Constellation Design for Color-Shift (CSK) Keying Using Convex Optimization Methods

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Contributions

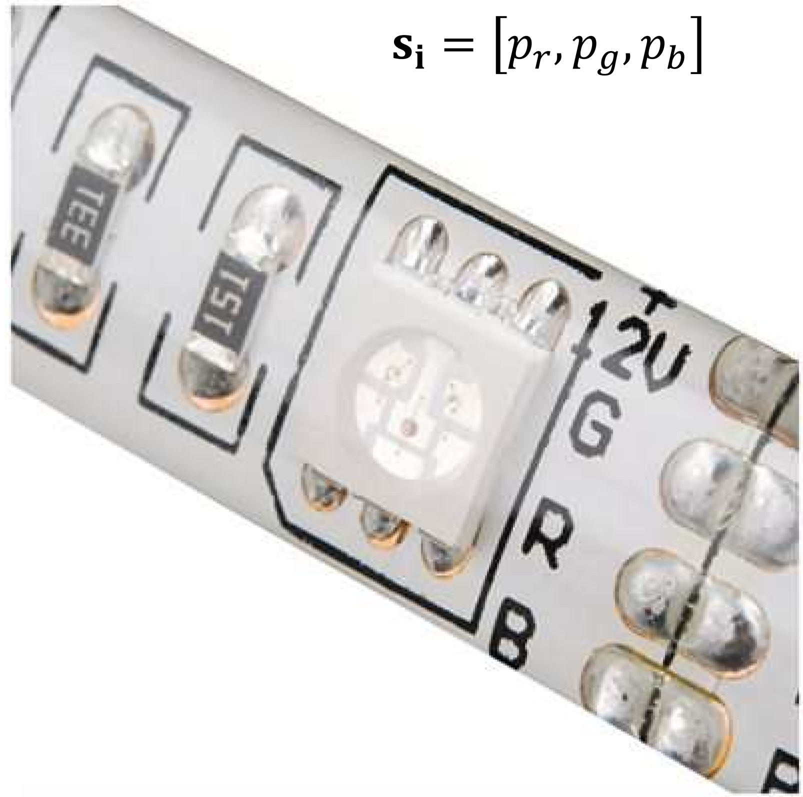
- Differentiable non-convex formulation solvable by interior point methods
  - Can specify perceived color of light source
- Efficient design heuristic for large constellations
  - Can NOT specify perceived color of light source

Color-Shift Keying (CSK)

Advantages:

- Zero flicker
- Reduced Inrush Currents

$\mathbf{s_i} = [p_r, p_g, p_b]$

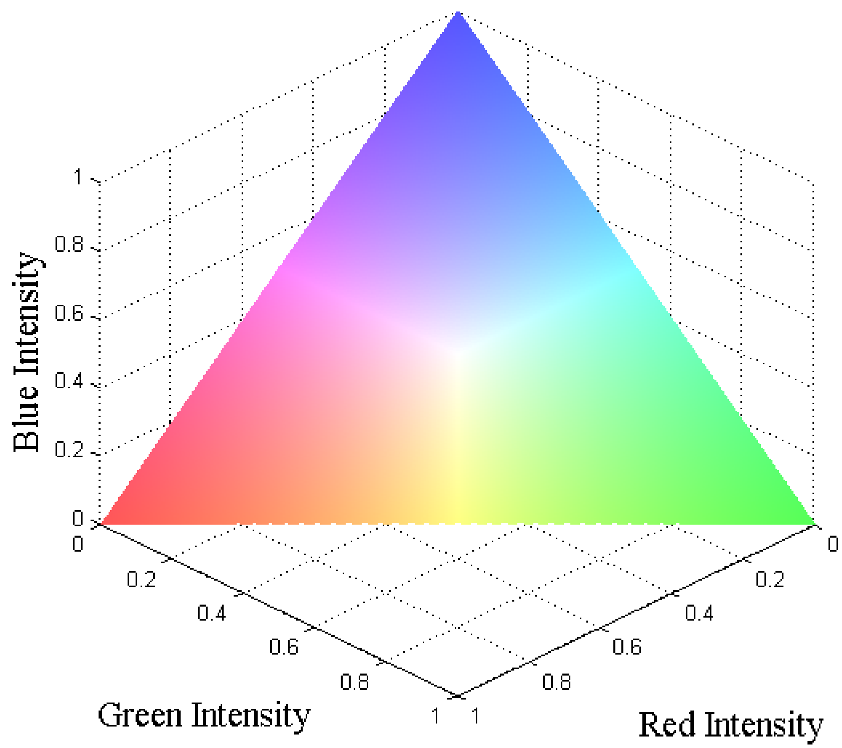


Constraints:

- $p_r + p_g + p_b = I$
- $0 \le p_{r,g,b} \le P_{r,g,b}$
- $\sum w_i \mathbf{s_i} = \mathbf{c_{avg}}$

