

## **Flat Plate Boundary Layer Investigation**

by

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Note: The figures in this lab report should have a larger font size for the axis labels and the legend. You will see that the information is barely readable currently. Otherwise, the report format and use of tables and figures stands as a good example of what is expected for ME@BU lab reports.

## **1. Introduction**

When considering the flow of a fluid past any object, friction plays an important role. As air passes over the surface, the flow adheres to the surface due to friction between the air and solid material of the plate (Anderson, 2012). The flow velocity is zero at the surface, the no-slip conditions, and therefore near the surface there is a region in which the flow is retarded. This region of the flow that is retarded is called the boundary layer (Flat Plate Lab Manual, 2013). Dimensionally, the boundary layer is described by the boundary layer thickness. It is the distance from the plate to the point where the flow speed is either 95% or 99% of the outer flow velocity. A velocity profile, which shows the variation of flow speed with vertical distance from the plate is used to describe the boundary layer.

There are two types of boundary layers: laminar and turbulent. The type of boundary layer that will occur depends upon the Reynolds number as well as the surface conditions. The different boundary types have different profiles and different growth rates. The objective of this experiment is to measure the velocity profile for different flat plates with differing surface roughness, compare the experimental values to those from theory, and determine which type of boundary layer is present.

## **2. Theory**

As the flow travels along the flat plate a boundary layer forms. Near the leading edge of the plate, which refers to the first point of flow contact, the flow will always be laminar. The boundary layer is very thin, and the fluid particles move in parallel planes which do not interact with each other (Anderson, 2012). As the flow travels further in the horizontal direction, the boundary layer grows and if the flow travels along a long enough plate, the boundary layer will transition become turbulent. In this case, the fluid particles are no longer flowing in the streamwise direction, instead the fluid elements begin to move in a random and irregular fashion (Anderson, 2012). In the turbulent boundary layer, because there is fluid moving perpendicular to the plate, there is more mixing. The transition from laminar to turbulent flow will happen naturally, however, one can also place a 'trip' at some point on the plate which forces the flow into a turbulent state.

The state of the boundary layer: laminar or turbulent, has a considerable effect on drag and therefore it is important to know which type of boundary layer will be present (Pritchard, 2011). It has been shown that the state of the boundary layer is dependent upon the Reynolds number given by

$$\text{Re}_x = \frac{Ux}{\nu} \quad (1)$$

where  $U$  is the velocity of the flow outside the boundary layer,  $x$  is the distance along the plate and  $\nu$  is the kinematic viscosity of the fluid. The Reynolds number describes the ratio of inertial to viscous forces. A Reynolds Number of 500,000 is the standard approximation for the point where the flow changes from laminar to turbulent (Flat Plate Lab Manual, 2013).

The shape of the boundary layer is different for laminar and turbulent boundary layers and approximations to the shape are given by:

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \text{for laminar flow} \quad (2)$$

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad \text{for turbulent flow} \quad (3)$$

where  $u$  is the local velocity,  $U$  is the velocity of the flow outside the boundary layer,  $y$  is the height above the plate and  $\delta$  is the thickness of the boundary layer. These numerical approximations can be compared to the actual experimental values.

The boundary layer thickness grows with distance from the leading edge of the plate. This growth rate is different for laminar and turbulent boundary layers. Approximations for the growth rate are given by:

$$\frac{\delta}{x} = \frac{4.6}{\text{Re}_x^{1/2}} \quad \text{Laminar Approximation} \quad (4)$$

$$\frac{\delta}{x} = \frac{0.37}{(\text{Re}_x)^{1/5}} \quad \text{Turbulent Approximation} \quad (5)$$

The approximations for the boundary layer profile and growth rate will be used to determine which type of boundary layer is present in the experiment.

