Qualifying Exam MA 582 problems **April 2017**

Problem 1.

Consider the Exp(β) family, where $\beta > 0$; note that $E(X) = \beta$ if $X \sim \text{Exp}(\beta)$.

a. Does this family satisfy all the regularity conditions for MLE?
b. Find the maximum likelihood estimator (MLE) of β, call it Y_n.
c. Show whether or not Y_n is unbiased for β.
d. Show whether or not Y_n is a consistent estimator for β.
e. Show whether or not Y_n is asymptotically normal, and if it is, identify its asymptotic normal variance.
f. Find the MLE of log β. Show whether or not it is biased.
g. In part (f) here, show the MLE of log β is CAN for log β; identify its asymptotic normal variance.
h. Find a function g so that $n^{1/2}(g(Y_n) - g(\beta))$ is asymptotically standard normal for all values of $\beta > 0$.
i. Compute Fisher's Information I(β) for this family of distributions.
j. Is your MLE in part (b) here efficient?

Problem 2.

We say the rv X has the D distribution with parameter $\theta > 0$ (written $X \sim D(\theta)$) if X has pdf

$$f(x, \theta) = 4x^3/\theta^4, \text{ for } 0 < x < \theta, \text{ and } f(x) = 0, \text{ elsewhere.}$$

Consider the parameterized D family $\{D(\theta) : \theta > 0\}$.

a. Show that the MLE of θ is the sample maximum.
b. Let Y_n be the maximum of the random sample of size n. Show that Y_n is a consistent estimator of θ.
c. Find the pdf of Y_n. (*Hint:* Find the cdf first.)
d. Show that Y_n is NOT an unbiased estimator of θ.
e. Show that $n(\theta - Y_n)$ converges in distribution, and find its asymptotic distribution explicitly.
f. Find an unbiased estimator of θ, call it T_n. Show that T_n is a consistent estimator of θ.
g. Show that $n(\theta - T_n)$ converges in distribution, and find its asymptotic distribution explicitly. (Hint: Use parts (e) and (f) here.)