The Shock Tube Problem

ME702 Final project

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The Shock Tube problem

- Studied by Gary A. Sod in 1978
- 1D problem
- analytical solutions are known
- used to test and validate computational fluid models

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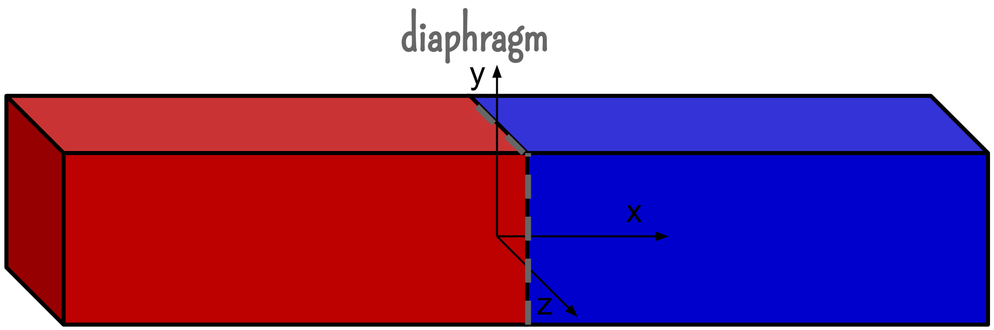

$p = 100$ kPa
$u = 0$ m/s
$\rho = 1.000$ kg/m^3

$p = 10$ kPa
$u = 0$ m/s
$\rho = 0.125$ kg/m^3

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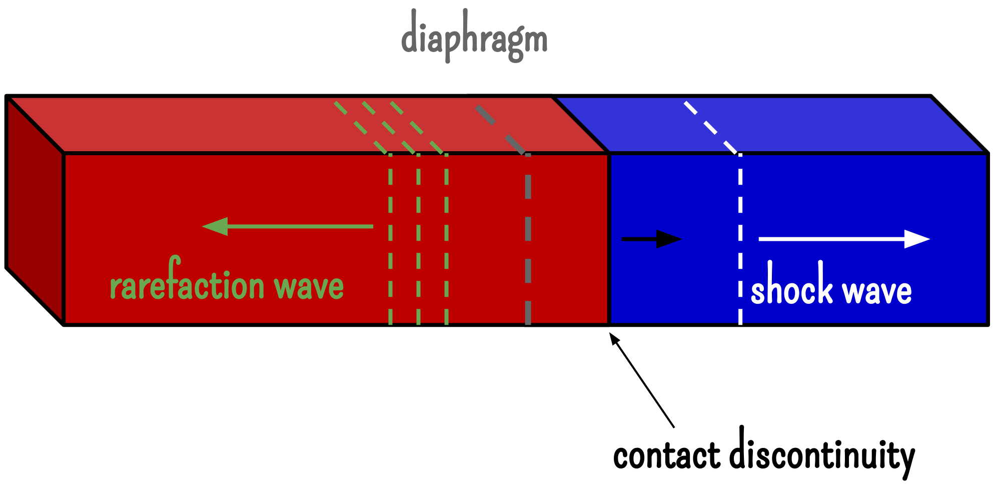

Governing Equations

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Conservation of momentum

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot [\vec{u}(\rho \vec{u})] + \nabla p + \nabla \cdot \vec{T} = 0$$

Conservation of energy

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot [E(\rho \vec{u})] + \nabla \cdot (\rho \vec{u}) + \nabla \cdot (\vec{T} \cdot \vec{u}) + \nabla \cdot \vec{j} = 0$$

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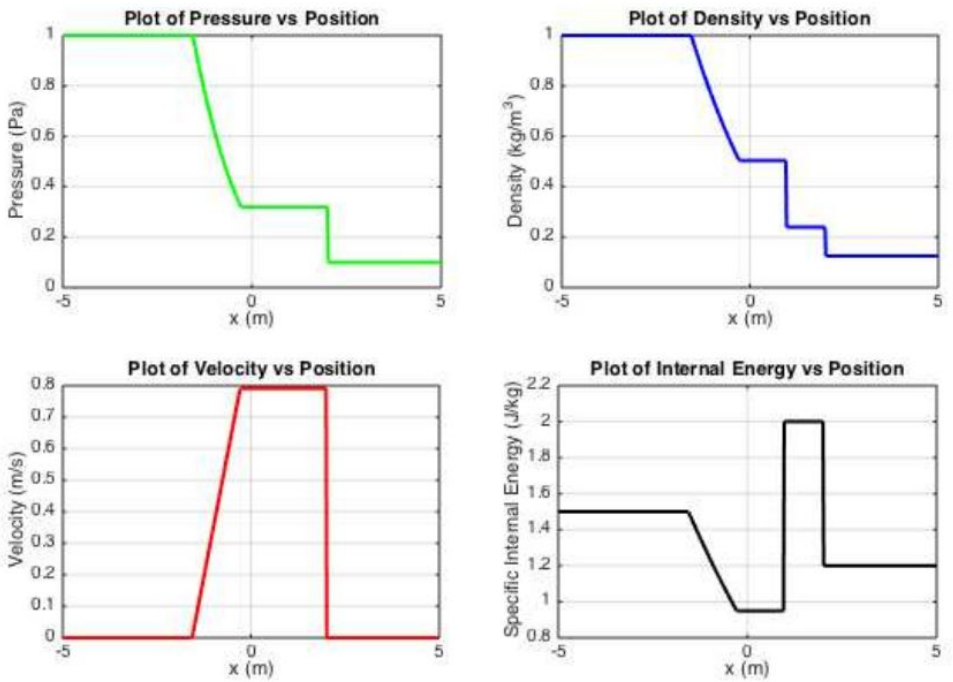
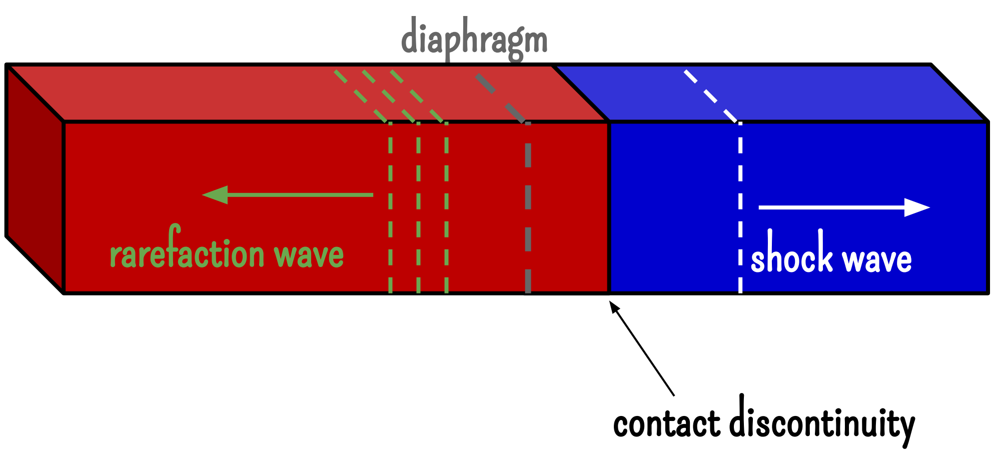
Assume:

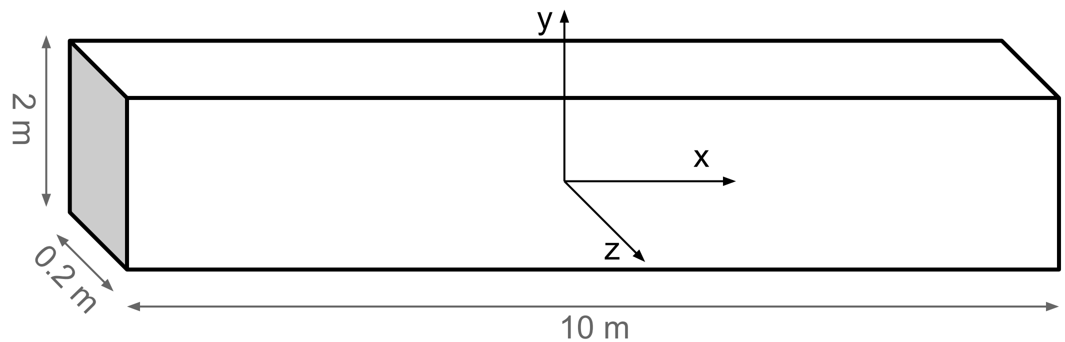
- inviscid flows (no viscosity)
- no heat transfer

