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Spontaneous emission near an absorbing dielectric surface

M. S. Yeung* and T. K. Gustafson
Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, California 94720

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The lifetime of an excited atom near an absorbing dielectric surface is calculated from an exact solution of a microscopic Hamiltonian model, which includes the effects of dispersion, local-field correction, and near-field Coulomb interaction. Results for the total decay rate are shown to be in excellent agreement with those based on classical electromagnetic theory and to yield the well-known result for the rate of nonradiative energy transfer in the limit of very small distance from the surface. [S1050-2947(96)01811-2]

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I. INTRODUCTION

The lifetime of an excited molecule has been known for a long time to be significantly affected by a partially reflecting mirror in its vicinity [1]. Early attempts to explain the experimental results using classical electromagnetic theory [2,3] have been quite successful. Nevertheless, the validity of these theoretical results has so far not been verified by calculation based on a fully canonical quantum theory. Recently, spontaneous emission by an excited atom near a lossless dielectric surface was analyzed from the viewpoint of quantization of macroscopic spatial modes [4]. However, such an approach cannot easily be extended to include the effects of losses in the dielectric. Thus, up to now, an analysis of the lifetime of an excited atom near an absorbing dielectric surface based on a fully canonical quantum theory has been lacking. In this paper, we present one such analysis based on an exact solution of a microscopic Hamiltonian model.

After discussing the Hamiltonian formulation in Sec. II, we diagonalize the matter part of the Hamiltonian density to obtain the dressed matter field in Sec. III. In Sec. IV, the self-energy of an excited atom near the dielectric surface is obtained to second order of perturbation theory by considering the Green function of the excited atom to this order. The decay rate of the excited atom, which is proportional to the imaginary part of the atom self-energy, is then expressed in terms of the instantaneous Coulomb interaction, the Green function for the transverse photons, and the Green function for the harmonic-oscillator field [Eq. (70)]. In Sec. V, the latter two Green functions are obtained by solving the corresponding Dyson equations *exactly*. This involves a three-step procedure. First, the Green function for the harmonic-oscillator field is solved exactly by ignoring the coupling to the transverse photons [Eq. (90)]. Then, the Green function for the transverse photons is solved exactly by including both the bulk and the surface contributions to the photon self-energy (Appendix). Finally, the Green function for the harmonic-oscillator field is corrected by including the coupling to the transverse photons [Eq. (122)]. Numerical results for the decay rate of the excited atom obtained from the above theory are compared with those obtained from the classical theory in Sec. VI.

II. THE MICROSCOPIC HAMILTONIAN MODEL

Our microscopic model of the absorbing dielectric is the Hopfield model with losses introduced by Huttner and Barnett [5]. This model has been used in the study of spontaneous emission in an *infinite* absorbing dielectric medium [6]. Here we apply it instead to an absorbing dielectric occupying the half space $z<0$.

In the absence of the excited atom, the Hamiltonian density of the system consisting of the lossy dielectric half space and the radiation field is

$$\mathcal{H}=\mathcal{H}_{\text{em}}^{A}+\mathcal{H}_{\text{mat}}+\mathcal{H}_{\text{res}}+\mathcal{H}_{\text{int}}^{\text{res}}+\mathcal{H}_{\text{int}}^{A}+\mathcal{H}_{\text{int}}^{U}. \tag{1}$$

The various parts of this Hamiltonian density are the following.

(i) $\mathcal{H}_{\text{em}}^{A}$ is the Hamiltonian density of the free radiation field

$$\mathcal{H}_{\text{em}}^{A}=\frac{\epsilon_0}{2}(\dot{\mathbf{A}})^2+\frac{1}{2\mu_0}(\nabla\times\mathbf{A})^2, \tag{2}$$

where $\mathbf{A}$ is the vector potential.

