Instantons and monopoles are locally correlated with the chiral condensate

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Abstract

We perform a mutual analysis of the topological and chiral vacuum structure of four-dimensional QCD on the lattice at finite temperature. Concerning the topological sector, correlation functions between the distributions of color magnetic monopoles in maximum abelian gauge and the densities of topological charge are computed. An enhanced probability for monopoles inside the core of an instanton is found on gauge field average and in specific configurations. In the chiral sector, clear evidence is found that monopole loops and instantons are locally correlated with the quark condensate.

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The chiral condensate $<\bar{\psi}\psi>$ is a key quantity in gauge theories including fermions. The fact that $<\bar{\psi}\psi>$ attains a non-vanishing value is a sign for the breakdown of chiral symmetry being present for massless quarks. The physical value of the chiral condensate has been applied to a variety of crucial physical problems, such as the estimation of current quark masses, PCAC and pion phenomenology [1]. It is well known that the zero eigenvalues of the fermionic determinant are related to the global topological charge Q of a gauge field configuration via the Atiyah-Singer index theorem [2, 3]. It is believed that the instantons as carriers of the topological charge might play a crucial role in understanding the confinement mechanism of four-dimensional QCD, if one assumes that they form a so-called instanton liquid [4].

In this letter we present first evidence that a non-vanishing value of the quark-antiquark density is predominantly found at the locations of instantons both on gauge average and in single field configurations. We also show that there are nontrivial correlations between regions of non-vanishing quark condensate and the occurrence of color magnetic monopoles, which are important for the confinement problem in the dual superconductor hypothesis and the dual Meissner effect [5]. Recently, it was demonstrated that monopole currents appear preferably in the regions of non-vanishing topological charge density [6, 7]. It has been also conjectured that both instantons and monopoles are related to chiral symmetry breaking [4, 8, 9]. This idea is further supported by the following results of a direct investigation of the local correlations of the quark condensate, the topological charge density, and the monopole density. The chiral condensate as well as the topological excitations are of non-perturbative character and, therefore, we adopt the lattice approach to gain insight into the underlying low energy physics. This allows for a model independent analysis of four-dimensional QCD with $SU(3)$ gluon fields and inclusion of the fermionic determinant.

For the implementation of the topological charge on a euclidian lattice there exists no unique discretization. In this work we restrict ourselves to the so-called field theoretic definitions which approximate the topological charge density in the continuum,

$q(x) = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}$ Tr $\left(F_{\mu\nu}(x)F_{\rho\sigma}(x)\right)$, in the following ways [10]:

$$q^{(P,H)}(x) = -\frac{1}{2^4 32\pi^2} \sum_{\mu,\ldots=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \mathrm{Tr}\ O^{(P,H)}_{\mu\nu\rho\sigma}, \tag{1}$$

with

$$O^{(P)}_{\mu\nu\rho\sigma} = U_{\mu\nu}(x)U_{\rho\sigma}(x)\ , \tag{2}$$

for the plaquette prescription and

$$\begin{aligned} O^{(H)}_{\mu\nu\rho\sigma} &= U(x,\mu)U(x+\hat{\mu},\nu)U(x+\hat{\mu}+\hat{\nu},\rho) \\ &\times\ U(x+\hat{\mu}+\hat{\nu}+\hat{\rho},\sigma)U^{\dagger}(x+\hat{\nu}+\hat{\rho}+\hat{\sigma},\mu) \\ &\times\ U^{\dagger}(x+\hat{\rho}+\hat{\sigma},\nu)U^{\dagger}(x+\hat{\sigma},\rho)U^{\dagger}(x,\sigma), \end{aligned} \tag{3}$$

for the hypercube prescription. The lattice and continuum versions of the theory represent different renormalized quantum field theories, which differ by finite, non-negligible renormalization factors. A simple procedure to get rid of renormalization constants, while preserving physical information contained in lattice configurations, is the cooling method. The cooling procedure systematically reduces quantum fluctuations and so reveals the topological structure.

