Achievable Data Rate Analysis of Clipped Flip-OFDM in Optical Wireless Communication

Zhenhua Yu[1] Robert J. Baxley[2] G. Tong Zhou[1]

[1]Center for Signal and Image Processing
Georgia Institute of Technology

[2]Georgia Tech Research Institute

zhenhuayu@ece.gatech.edu

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Outline

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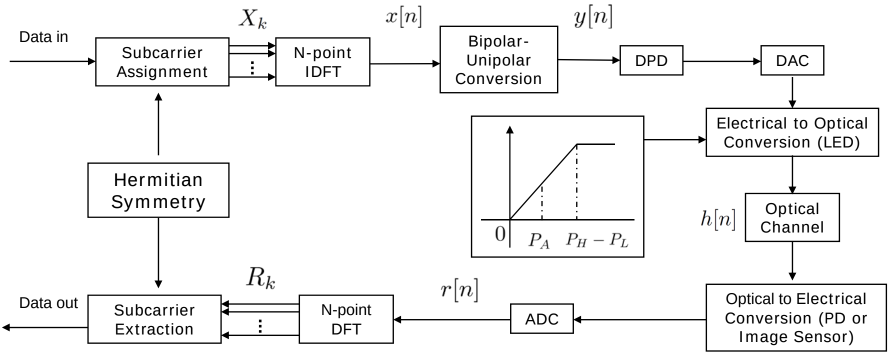
1. System model
2. Achievable Data Rate Analysis
3. Numerical Results
4. Conclusion

OFDM for Optical Wireless Communication (OWC)

- Intensity modulation (IM) and direct detection (DD) schemes require the electric signal to be real-valued and unipolar (positive-valued)
  - Hermitian symmetric
  - Bipolar-unipolar conversion
- Real-valued unipolar OFDM signal for OWC
  - DC biased optical OFDM (DCO-OFDM) [Hranilovic, 2005]
  - Asymmetrically clipped optical OFDM (ACO-OFDM) [Armstrong and Lowery, 2006]
  - Flip-OFDM [Yong, 2007] [Fernando et al., 2011]
  - Unipolar OFDM (U-OFDM) [Tsonev et al., 2012]
- Disadvantage of OFDM: high peak-to-average-power ratio (PAPR)
  - The OFDM signal often has to be double-sided clipped in order to fit the linear range of LED [Mesleh et al., 2011][Dimitrov et al., 2011] [Yu et al., 2012][Dimitrov et al., 2012]
  - Introduces nonlinear distortions

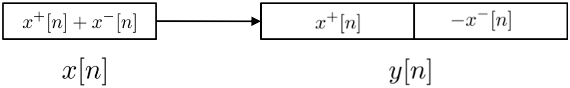
OFDM for OWC (Continued)



$$r[n] = y[n] \otimes h[n] + w[n]$$

- Assume that the digital pre-distortion (DPD) has perfectly linearized the LED between the interval $[P_L, P_H]$ [Elgala et al., 2009].

Flip-OFDM [Fernando et al., 2011]



- The unipolar signal $y[n]$ is composed of a positive part and a negative part from $x[n]$

$$x[n] = x^+[n] + x^-[n],$$

where the positive part $x^+[n]$ and the negative part $x^-[n]$ are obtained as

$$x^+[n] = \begin{cases} x[n], & x[n] > 0 \\ 0, & x[n] < 0 \end{cases}$$

$$x^-[n] = \begin{cases} x[n], & x[n] < 0 \\ 0, & x[n] > 0 \end{cases}$$

Flip-OFDM (Continued) [Fernando et al., 2011]

