PREDICTING UNSTEADY AERODYNAMIC PARAMETERS FROM AIRFOIL SURFACE PRESSURE MEASUREMENTS

Sheryl Patrick Grace †
Aerospace and Mechanical Engineering Department
Boston University
Boston, MA 02215

ABSTRACT

This paper explores the inverse unsteady aerodynamic problem for a flat-plate airfoil. The goal of the aerodynamic inversion is to calculate the parameters which describe the nature of the unsteadiness in the problem from information on the surface of the airfoil. Interest in the aerodynamic inversion stems from the fact that it, in conjunction with the much studied aeroacoustic inversion, offers a method for nonintrusive and nondestructive measurements. Using a linearized model, it is shown that only the component of the velocity disturbance normal to the airfoil can be recovered uniquely from the unsteady surface pressure; in addition, unique recovery of aeroelastic parameters describing heaving and pitching of the airfoil is feasible provided some information concerning the rotation axis is known.

INTRODUCTION

Until now, most research on inverse problems in unsteady aerodynamics and aeroacoustics has been focussed on the inverse aeroacoustic portion of the problem (Grace, Atassi, & Blake 1997; Patrick & Atassi 1996; Minniti & Mueller 1996). It has been shown, both theoretically and experimentally, that using far-field acoustic measurements to infer unsteady surface pressure on a flat-plate airfoil in unsteady subsonic flow is feasible. Theoretically, the aeroacoustic inversion is valid for several unsteady airfoil problems including: an oscillating airfoil; an airfoil in vortical flow; and an airfoil subject to incident acoustic waves.

The associated inverse aerodynamic problem for these three problems is discussed in this paper. The goal of the aerodynamic inversion is to calculate the parameters which describe the unsteady part of the flow from information about the unsteady pressure on the surface of the airfoil. If this aerodynamic inversion, along with the aeroacoustic inversion, can be shown to work effectively in actual measurement situations, together they offer a very powerful nonintrusive and nondestructive measurement method.

In its most advanced form, such an inversion method would be able to use acoustic field measurements to predict the amplitude of rigid oscillations of a streamlined body, as well as the magnitude of any flow disturbances in the vicinity of the body. Thus far, research has shown that, for very simple streamlined geometries, the acoustic field can be used to predict the unsteady surface pressure on the body. However, to complete this complex acoustic inversion, the inverse aerodynamic problem must be considered. This paper will describe the inversion for the flat-plate airfoil geometry and 3 unsteady disturbance types.

First, the equation which governs the inversion is briefly described. It is seen that, whereas, the inverse aeroacoustic problem is ill-posed, the inverse aerodynamic problem can be well-posed, depending on what quantities are being recovered using the technique. Any

† Assistant Professor

ill-posedness of the inverse aerodynamic problem will come from the nonuniquess of a solution. This is very different from the ill-posedness of the inverse aeroacoustic problem which is very sensitive to noise in the input data. The inversion for each unsteady case is discussed separately. Finally, a study of the combination of unsteady phenomenon shows that another issue of nonuniqueness may arise when both incident vortical waves and incident acoustic waves impinge on the airfoil simultaneously.

GOVERNING EQUATION

The governing equation that was developed for analysis of the direct problem is used to study the inverse unsteady aerodynamic problem for a flat-plate airfoil in unsteady compressible flow. A brief formulation of this equation is included here. After assuming that the fluid is inviscid and isentropic, the Euler's equations are linearized about the constant mean flow quantities

$$
\begin{aligned}
\vec{V}^*(\vec{x}^*, t^*) &= U^*_\infty + \vec{u}^*(\vec{x}^*, t^*) && (1)\\
p^*(\vec{x}^*, t^*) &= p^*_\infty + p'^*(\vec{x}^*, t^*) && (2)\\
\rho^*(\vec{x}^*, t^*) &= \rho^*_\infty + \rho'^*(\vec{x}^*, t^*) && (3)\\
& && (4)
\end{aligned}
$$