In order to investigate monopole currents in nonabelian gauge theories one has to project $SU(N)$ onto its abelian degrees of freedom, such that an abelian $U(1)^{N-1}$ theory remains [11]. We employ the so-called maximum abelian gauge [12]. To extract abelian parallel transporters $u(x,\mu)$ after imposing the maximum abelian gauge one has to perform the decomposition

$$U(x,\mu) = c(x,\mu)u(x,\mu)\ , \quad \text{with} \quad (N=3) \tag{4}$$

$$\begin{aligned} u(x,\mu) &= \mathrm{diag}\ [u_1(x,\mu), u_2(x,\mu), u_3(x,\mu)]\ , \\ u_i(x,\mu) &= \exp\left[i\ \arg\ \tilde{U}_{ii}(x,\mu) - \frac{1}{3}i\phi(x,\mu)\right]\ , \\ \phi(x,\mu) &= \sum_i \arg\ \tilde{U}_{ii}(x,\mu)\Big|_{\mathrm{mod}\ 2\pi} \in (-\pi,\pi]. \end{aligned}$$

Since the maximum abelian subgroup $U(1)^{N-1}$ is compact, there exist topological excitations. These are color magnetic monopoles which have integer-valued magnetic currents on the links of the dual lattice:

$$m_i(x,\mu) = \frac{1}{2\pi} \sum_{\Box \ni \partial f(x+\hat{\mu},\mu)} \arg\, u_i(\Box) \ , \tag{5}$$

where $u_i(\Box)$ denotes a product of abelian links $u_i(x,\mu)$ around a plaquette $\Box$ and $f(x+\hat{\mu},\mu)$ is an elementary cube perpendicular to the μ direction with origin $x+\hat{\mu}$. The magnetic currents form closed loops on the dual lattice as a consequence of monopole current conservation. From the monopole currents we define the local monopole density as $\rho_m(x) = \frac{1}{3\cdot 4V_4}\sum_{\mu,i}|m_i(x,\mu)|$. We mention here that the topological charge densities employed are locally gauge invariant, whereas the monopole currents are not.

We are interested in correlation functions between fundamental quantities of the vacuum of QCD. The quark condensate represents such an important observable having several significances. As the expectation value $<\bar{\psi}\psi>$ in the limit of infinite volume and zero quark mass it is a signature for spontaneous chiral symmetry breaking. Its local distribution $\bar{\psi}\psi(x)$ is the density of virtual quark-antiquark pairs in a specific configuration of QCD. This interpretation is adopted for our studies. Mathematically and numerically the local chiral condensate $\bar{\psi}\psi(x)$ is a diagonal element of the inverse of the fermionic matrix of the QCD action.

We compute correlation functions between two observables $\mathcal{O}_1(x)$ and $\mathcal{O}_2(y)$

$$g(y-x) = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle - \langle \mathcal{O}_1\rangle\langle \mathcal{O}_2\rangle \tag{6}$$

and normalize them, $c(y-x) = g(y-x)/g(d_{\rm min})$, to the smallest lattice separation $d_{\rm min}$. Since the hypercube prescription (3) and the monopole definition (5) assign the densities to the center of a (hyper)cube, $d_{\rm min}$ need not be zero. Because topological objects with opposite sign are equally distributed, we correlate the monopole density and the quark-antiquark density with the square of the topological charge density.

Our simulations were performed for full $SU(3)$ QCD on an $8^3 \times 4$ lattice with periodic boundary conditions. Applying a standard Metropolis algorithm has the advantage that tunneling between sectors of different topological charges occurs at reasonable rates. Dynamical quarks in Kogut-Susskind discretization with 3 flavors of degenerate mass $m = 0.1$ were taken into account using the pseudofermionic method. We performed runs both in the confinement phase at $\beta = 5.2$ and in the deconfinement phase at $\beta = 5.4$. Measurements were taken on 1000 configurations separated by 50 sweeps. Each configuration was cooled and then subjected to 300 gauge fixing steps enforcing the maximum abelian gauge.