In the lab, a pitot probe will be used to measure the local pressure,  $p$ . The pitot measures the difference in total pressure,  $p_T$ , and static pressure,  $p$ , at the location of the probe. This

pressure difference is given by the difference of the height of the water in the manometer:

$$\Delta p = p_T - p = \rho_{water} g \Delta h \quad \text{Manometer equation} \quad (6)$$

The pressure must then be used to compute the local velocity in the flow,  $u$ . Bernoulli's equation is used for this calculation:

$$p_T = p + \frac{1}{2} \rho u^2 \quad \text{Bernoulli's equation} \quad (7)$$

### 3. Measurement method

#### Apparatus

**Micrometer** - used to adjust distance between pitot static tube and flat plate

**Computer** - Lab-view Software on computer allows for pressure readings to be taken

**Flat Plate** - contains rough and smooth side over which air flows

**Pitot Static Probe** - contains inlet that takes pressure readings for dynamic pressure

**Air Bench** - provides flow of air for analysis of flat plate boundary layer

**Strip of Paper** - used to ensure pitot static tube was adjust tightly against flat plate

**Thermometer** - used to measure ambient temperature

**Bourdon-type Pressure gauge** - Provides reading for ambient pressure

#### Procedures

1. The speed at which the tunnel should be run in order to ensure laminar flow is calculated.
2. Room Temperature and Pressure are recorded.
3. The plate is put in the test section of a wind tunnel such that the smooth side of the plate faced the pitot probe and micrometer. The micrometer is adjusted so that the probe just touched the plate's surface. A thin strip of paper was placed between the plate and probe in order to ensure that it is touching the surface. The proper location of the probe is found when a slight resistance was felt when the paper was removed.
4. The manometer is leveled using the screws on its base.
5. The flow of air is turned on, and it is allowed to run for a couple of seconds to ensure a steady flow of air is being produced. Three readings of pressure are taken at 0.2 mm from the surface

of the plate until the probe is out of the boundary layer (the velocity does not change). The measurements are done 3 times in order to assure accuracy.

**Spot check:** When the probe is as close to the plate surface as possible, the velocity should almost be 0, so the pressure should equal the stagnation or total pressure.

Therefore the manometer should show no height differential in the water.

6. The rough side of the plate is placed in the test section and steps 3 – 5 are repeated.

#### 4. Results and Analysis

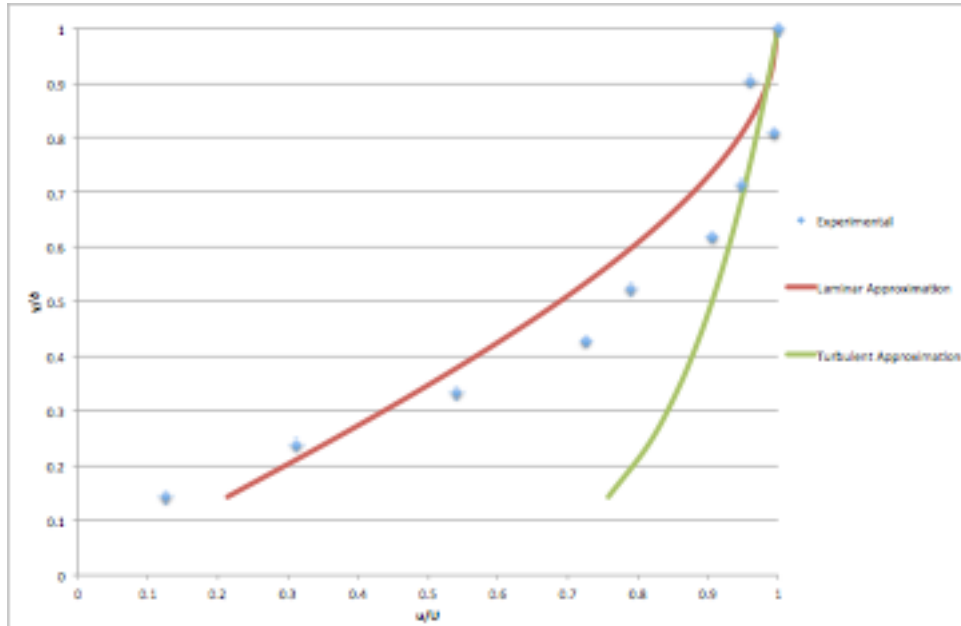
The readings of atmospheric conditions in the lab on the day of the experiment are given in Table 1.