- At the receiver, assume the channel $h[n]$ is constant over two frames, the two received components can be expressed as

$$r^{+}[n] = x^{+}[n] \otimes h[n] + w^{+}[n],$$

$$r^{-}[n] = -x^{-}[n] \otimes h[n] + w^{-}[n]$$

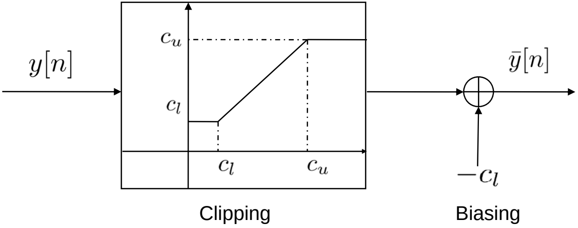
- Then the bipolar signal is reconstructed as

$$\begin{aligned} r[n] &= r^{+}[n] - r^{-}[n] \\ &= (x^{+}[n] + x^{-}[n]) \otimes h[n] + w^{+}[n] + w^{-}[n] \\ &= x[n] \otimes h[n] + w^{\star}[n], \end{aligned}$$

- $w^{\star}[n] = w^{+}[n] + w^{-}[n]$: has power $2\sigma_w^2$; can be further reduced with noise filtering scheme [Fernando et al., 2012].

Outline

Clipping and Biasing Model [Dimitrov et al., 2011, Yu et al., 2012]



- In order to fit the signal within the optical power constraints of the transmitter

$$\bar{y}[n] = \begin{cases} c_u - c_l, & y[n] > c_u \\ y[n] - c_l, & c_l \leq x[n] \leq c_u \\ 0, & x[n] < c_l \end{cases}$$

Definitions

- Clipping ratio γ and Biasing ratio ς

$$\gamma \triangleq \frac{(c_u - c_l)/2}{\sigma}, \quad \varsigma \triangleq \frac{-c_l}{c_u - c_l}$$

  where σ denotes the standard deviation of $x[n]$
- Average optical power

$$O_{\bar{y}} \triangleq \mathcal{E}\{\bar{y}[n]\}$$

- Optical signal to noise power ratio (OSNR)

$$\text{OSNR} \triangleq \frac{O_{\bar{y}}}{\sigma_w}$$

- Dynamic optical power

$$G_{\bar{y}} \triangleq \max\left(\bar{y}[n]\right) - \min\left(\bar{y}[n]\right)$$

- Dynamic signal to noise power ratio (DSNR)

$$\text{DSNR} \triangleq \frac{G_{\bar{y}}}{\sigma_w}$$

Clipping and Biasing for Flip-OFDM

- After the clipping and biasing, the two components of Flip-OFDM can be expressed as

$$\bar{x}^+[n] = \begin{cases} c_u - c_l, & x^+[n] > c_u \\ x^+[n] - c_l, & c_l \leq x^+[n] \leq c_u \\ 0, & x^+[n] < c_l \end{cases}$$

$$-\bar{x}^-[n] = \begin{cases} c_u - c_l, & -x^-[n] > c_u \\ -x^-[n] - c_l, & c_l \leq -x^-[n] \leq c_u \\ 0, & -x^-[n] < c_l \end{cases}$$

- Average optical power of $\bar{y}[n]$ is reduced to

$$\begin{aligned} O_{\bar{y}} &= \sigma\Big(\phi(2\gamma\varsigma) - \phi\left(2\gamma(1-\varsigma)\right) - 2\gamma\varsigma\Phi\left(-2\gamma\varsigma\right) \\ &\quad +2\gamma(1-\varsigma)\Phi\left(-2\gamma(1-\varsigma)\right) + 2\gamma\varsigma\Big) \end{aligned}$$

Clipping and Biasing for Flip-OFDM (Continued)

- Dynamic optical power of $\bar{y}[n]$ is reduced to

$$G_{\bar{y}} = \max\left(\bar{y}[n]\right) - \min\left(\bar{y}[n]\right) = c_u - c_l = 2\sigma\gamma$$

- At the receiver, we can obtain the reconstructed signal

$$\bar{r}[n] = \bar{x}[n] \otimes h[n] + w^{\star}[n],$$

where

$$\begin{aligned} \bar{x}[n] &= \bar{x}^{+}[n] + \bar{x}^{-}[n] \\ &= \begin{cases} c_u - c_l, & x[n] > c_u \\ x[n] - c_l, & c_l \leq x[n] \leq c_u \\ 0, & -c_l < x[n] < c_l \\ x[n] + c_l, & -c_u \leq x[n] \leq -c_l \\ c_l - c_u, & x[n] \leq -c_u \end{cases} \end{aligned}$$