(ii) $\mathcal{H}_{\text{mat}}$ is the Hamiltonian density of the bare dielectric occupying the half space $z<0$, modeled by a harmonic-oscillator field

$$\mathcal{H}_{\text{mat}}=\theta(-z)\left(\frac{1}{2\rho}\mathbf{P}_X^2+\frac{\rho\tilde{\omega}_0^2}{2}\mathbf{X}^2\right), \tag{3}$$

where $\mathbf{X}$ is the bare matter field operator and $\mathbf{P}_X$ is the momentum conjugate to $\mathbf{X}$.

(iii) $\mathcal{H}_{\text{res}}$ is the Hamiltonian density of the reservoir associated with the dielectric, modeled by a continuum of harmonic oscillators

$$\mathcal{H}_{\text{res}}=\theta(-z)\int_0^\infty d\omega\left(\frac{1}{2\rho}\mathbf{P}_\omega^2+\frac{\rho\omega^2}{2}\mathbf{Y}_\omega^2\right), \tag{4}$$

where $\mathbf{Y}_\omega$ is the reservoir field operator and $\mathbf{P}_\omega$ is the momentum conjugate to $\mathbf{Y}_\omega$.

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*Present address: Boston University, 15 St. Mary's Street, Boston, MA 02215.

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(iv) $\mathcal{H}_{\text{int}}^{\text{res}}$ is the coupling between the bare dielectric and the reservoir leading to losses in the dielectric

$$\mathcal{H}_{\text{int}}^{\text{res}}=\theta(-z)\int_0^\infty d\omega\frac{v(\omega)}{\rho}\mathbf{X}\cdot\mathbf{P}_\omega, \tag{5}$$

where $v(\omega)$ is a square-integrable function with the properties that the analytic continuation of $v(\omega)^2$ to negative frequencies is an even function and $v(\omega)\neq 0$ for all nonzero frequencies.

(v) $\mathcal{H}_{\text{int}}^{A}$ is the interaction between the bare dielectric and the vector potential

$$\mathcal{H}_{\text{int}}^{A}=\theta(-z)\left(-\frac{e}{\rho}\mathbf{A}\cdot\mathbf{P}_X+\frac{e^2}{2\rho}\mathbf{A}^2\right). \tag{6}$$

(vi) $\mathcal{H}_{\text{int}}^{U}$ is the interaction between the bare dielectric and the scalar potential U

$$\mathcal{H}_{\text{int}}^{U}=-eU\boldsymbol{\nabla}\cdot[\mathbf{X}\theta(-z)]-\frac{\epsilon_0}{2}(\boldsymbol{\nabla}U)^2-\frac{e^2}{6\epsilon_0}\mathbf{X}^2\theta(-z), \tag{7}$$

where the third term on the right-hand side (RHS) of Eq. (7) is introduced to model the effects of local-field correction.

For simplicity, we have omitted the dependence of the quantities $\mathbf{A}$, U, $\mathbf{X}$, $\mathbf{P}_X$, $\mathbf{Y}_\omega$, and $\mathbf{P}_\omega$ in the above expressions on $(\mathbf{r},t)$. Also, the Heaviside unit function $\theta(-z)$ in the above expressions is used to indicate that the dielectric is confined to the half space $z<0$.

In our quantization scheme, U is not treated as an independent field. Instead, U is eliminated from the Hamiltonian density by means of its Euler-Lagrange equation of motion. In the Coulomb gauge $\boldsymbol{\nabla}\cdot\mathbf{A}=0$, which we shall use in this paper, the solution for U is the instantaneous Coulomb potential for the charge density $-e\boldsymbol{\nabla}\cdot[\mathbf{X}\theta(-z)]$:

$$U(\mathbf{r},t)=\int_\infty d^3r'\,\frac{-e\boldsymbol{\nabla}'\cdot[\mathbf{X}(\mathbf{r}',t)\theta(-z')]}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}, \tag{8}$$

in which the same value of time t appears on both sides of the equation.

