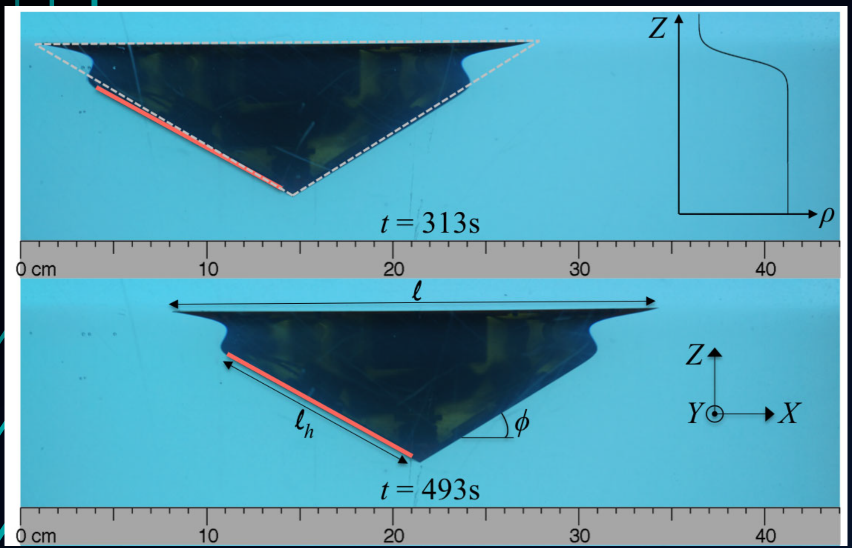
Natural Convection-Driven Propulsion

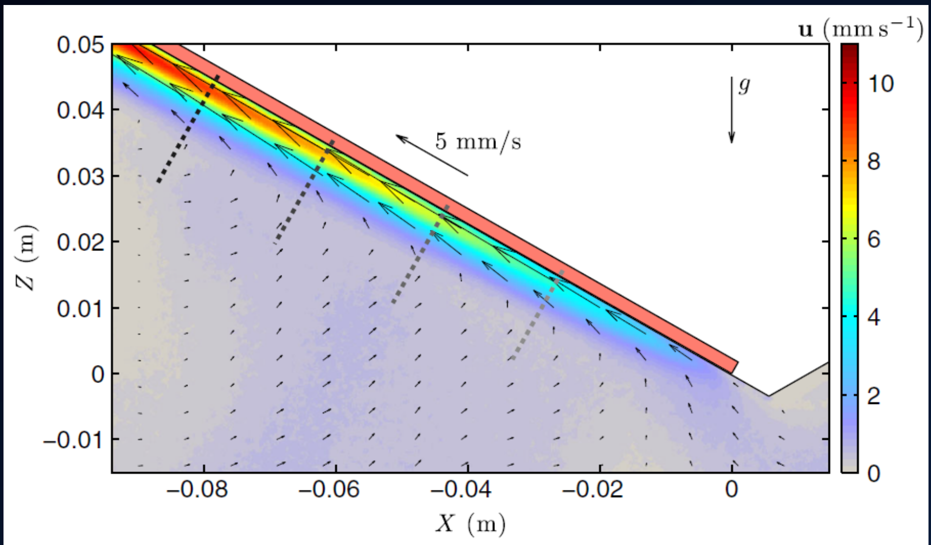
MODELING AND SIMULATION OF HEAT TRANSFER BY MIXED CONVECTION

Overview

- Experimental data taken from Mercier, et al. (2014) "*Self-Propulsion of Immersed Objects via Natural Convection*." Physical Review Letters 12:204501
- The system is analytically complicated, requiring coupled governing equations for both fluids and heat transfer
- Our goal is to construct a faithful simulation using OpenFOAM CFD software, as well as an accurate but tractable model that we can use to approximate the experimental and numerical results

Experiment

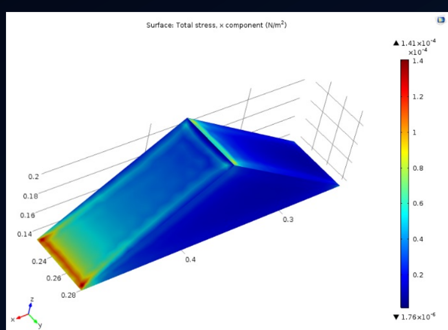




Experimental setup, left, and experimentally measured velocity profile near heated edge, right. Figures taken from Mercier, et al.