System Constraints

- Average optical power constraint P_A (limit on power consumption, eye safety regulation, dim illumination requirement, etc.)

$$O_{\bar{y}} \leq P_A$$

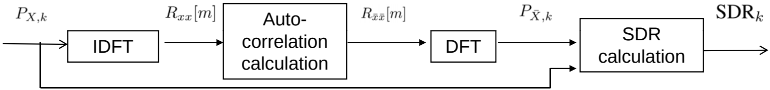
- OSNR constraint $\eta_{OSNR} \triangleq P_A/\sigma_w$
- Dynamic optical power constraint $P_H - P_L$ (dynamic range of the LED)

$$G_{\bar{y}} \leq P_H - P_L$$

- DSNR constraint $\eta_{DSNR} \triangleq (P_H - P_L)/\sigma_w$
- The maximum σ/σ_w value can be obtained as

$$\frac{\sigma}{\sigma_w} = \min\left(\frac{\eta_{OSNR}}{O_{\bar{y}}/\sigma}, \quad \frac{\eta_{DSNR}}{G_{\bar{y}}/\sigma}\right)$$

Signal to Distortion Ratio



- Bussgang's Theorem [Bussgang, 1952]

$$\bar{x}[n] = \alpha \cdot x[n] + d[n], \quad n = 0, \ldots, N-1$$

- The output auto-correlation function $R_{\bar{x}\bar{x}}[m]$ is related to the input auto-correlation function $R_{xx}[m]$ via [Davenport and Root, 1987]

$$R_{\bar{x}\bar{x}}[m] = \sum_{\ell=0}^{\infty} \frac{b_\ell^2}{\ell!} \left[\frac{R_{xx}[m]}{\sigma^2} \right]^\ell$$

- The SDR at the kth subcarrier

$$\mathrm{SDR}_k = \frac{\mathcal{E}[|\alpha \cdot X_k|^2]}{\mathcal{E}[|D_k|^2]} = \frac{\alpha^2 P_{X,k}}{P_{D,k}} = \frac{\alpha^2 P_{X,k}}{P_{\bar{X},k} - \alpha^2 P_{X,k}}$$

Achievable Data Rate

- The signal-to-noise-and-distortion ratio (SNDR) for the kth subcarrier

$$\mathsf{SNDR}_k = \left((\mathsf{SDR}_k)^{-1} + \frac{2\beta(N-2)}{N\alpha^2|H_k|^2} \cdot \max\left(\frac{O_{\bar{y}}^2/\sigma^2}{\eta_{OSNR}^2}, \quad \frac{G_{\bar{y}}^2/\sigma^2}{\eta_{DSNR}^2} \right) \right)^{-1}$$

- The achievable data rate

$$\begin{aligned} &\mathcal{R}\left(\gamma, \varsigma, \eta_{OSNR}, \eta_{DSNR}, \mathbf{H}\right) \\ = \quad & \frac{1}{2N} \sum_{k=1}^{N/2-1} \log_2\left(1 + \mathsf{SNDR}_k\right) \frac{\text{bits}}{\text{subcarrier}} \end{aligned}$$

- For given η_{OSNR}, η_{DSNR} values and channel response $\mathbf{H}$, we can obtain a pair of optimum clipping ratio $\gamma^\ddagger$ and optimum biasing ratio $\varsigma^\ddagger$ that maximize the achievable data rate by

$$(\gamma^\ddagger, \varsigma^\ddagger) = \underset{(\gamma,\varsigma)}{\text{argmax}} \quad \mathcal{R}|_{\eta_{OSNR}, \eta_{DSNR}, \mathbf{H}},$$

  and the corresponding achievable data rate.