Transmittable Set

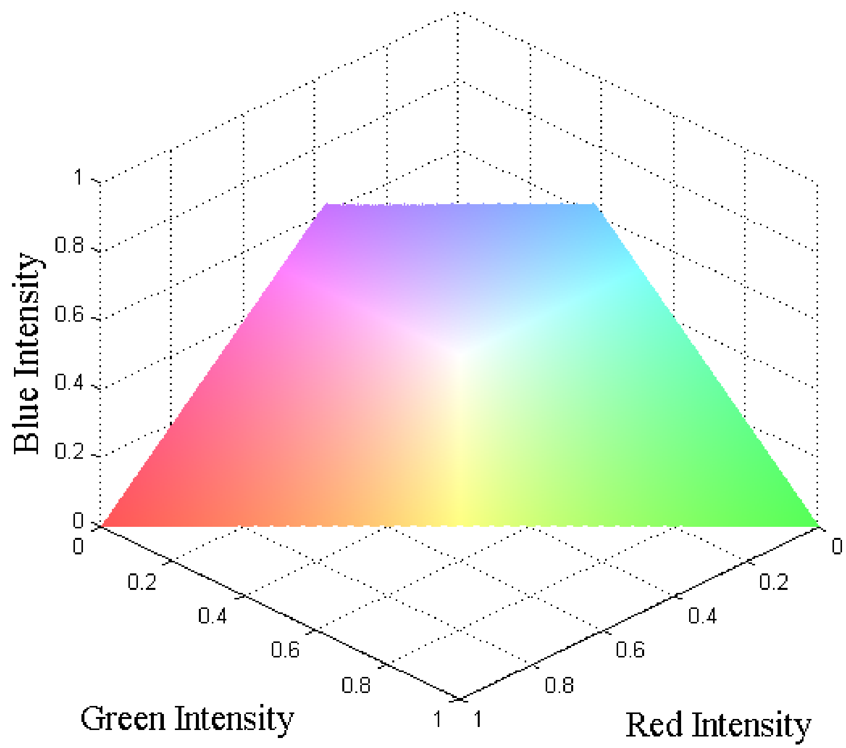
$$p_r + p_g + p_b = I$$



$$P_{r,g,b} > I$$

Transmittable Set

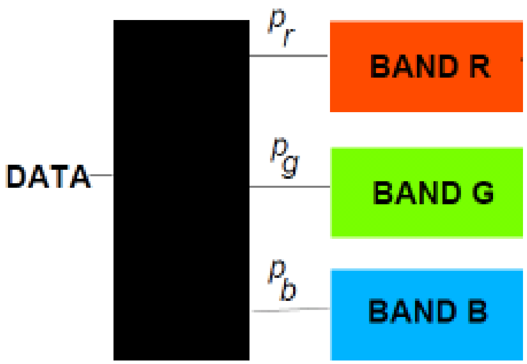
$$p_r + p_g + p_b = I$$



$$P_{r,g} > I$$
$$P_b < I$$

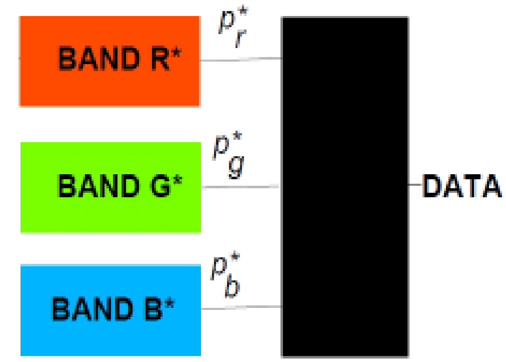
System Model

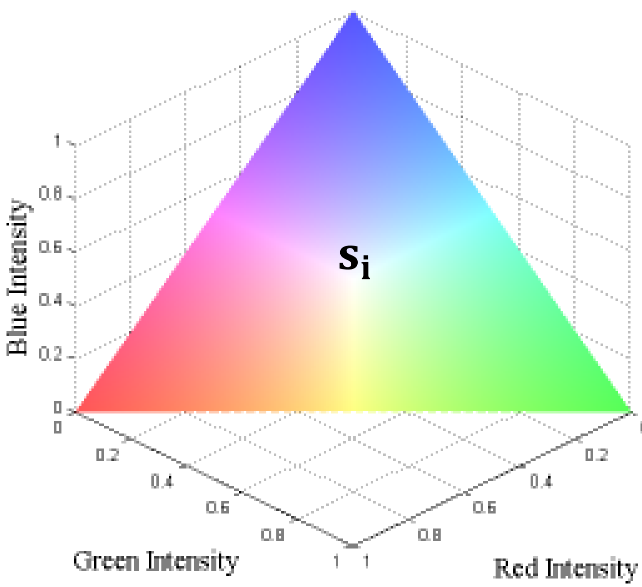
RGB Light Emitting Diode



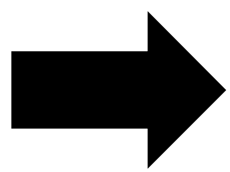
$$\mathbf{H} = \begin{bmatrix} h_{r,r} & h_{r,g} & h_{r,b} \\ h_{g,r} & h_{g,g} & h_{g,b} \\ h_{b,r} & h_{b,g} & h_{b,b} \end{bmatrix}$$