Developing the Analytical Model

- Begin with essential correlations
- Correlate drag force with thermal expansion
- Resultant Nu for convection-driven flow over a body



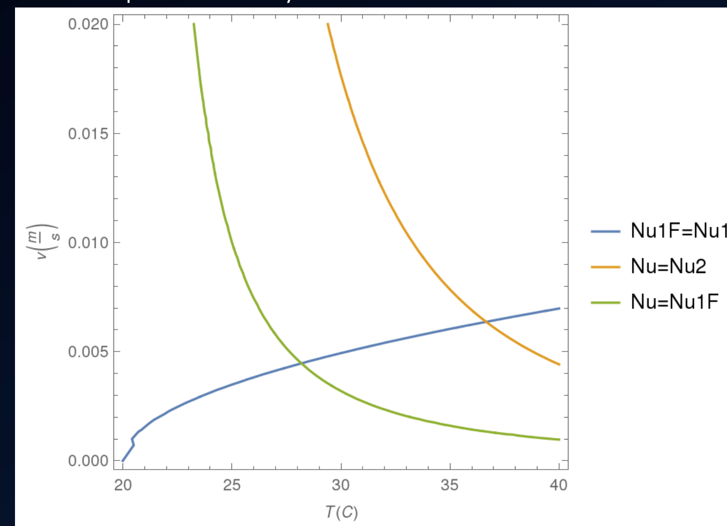
Nusselt Condition		Correlation	Constituent Terms
Definition	(1)	$\frac{hL_c}{k_f}$	$h = \frac{q}{(T - T_\infty)}$
Forced Convection: laminar flow over a flat plate	(2)	$0.6795 * Re^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$Re = \frac{vL_c}{\upsilon}$ $Pr = \frac{\upsilon}{\alpha}$
Natural Convection: heated bottom of an inclined wall	(3)	$\left[0.825 + \frac{0.387 * Ra^{\frac{1}{6}}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2$	$Ra = Gr * Pr$ $Gr = \frac{g\cos\theta * \beta(T - T_\infty)L_c^3}{\upsilon^2}$

Extension of Natural Convection to Velocity		
Drag Force	$D = \frac{1}{2}\rho v_f^2 C_D A_D$	(4)
Pressure Gradient	$F = \Delta P \sin\theta * A_S$	(5)
Hydrostatic Pressure	$\Delta P = gH(\rho - \rho_0)$	(7)
Thermal Expansion	$(\rho - \rho_0) = \beta(T - T_\infty)$	(8)
Natural convection velocity correlation➔	$Nu = \left[0.825 + \frac{0.387 * \left(\frac{\cos\theta * \rho v_f^2 C_D A_D L_c^3}{2H\sin\theta * A_S v\alpha}\right)^{\frac{1}{6}}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2$	(12)

Comparing Nu Formulations

- Adding the forced convection relation, we arrive at an upper bound for T and v
- The forced correlation only accounts for forced velocity; the actual velocity includes free convection and so should be larger
- Therefore the T estimate is an upper bound

Temperature-Velocity Correlation for several Nu formulations



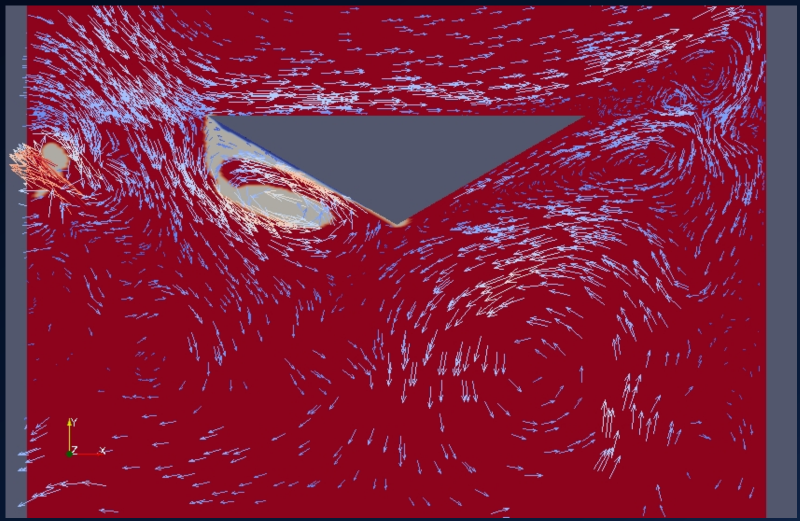
Analytic Model Results	T [°C]	v [$^{mm}/_{s}$]
Natural Convection	28.18	4.46
Forced Convection	36.65	6.36
Experimental	32.42	5.0

Modeling Heat Transfer

GOVERNING EQUATIONS

- $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
- $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$
- $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \boldsymbol{g\beta(T - T_\infty)}$
- $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$