**Table 1:** Atmospheric Pressure and Temperature

Atmospheric Pressure (kPa)	Atmospheric Temperature (°C)
100.8	19.1

The flow measurements near the plate were taken 12cm downstream of the leading edge of the plate. The raw data from the lab that give the distance from the surface and the pressure for three measurement runs for each surface case are given in Appendix 1. The manometer height values can be converted to velocity values and then the ratio with the free stream flow value can be obtained. Sample calculations are shown in Appendix II. The final values are given in Table 4 in Appendix III. It was observed that, after some time, as the pitot static tube was moved further and further away from the flat plate, the readings for dynamic pressure began to fluctuate around some centralized value. Physically, this meant that the flow velocity was the same at each point, and that the pitot static tube had exited the boundary layer and was now in potential flow. For the smooth surface, as seen in Table 1, distances of 1.4 mm to 2.0 mm recorded similar pressure values ranging between 0.350 and 0.400 inches of water with most values being grouped around 0.400 inches of water. As a result, the distance of 1.80 mm was chosen as the boundary layer thickness. From the established boundary layer thickness, and average pressure reading, the free stream velocity was calculated to be 12.82 m/s.

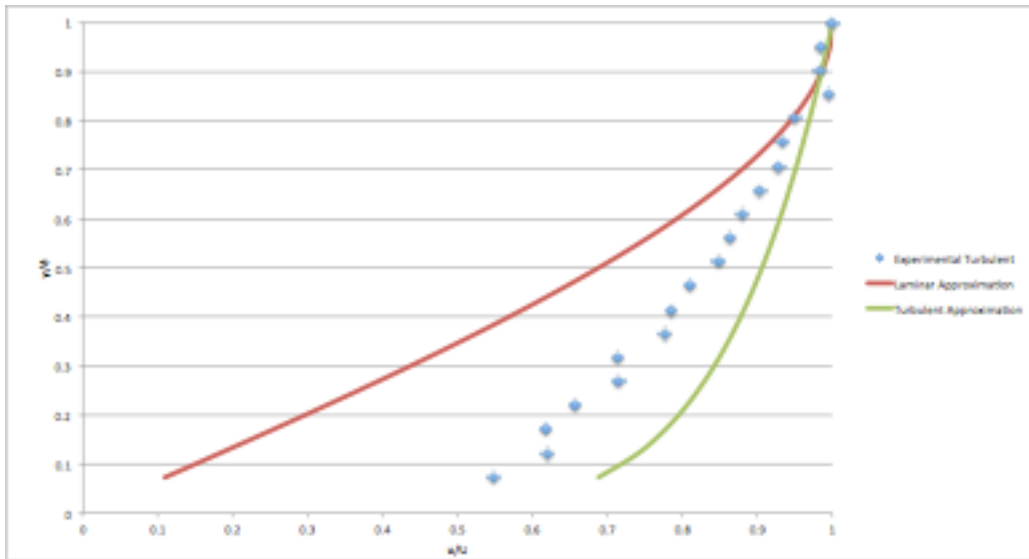
Figure 1 displays the velocity profile obtained for the smooth flat plate along with the laminar and turbulent approximations.



**Figure 1:** Velocity Profile for Smooth Flat Plate

The experimental values compare more closely with the laminar approximation of the boundary layer. Quantitatively, the average difference between the experimental values and laminar approximations is 6.77% as seen in Table 6 of Appendix III. The non-dimensional boundary layer thickness was also computed. For the smooth plate, it is found to be 0.015 (as demonstrated in Appendix II). This differs from the theoretical laminar value of 0.0144 by 4.2%.

Figure 2 shows the measured velocity profile obtained for the rough side of the flat plate in comparison to theoretical laminar and turbulent approximations. The boundary layer thickness was determined to be 3.80 mm for this case with a free stream velocity of 12.45 m/s. As expected, this is basically the same free stream value as for the smooth plate case.



**Figure 2:** Velocity Profile for Rough Flat Plate Area

The experimental values for this case lie closer to the turbulent approximations which is expected because the roughness of the plate trips the boundary layer to the turbulent state. The experimental values differed from the laminar approximations on average by 17.13% and different from turbulent values on average by 7.39% as seen in table 7 of Appendix III. The other parameter that is calculated is non-dimensional boundary layer thickness ( $\delta/x$ ). For the rough side of the flat plate, the experimental value of this ratio was found to be 0.0342. It differed from its laminar approximation by 1.98% and by its turbulent approximation by 0.290%. All comparisons indicate that the boundary layer along the rough wall was turbulent.

## 6. Discussion and Conclusion

In this experiment, the velocity profiles in the boundary layer of a flat plate were measured for a flat plate with both a smooth and rough surfaces. In order to discern the type of flow in each case, theoretical approximations for laminar flow and turbulent flow were compared with the experimental values obtained.