where the asterix denotes dimensional quantities and p'^* and ρ'^* are the unsteady perturbation quantities. The linearized Euler's equations are then given as

$$
\begin{aligned}
\frac{D_\infty \rho'^*}{Dt^*} + \rho_\infty \vec{\nabla} \cdot \vec{u}^* &= 0 && (5)\\
\rho^*_\infty \frac{D_\infty \vec{u}^*}{Dt^*} &= -\vec{\nabla} p'^* && (6)
\end{aligned}
$$

where $\frac{D_\infty}{Dt} = \frac{\partial}{\partial t^*} + U^*_\infty \frac{\partial}{\partial x^*_1}$. The equations are nondimensionalized using

$$
\begin{array}{ccc}
x^*_1, x^*_2, x^*_3 & \text{by} & c/2 \\
U^*_\infty, c^*_\infty & \text{by} & U^*_\infty \\
\rho^*_\infty & \text{by} & \rho^*_\infty \\
\rho'^* & \text{by} & \rho^*_\infty a / U^*_\infty \\
p'^* & \text{by} & \rho_\infty U^*_\infty a \\
t & \text{by} & c/(2U^*_\infty) \\
\vec{u}^* & \text{by} & a
\end{array}
$$

where a is a quantity associated with the unsteady disturbance in the problem. The nondimensional quantities will be denoted with no asterix. Combining equations (5) and (6), and using the isentropic relationship, $M^2 p' = \rho'$, one obtains a convective wave equation for the unsteady pressure. (Note that $M = U^*_\infty / c_\infty$, where c_∞ is the speed of sound in the fluid, is the Mach number.) In nondimensional form, this gives

$$
\left(M^2 \frac{D^2_\infty}{Dt^2} - \nabla^2 \right) p' = 0 \qquad (7)
$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$.

The boundary conditions which complete this boundary value problem arise from the impermeability condition on the airfoil surface, and from the continuity of pressure in the fluid. The impermeability condition on the flat plate states that $\vec{u} \cdot \vec{n} = 0$ where $\vec{n}$ is the unit normal to the flat plate. The normal is $(0, 1, 0)$ on the top surface and $(0, -1, 0)$ on the bottom surface and the flat plate in nondimensional coordinates lies between -1 and 1, so that in nondimensional form the boundary condition becomes

$$
u_2(x_1, 0, x_3) = 0 \quad \text{for} \quad -1 < x_1 < 1. \qquad (8)
$$

Also, the pressure must be continuous upstream and downstream of the flat-plate airfoil, which leads to

$$
\triangle p' = 0 \quad x_1 > 1 \ \text{ or } \ x_1 < -1; \quad \text{and} \quad x_2 = 0 \qquad (9)
$$

where $\triangle$ signifies a jump, i.e. $\triangle p' = p'_+ - p'_-$.

The convective wave equation reduces to the Helmholtz equation in the Prandtl-Glauert coordinate system by defining

$$p' = P(\tilde{x}_1, \tilde{x}_2) e^{i(\gamma t + M^2\gamma\tilde{x}_1/\beta^2 - k_3\tilde{x}_3/\beta)} \quad (10)$$

with $\beta = \sqrt{1-M^2}$, and γ is the nondimensional angular frequency: $\gamma = \omega^* c/2U_\infty^*$, and the Prandtl-Glauert coordinates are defined by $\tilde{x}_1 = x_1, \tilde{x}_2 = \beta x_2, \tilde{x}_3 = \beta x_3$. The governing equation is then written as

$$\left(\tilde{\nabla}^2 + K^2\right) P = 0 \quad (11)$$

where $K^2 = (M\gamma/\beta^2)^2 - \frac{k_3^2}{\beta^2}$ and $\tilde{\nabla}^2 = \frac{\partial^2}{\partial\tilde{x}_1^2} + \frac{\partial^2}{\partial\tilde{x}_2^2}$.