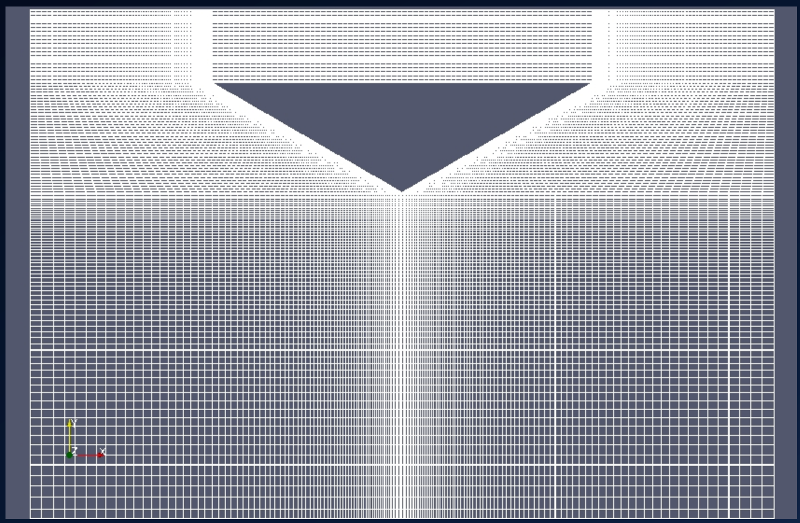
SIMULATED VELOCITY



Meshing

- Modeled a stationary wedge with fluid moving around it
- Block Mesh
- Divided Region into seven blocks
- Finer Meshes around the wedge

FINAL MESH



Solvers

BUOYANT SIMPLE FOAM

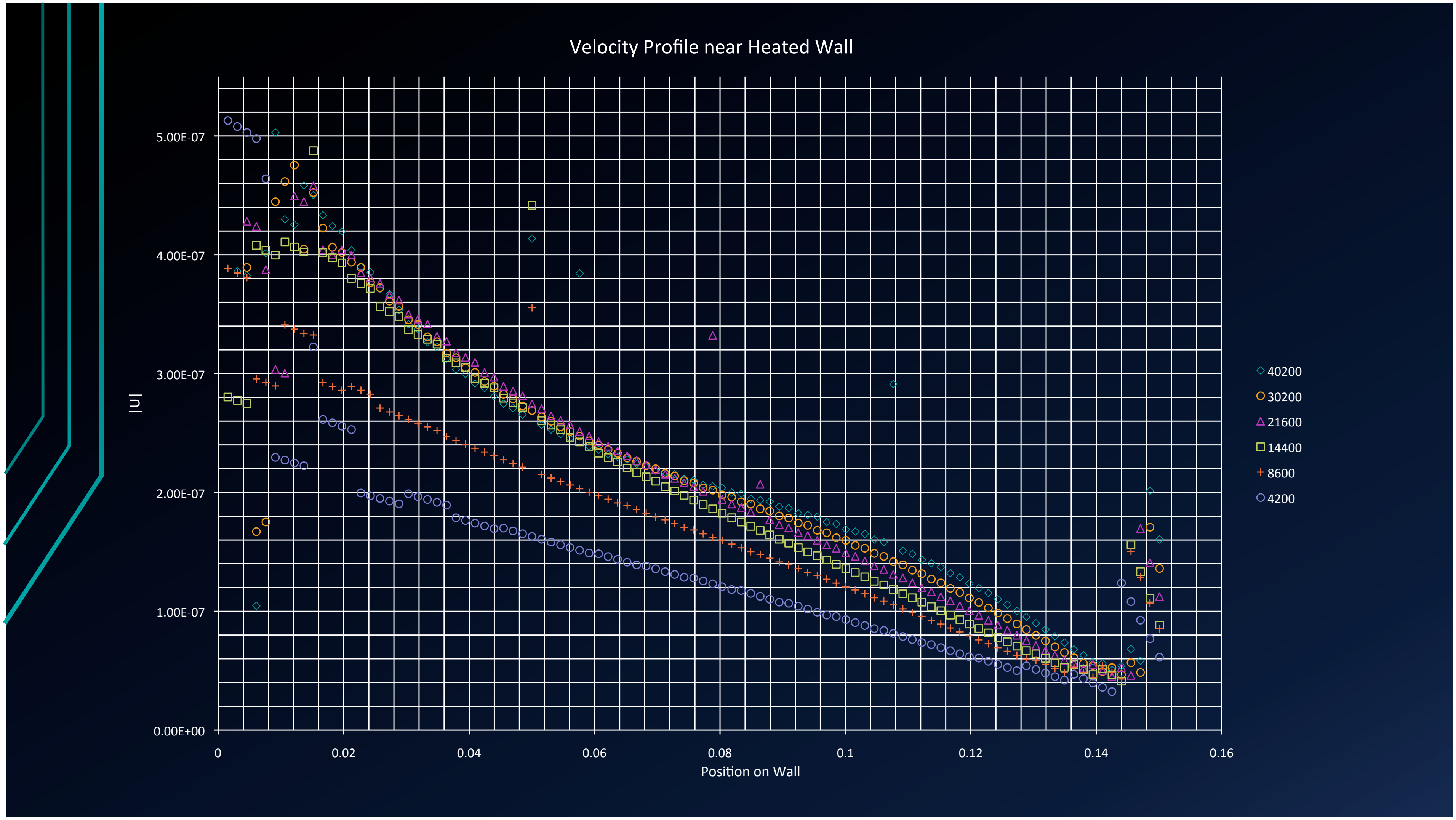
- Steady State: no dt term
- Added relaxation factor to help converge
  - Reduces the amount p can change over iterations
- Polynomial fit of thermal properties
  - Another option is a Boussineq model with β constant