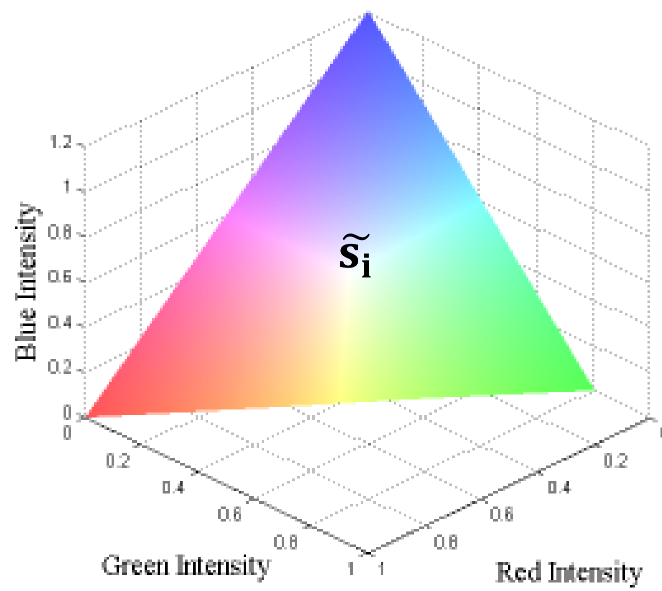
Filtered Photo-detectors



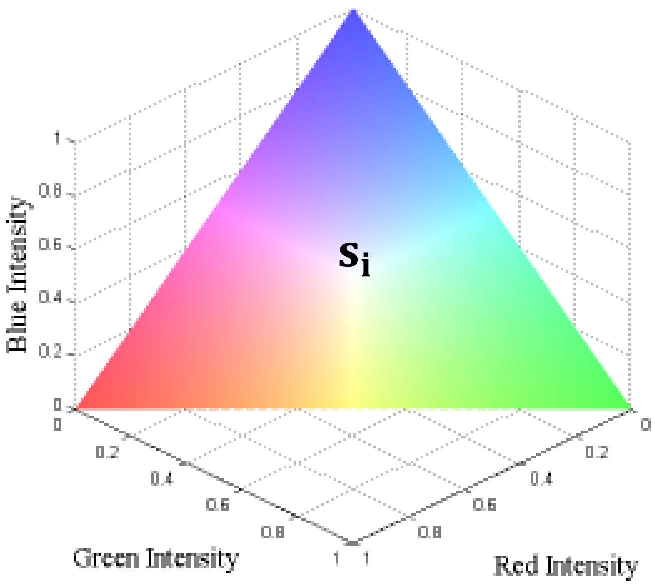


$$\widetilde{\mathbf{s_i}} = \mathbf{H}\mathbf{s_i}$$

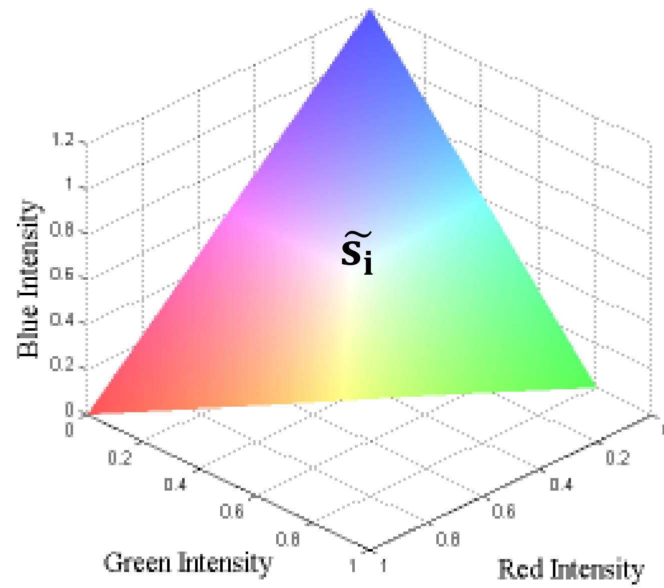




System Model

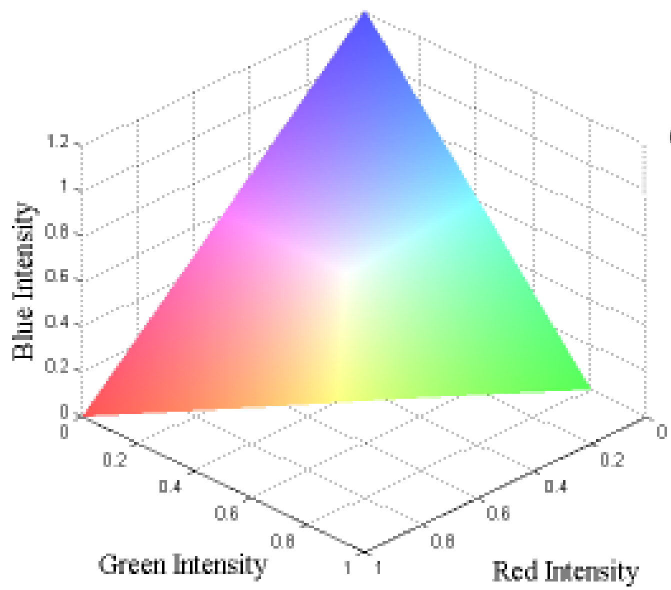


$$\widetilde{\mathbf{s}}_{\mathbf{i}} = \mathbf{H}\mathbf{s}_{\mathbf{i}}$$

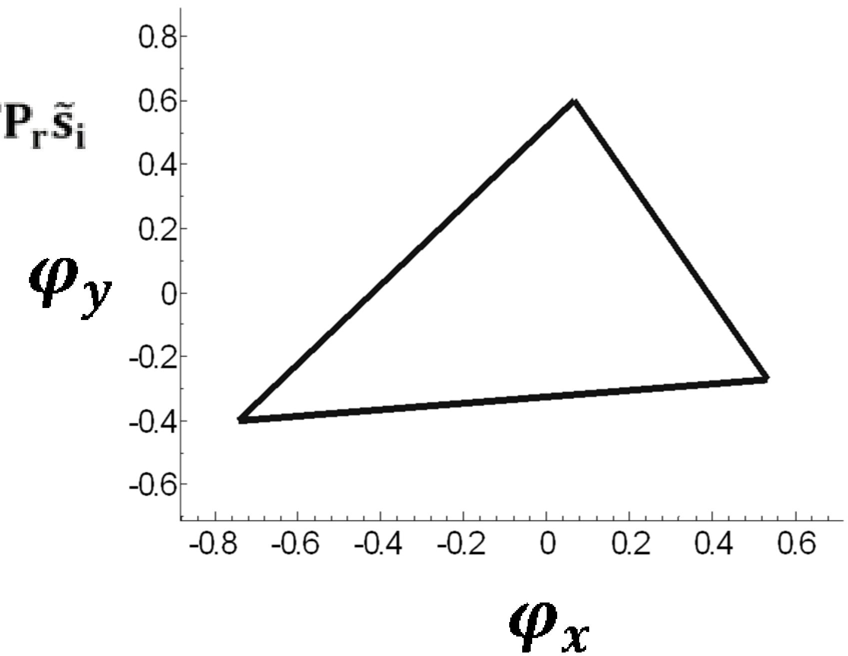


$$\mathbf{r_i} = \mathbf{H}\mathbf{s_i} + \mathbf{n_i}$$

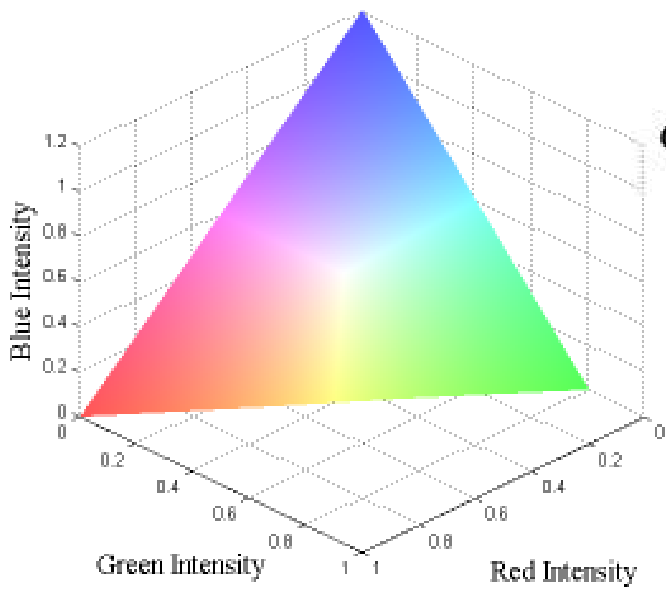
Basis Change



$$\phi_i = \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \frac{1}{I}\mathbf{T}\mathbf{P}_\mathbf{r}\tilde{\mathbf{s}}_\mathbf{i}$$

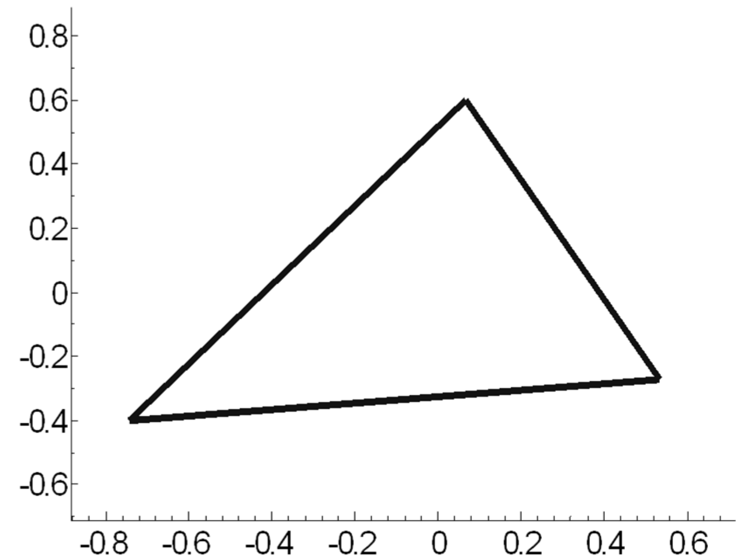


Basis Change



$$\boldsymbol{\phi}_i = \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \frac{1}{I}\mathbf{T}\mathbf{P}_\mathbf{r}\tilde{\mathbf{s}}_\mathbf{i}$$

$\boldsymbol{\varphi_y}$



$\boldsymbol{\varphi_x}$

$$\overline{\boldsymbol{\Phi}} = \mathrm{vec}(\boldsymbol{\phi}_i)$$

$$0 \le p_{r,g,b} \le P_{r,g,b}$$
$$p_r + p_g + p_b = I \quad \rightarrow \quad \boldsymbol{A}\overline{\boldsymbol{\Phi}} \le \mathbf{b}$$

$$\sum w_i \mathbf{s}_\mathbf{i} = \mathbf{c}_\mathbf{avg} \quad \rightarrow \quad \mathbf{C}\overline{\boldsymbol{\Phi}} = \overline{\mathbf{c}_\mathbf{avg}}$$

Objective

$$\max_{\{\boldsymbol{\phi}_i\}} \min_{i \neq j} \left\{ \left\| \boldsymbol{\phi}_i - \boldsymbol{\phi}_j \right\|_2^2 \right\}$$

- Non-Convex
- Discontinuous

Optimization Methods

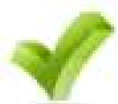
Stochastic

- ✓ Works with any objective
- ✓ Avoids poor local minima
- ✗ Brute force/inefficient
- ✗ Converge in probability

Deterministic

- ✓ Well defined stopping criteria
- ✓ Comparatively fast (per trial)
- ✓ Easy to implement
- ✗ Sensitive to starting point

Interior Point Methods

- Intended for Convex Optimization
  - Finds local minima of non-convex problems
- Assumptions:
  - Linear inequality constraints 
  - Affine Equality Constraints 
  - Continuous Objective 

Continuous Approximation

- Minimum approximation:

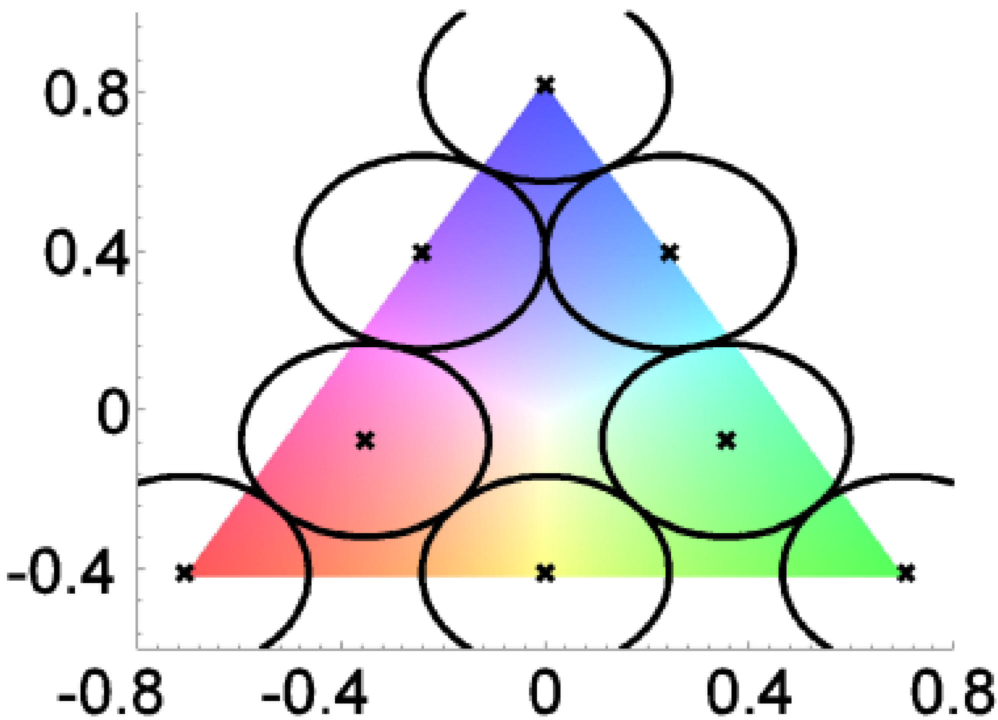
$$\min_i\{d_i\} \approx -ln\left(\sum_i e^{-\beta d_i}\right)/\beta$$

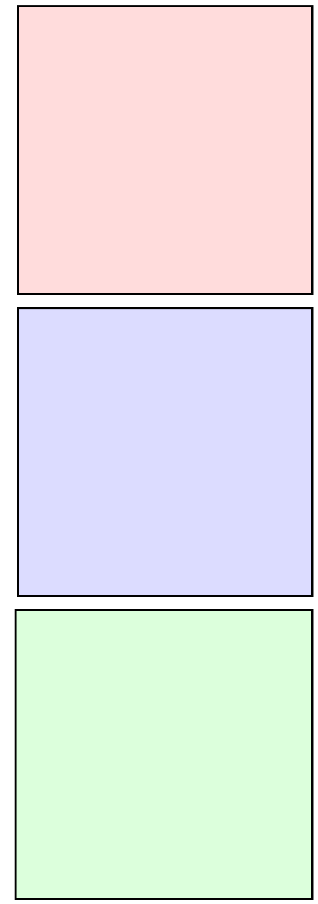
- New Objective:

$$\max_{\{\phi_i\}} -ln\left(\sum_{i \neq j} e^{-\beta\|\boldsymbol{\phi}_i - \boldsymbol{\phi}_j\|_2^2}\right)/\beta$$

Results: Validity

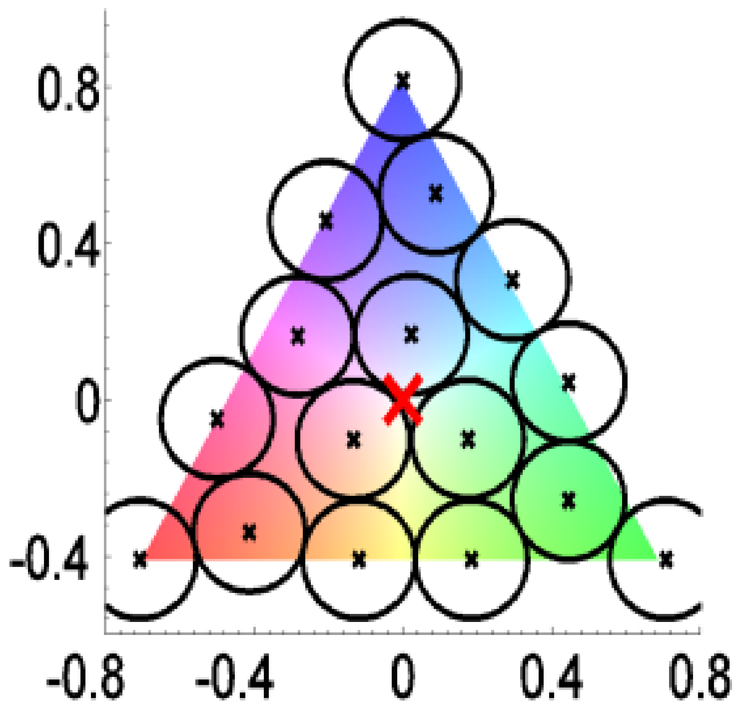
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad N = 8$$