An overview of the general analysis indicates several errors that could have occurred. In terms of atmospheric pressure and temperature readings, sources of parallax error may have been an issue in taking these readings for two main reasons. Firstly, the scale on the pressure gage and thermometer were in very small increments, and thus extreme care was taken in ensuring proper readings were taken. Secondly, the design of the flow bench in relation to the

thermometer was obstructive. The flow bench physically blocked direct view of the thermometer, and thus, temperature readings had to be taken at an angle making these readings prone to some sort of parallax error. Every effort was still made to ensure that proper values for these physical properties were obtained as it would affect all data calculations in this experiment.

Pertinent to the determination of the velocity, the pressure value taken at a particular distance was not the same for each repetition. The three readings allowed for an average value to be calculated to offset the effects of random error. It is noted that initial results obtained included a negative value for the pressure. This could have been as a result of experimental error in that the experimenter may have hit the ‘tare’ button on the software before the flow was turned on. As a result, some of the values may have oscillated below zero. In order to correct this error, the absolute value of the lowest negative value was added to all data points collected in that set. In summary, the measurement from the pitot probe was not precise.

In addition, the method for determining the boundary layer thickness was not precise. Therefore the values of  $\delta$  and  $U$  used to nondimensionalize the local position and velocity are not precise. This could be the source of some of the difference in both the profile and the boundary layer thickness values between the experimental and theoretical values.

Even though there is error associated with the measurements, the boundary layer that was suppose to be laminar indeed matched more closely to the theoretical approximation for the laminar boundary layer. The average difference was 6.77% while if one compared it to the turbulent theoretical approximation it had an average difference of 20.14%. The 4% difference in the boundary layer thickness measurement from theory for a laminar boundary layer also shows quite good agreement.

For the rough plate, where the boundary layer should be turbulent, it was seen that the experimental measurements compared more closely with the theoretical approximations for the turbulent case than the laminar case. For the rough plate, the average profile difference between the measured and theoretical turbulent case was 7.39% and the boundary layer thickness difference was just 0.29%. For the turbulent case, additional measurement errors may have occurred because of the nature of turbulent flow. The vortices and eddies that developed within the flow, may have influenced the pressure readings. The variations of the data as shown in figure 2 might be explained by this.



The difference between the boundary layer for the laminar and turbulent cases is clear. The boundary layer thickness for the laminar case was 1.8mm and for the turbulent case it was 3.8mm. This is one of the major characteristics of a turbulent boundary layer which is that turbulent boundary layers are thicker than laminar boundary layers. Also, the turbulent boundary layer profile has a steeper gradient near plate than the laminar boundary layer which is expected. The steeper gradient of velocity means that the shear stress would be greater for the turbulent case. Therefore, from an applications standpoint, if the shear stress is to be decreased, the preferable flow would be laminar flow.

In conclusion, comparisons made between experimental results and theoretical data allow the determination to be made that the boundary layer over the smooth plate was a laminar boundary layer, and the boundary layer over the rough side of the plate was turbulent. The differences between these two types of boundary layers was clearly demonstrated.

## **7. References**

Anderson, J. (2012). Introduction to Flight. New York: McGraw-Hill Companies

Pritchard, P. (2011). Fox and McDonald's Introduction to Fluid Mechanics. New Jersey: Wiley and Sons Inc.

Saint Louis University. (2013). Fluid Dynamics Laboratory Manual - Flat Plate Boundary Layer.

## Appendix I : Raw Data

**Table 2:** Pressure Readings at Various Distances above Flat Plate for Smooth Surface

Distance (mm)	Pressure 1 (inches of water)	Pressure 2 (inches of water)	Pressure 3 (inches of water)
0	0.0176	0	0.0013
0.2	0.0485	0.0406	0.0261
0.4	0.109	0.124	0.113
0.6	0.213	0.23	0.182
0.8	0.241	0.25	0.25
1	0.316	0.357	0.301
1.2	0.345	0.366	0.356
1.4	0.388	0.386	0.399
1.6	0.354	0.39	0.35
1.8	0.39	0.397	0.4
2	0.392	0.394	0.397
2.2	0.417	0.402	0.404
2.4	0.407	0.382	0.381