PRESSURE

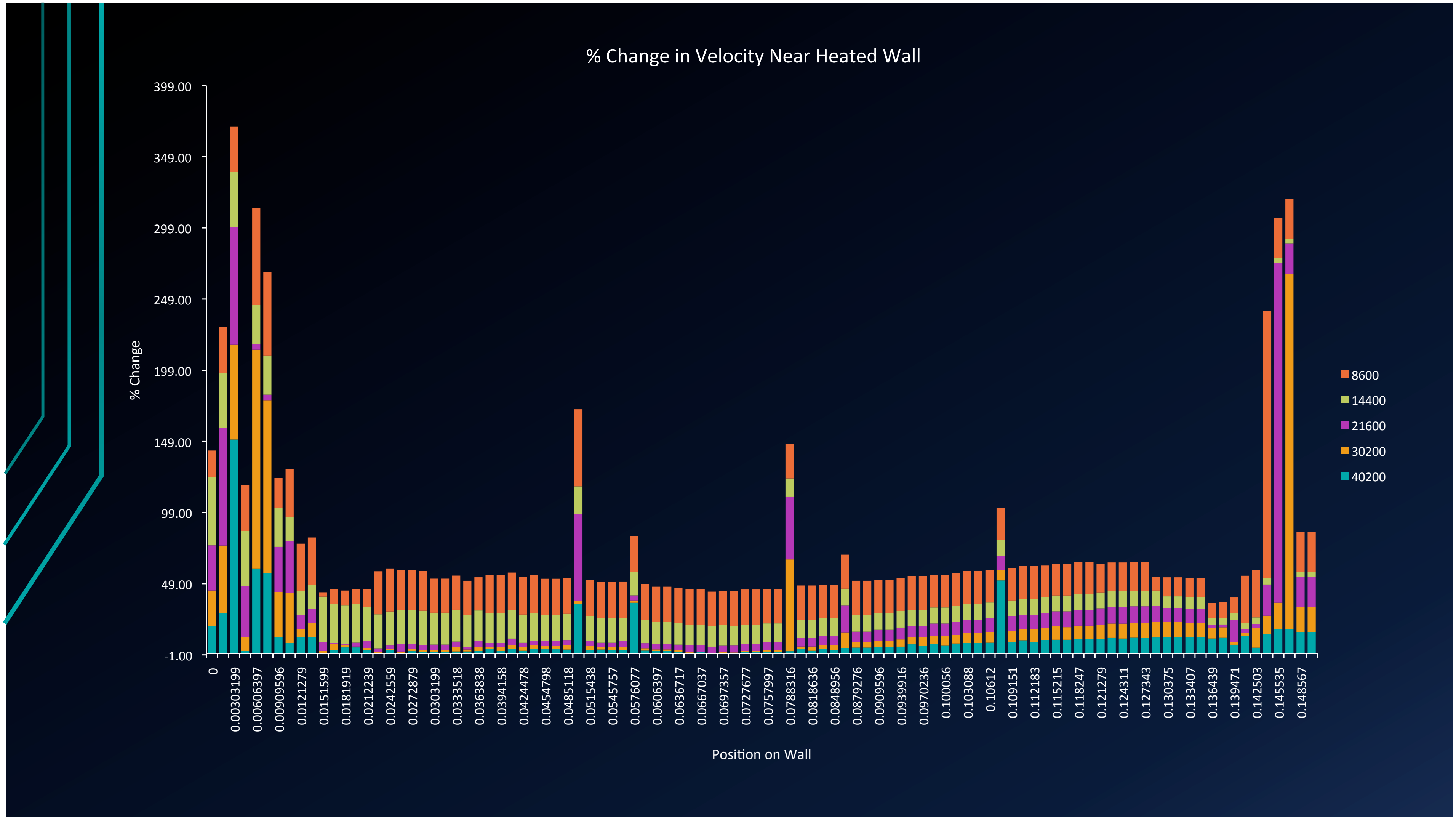
- Pressure flux model
  - Solves for pressure without gravitational influence
  - Allows constant boundary condition on verticel walls
- Used a Gauss Siedel solver for pressure
  - Can solve both symmetric and asymmetric matricies