In two-dimensions, the appropriate separable solution to the Helmholtz equation for the flat-plate geometry includes an infinite series of Mathieu functions. In lieu of this solution method, an integral equation solution was derived by Possio (1938) for cases when $K^2 > 0$. Later, Graham (1970) extended the solution to include all possible values of K^2

The method of Possio and Graham uses a plane wave expansion for the transformed pressure, P, to express the solution to the Helmholtz equation (11) as

$$P(\vec{\tilde{x}}) = \int_{-\infty}^{\infty} f(\alpha) e^{-i\alpha\tilde{x}_1 \pm i\sqrt{K^2-\alpha^2}\tilde{x}_2} d\alpha \quad (12)$$

Now, by using the momentum equation, Eq. (6), and the boundary conditions with the addition of a far-field radiation condition for the scattered sound field, the unsteady pressure on the airfoil surface can be related to the unsteady velocity. The relationship is

$$u_2(\tilde{x}_1) = \frac{\beta}{4\pi}\int_{-1}^{1} \Delta p'(\xi)\mathcal{K}(\xi - \tilde{x}_1) d\xi \quad (13)$$

where

$$\mathcal{K}(\xi - \tilde{x}_1) = \int_{-\infty}^{\infty} \frac{\sqrt{K^2-\alpha^2}}{(\alpha + \gamma/\beta^2)} e^{-i(\alpha+\gamma/\beta^2)(\xi-\tilde{x}_1)} d\alpha \quad (14)$$

BOUNDARY CONDITIONS

In the last section, a general equation which relates the unsteady surface pressure jump along a flat-plate airfoil and the unsteady velocity along the airfoil was derived. This section will describe the surface velocity condition for the cases of an oscillating airfoil and an airfoil interacting with either vortical or acoustic waves.

Oscillating Airfoil

For an oscillating airfoil, the airfoil position is usually expressed in terms of the magnitude of the rotational oscillation, α, the magnitude of the plunging oscillation, h, and the center of rotation, x_0. If the rotational and plunging oscillations occur at the same frequency, ω, the x_2^* position of the airfoil can be expressed as

$$x_2^* = [\alpha^*(x_1^* - x_0^*) + h^*] e^{i\omega^* t^*} \quad (15)$$

Recall that the asterix indicates physical variables. The surface normal velocity then, which is given by $D/Dt^* x_2^*$, is

$$u_2^*(x_1^*) = (i\omega^*\alpha^*(x_1^* - x_0^*) + \alpha^* U_\infty^* + i\omega^* h) e^{i\omega^* t} \quad (16)$$

$$-c/2 \le x_1^* \le c/2$$

In nondimensional form the normal velocity is given by

$$u_2(x_1) = a\left[i\gamma\alpha(x_1 - x_0) + \alpha + i\gamma h\right] e^{i\gamma t} \quad (17)$$

$$-1 \le x_1 \le 1$$

In this description of the normal velocity, the magnitude of the rotational oscillation, which is nondimensional originally, remains unchanged. For the *direct problem*, u_2 is thus prescribed and is used as the left hand side of Equation (13). The integral equation is then solved for the corresponding unsteady surface pressure jump.

Vortical Disturbance

For the case of an airfoil interacting with a vortical disturbance, arriving at the boundary condition along the airfoil is a bit different than for the case of the oscillating airfoil. First, the unsteady velocity field is split into two parts: rotational and irotational. It was shown that for the uniform flow case, the rotational component of the unsteady velocity, which is associated with the incoming vorticity disturbance, is pressure free (Atassi & Grzedzinski 1989). Moreover, the only place where the acoustic and vortical velocities couple is at the airfoil surface. If the acoustic velocity is denoted $\vec{u}_a$, and the vortical velocity is denoted $\vec{u}_g$, the impermeability condition on the airfoil states

$$u_{a_2^*}(x_1^*, 0, x_3^*) = -u_{g_2}^*(x_1^*, 0, x_3^*) \qquad (18)$$
$$-c/2 \le x_1^* \le c/2$$

Since all of the pressure is associated with the acoustic velocity, it is appropriate that the left hand side of Equation (13) be replaced by $u_{a_2}^*$.