The Hamiltonian H of the system is obtained by integrating the Hamiltonian density $\mathcal{H}$ over all space. For the Coulomb interaction $\mathcal{H}_{\text{int}}^{U}$ we can eliminate U using Eq. (8). After integration by parts, we obtain the contribution to H due to the Hamiltonian density $\mathcal{H}_{\text{int}}^{U}$,

$$H_{\text{int}}^{U}(t)=\frac{1}{2}\int_{z<0}\int_{z'<0}d^3r\,d^3r'X_i(\mathbf{r},t)X_j(\mathbf{r}',t)F_{ij}(\mathbf{r}-\mathbf{r}'), \tag{9}$$

where

$$F_{ij}(\mathbf{r}-\mathbf{r}')\overset{\text{def}}{=}\frac{e^2}{\epsilon_0}\left[\frac{1}{4\pi}\frac{\partial^2}{\partial x_i\partial x_j'}\frac{1}{|\mathbf{r}-\mathbf{r}'|}-\frac{1}{3}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\right]. \tag{10}$$

The fields are quantized in the usual way by imposing equal-time commutation relations between the field operators $\mathbf{A}$, $\mathbf{X}$, and $\mathbf{Y}_\omega$ and their conjugate momenta

$$[A_i(\mathbf{r},t),\dot{A}_j(\mathbf{r}',t)]=\frac{i\hbar}{\epsilon_0}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{11}$$

$$[X_i(\mathbf{r},t),P_{X,j}(\mathbf{r}',t)]=i\hbar\,\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{12}$$

$$[Y_{\omega,i}(\mathbf{r},t),P_{\omega',j}(\mathbf{r}',t)]=i\hbar\,\delta_{ij}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}'). \tag{13}$$

III. DIAGONALIZATION OF THE MATTER PART

Ideally, one would like to diagonalize the Hamiltonian H by means of a canonical transformation from the field operators $\mathbf{A}$, $\mathbf{X}$, and $\mathbf{Y}_\omega$ to some other set of operators. This proved to be too ambitious a task. Instead, we seek an exact solution of our Hamiltonian model using diagrammatic perturbation theory.

We first diagonalize the Hamiltonian density of the subsystem consisting of the bare matter field, the reservoir, and the coupling between the two,

$$\mathcal{H}_{\text{mat}}^{(0)}=\mathcal{H}_{\text{mat}}+\mathcal{H}_{\text{res}}+\mathcal{H}_{\text{int}}^{\text{res}}. \tag{14}$$

Our approach follows closely that of Huttner and Barnett [5]. Whereas these authors performed the diagonalization in reciprocal space, as is appropriate for an *infinite* dielectric medium, we perform the diagonalization in real space, since we are dealing with a dielectric *half space*. The annihilation operators $\mathbf{b}(\mathbf{r},t)$ and $\mathbf{b}_\omega(\mathbf{r},t)$ for the bare matter and reservoir fields are defined in terms of $\mathbf{X},\mathbf{Y}_\omega$, and their conjugate momenta by

$$\mathbf{b}=\sqrt{\frac{\rho\tilde{\omega}_0}{2\hbar}}\left(\mathbf{X}+i\frac{\mathbf{P}_X}{\rho\tilde{\omega}_0}\right), \tag{15}$$

$$\mathbf{b}_\omega=\sqrt{\frac{\rho\omega}{2\hbar}}\left(-i\mathbf{Y}_\omega+\frac{\mathbf{P}_\omega}{\rho\omega}\right), \tag{16}$$

where, for simplicity, we have omitted the dependence of the operators on $(\mathbf{r},t)$. Equations (15) and (16) may be inverted, using the fact that $\mathbf{X},\mathbf{Y}_\omega$, and their conjugate momenta are Hermitian,

$$\mathbf{X}=\sqrt{\frac{\hbar}{2\rho\tilde{\omega}_0}}(\mathbf{b}^\dagger+\mathbf{b}), \tag{17}$$

$$\mathbf{P}_X=i\sqrt{\frac{\hbar\rho\tilde{\omega}_0}{2}}(\mathbf{b}^\dagger-\mathbf{b}), \tag{18}$$

$$\mathbf{Y}_\omega=-i\sqrt{\frac{\hbar}{2\rho\omega}}(\mathbf{b}_\omega^\dagger-\mathbf{b}_\omega), \tag{19}$$

$$\mathbf{P}_\omega=\sqrt{\frac{\hbar\rho\omega}{2}}(\mathbf{b}_\omega^\dagger+\mathbf{b}_\omega). \tag{20}$$