Boundary Conditions

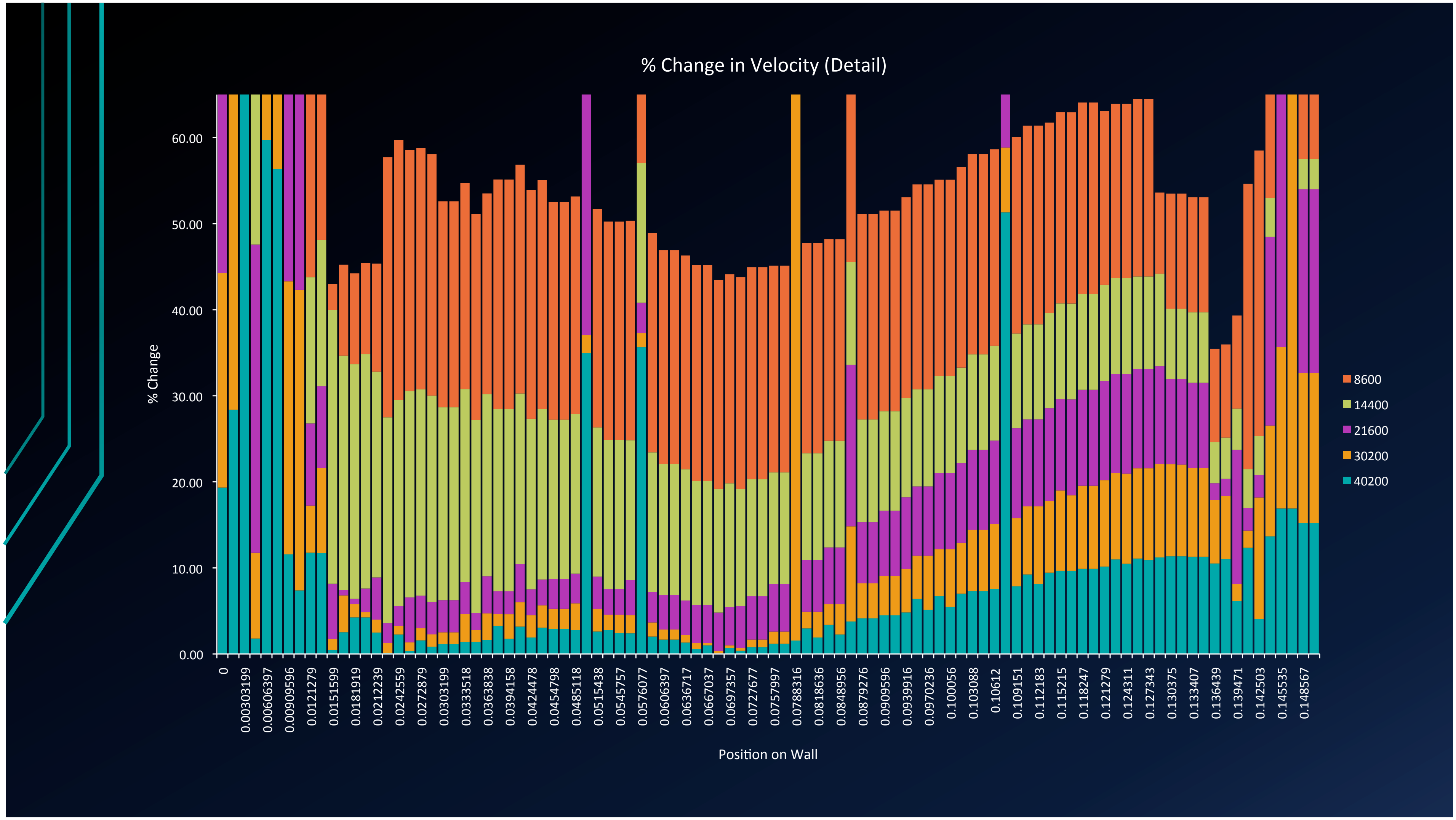
- Approximations that need to fit the data
- Temperature:
  - Zero Gradient
- Inlet:
  - Constant velocity
  - Variable Pressure (constant flux)
- Outlet:
  - inletOutlet Condition
  - Free movement of fluid, with average velocity
  - Constant pressure
- Surface:
  - 0 total velocity
  - Free movement of fluid
  - Constant pressure



