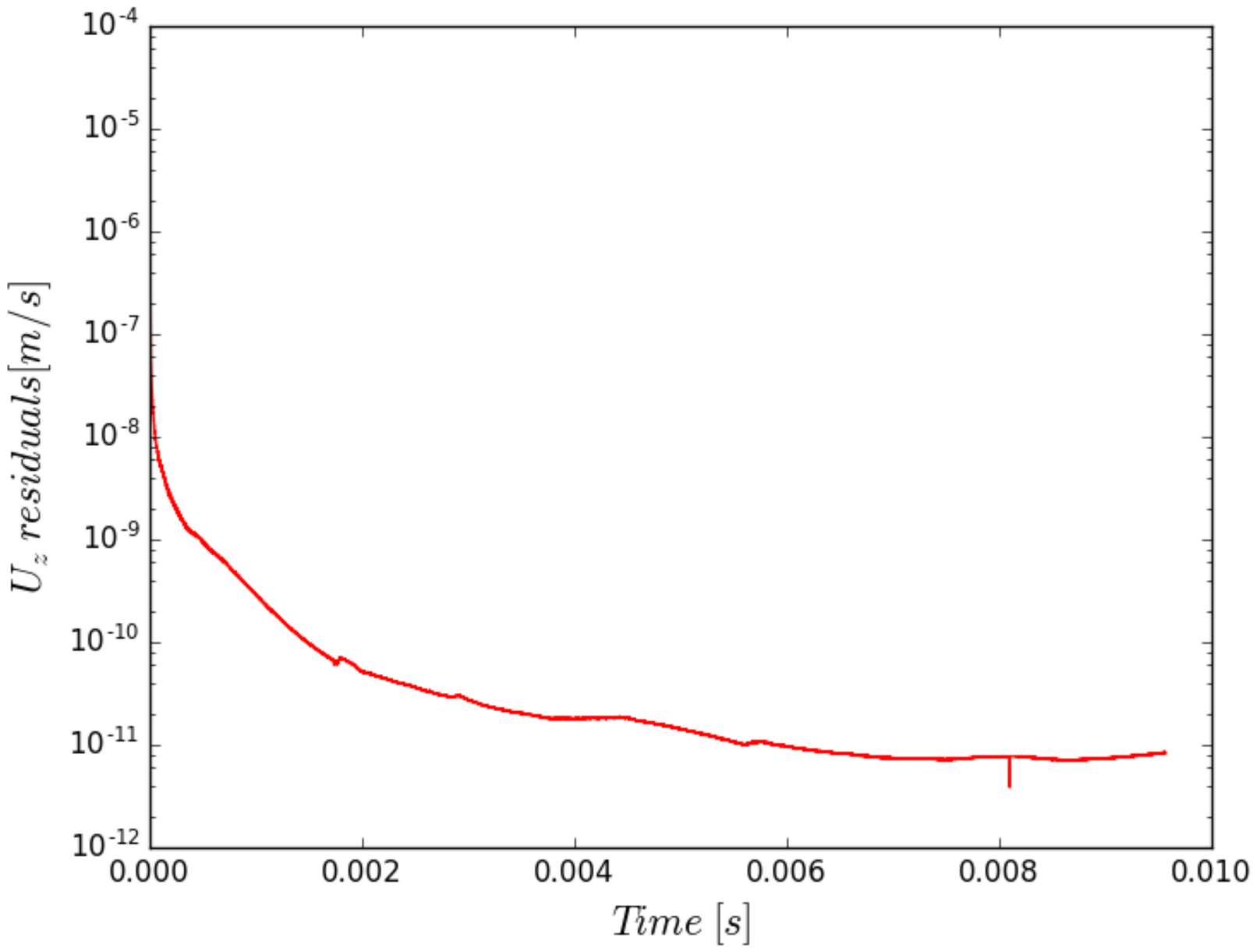


We can compare the simulation results to analytic inviscid solutions





Looking at residuals tells us if the simulation has converged



Summary

- Nozzles can transition flow from subsonic to supersonic
- Built a flexible Python code that generates meshes that have cylindrical symmetry
- Using that mesh, set up a simulation of a supersonic de Laval nozzle that agrees with the analytic results quite well