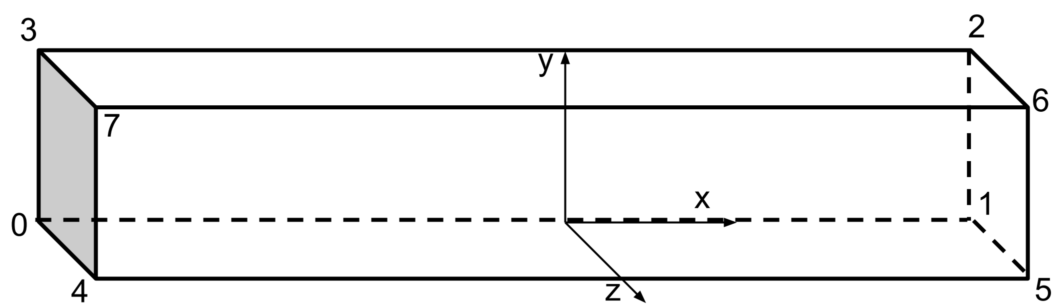
Analytical solutions





OpenFOAM 2D shock tube set-up





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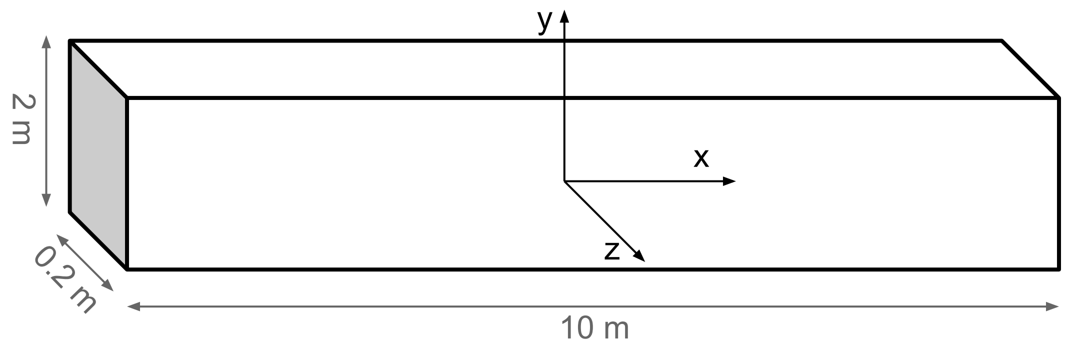

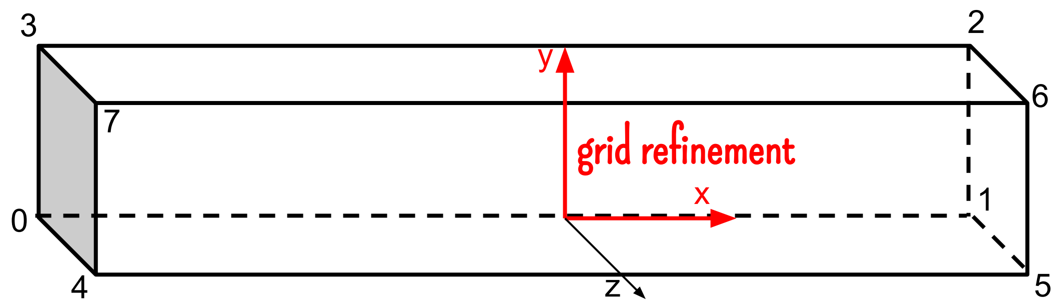

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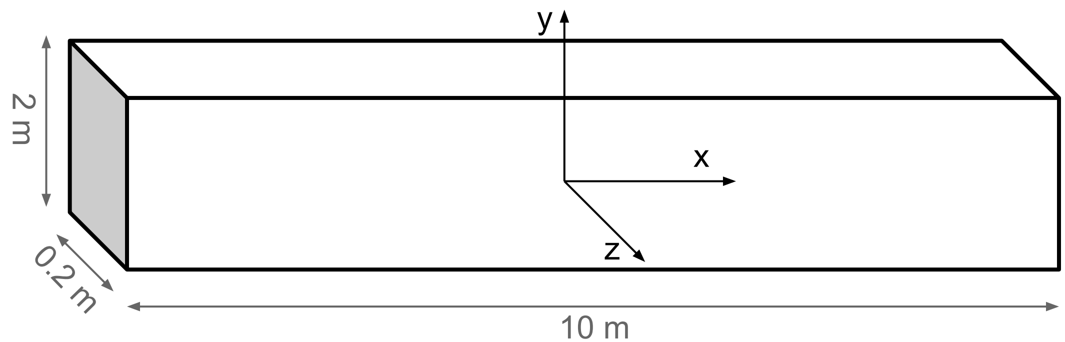

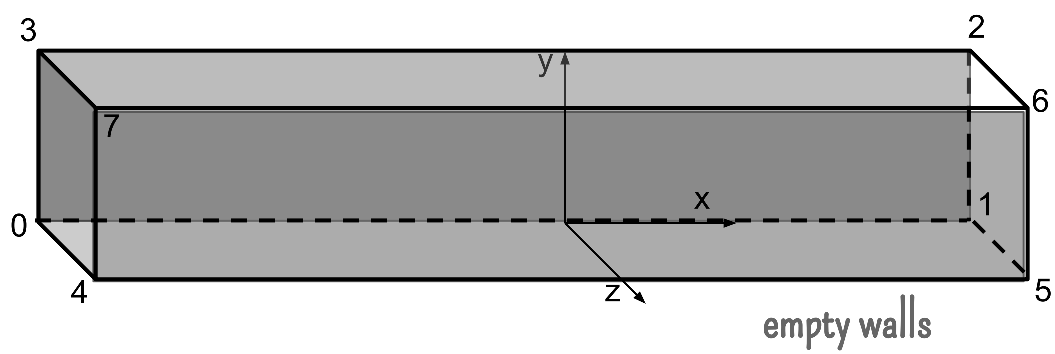

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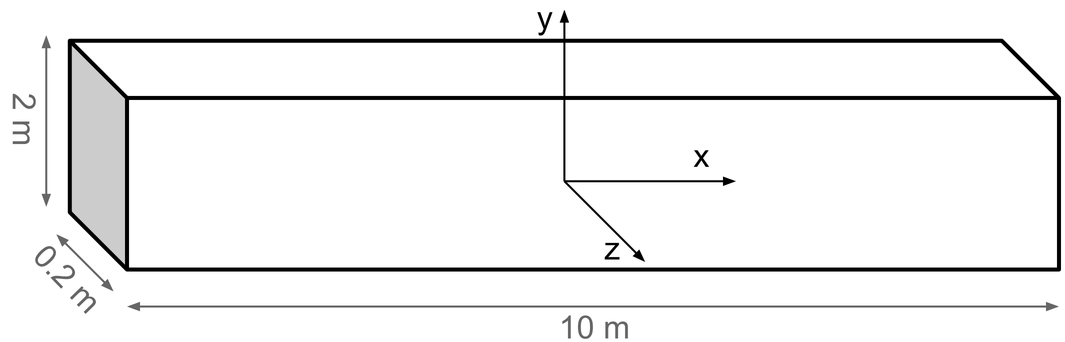
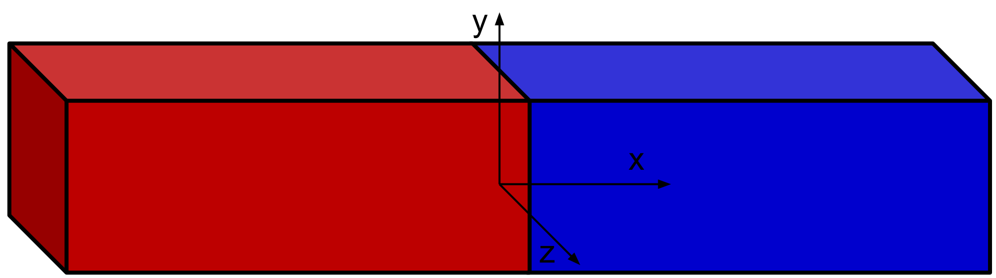
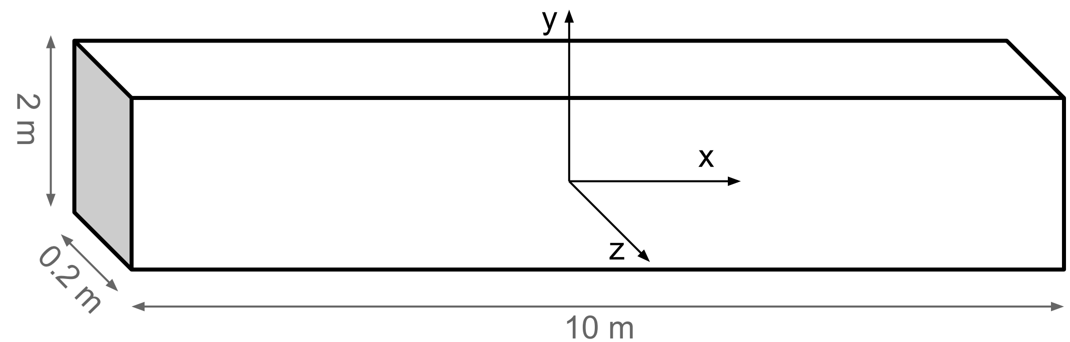