Using Eqs. (15), (16), (12), and (13) we readily obtain

$$[b_i(\mathbf{r},t),b_j^\dagger(\mathbf{r}',t)]=\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{21}$$

$$[b_{\omega,i}(\mathbf{r},t),b^\dagger_{\omega',j}(\mathbf{r}',t)]=\delta_{ij}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}'), \tag{22}$$

while all other commutators between $\mathbf{b},\mathbf{b}_\omega$ and their Hermitian adjoints vanish. Substituting Eqs. (17)–(20) into Eq. (14) and using Eqs. (3)–(5), we obtain

$$\mathcal{H}^{(0)}_{\text{mat}}=\theta(-z)\bigg[\hbar\bar{\omega}_0\mathbf{b}^\dagger\cdot\mathbf{b}+\int_0^\infty d\omega\;\hbar\omega\mathbf{b}^\dagger_\omega\cdot\mathbf{b}_\omega$$
$$+\frac{\hbar}{2}\int_0^\infty d\omega\;V(\omega)(\mathbf{b}^\dagger+\mathbf{b})\cdot(\mathbf{b}^\dagger_\omega+\mathbf{b}_\omega)\bigg], \tag{23}$$

where $V(\omega)=[v(\omega)/\rho]\sqrt{\omega/\bar{\omega}_0}$ and we have omitted an infinite zero-point energy term.

Next, we diagonalize the Hamiltonian density Eq. (23) by defining the annihilation operator $\mathbf{B}_\omega(\mathbf{r},t)$ for the dressed matter field

$$\mathbf{B}_\omega=\alpha_0(\omega)\mathbf{b}+\beta_0(\omega)\mathbf{b}^\dagger$$
$$+\int_0^\infty d\omega'[\alpha_1(\omega,\omega')\mathbf{b}_{\omega'}+\beta_1(\omega,\omega')\mathbf{b}^\dagger_{\omega'}], \tag{24}$$

the dependence of the operators on $(\mathbf{r},t)$ being understood. The coefficients $\alpha_0(\omega)$, $\beta_0(\omega)$, $\alpha_1(\omega,\omega')$, and $\beta_1(\omega,\omega')$ are to be chosen so that $\mathcal{H}^{(0)}_{\text{mat}}$ is diagonalized,

$$\mathcal{H}^{(0)}_{\text{mat}}(\mathbf{r},t)=\theta(-z)\int_0^\infty d\omega\;\hbar\omega\mathbf{B}^\dagger_\omega(\mathbf{r},t)\cdot\mathbf{B}_\omega(\mathbf{r},t), \tag{25}$$

and furthermore that the transformation Eq. (24) is canonical,

$$[B_{\omega,i}(\mathbf{r},t),B^\dagger_{\omega',j}(\mathbf{r}',t)]=\delta_{ij}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}'). \tag{26}$$