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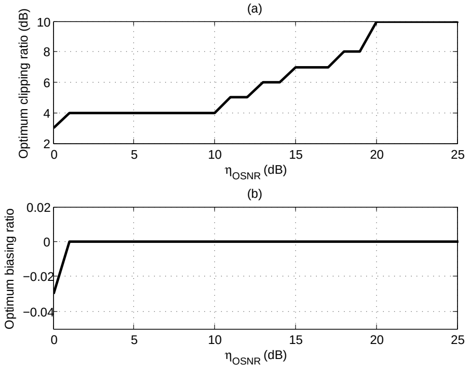
Optimal clipping ratio and biasing ratio



- $N = 512$. AWGN channel.
- $\eta_{DSNR}/\eta_{OSNR} = (P_H - P_L)/P_A = 18$ dB

Achievable data rate ($\eta_{DSNR}/\eta_{OSNR} = 6$ dB)

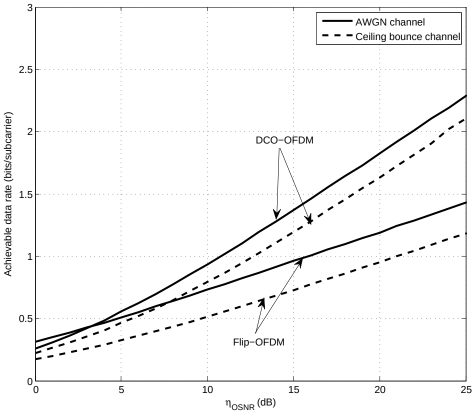


Figure : Achievable data rate with optimal clipping ratio and optimal biasing ratio for $\eta_{OSNR} = 0, 1, ..., 25$ dB (in step size of 1 dB), and $\eta_{DSNR}/\eta_{OSNR} = 6$ dB.

Achievable data rate ($\eta_{DSNR}/\eta_{OSNR} = 18$ dB)

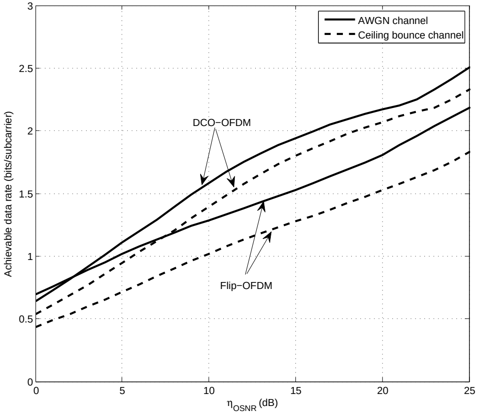


Figure : Achievable data rate with optimal clipping ratio and biasing ratio for $\eta_{OSNR} = 0, 1, ..., 25$ dB (in step size of 1 dB), and $\eta_{DSNR}/\eta_{OSNR} = 18$ dB.

Achievable data rate (no η_{DSNR} constraint)

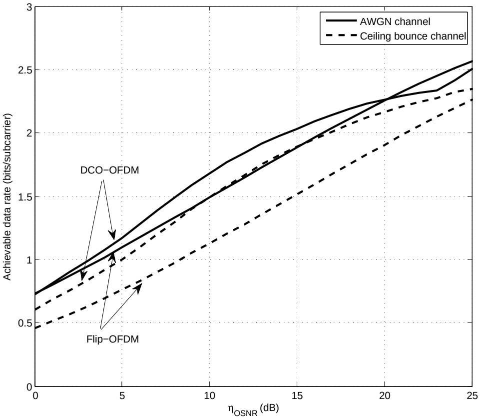


Figure : Achievable data rate with optimal clipping ratio and biasing ratio for $\eta_{OSNR} = 0, 1, ..., 25$ dB (in step size of 1 dB), and no η_{DSNR} constraint.

Outline

Summary

- Derived the achievable data rate of clipped Flip-OFDM
- Investigated the trade-off between the optical power constraint and distortion
- Analyzed the optimum clipping ratio and biasing ratio and compared the performance of Flip-OFDM and DCO-OFDM techniques
- Numerical results showed that DCO-OFDM outperforms the Flip-OFDM for most of the optical power constraint scenarios

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