non-uniform internal flow



p = 100 kPa	p = 10 kPa
u = 0 m/s	u = 0 m/s
T = 348.432 K	T = 278.746 K

OpenFOAM 2D shock tube set-up



$\Delta t = 10^{-6}$ s

t = 10 ms

sampling rate: 0.5 ms

resolution cases: 20x20x1

30x30x1

50x50x1

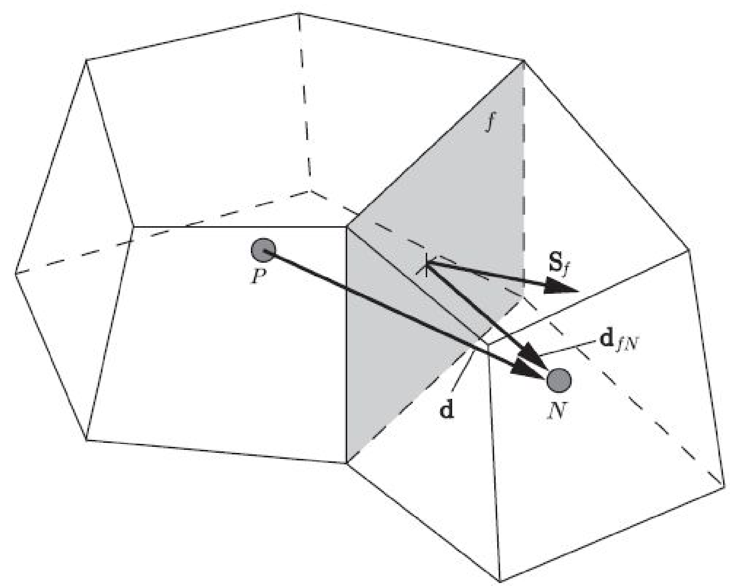
100x100x1

300x300x1

400x400x1

OpenFOAM rhoCentralFoam solver

- Greenshields et al., 2010
- compressible fluids $\nabla \cdot (\rho \ \vec{u}) \neq 0$
- finite volumes method
- values are provided at the centroid of the volume cell
- cells are contiguous polyhedral volumes
- volume integrals in divergence and gradient terms are converted to surface integrals via Gauss's theorem



Results: time evolution

line cuts along x-axis

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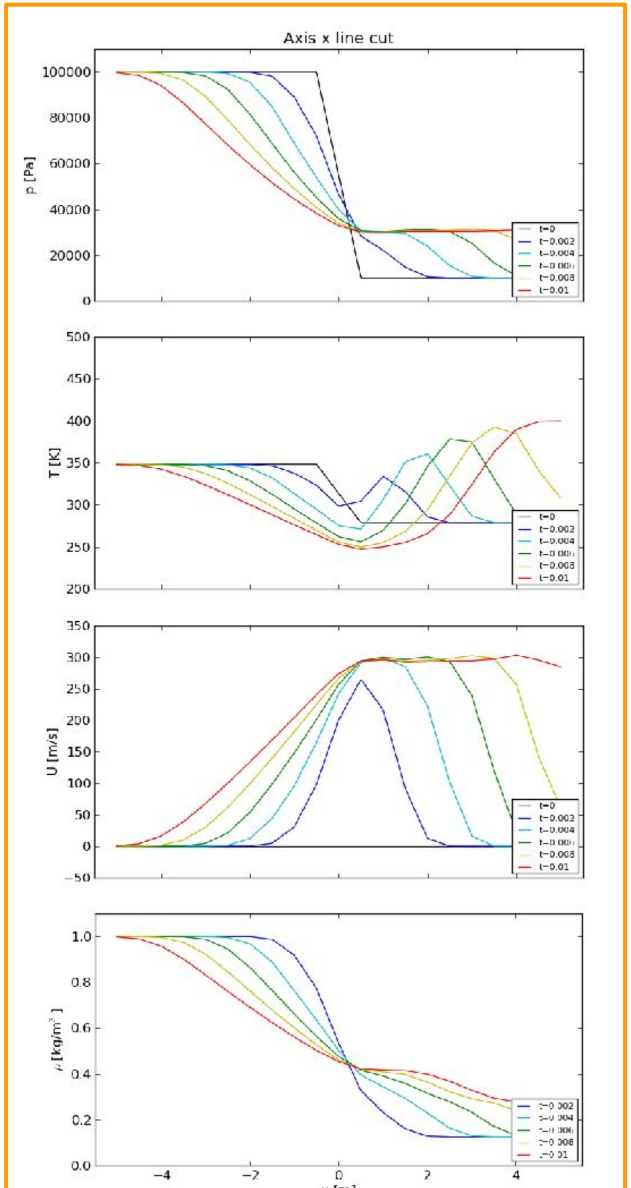
line cuts along x-axis