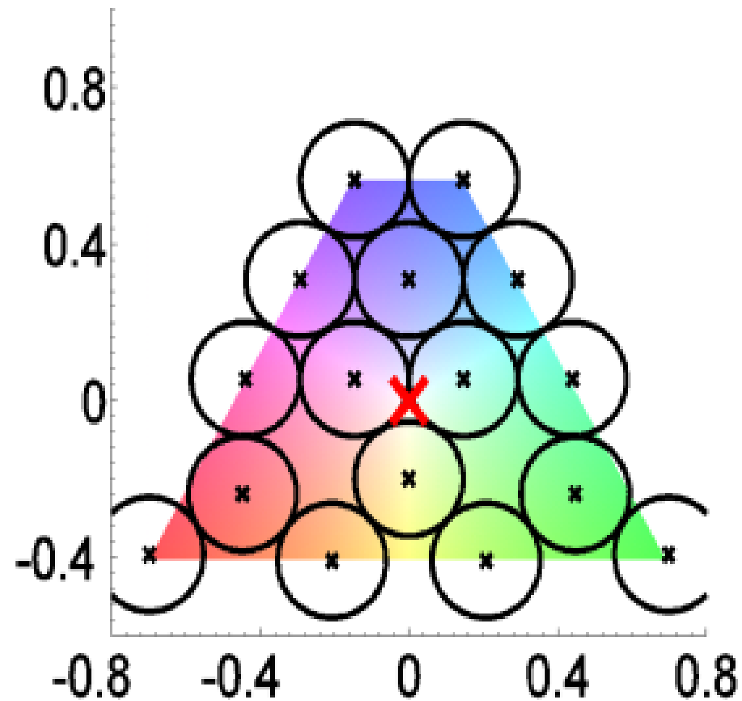


Results: Arbitrary Regions

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad N = 16$$

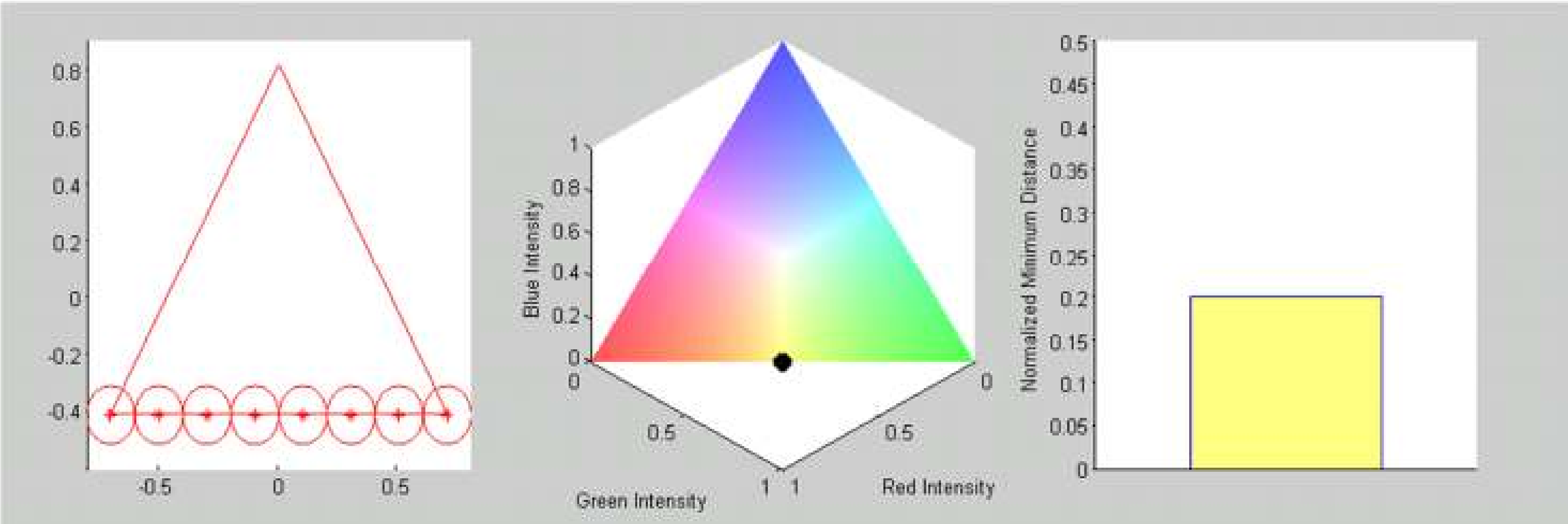


$d_{min} = 0.3021$



$d_{min} = 0.2915$

Results: Color Balance

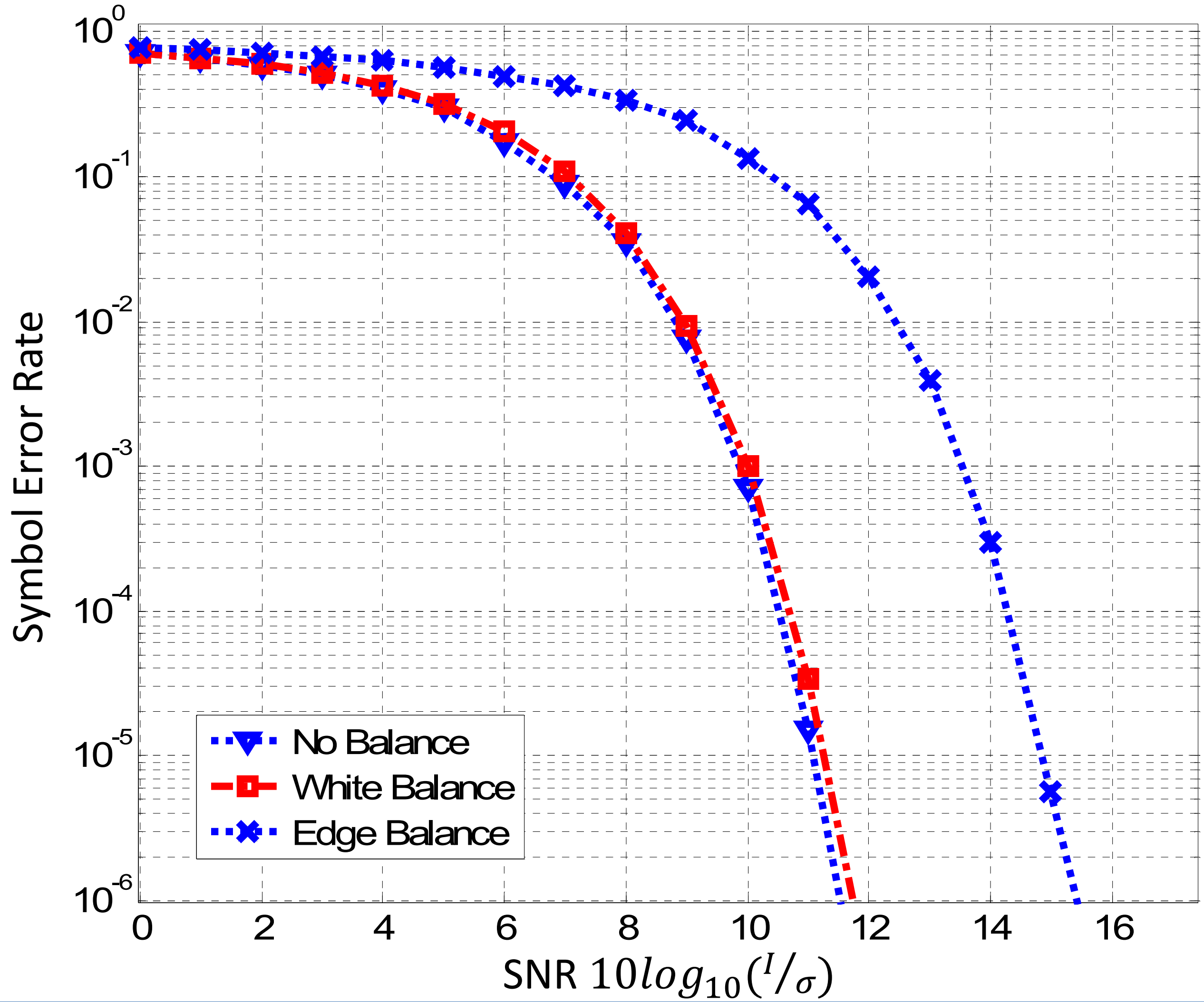


$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad N = 8$$

Results: Color Balance

$N = 8$

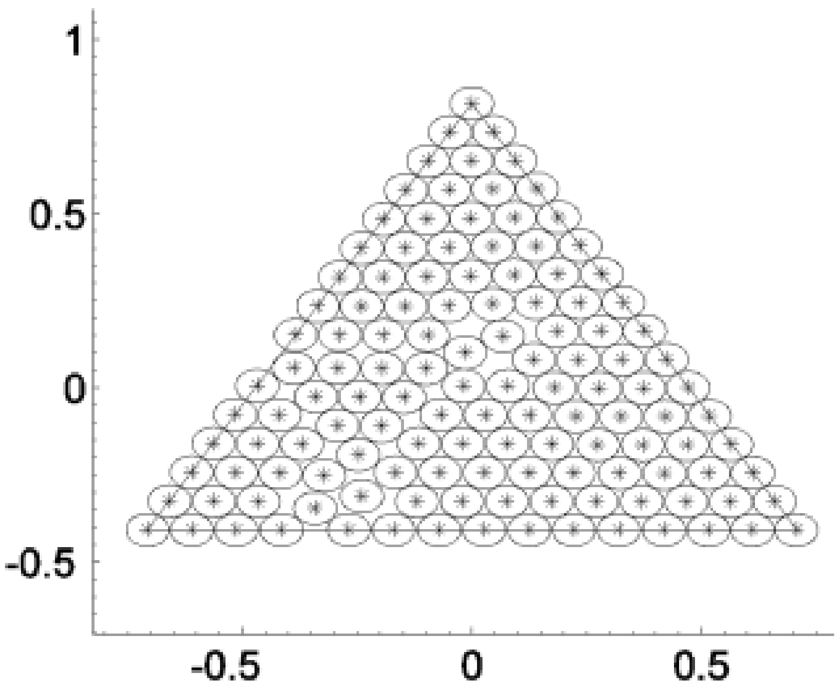
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