In the gust problem, it is common to treat distinct Fourier components of the gust, i.e. the disturbance can be written in dimensional form as

$$\hat{\vec{u}}_g^*(\omega, \vec{k}^*) e^{i(\omega^* t^* - \vec{k}^* \cdot \vec{x}^*)} \qquad (19)$$

Further, since the vorticity can be shown to be purely convected by the mean flow, the disturbance becomes

$$\hat{\vec{u}}_g^*(\omega^*, k_2^*, k_3^*) e^{i(\omega^* t^* - \omega^*/U_\infty^* x_1^* - k_2^* x_2^* - k_3^* x_3^*)} \qquad (20)$$

In nondimensional form, the second component of this along the airfoil is given as

$$\hat{u}_{g_2}(\gamma, k_2, k_3) = e^{i(\gamma t - \gamma x_1 - k_3 x_3)} \qquad (21)$$
$$-1 \le x_1 \le 1$$

Here, we can identify γ with the reduced frequency which is usually denoted as k_1.

In the direct problem, $\vec{u}_g$ is specified and the left hand side of Equation (13) is set equal to $-u_{g_2}$. One then obtains the unsteady surface pressure response to the vortical disturbance by solving the integral equation.

Acoustic Disturbance

For the case when acoustic waves are incident on the airfoil, the unsteady velocity field is again split. Here however, the velocity is made up of an incident velocity field, $\vec{u}^i$ and a scattered velocity field, $\vec{u}^s$. Unlike the splitting for the gust problem, now both fields have pressure associated with them. Since a far-field radiating boundary condition was used to develop the integral equation, Eq. (13), the left hand side, must coincide with $\vec{u}^s$; and, the associated $\Delta p'$ must coincide with the scattered pressure field. There is again a coupling of the two unsteady velocity fields along the airfoil such that in nondimensional form

$$u_2^i(x_1, 0, x_3) = -u_2^s(x_1, 0, x_3) \qquad (22)$$

As in the case of the incident vortical gust, a single Fourier component of the incident acoustic wave is considered, i.e.

$$\hat{\vec{u}}^i(\gamma, k_1, k_2, k_3) e^{i(\gamma t - \vec{k} \cdot \vec{x})} \qquad (23)$$

There is no explicit relation between ω^* and k_1^* in the case of acoustic scattering; still, the direct problem follows the exact methodology as explained for the gust problem.

INVERSION

As the last section showed, one obtains solutions to the direct unsteady aerodynamic problem by solving an integral equation. Hence, to obtain solutions to the inverse problem, one must simply compute an integral. It seems then, that for unsteady aerodynamic problems, it is actually easier to consider the inverse rather than the direct problem. To analyze this statement further, the inverse problem is considered in terms of the Hadamard criteria for well-posedness. In order to analyze the existence, uniqueness and sensitivity of solutions to noise, the definition of what constitutes an acceptable solution to the inverse aerodynamic problem must be given.

If the inverse aerodynamic problem is defined as determining the normal velocity component associated with a given unsteady pressure jump on the flat-plate airfoil, then the problem is well posed. A solution will exist, since it is always possible to integrate the right hand side of Equation (13). The solution will be unique, since again, the integration is unique. And finally, slight variations in the unsteady surface pressure will translate to slight variations in the normal velocity, since there is no means by which to amplify these variations.

The well-posedness of the inverse aerodynamic problem just discussed, rests on the facts that a satisfactory solution to the inverse problem is the total normal velocity and that the unsteady surface pressure is described along the entire airfoil. In practice, however, the unsteady pressure may not be known on the entire airfoil surface. Moreover, it is of more interest to determine the amplitude of the disturbances, i.e. α and h; or the total velocity fields $\vec{u}_g(\gamma, \vec{k})$; or $\vec{u}^s(\gamma, \vec{k})$, rather than simply the total normal velocity.