20x20x1



pressure

temperature

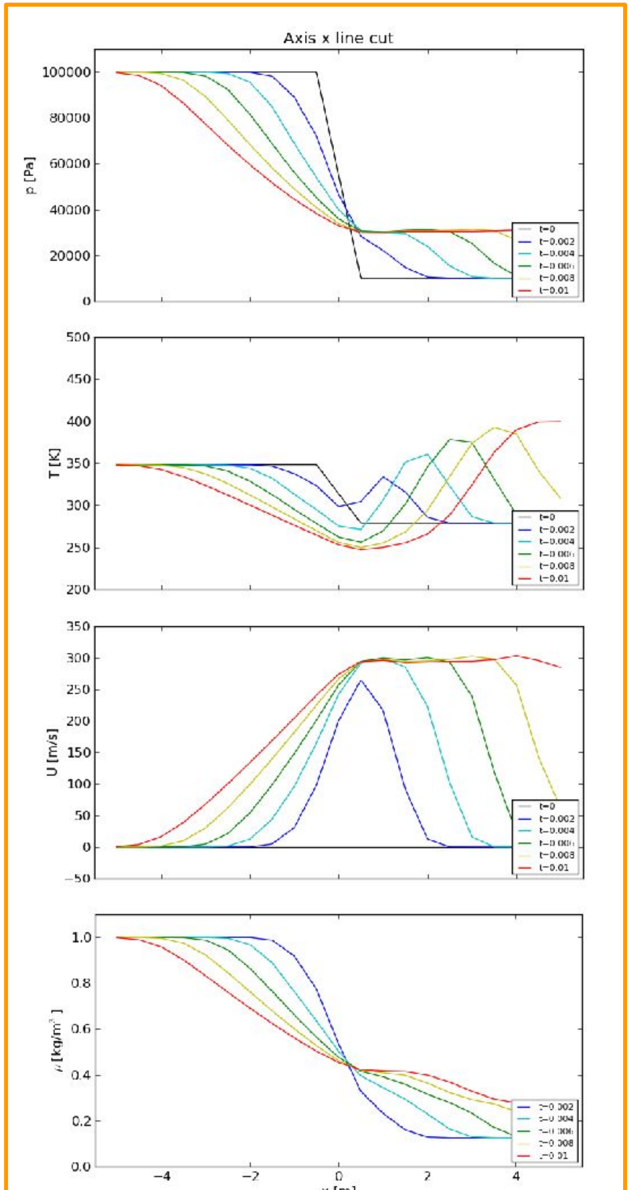
speed

density

Results: time evolution

line cuts along x-axis

20x20x1



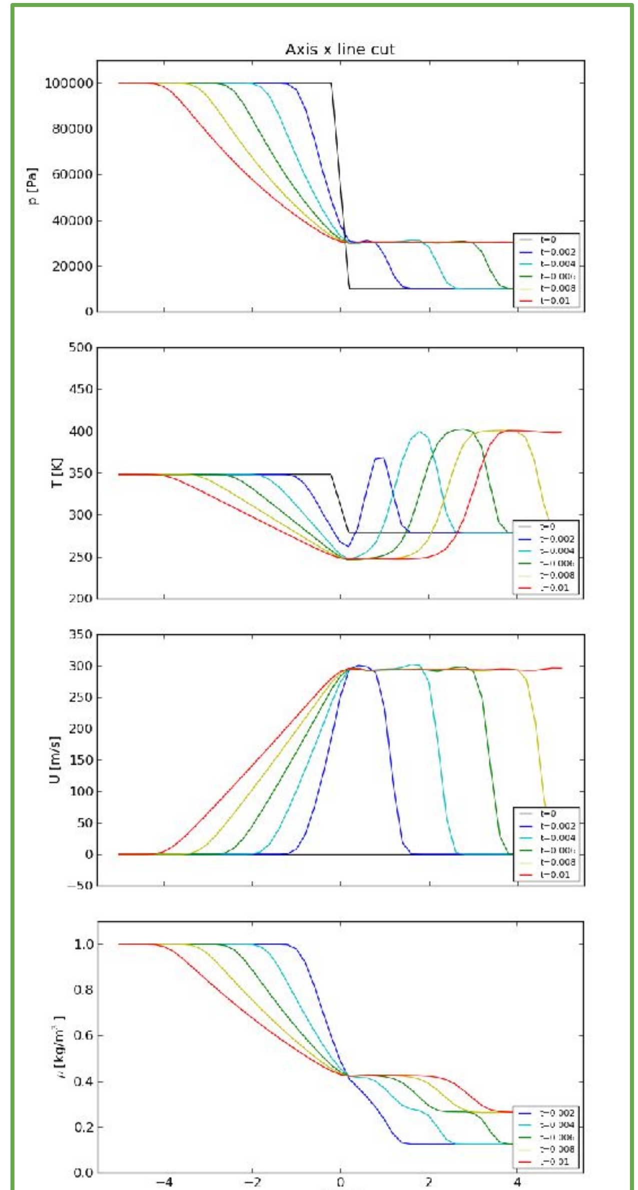
pressure

temperature

speed

density

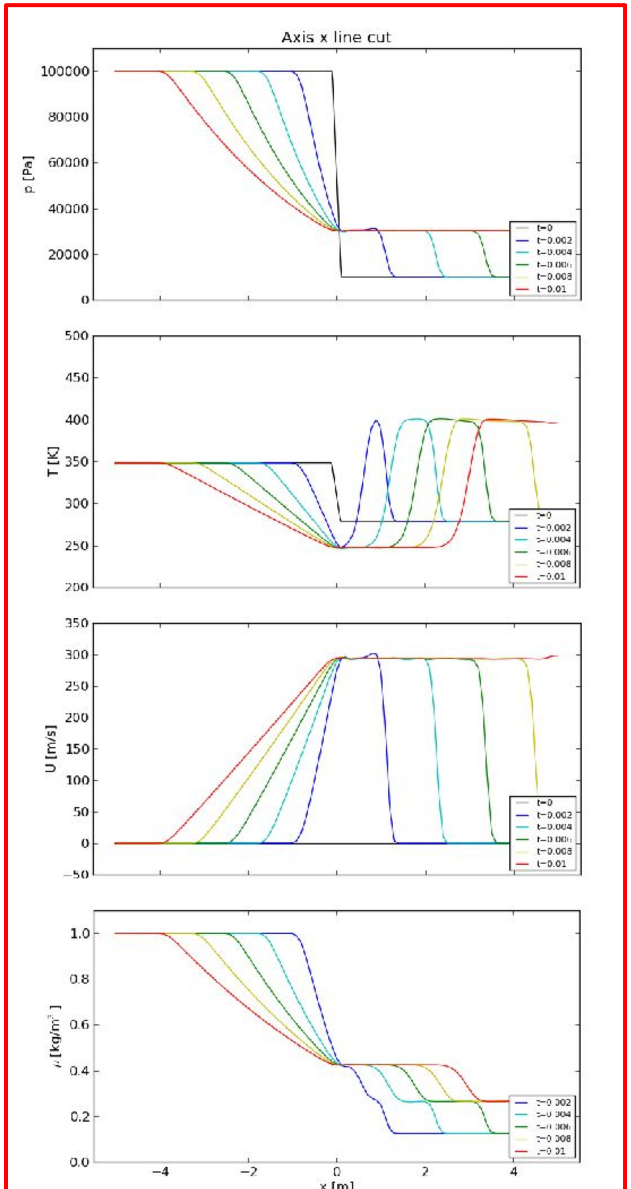
50x50x1



Results: time evolution

line cuts along x-axis

100x100x1



pressure

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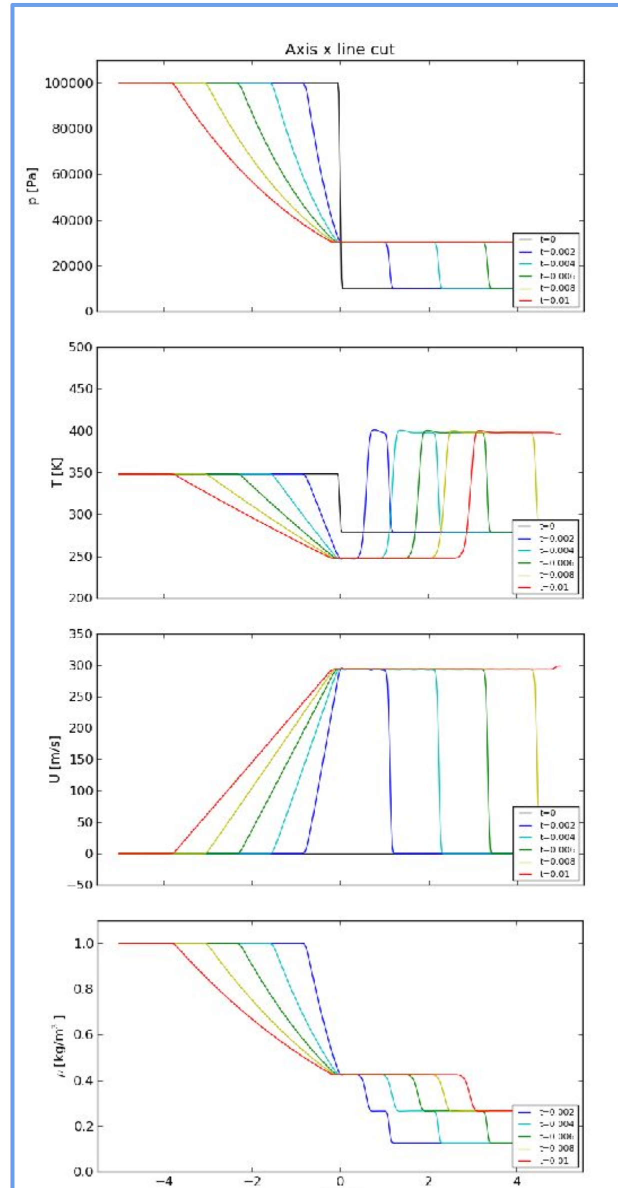
speed