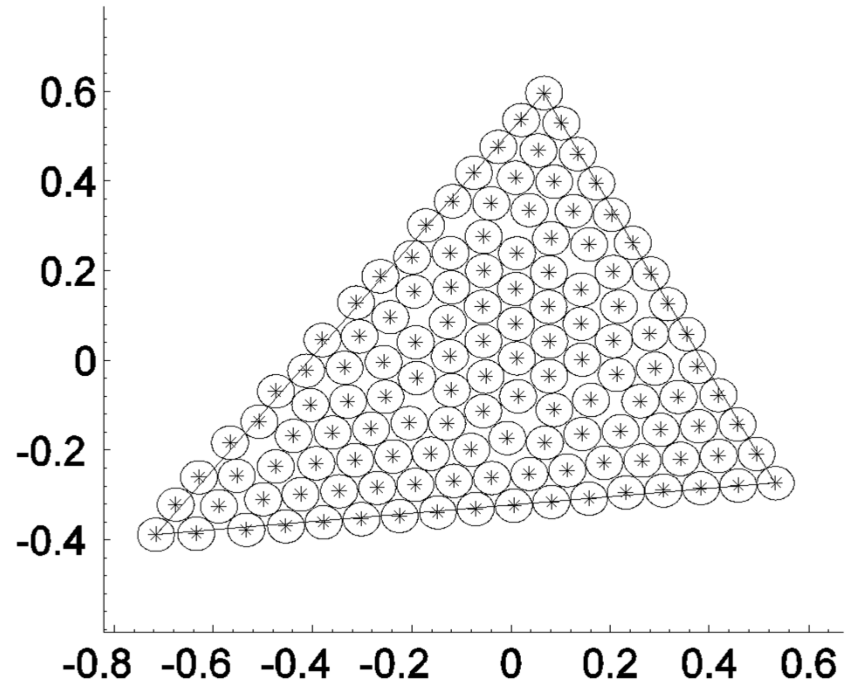


Results: Large Constellations

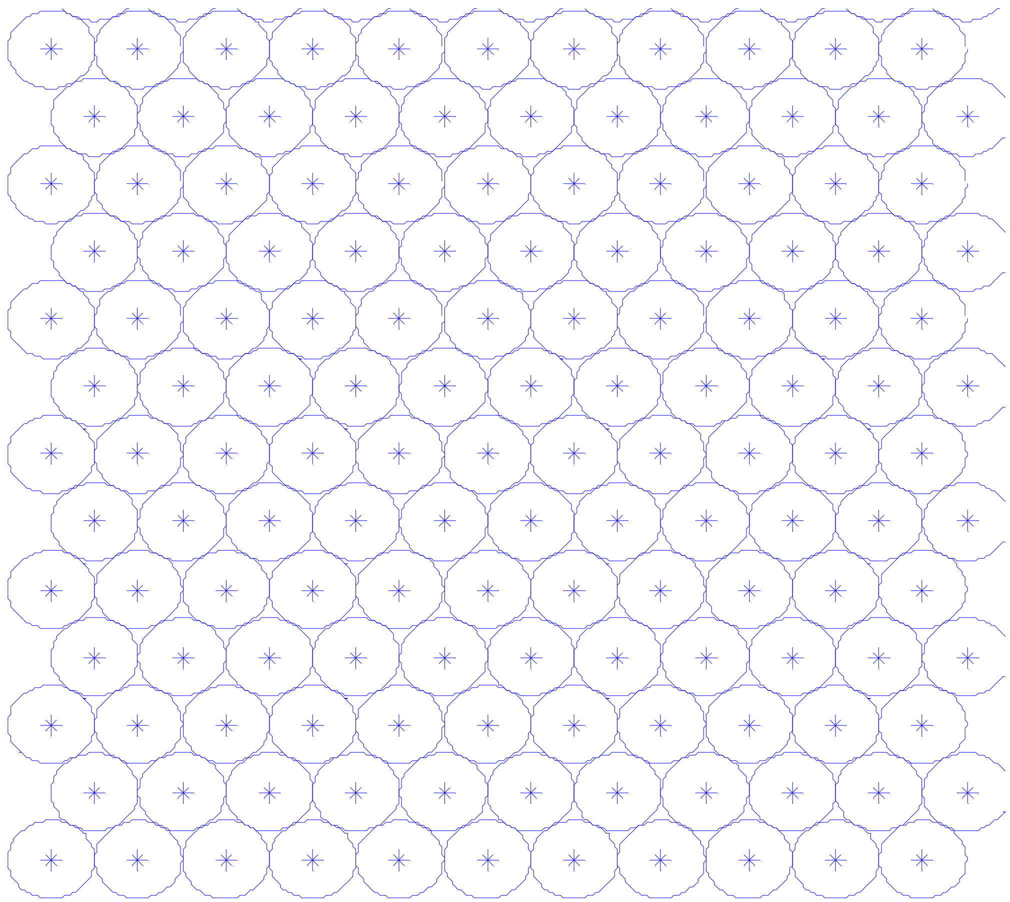
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad N = 128$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix} \quad N = 128$$