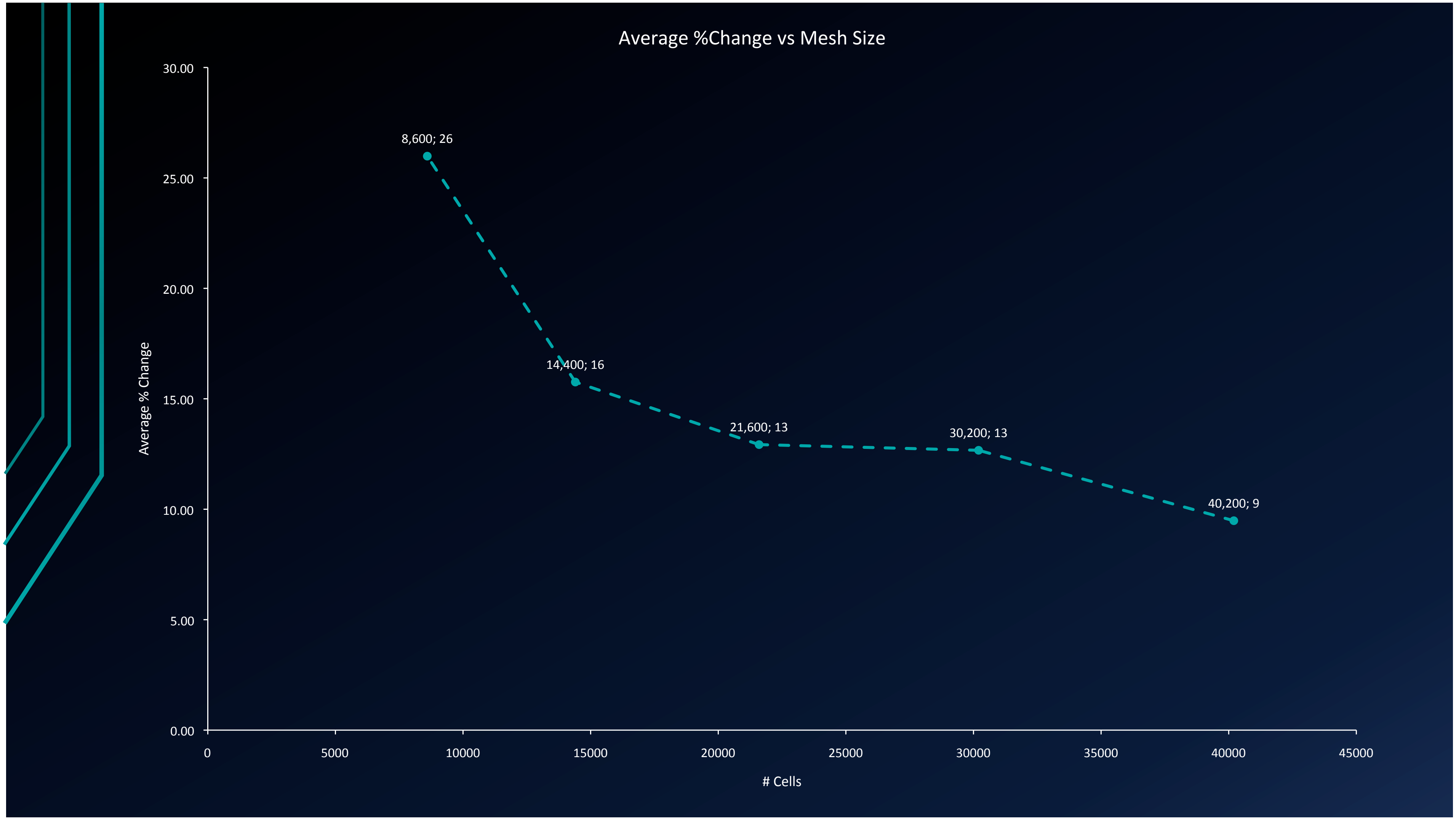
Velocity Profile near Heated Wall
|U|
5.00E-07
4.00E-07
3.00E-07
2.00E-07
1.00E-07
0.00E+00
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
Position on Wall
40200
30200
21600
14400
8600
4200



% Change in Velocity Near Heated Wall
% Change
Position on Wall
399.00
349.00
299.00
249.00
199.00
149.00
99.00
49.00
-1.00
8600
14400
21600
30200
40200