For the case of an oscillating airfoil, it is noted that if the rotation axis, x_0, is known, then the problem remains well-posed. In this instance, knowing the normal velocity at any two points along the airfoil is sufficient to calculate both α and h. If however, x_0 is not known, the solution for α, h, and x_0 may not be unique. For the cases of incident vortical or acoustic waves, it is noted that while the normal component of the disturbance can be determined, the other two components remain completely unknown. This again can be considered a nonuniqueness in the inversion.

Finally, the fact that $\Delta p'$ may not be known at every point on the airfoil must be addressed. If $\Delta p'$ is known discretely, the above discussion of the Hadamard criteria remains unchanged, although the solution method must obviously be modified.

The proposed modification to the inversion method will allow $\Delta p'$ to be known at only discrete points along the airfoil. First, for each measurement of $\Delta p'$ along the airfoil, denoted here $\Delta p_{measured}(x_{1i})$, $i = 1, 2, \cdots, n$, n equations are constructed by using solutions to the integral Equation (13) for unit amplitude disturbances. For instance, the solutions Δp_α and Δp_h can be determined by solving the integral equations:

$$x_1 e^{i(k_1 t - k_3 x_3)} = \frac{\beta}{4\pi} \int_{-1}^{1} \Delta p'_\alpha(\xi) \mathcal{K}(\xi - \tilde{x}_1) d\xi \qquad (24)$$

and

$$e^{i(k_1 t - k_3 x_3)} = \frac{\beta}{4\pi} \int_{-1}^{1} \Delta p'_h(\xi) \mathcal{K}(\xi - \tilde{x}_1) d\xi \qquad (25)$$

Then, by using the x_{1i}, $i = 1, 2, \cdots, n$ locations where the measured pressure jump is available, one can derive n equations valid for an oscillating airfoil. The n equation are

$$\begin{aligned} \Delta p_{measured}(x_{1i}) &= i\gamma\alpha\Delta p_\alpha(x_{1i}) \\ &\quad - i\gamma\alpha x_0 \Delta p_h(x_{1i}) \\ &\quad + \alpha U_\infty \Delta p_h(x_{1i}) + i\gamma h \Delta p_h(x_{1i}) \\ &\qquad i = 1, 2, \cdots, n \end{aligned} \qquad (26)$$

To recover the values of α, h and x_0, at least 3 measurement locations, are needed. Another requirement is that $\Delta p_{measured}(x_i)$ must contain phase information.

Still however, this equation does not lead to a linear system. Hence, a technique like Newton's method for several variables must be used to find the quantities α, h and x_0. As mentioned before, there is no guarantee that the solution obtained for α, h and x_0 is unique. In application, it is possible that x_0 is fixed by geometrical constraints. When this is the case, α and h can be uniquely determined.

The technique for determining the normal component of an incident vortical or acoustic wave from a discrete set of unsteady pressure measurements follows readily. In these cases, the integral equation

$$e^{i(\gamma t - \gamma x_1 - k_3 x_3)} = \frac{\beta}{4\pi} \int_{-1}^{1} \Delta p'_g(\xi) \mathcal{K}(\xi - \tilde{x}_1) d\xi \qquad (27)$$

or

$$e^{i(\gamma t - \vec{k}\cdot\vec{x})} = \frac{\beta}{4\pi} \int_{-1}^{1} \Delta p'_a(\xi) \mathcal{K}(\xi - \tilde{x}_1) d\xi \qquad (28)$$

are solved *a priori* for $\Delta p'_g$ or $\Delta p'_a$ respectively.