**Table 3:** Pressure Readings at Various Distances above Flat Plate for Rough Surface

Distance (mm)	Pressure 1 (inches of water)	Pressure 2 (inches of water)	Pressure 3 (inches of water)
0.00	0.1194	0.1094	0.1076
0.20	0.1381	0.1337	0.1586
0.40	0.1416	0.1406	0.1454
0.60	0.1574	0.1587	0.1659
0.80	0.22	0.1695	0.1829
1.00	0.2014	0.1882	0.1804
1.20	0.2263	0.2131	0.235
1.40	0.2163	0.2487	0.223
1.60	0.2451	0.2375	0.2506
1.80	0.2947	0.2573	0.2545
2.00	0.2706	0.2987	0.2637
2.20	0.2861	0.3189	0.2605
2.40	0.3026	0.3016	0.3078
2.60	0.3439	0.2793	0.3413
2.80	0.3256	0.3297	0.3178
3.00	0.3413	0.3475	0.321
3.20	0.3735	0.3554	0.3794
3.40	0.3701	0.3614	0.35
3.60	0.3574	0.3762	0.3518
3.80	0.3601	0.3905	0.3693
4.00	0.4115	0.3695	0.3763
4.20	0.3861	0.3879	0.3774

## Appendix II - Sample Calculations

### Temperature Conversion

$$^{\circ}K = ^{\circ}C + 273.15$$

$$\begin{aligned}^{\circ}K &= 19.1^{\circ}C + 273.15 \\ &= 292.25^{\circ}K\end{aligned}$$

### Pressure Conversion

$$1000Pa = 1kPa$$

$$\begin{aligned}Pa &= 100.8kPa \times \frac{1000Pa}{1kPa} \\ &= 100800Pa\end{aligned}$$

### Length Conversion

$$\begin{aligned}m &= cm \times \frac{1m}{100cm} \\ &= 12cm \times \frac{1m}{100cm} \\ &= .12m\end{aligned}$$

### Density

$$\begin{aligned}\rho &= \frac{P_{total}}{R_{total}T} \\ &= \frac{100800Pa}{\left(287 \frac{J}{kgK}\right)(292.25K)} \\ &= 1.20 \frac{kg}{m^3}\end{aligned}$$

Average Pressure

$$\begin{aligned} \text{Average} &= \frac{1}{n} \sum_{i=1}^n x_i \\ &= \frac{1}{3}(0.0485 + 0.0406 + 0.0261) \text{ inches of water} \\ &= 0.0384 \text{ inches of water} \end{aligned}$$

Pressure Conversion

$$\begin{aligned} Pa &= \text{inches of water} \times \frac{1 \text{ kPa}}{4.01463 \text{ inches of water}} \times \frac{1000 \text{ Pa}}{1 \text{ kPa}} \\ &= 0.0384 \text{ inches of water} \times \frac{1 \text{ kPa}}{4.01463 \text{ inches of water}} \times \frac{1000 \text{ Pa}}{1 \text{ kPa}} \\ &= 9.57 \text{ Pa} \end{aligned}$$

Velocity from Dynamic Pressure

$$\begin{aligned} p &= \frac{1}{2} \rho u^2 \\ u &= \sqrt{\frac{2p}{\rho}} \\ &= \sqrt{\frac{2(9.57 \text{ Pa})}{1.20 \frac{\text{kg}}{\text{m}^3}}} \\ &= 3.99 \text{ m/s} \end{aligned}$$

Velocity Ratio

$$\begin{aligned} \frac{u}{U_j} &= \frac{3.99}{12.8} \\ \frac{u}{U_j} &= 0.312 \end{aligned}$$

### Distance Correction

$$y = h + \frac{t}{2}$$

$$y = 0.2mm + \frac{0.6mm}{2}$$

$$y = 0.5mm$$

### Distance to Boundary Layer Thickness Ratio

$$\frac{y}{\delta} = \frac{0.5mm}{2.1mm}$$

$$\frac{y}{\delta} = 0.238$$

### Velocity Ratio Laminar Approximation

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{0.5mm}{2.1mm}\right) - \frac{1}{2}\left(\frac{0.5mm}{2.1mm}\right)^3$$

$$\frac{u}{U_\infty} = 0.350$$

### Velocity Ratio Turbulent Approximation

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$\frac{u}{U_\infty} = \left(\frac{0.5mm}{2.1mm}\right)^{\frac{1}{7}}$$