density

300x300x1



Results: resolution

line cuts along x-axis

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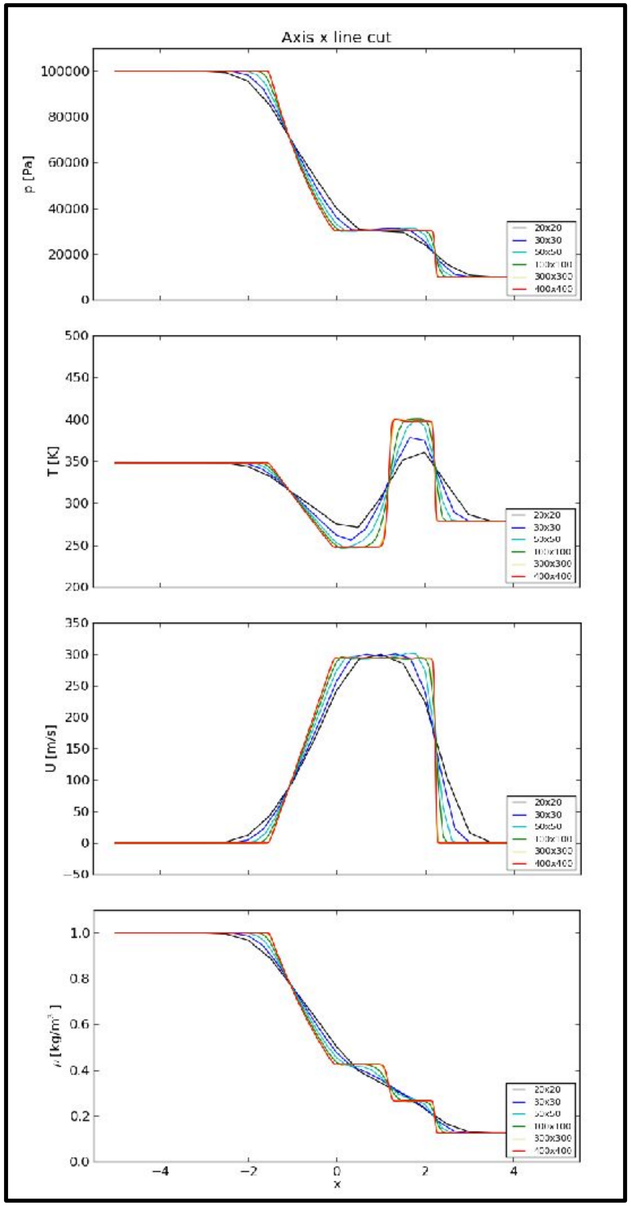


pressure

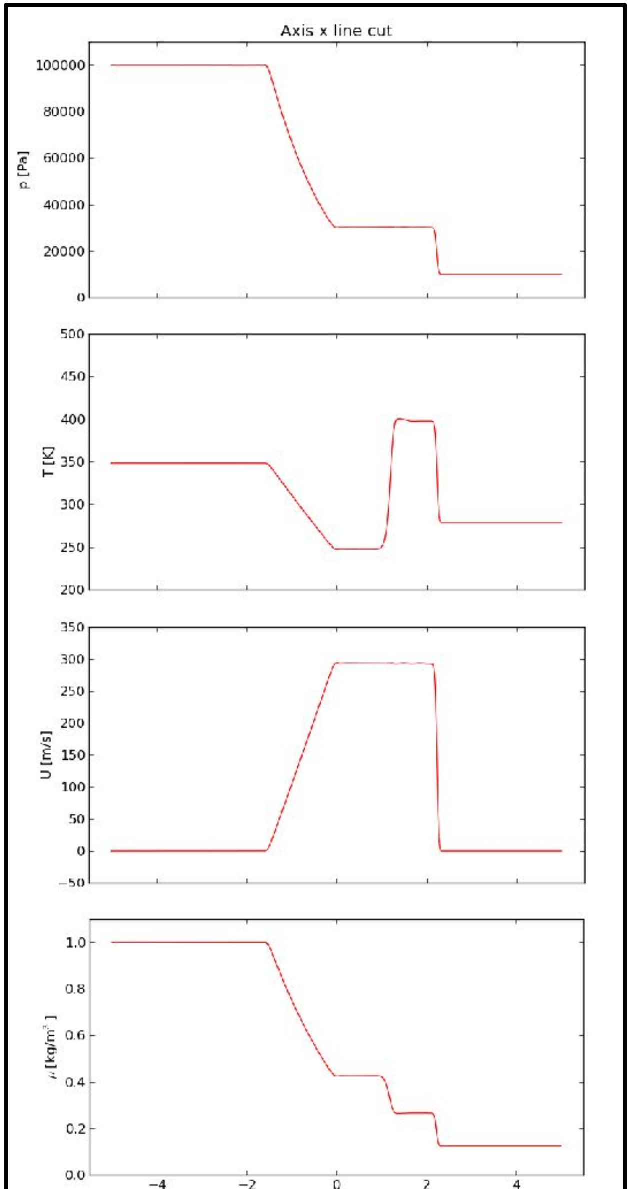
temperature

speed

density

Results

line cuts along x-axis for 300x300x1 resolution grid



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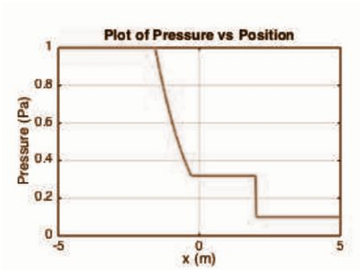
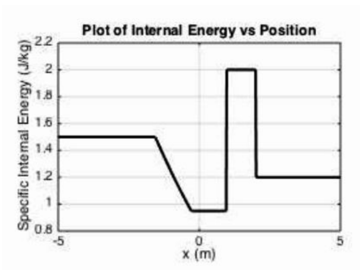
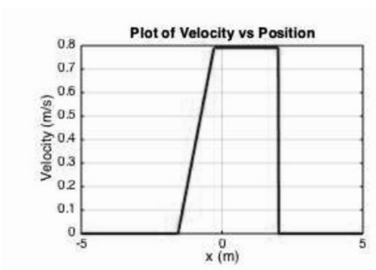
density

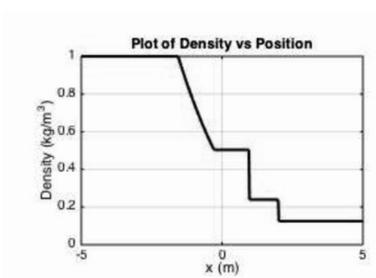
Results

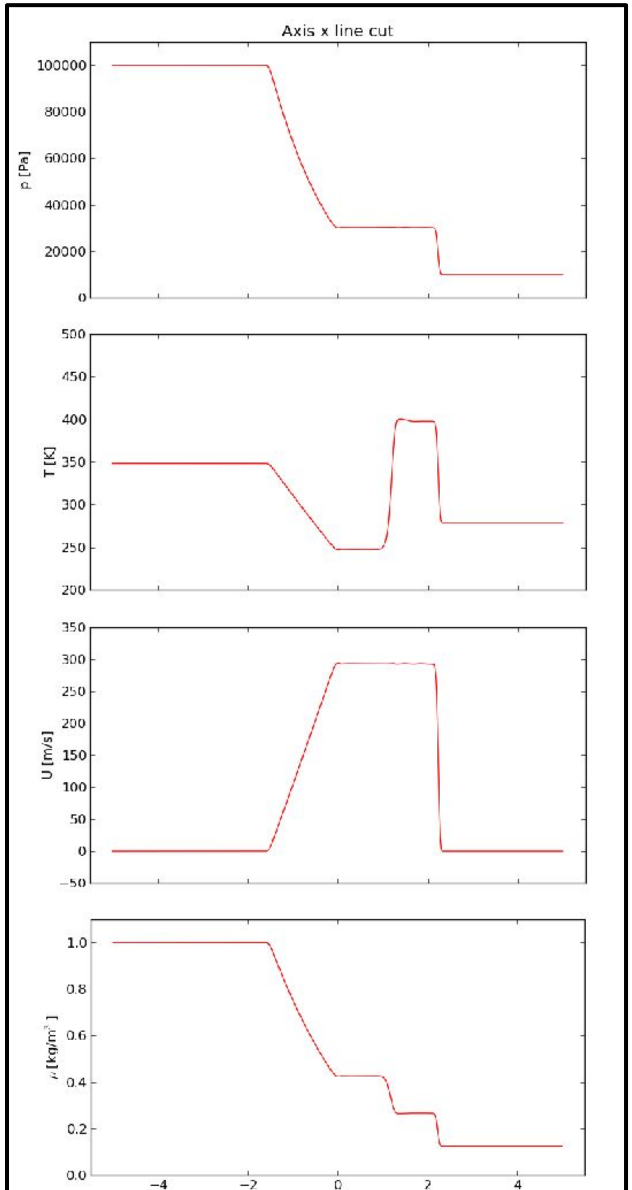
line cuts along x-axis for 300x300x1 resolution grid