Using the measured pressure jump then, it seems that only one measurement point is needed to determine the component of the disturbance normal to the airfoil, i.e.

$$\Delta p_{measured}(x_{1i}) = \hat{u}_{g_2}(\gamma, k_3) \Delta p_g(x_{1i}) \qquad (29)$$

$$\Delta p_{measured}(x_{1i}) = \hat{u}_{a_2}(\gamma, \vec{k}) \Delta p_a(x_{1i}) \qquad (30)$$

$$(31)$$

Finally, if all three of the unsteady phenomena are present, the following system of equations must be solved

$$\Delta p_{measured}(x_{1i}) = i\gamma\alpha\Delta p_{\alpha}(x_{1i}) - i\gamma\alpha x_0 \Delta p_h(x_{1i})$$

$$+ \alpha U_{\infty}\Delta p_h(x_{1i}) + i\gamma h \Delta p_h(x_{1i}) \qquad (32)$$

$$+\hat{u}_{g_2}(\gamma, k_3)\Delta p_g(x_{1i}) + \hat{u}_{a_2}(\gamma, \vec{k})\Delta p_a(x_{1i}) \quad i = 1, 2, \cdots, n$$

It is noted that if the reduced frequencies of the vortical and acoustic disturbances coincide, Δp_g and Δp_a will be identical. In this case, the values of $\hat{u}_{g_2}$ and $\hat{u}_{a_2}$ cannot be resolved independently. This creates an additional nonuniqueness in the inversion.

CONCLUSIONS

This paper has shown that the most important issue for the inverse unsteady aerodynamic problem is uniqueness of the solution. The other Hadamard criteria for well-posed problems are easily satisfied. It is shown that the normal velocity of an unsteady disturbance responsible for creating an unsteady surface pressure on a flat-plate airfoil in subsonic flow, can always be determined uniquely.

If however, all of the parameters characterizing the unsteady disturbance must be recovered, then the solutions may be nonunique. For instance, if one would like to recover an entire vortical disturbance, which constitutes unsteady velocities in 3 directions, the inversion is nonunique. One can only uniquely determine the normal component.

The conclusion must be then, that if other information concerning the disturbances is known, perhaps the full inversion is possible and unique. Moreover, if the desired parameter set is not full, the inversion may be unique.

The steps to perform the aerodynamic inversion which are outlined in this paper, may seem quite extensive especially for the case of an oscillating airfoil. For instance, accelerometers on the flat-plate airfoil could easily determine the magnitude of the rotational and plunging oscillations as well as the center of rotation. However, if the aerodynamic inversion is just one part of a complex acoustic inversion, in which far-field acoustic measurements are used to define airfoil excitation parameters, the methods developed in this paper are essential. At this point, when certain combinations of unsteady parameters must be recovered, the total complex acoustic inversion is feasible. For the other cases, only certain properties of the unsteady parameters can be determined uniquely from the scattered acoustic field.

REFERENCES

Atassi, H. M. & Grzedzinski, J. 1989. "Unsteady Disturbances of Streaming Motions Around Bodies,". *Journal of Fluid Mechanics*, Vol. 209, pp. 385–403.

Grace, S. M. P., Atassi, H. M., & Blake, W. K. 1997. "Inverse Aeroacoustic Problem for a Streamlined Body: Part I: Basic Formulation,". *to be published in AIAA Journal, September.*

Graham, J. M. R. 1970. "Similarity Rules for Thin Airofils in Non-Stationary Subsonic Flows,". *Journal of Fluid Mechanics*, Vol. 43, pp. 753–766.

Minniti, R. & Mueller, T. 1996. "Experimental Investigations of Inverse Problems for Thin Airfoils Exposed to Periodic Gusting,". In *Proceedings of the 2nd AIAA/CEAS Aeroacoustics Conference*, May 6-8, State College, PA.

Patrick, S. M. & Atassi, H. M. 1996. "Inverse Aeroacoustic Problem for a Rectangular Wing Interacting with a Gust,". In *Proceedings of the 2nd AIAA/CEAS Aeroacoustics Conference*, May 6-8, State College, PA.

Possio, D. 1938. "L'Azione Aerodinamica sul Prodilo Oscillante in un Fluido Compressible a Velocita Ipsonora,". *L'Aerotecnica*, Vol. XVIII, 4.