Qualifying Exam: CAS MA 583

Boston University, Spring 2010

Problem 1. Assume that when the USA plays Brazil in soccer, each team scores independently as a homogeneous Poisson process with rates $\lambda_{\text{USA}} = 1/90$ and $\lambda_{\text{Brazil}} = 2/90$ goals per minute. Assume that after the first 90 minutes, the game is tied. Now the two teams will play a sudden death overtime period. The first team to score will win the game. If no team scores within 30 minutes, the game ends in a tie. Compute:

(a) The probability that the USA wins.

(b) The probability that the game ends in a tie.

(c) The probability that the USA wins given that the USA does not lose the game in overtime.

Problem 2. Assume you have a 3-state Markov Chain with $P_{12} = .5$, $P_{13} = .5$, $P_{23} = 1$, and $P_{31} = 1$.

(a) Is the chain irreducible? Classify the states as transient, null recurrent, or positive recurrent. What is the period of each state?

(b) How many stationary distributions will this Markov chain have? Write down a stationary distribution if at least one exists. If not, prove that no stationary distribution exists.

(c) Compute the expected amount of time to reach state 2 from state 1.

(d) Is this Markov chain time-reversible?