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Results: 2D plots

xy plane cut

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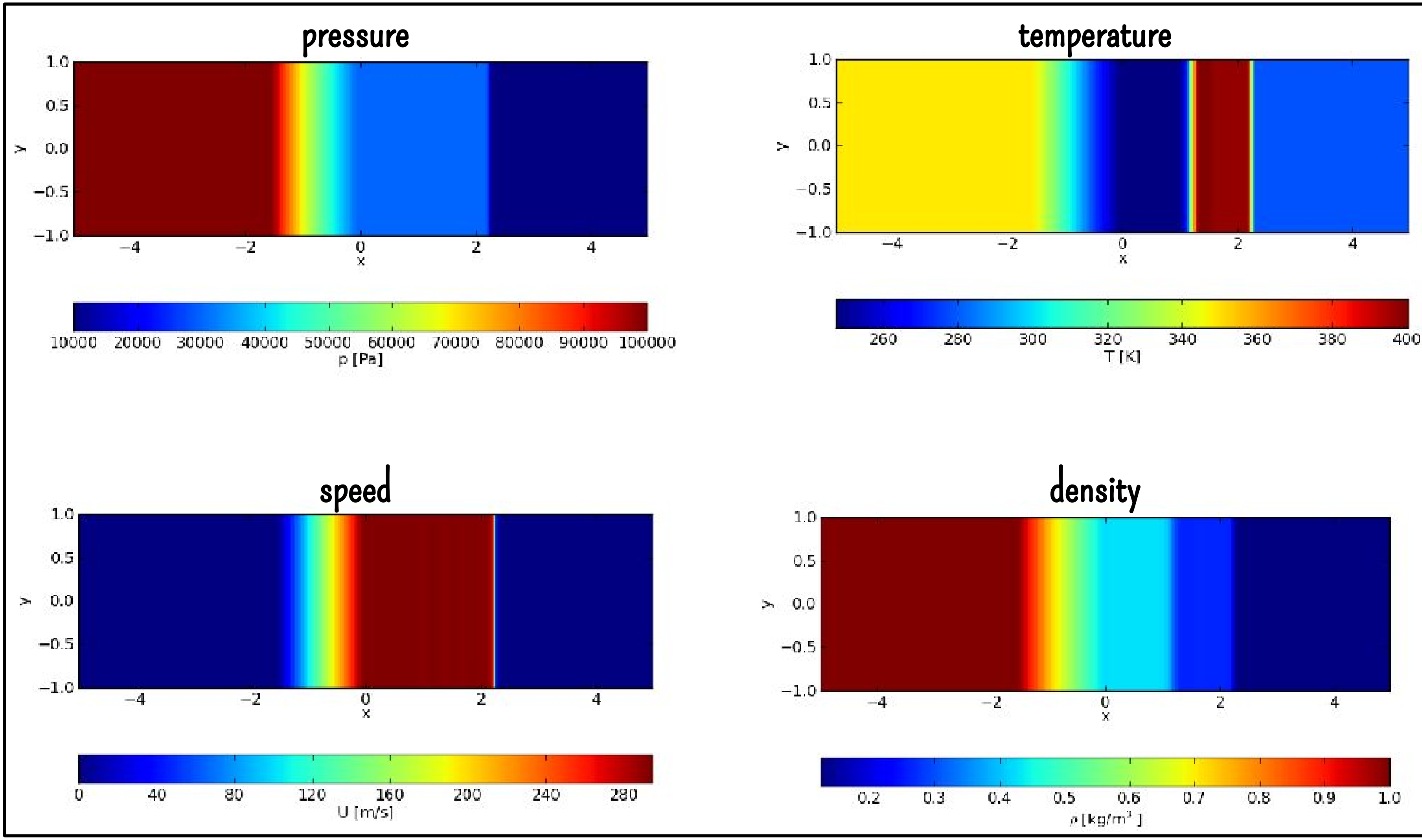
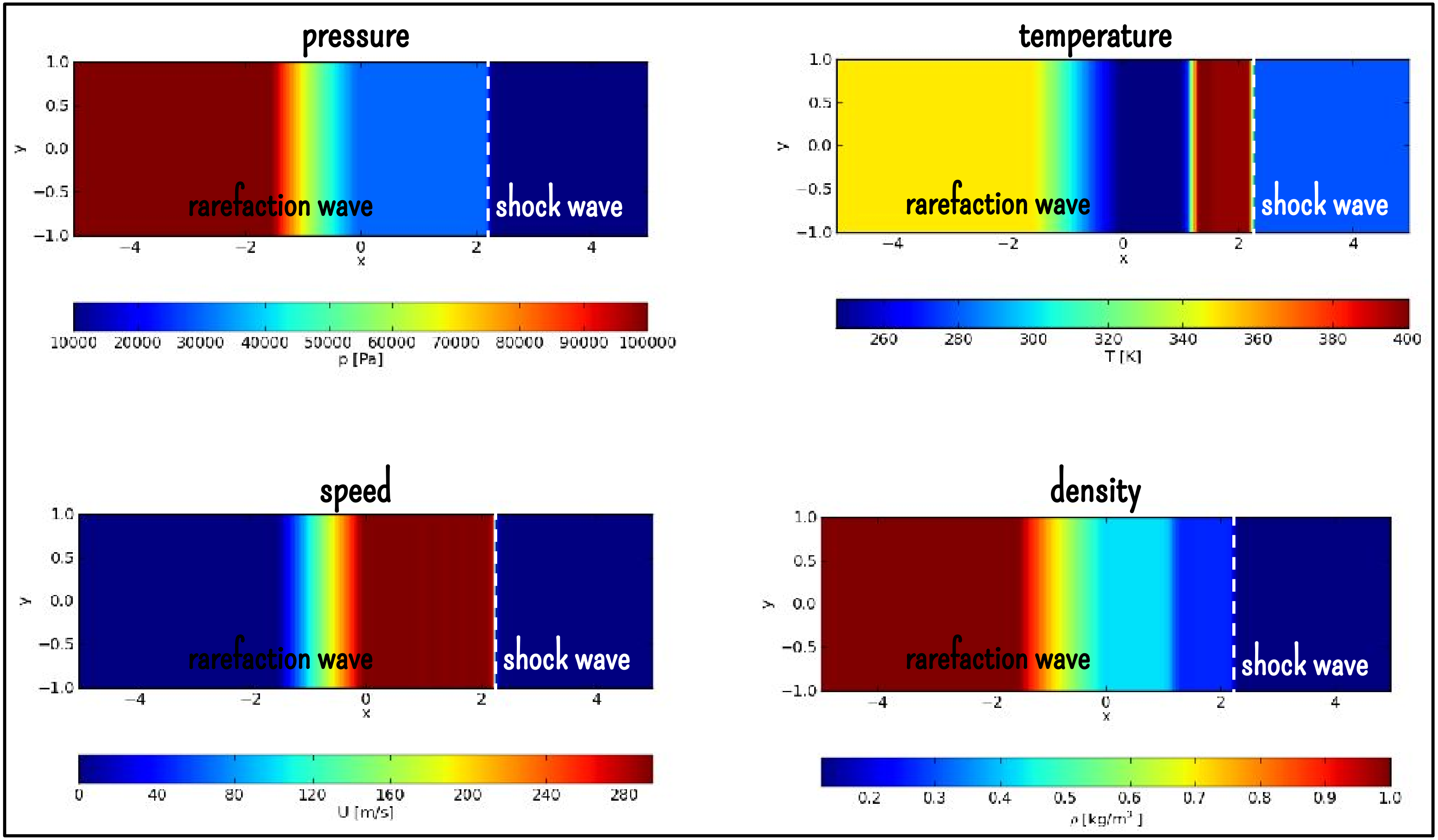