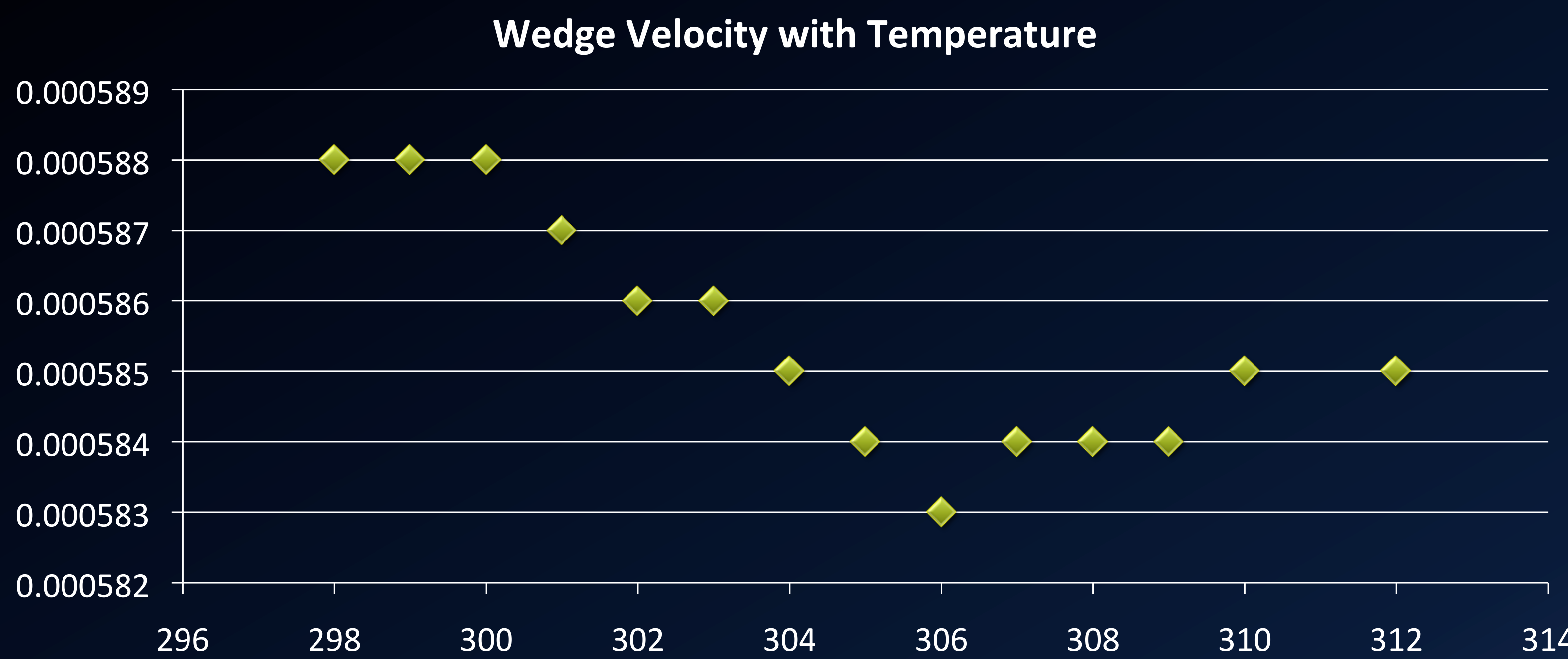


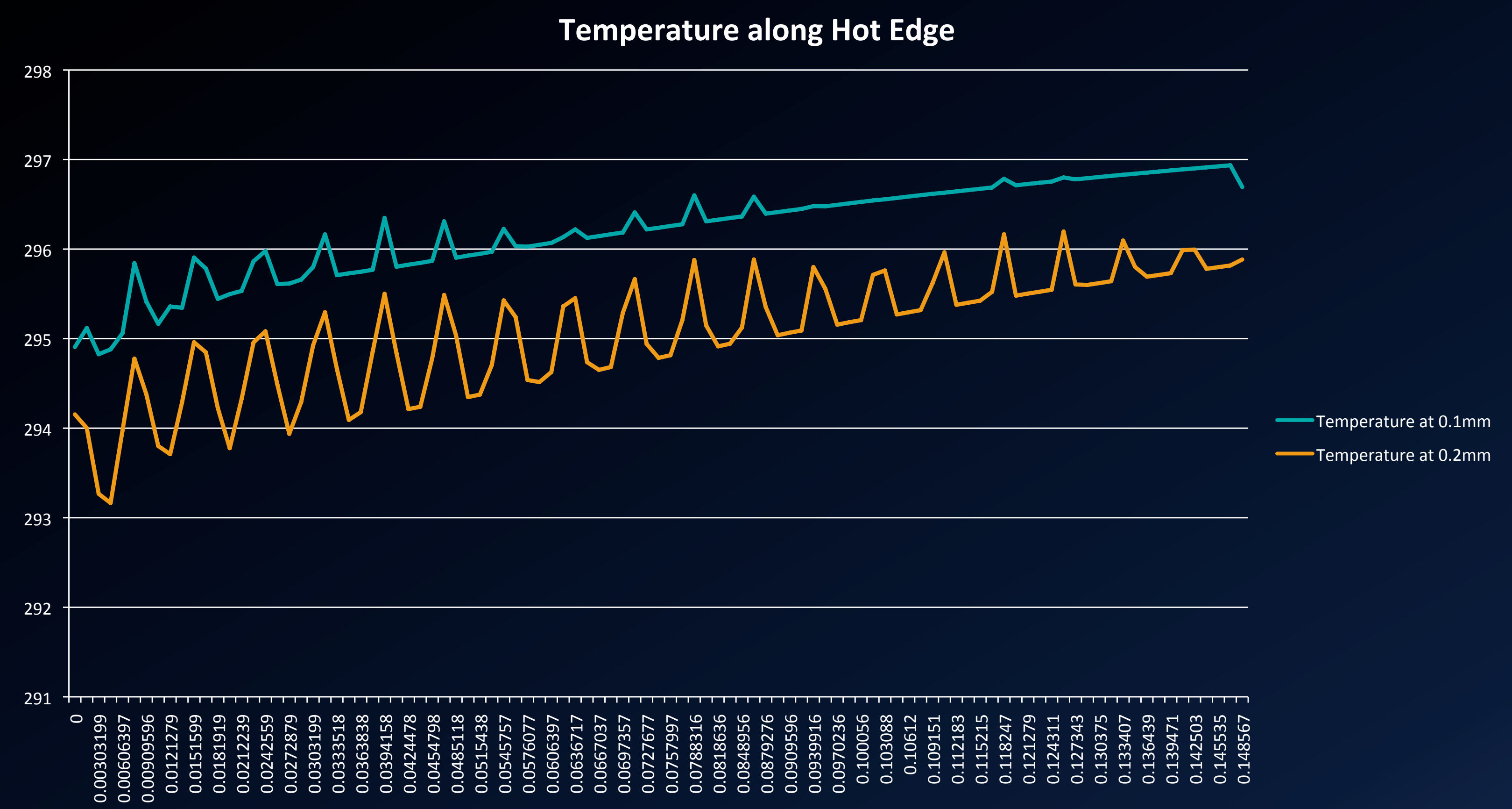
% Change in Velocity (Detail)
% Change
0.00
10.00
20.00
30.00
40.00
50.00
60.00
0
0.00303199
0.00606397
0.00909596
0.0121279
0.0151599
0.0181919
0.0212239
0.0242559
0.0272879
0.0303199
0.0333518
0.0363838
0.0394158
0.0424478
0.0454798
0.0485118
0.0515438
0.0545757
0.0576077
0.0606397
0.0636717
0.0667037
0.0697357
0.0727677
0.0757997
0.0788316
0.0818636
0.0848956
0.0879276
0.0909596
0.0939916
0.0970236
0.100056
0.103088
0.10612
0.109151
0.112183
0.115215
0.118247
0.121279
0.124311
0.127343
0.130375
0.133407
0.136439
0.139471
0.142503
0.145535
0.148567
Position on Wall
8600
14400
21600
30200
40200



