Qualifying Exam: CAS MA 583

Boston University, Spring 2015

Problem 1. Let X, Y be independent $N(0,1)$ random variables.

i). Check weather $X+Y$ and $X-Y$ are independent.

ii). Calculate $E[X+2Y|X-Y]$.

iii). Compute $E[Y|Y>0]$.

Problem 2. Consider the Markov Chain $\{X_n, n \in \mathbb{N}\}$ on the state space $\mathcal{X} = \{1,2,3\}$ with transition probability matrix

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & 5/12 & 5/12 \end{pmatrix}$$

i). Does this Markov chain have a limiting distribution and why? Justify your answer by appealing to the statements of generally applicable theorems.

ii). If it has a stationary distribution, find it explicitely.

iii). Let ν_N be the number of transitions from State 3 to State 2 in the first N steps. Caclulate $\lim_{N\to\infty} \frac{\nu_N}{N}$.