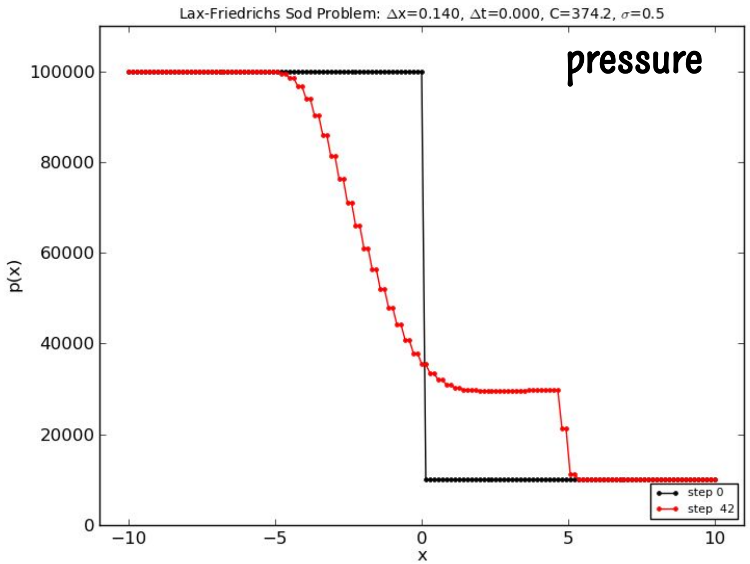
Results: 2D plots

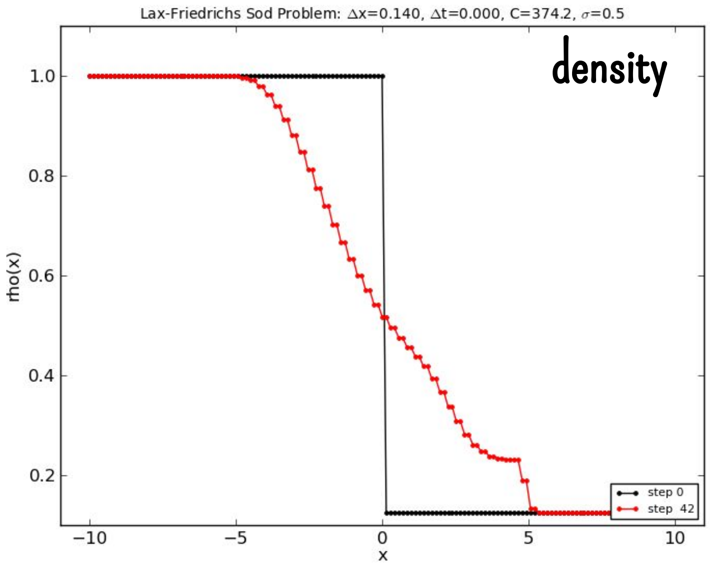
xy plane cut

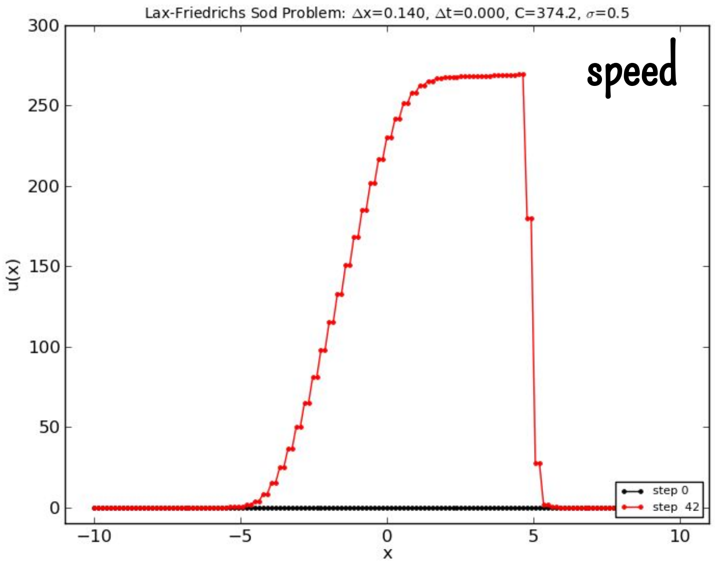


Python solvers

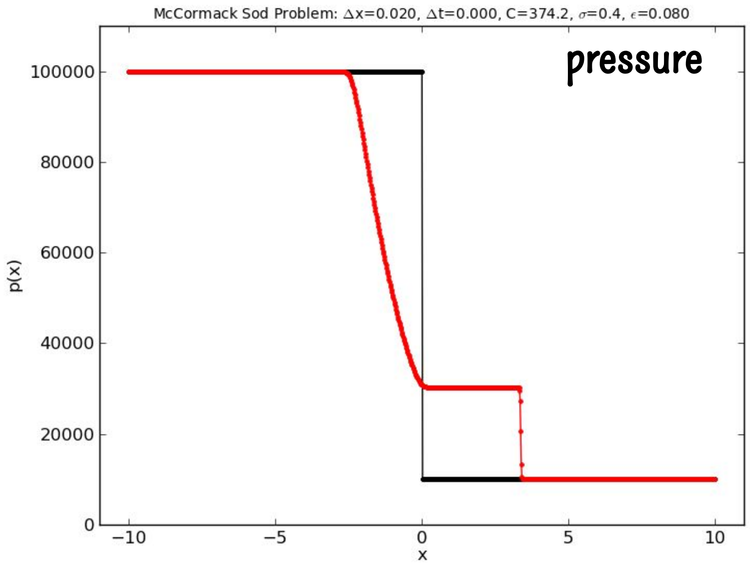
Python solvers: Lax-Friedrichs (1-step)