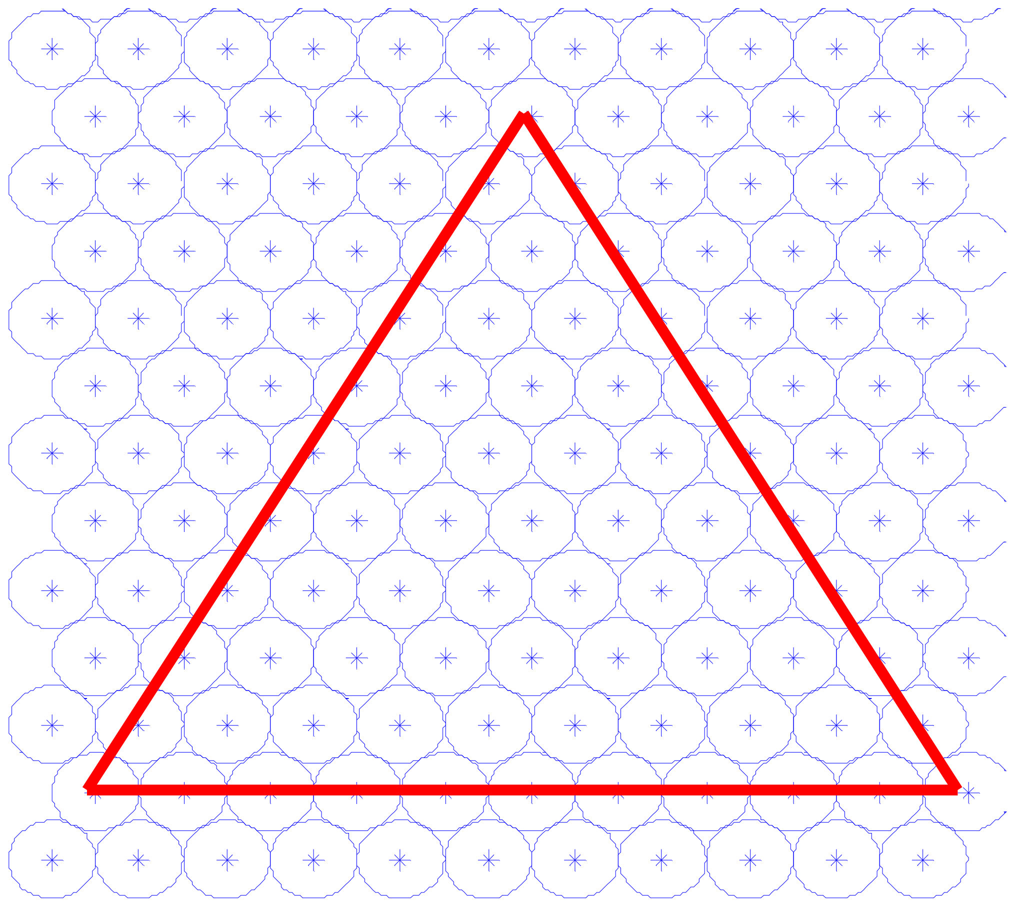




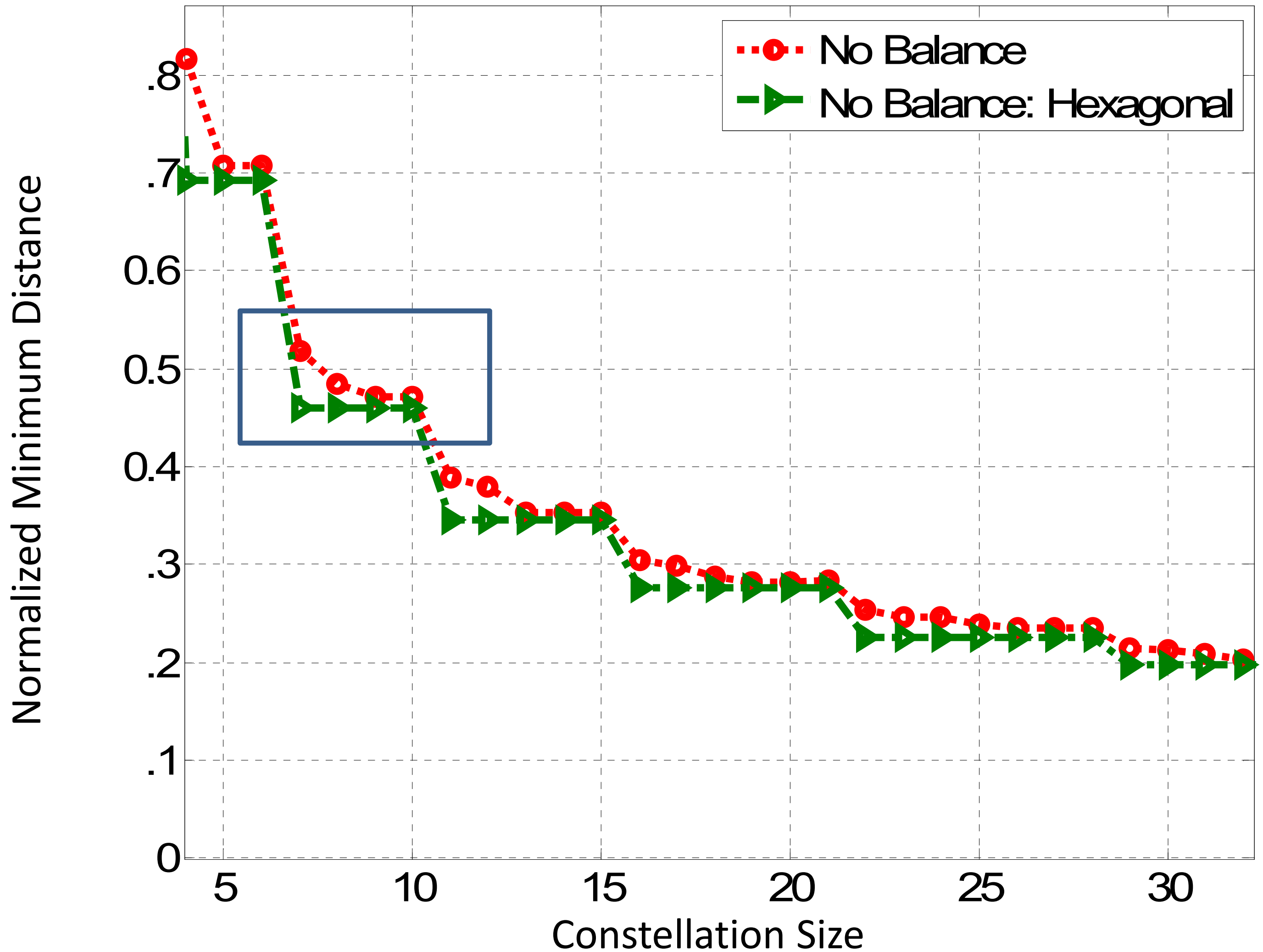
Heuristic



Heuristic

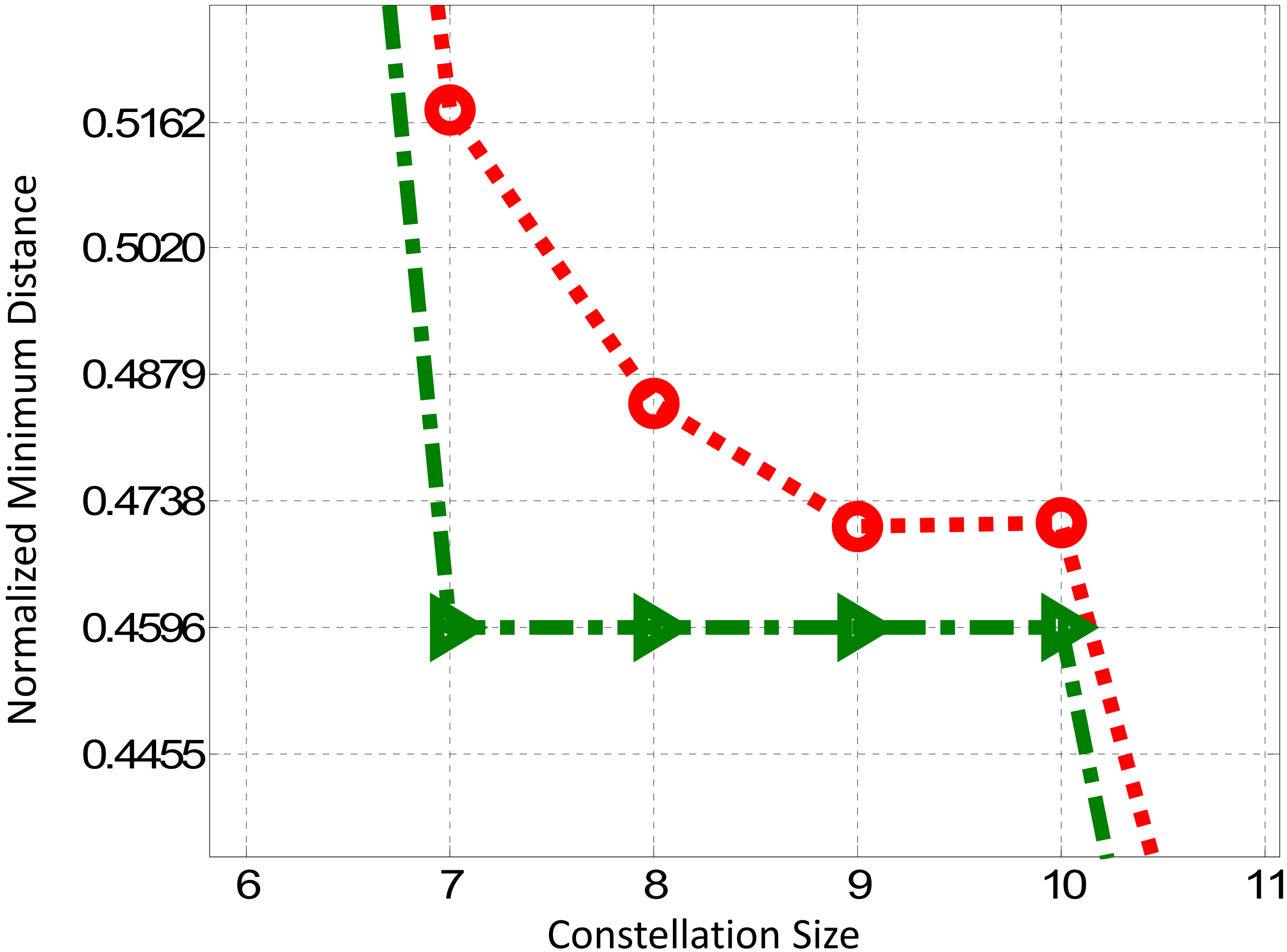


Comparison



No Balance
No Balance: Hexagonal
Normalized Minimum Distance
.8
.7
0.6
0.5
0.4
.3
.2
.1
0
5
10
15
20
25
30
Constellation Size