Figure 1 shows results for the correlation functions of Eq. (6) in the confinement region. The $\bar{\psi}\psi q^2$-correlation function (a) exhibits an extension of more than two lattice spacings, indicating nontrivial correlations. The topological charge density is taken from the plaquette prescription so that $d = 0$ is a valid lattice separation. The diamonds indicate a non-vanishing correlation function at $d = 4$ although the expectation values of the individual operators were subtracted which means that the asymptotic value is not yet reached on the finite lattice. To gain information about the correlation lengths exponential fits to the tails of the correlation functions were performed. They show that an increasing number of cooling steps yields shorter correlation lengths. The corresponding screening masses are $\zeta = 0.59$ and $\zeta = 1.56$ in inverse lattice units for 0 and 11 cooling steps, respectively. They have to be interpreted as effective masses and reflect the effective gluon exchange and the vacuum structure of QCD.

It is assumed that the size ρ of a t'Hooft instanton $q_\rho(x) \sim \rho^4\,(x^2+\rho^2)^{-4}$ centered around the origin enters also into the associated distribution of the chiral condensate $\bar{\psi}\psi_\rho(x) \sim \rho^2\,(x^2+\rho^2)^{-3}$ [13]. To estimate ρ we fitted a convolution of the functional form $f(x) = \int \bar{\psi}\psi_\rho(t) q_\rho^2(x-t)\,dt$ to the data points. This was evaluated after 11 cooling steps where the configurations are reasonably dilute. Our fit yields $\rho(\bar{\psi}\psi q^2) = 1.8$ in lattice spacings. To check consistency we extracted from the qq-correlation a value of $\rho(qq) = 2.05$ which is in good agreement.

The $\bar{\psi}\psi\rho_m$-correlation (b) is not sensitive to cooling and has approximately the same exponential decay as the $\bar{\psi}\psi q^2$-correlation after some cooling steps. The $\rho_m q^2$-correlation

(c) - here with the hypercube definition of the topological charge density - looks similar to the corresponding function in pure QCD presented in [6]. The same holds for the instanton and monopole autocorrelation functions (not shown).

We now visualize densities of the quark condensate and topological quantities from individual gauge fields. The topological content of typical individual gauge field configurations was already studied for pure SU(2) and SU(3) theory [6]. We found that at the local regions of clusters of topological charge density, which are identified with instantons, there are monopole trajectories looping around in almost all cases.

In Fig. 2 a time slice of a typical configuration from $SU(3)$ theory with dynamical quarks on the $8^3 \times 4$ lattice in the confinement phase is shown. We display the positive instanton density by blue colors and the negative density by red colors if the absolute value $|q(x)| > 0.003$. The quark-antiquark density is indicated by green colors if a threshold for $\bar{\psi}\psi(x) > 0.066$ is exceeded. By analyzing dozens of gluon and quark field configurations we found the following results. The topological charge is covered by quantum fluctuations and becomes visible by cooling of the gauge fields. For 0 cooling steps no structure can be seen in $q(x)$, $\bar{\psi}\psi(x)$ or the monopole currents, which does not mean the absence of correlations between them. After 5 cooling steps clusters of nonzero topological charge density and quark condensate are resolved. This particular configuration possesses a cluster with a positive and a negative topological charge corresponding to an instanton and an antiinstanton, respectively. For more than 10 cooling steps both topological charge and chiral condensate begin to die out and finally vanish. Combining the above finding of Fig. 1 showing that the correlation functions between $< \bar{\psi}\psi(x) >$ and $< q^2(y) >$ (or $< \rho_m(y) >$) are rather insensitive under cooling together with the cooling history of the 3D images in Fig. 2, we conclude that instantons go hand in hand with clusters of $\bar{\psi}\psi(x) \neq 0$ also in the uncooled QCD vacuum [3].