$$\frac{u}{U_\infty} = 0.815$$

### Difference between Experimental and Numerical Approximations

$$\text{Difference} = (0.350 - 0.312) \times 100$$

$$\text{Difference} = 3.80\%$$

### Coefficient of Dynamic Viscosity

$$\mu = 1.458 \times 10^{-6} \frac{T^{1.5} \text{ kg}}{T + 110.4 \text{ ms}}$$

$$\mu = 1.458 \times 10^{-6} \frac{292.1^{1.5} \text{ kg}}{292.1 + 110.4 \text{ ms}}$$

$$\mu = 1.808 \times 10^{-5} \frac{\text{kg}}{\text{ms}}$$

### Kinematic Viscosity

$$\nu = \frac{\mu}{\rho}$$

$$\nu = \frac{1.808 \times 10^{-5} \text{ kg/ms}}{1.20 \text{ kg/m}^3}$$

$$\nu = 1.507 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$$

### Reynolds Number

$$\text{Re}_x = \frac{ux}{\nu}$$

$$\text{Re}_x = \frac{(12.82 \text{ m/s})(0.12 \text{ m})}{1.507 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\text{Re}_x = 102083.6$$

### Laminar Cubic Approximate of $\delta/x$

$$\frac{\delta}{x} = \frac{4.6}{\text{Re}_x^{1/2}}$$

$$\frac{\delta}{x} = \frac{4.6}{(102083.6)^{0.5}}$$

$$\frac{\delta}{x} = 0.0144$$

Turbulent Power Law Approximate of  $\delta/x$

$$\frac{\delta}{x} = \frac{0.37}{(\text{Re}_x)^{1/5}}$$

$$\frac{\delta}{x} = \frac{0.37}{(102083.6)^{1/5}}$$

$$\frac{\delta}{x} = 0.0368$$

Experimental Value of  $\delta/x$  for Smooth Plate

$$\frac{\delta}{x} = \frac{(1.8\text{mm}) \times (1\text{m}/1000\text{mm})}{12\text{cm} \times (1\text{m}/100\text{cm})}$$



### Appendix III - Supplementary Tables

**Table 4:** Velocity Profiles for Smooth Flat Plate Area with Laminar and Turbulent Approximations

Experimental Values	Laminar Approximation	Turbulent Approximation
0.126	0.213	0.757
0.312	0.350	0.815
0.541	0.481	0.855
0.726	0.603	0.886
0.790	0.714	0.912
0.906	0.810	0.934
0.948	0.889	0.953
0.994	0.949	0.970
0.960	0.987	0.986
1.000	1.000	1.000

**Table 5:** Velocity Profiles for Rough side of Flat Plate with Laminar and Turbulent Approximations

Experimental Values	Laminar Approximation	Turbulent Approximations
0.548	0.110	0.688
0.620	0.182	0.740
0.618	0.254	0.777
0.656	0.324	0.805
0.715	0.393	0.829
0.713	0.460	0.849
0.776	0.524	0.866
0.784	0.586	0.882
0.809	0.645	0.896
0.849	0.701	0.909
0.862	0.753	0.921
0.879	0.801	0.932
0.902	0.845	0.942
0.928	0.884	0.952
0.932	0.918	0.961
0.950	0.947	0.969
0.995	0.969	0.978
0.983	0.986	0.985
0.984	0.996	0.993
1.000	1.000	1.000

**Table 6:** Percentage Difference between Smooth Velocity Profile and Laminar and Turbulent Approximations

Smooth and Laminar Difference (%)	Smooth and Turbulent Difference (%)
8.66	63.11
3.89	50.31
5.90	31.43
12.24	16.01
7.59	12.20
9.59	2.79
5.88	0.51
4.48	2.35
2.69	2.59
<b>Average = 6.77</b>	<b>Average = 20.14</b>

**Table 7:** Percentage Difference between Rough Plate Velocity Profile and Laminar and Turbulent Approximations

Rough and Laminar Difference (%)	Rough and Turbulent Difference (%)
43.843	14.028
43.799	12.037
36.423	15.899
33.213	14.912
32.214	11.373
25.375	13.524
25.172	9.018
19.743	9.808
16.377	8.681
14.746	6.028
10.930	5.824
7.783	5.266
5.741	3.965
4.399	2.371
1.418	2.864
0.297	1.989
2.532	1.711
0.348	0.274
1.201	0.840
<b>Average = 17.13</b>	<b>Average = 7.390</b>