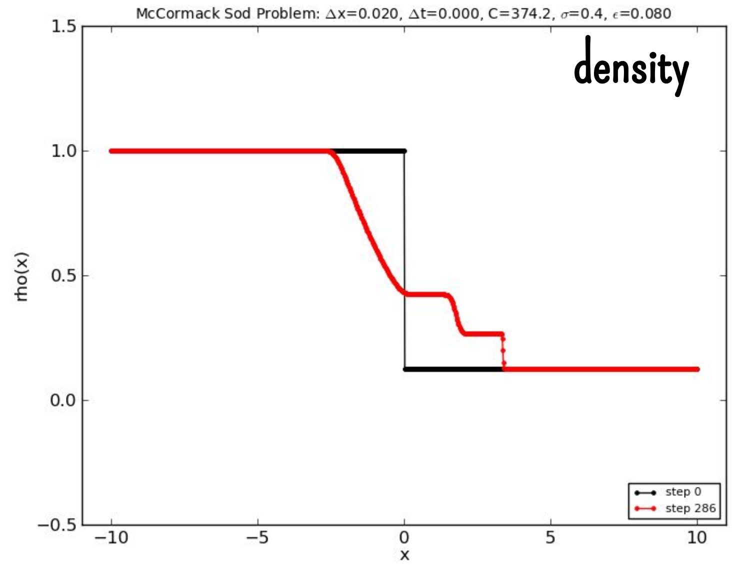


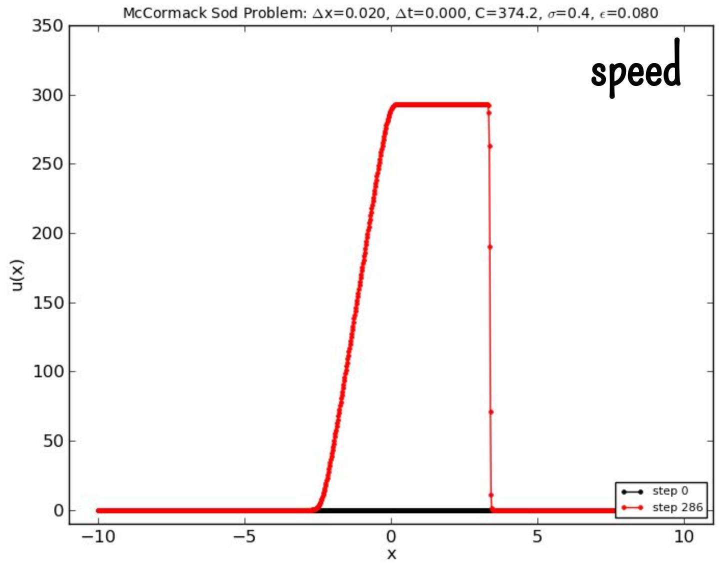




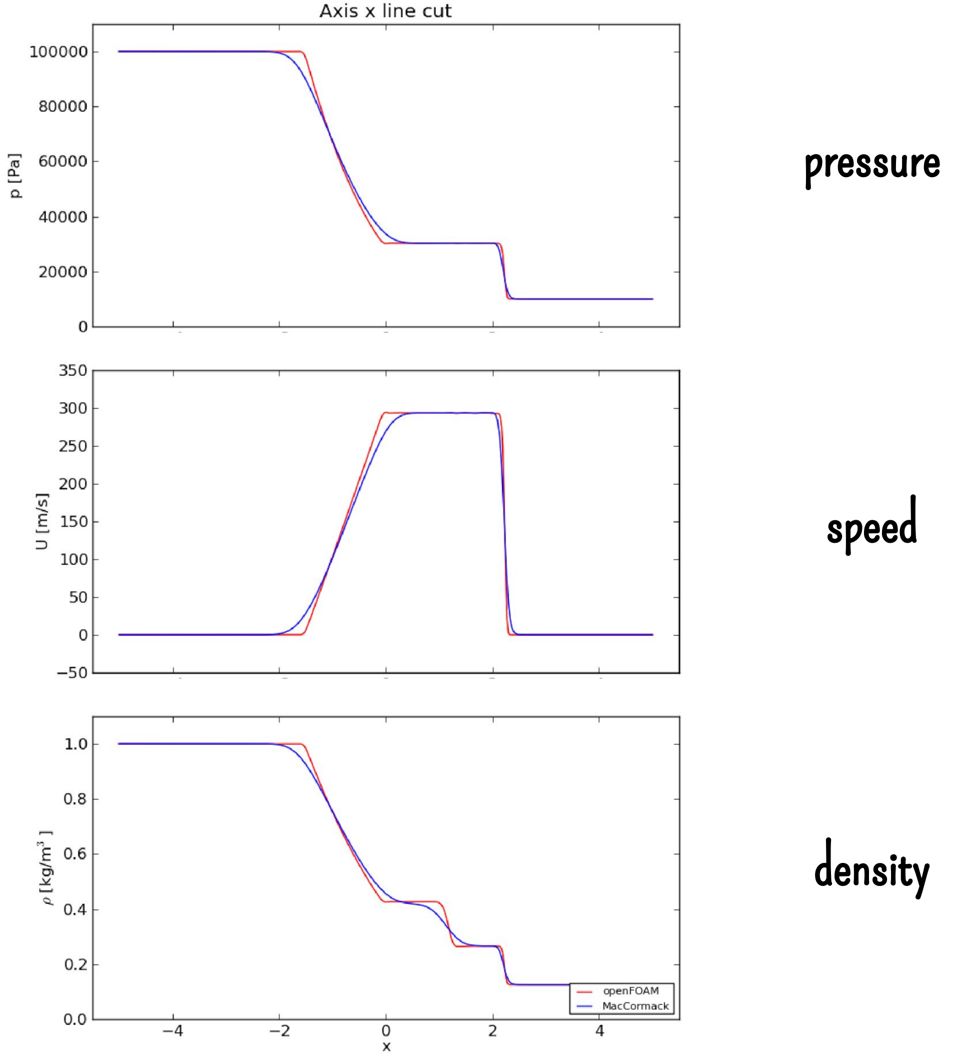
Python solvers: MacCormack (2-step)







OpenFOAM *vs.* MacCormack



Conclusions

- OpenFOAM can simulate the 2D shock tube problem
- solutions match to the analytical ones for resolution of at least 300x300
- 2D solutions are the same for the 1D case
- can extract 1D solutions to 2D and 3D case, if the diaphragm is along x only

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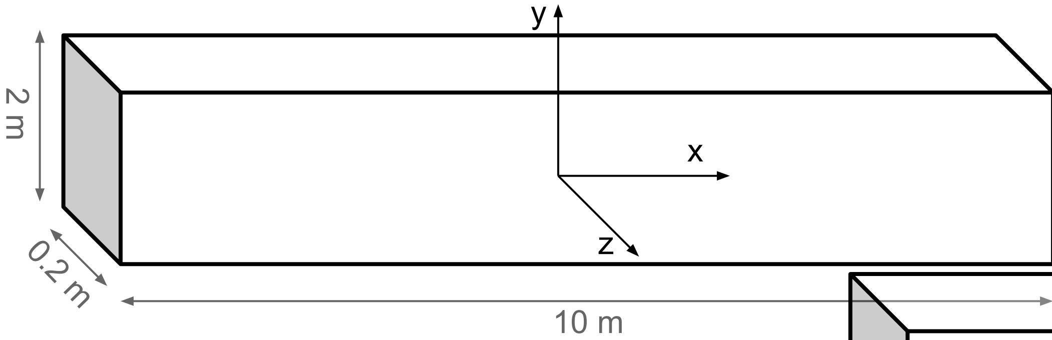
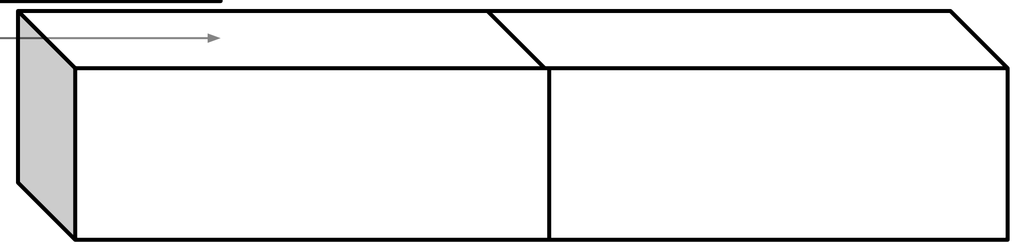
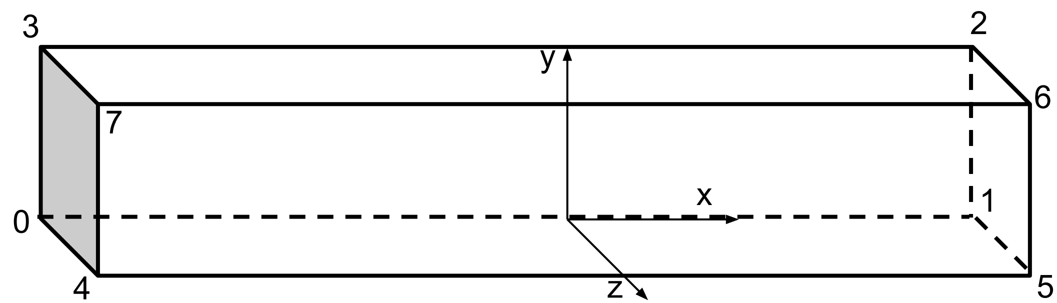
- MacCormack 2-step scheme also approximates well the analytical solutions, but additional artificial viscosity is needed
- MacCormack 2-step scheme has more diffusion

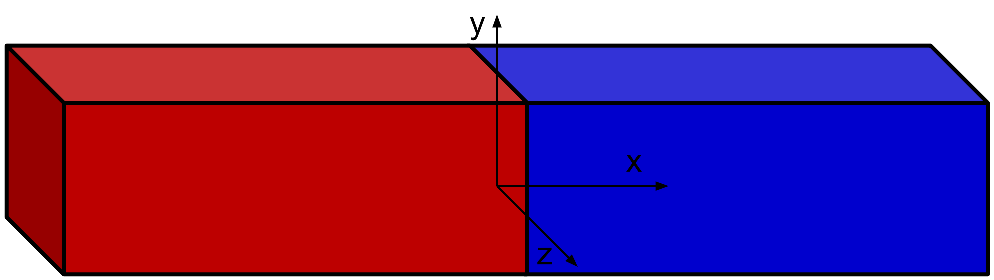
Temperature is obtained given that e = C_VT = (γ - 1)RT:

$$T = \frac{1}{C_V}\left(\frac{E}{\rho} - \frac{u^2}{2}\right)$$


y
x
z
2 m
0.2 m
10 m




3
2
y
6
7
x
1
0
z
4
5


y
x
z