Figure 2 has demonstrated that the quark-antiquark density attains its maximum values at the same positions as the maximum values of the topological charge density. This behavior is further substantiated in Fig. 3 where the $\bar{\psi}\psi(x)$-values are plotted against $q(x)$ for all points x in the same configuration at 10 cooling steps. At first sight a linear relationship between the absolute value of the topological charge density and the virtual quark density is suggested.

We comment on the behavior of the topological structure and the quark condensate when crossing the phase transition. The normalized correlation functions do not change qualitatively [6]. The strength of the correlations becomes, however, 2-3 orders of magnitude smaller. This means that the topological activity becomes much weaker in the deconfinement phase as expected. Most of the configurations have trivial net topological charge. This does not exclude the existence of instanton-antiinstanton pairs which become more difficult to be resolved at the smaller physical volume at $\beta = 5.4$. In less than one percent of the gauge field configurations instantons could be identified. The configurations we scanned did not show pronounced pictures of instantons and quark condensate. The quark-antiquark density is considerably lower and does not have the tendency to cluster anymore, $\bar{\psi}\psi(x)$ becomes uniformly distributed over the lattice sites. The maximum values of $\bar{\psi}\psi(x)$ in configurations in the deconfinement after 10 cooling steps are one order of magnitude smaller.

In summary, our calculations of correlation functions between topological objects and the quark condensate yield an extension of about two lattice spacings. The correlations suggest that the local chiral condensate takes a non-vanishing value predominantly in the regions of instantons and monopole loops. It was well known before that the chiral condensate is related to the topological charge and topological susceptibility. The visualization exhibited that the distribution of the "chiral condensate" concentrates around areas with enhanced topological activity (instantons, monopoles). We demonstrated that exactly at those places in euclidean space-time, where tunneling between the vacua occurs, amplified production of quark condensate takes place. To our knowledge this observation gives the first direct insight into the local interplay of chiral symmetry breaking and nontrivial topological structure. It must be emphasized that this represents the situation on a finite lattice with finite quark mass without the extrapolation to the thermodynamic and chiral limit. We found for full

$SU(3)$ QCD with dynamical quarks that the clusters of non-vanishing quark condensate have a size of about 0.4 fm, which corresponds to the instanton sizes observed in the same configurations. In the deconfinement phase $\bar{\psi}\psi(x)$ is significantly lower and looses its cluster property being present in the confinement regime. We propose two main directions for future investigations: Visualization of quark and gluon fields might be especially useful to decide if the instanton-liquid model is realized in nature and if instanton-antiinstanton pairs appear in the deconfined phase. It might also help to clarify the question of the existence of a disoriented chiral condensate with important consequences for heavy-ion experiments.

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Figure Captions

FIG. 1. Correlation functions in the presence of dynamical quarks in the confinement phase ($\beta = 5.2$). The correlation of the quark-antiquark density and the topological charge density (a) is cooling dependent, whereas the correlation between the quark-antiquark density and the monopole density (b) is not. The monopole-instanton density-correlation (c) is similar as in pure $SU(3)$ theory. All correlations extend over two lattice spacings and indicate local correlations of the quark condensate and topological objects. Error bars are in the size of the symbols and omitted for clarity of the plots.

FIG. 2. Cooling history for a time slice of a single gauge field configuration of $SU(3)$ theory with dynamical quarks. The blue and red colors represent the positive and negative topological charge density. The green colors the density of the quark condensate. It turns out that the quark condensate takes a non-vanishing value at the positions of the instantons.

FIG. 3. Scatter plot of $\bar{\psi}\psi(x)$ against $q(x)$ in the volume of the same gauge field configuration with 10 cooling steps as in Fig. 2. It reflects the separation into an instanton and antiinstanton.