Average %Change vs Mesh Size
Average % Change
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
Cells
8,600; 26
14,400; 16
21,600; 13
30,200; 13
40,200; 9

Wedge Velocity vs Temperature

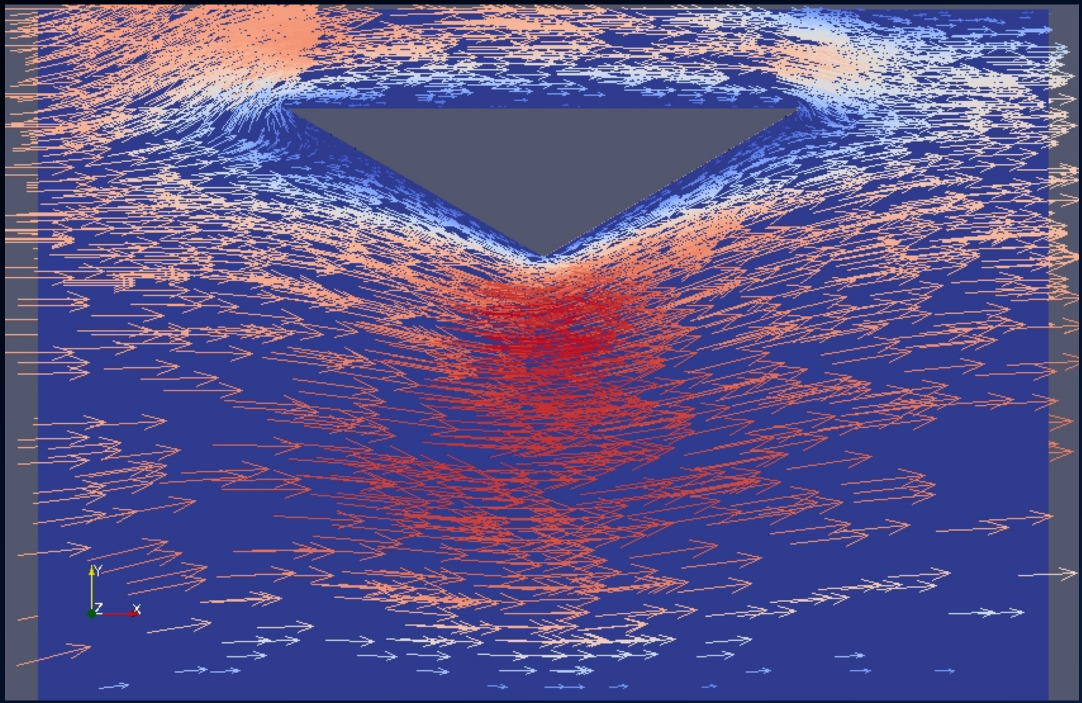




Heat Flux: 4200-6300W

Plate Temperature: 297K

Velocity Profile



Discussion and Future Work

- The concept works and the numerical data matches
- Lessons learned:
  - Relaxation factors are good for efficiency, but don't help until the code works
  - Be willing to try many boundary conditions, even if they seem unlikely to work
  - Finer meshes don't always work
- Future work:
  - Write in a shell code for iterative process
  - Try different shapes, temperature and heat fluxes