Qualifying Exam MA 582 problems April 2018

Problem 1.

Consider the N (0, θ) family, where $\theta > 0$ (note that $\mathrm{var}(X) = \theta$).

a. Find the maximum likelihood estimator (MLE) of θ, call it Y_n.
b. Show whether or not Y_n is unbiased for θ.
c. Show whether or not Y_n is a consistent estimator for θ.
d. Show whether or not Y_n is asymptotically normal, and if it is, identify its asymptotic normal variance.
e. Find $I(\theta)$, Fisher's Information for θ. –– Is MLE(θ) efficient?
f. Find the MLE of θ^3. Show whether or not it is biased.
g. Find a function g so that $n^{1/2}(g(Y_n) - g(\theta))$ is asymptotically standard normal.

Problem 2.

We say the rv X has the W distribution with parameter $\theta > 0$ (written $X \sim W(\theta)$) if X has pdf $f(x, \theta) = 4x^3/\theta^4$, for $0 < x < \theta$, and $f(x) = 0$, elsewhere.

Consider the parameterized D family $\{D(\theta) : \theta > 0 \}$.

a. Let Y_n be the maximum of the random sample of size n. Show that Y_n is a consistent estimator of θ.
b. Find the pdf of Y_n. (*Hint:* Find the cdf first.)
c. Show that Y_n is NOT an unbiased estimator of θ.
d. Show that $n(\theta - Y_n)$ converges in distribution, and find its asymptotic distribution explicitly.
e. Find an unbiased estimator of θ, call it T_n. Show that T_n is a consistent estimator of θ.
f. Show that $n(\theta - T_n)$ converges in distribution, and find its asymptotic distribution explicitly. (Hint: Use parts (d) and (e) here.)