Qualifying Exam: CAS MA 583

Boston University, Spring 2009

Problem 1. Let X_1 be the first arrival time of an inhomogeneous Poisson process with rate function $\lambda(t)$.

(a) Write down the density function for X_1.

(b) Compute the probability that there was exactly one arrival in the interval $[0,T]$.

(c) Compute the likelihood of observing one arrival at time x_1 and observing no other arrivals in the interval $[0,T]$.

(d) Compute the probability density function of X_1 given that there was exactly one arrival in the interval $[0,T]$.

Problem 2. You are going to successively flip a coin until the pattern *HHT* appears; that is, until you observe two successive heads followed by a tails. In order to calculate some properties of this process, you set up a Markov chain with the following states: *0*, *H*, *HH*, and *HHT*, where *0* represents the state where you just flipped a tail that was not preceded by two heads, *H* represents the state where you just flipped a head and the previous flip was tails, *HH* represents the state where you have flipped at least two heads in a row, and *HHT* represents the state that you flipped HHT and finished the game (you stay in this state perpetually once you finish)

(a) Construct a Markov chain describing this process by writing down a probability transition matrix. Is the chain irreducible? Classify the states as transient, null recurrent, or positive recurrent. What is the period of each state?

(b) How many stationary distributions will this Markov chain have? Write down a stationary distribution if at least one exists. If not, prove that no stationary distribution exists.

(c) Compute the expected amount of time to reach state *HHT* from state *0*.