Comparison



0.5162
0.5020
0.4879
0.4738
0.4596
0.4455
Normalized Minimum Distance
6
7
8
9
10
11
Constellation Size

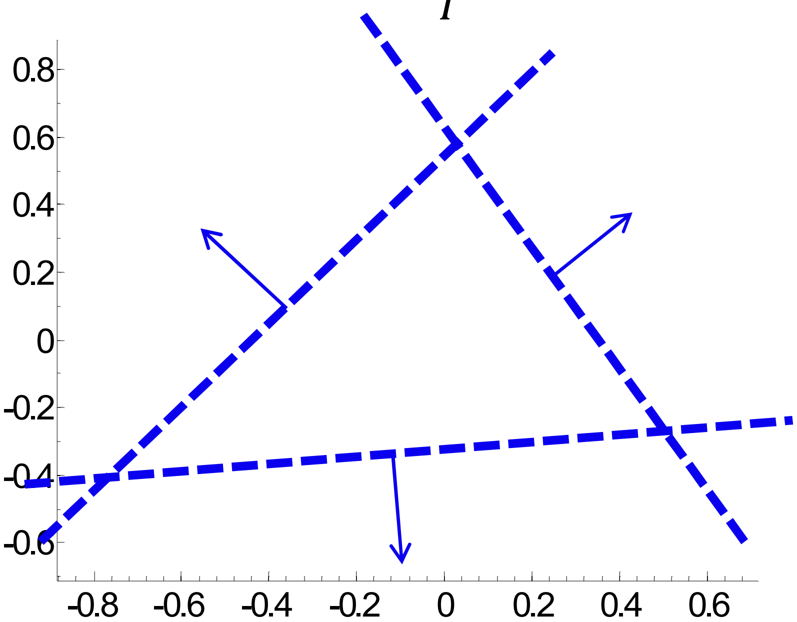
Conclusions

- A systematic optimization approach to CSK design has been presented, which functions under:
  - Any constellation size
  - Arbitrary constraint region and color balance
- Under no color balance, an efficient heuristic based on hexagonal lattices has been demonstrated

QUESTIONS?

Detailed Constraints

$$\boldsymbol{\phi}_i = \frac{1}{I}\mathbf{T}\mathbf{P}_{\mathbf{r}}\tilde{\mathbf{s}}_{\mathbf{i}}$$



$$A\boldsymbol{\phi}_i \leq \mathbf{b}$$

$$\overline{\boldsymbol{\Phi}} = \mathrm{vec}(\boldsymbol{\phi}_i)$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{A} \end{bmatrix} \qquad \mathbf{d} = \begin{bmatrix} \mathbf{b} \\ \vdots \\ \mathbf{b} \end{bmatrix}$$

$$\mathbf{C}\overline{\boldsymbol{\Phi}} = \overline{\mathbf{c}_{\mathbf{avg}}}$$

Minimum example

$let\ d_1 \ll\ d_2\ and\ \beta = 1$

$$\min_i\{d_1, d_2\} \approx -ln\left(e^{-d_1} + e^{-d_2}\right)$$
$$\approx -ln\left(e^{-d_1}\right) = d_1$$

Optimization Example

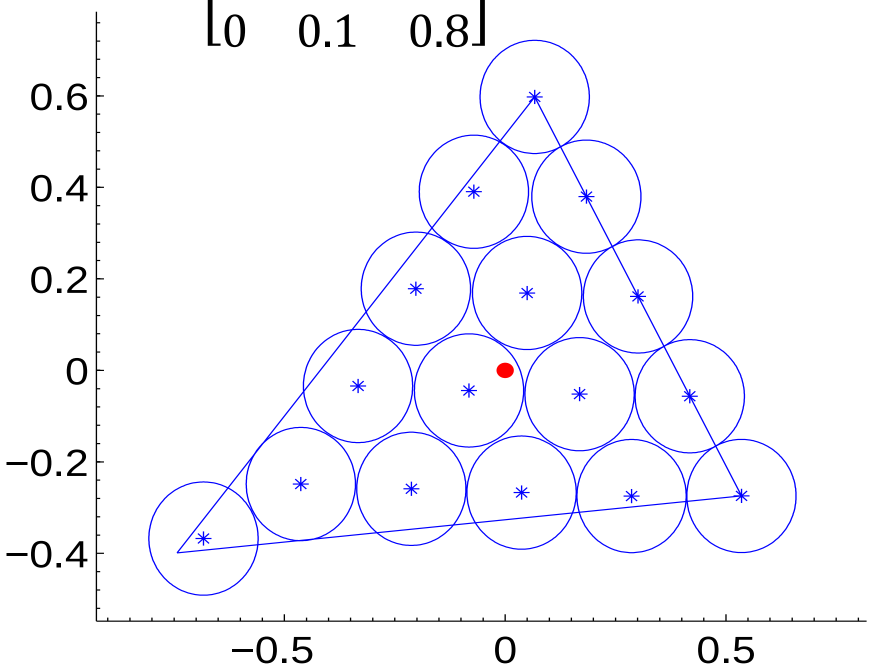
$\boldsymbol{\phi_i} \leftarrow random\ starting\ point$
$\beta \leftarrow 1$
while $\beta < \beta_{stop}$ **do**

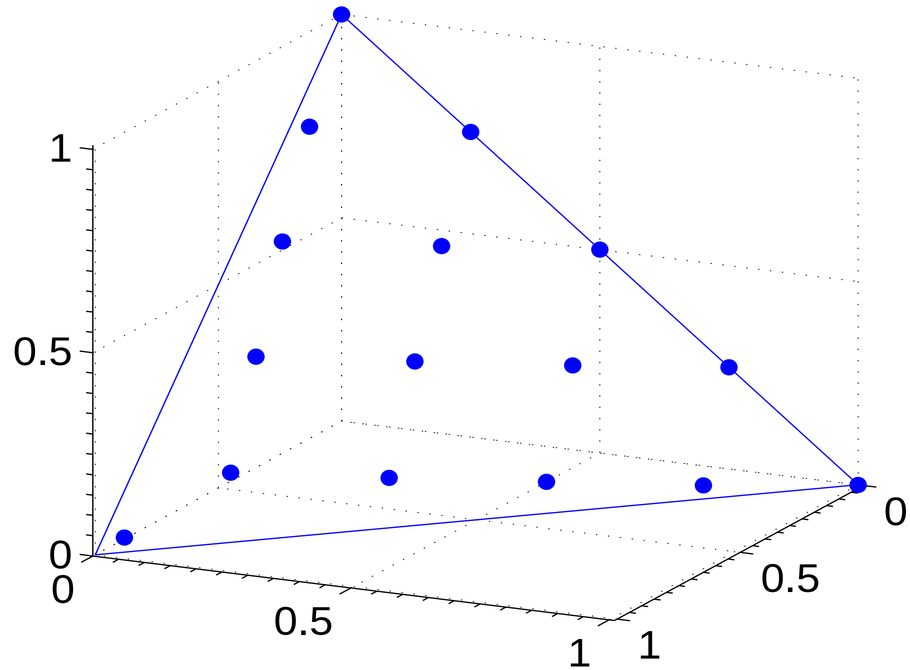
$$\boldsymbol{\phi_{opt}} \leftarrow \arg\max_{\{\boldsymbol{\phi_i}\}} -ln\left(\sum_{i \neq j} e^{-\beta \|\boldsymbol{\phi_i} - \boldsymbol{\phi_j}\|_2^2}\right)/\beta$$

$\beta \leftarrow 2\beta$
$\boldsymbol{\phi_i} \leftarrow \boldsymbol{\phi_{opt}}$
end while

Results: Cross-talk

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix}$$





$d_{min} = 0.2474$