As shown by Huttner and Barnett [5], the conditions (25) and (26) determine the coefficients up to a phase to be

$$\alpha_0(\omega)=\left(\frac{\omega+\bar{\omega}_0}{2}\right)\frac{V(\omega)}{\omega^2-\bar{\omega}_0^2z(\omega)}, \tag{27}$$

$$\beta_0(\omega)=\left(\frac{\omega-\bar{\omega}_0}{2}\right)\frac{V(\omega)}{\omega^2-\bar{\omega}_0^2z(\omega)}, \tag{28}$$

$$\beta_1(\omega,\omega')=\frac{\bar{\omega}_0}{2}\left[\frac{V(\omega')}{\omega+\omega'}\right]\frac{V(\omega)}{\omega^2-\bar{\omega}_0^2z(\omega)}, \tag{29}$$

$$\alpha_1(\omega,\omega')=\delta(\omega-\omega')+\frac{\bar{\omega}_0}{2}\left[\frac{V(\omega')}{\omega-\omega'-i\epsilon}\right]\frac{V(\omega)}{\omega^2-\bar{\omega}_0^2z(\omega)}, \tag{30}$$

where $z(\omega)$ is defined by

$$z(\omega)=1-\frac{1}{2\bar{\omega}_0}\int_{-\infty}^{\infty}d\omega'\frac{V^2(\omega')}{\omega'-\omega+i\epsilon}. \tag{31}$$

Furthermore, if the set of dressed operators $\mathbf{B}_\omega$ and $\mathbf{B}^\dagger_\omega$, $0<\omega<\infty$, is assumed to be complete, Eq. (24) can be inverted to give $\mathbf{b}$ and $\mathbf{b}_\omega$ as functions of the dressed operators

$$\mathbf{b}=\int_0^\infty d\omega'[\alpha_0^*(\omega)\mathbf{B}_\omega-\beta_0(\omega)\mathbf{B}^\dagger_\omega], \tag{32}$$

$$\mathbf{b}_\omega=\int_0^\infty d\omega'[\alpha_1^*(\omega',\omega)\mathbf{B}_{\omega'}-\beta_1(\omega',\omega)\mathbf{B}^\dagger_{\omega'}]. \tag{33}$$

Substituting Eq. (32) into Eqs. (17) and (18), we can express the bare matter field operator and its conjugate momentum in terms of the dressed operators

$$\mathbf{X}=\sqrt{\frac{\hbar}{2\rho\bar{\omega}_0}}\int_0^\infty d\omega[h(\omega)\mathbf{B}^\dagger_\omega+\text{H.c.}], \tag{34}$$

$$\mathbf{P}_X=\sqrt{\frac{\hbar\rho\bar{\omega}_0}{2}}\int_0^\infty d\omega[g(\omega)\mathbf{B}^\dagger_\omega+\text{H.c.}], \tag{35}$$

where

$$h(\omega)=\alpha_0(\omega)-\beta_0(\omega), \tag{36}$$

$$g(\omega)=i[\alpha_0(\omega)+\beta_0(\omega)], \tag{37}$$

and H.c. denotes the Hermitian conjugate of the immediately preceding term.

IV. SPONTANEOUS DECAY RATE

Up to now, we have only considered the system consisting of the lossy dielectric half space and the radiation field with which it interacts, as described by the Hamiltonian density $\mathcal{H}$ defined by Eqs. (1)–(7). When an excited atom is introduced into the system, there is an additional interaction Hamiltonian of the form

$$H_a=\int_\infty d^3r(-\mathbf{j}_a\cdot\mathbf{A}+\rho_aU), \tag{38}$$

where $\mathbf{j}_a$ and ρ_a are the current and charge densities of the atom. For simplicity, we assume the atom to be made up of a single electron of mass m and charge e in orbit around a fixed nucleus of charge $-e$ at a point $\mathbf{r}_a$ on the *air* side of the dielectric surface $z_a>0$. Then the current and charge densities of the atom are given by

$$\mathbf{j}_a(\mathbf{r},t)=-\frac{i\hbar e}{2m}\{\psi^\dagger(\mathbf{r},t)\nabla\psi(\mathbf{r},t)-[\nabla\psi^\dagger(\mathbf{r},t)]\psi(\mathbf{r},t)\}$$
$$-\frac{e^2}{2m}\mathbf{A}(\mathbf{r},t)\psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t), \tag{39}$$

$$\rho_a(\mathbf{r},t)=e\psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t)-e\delta(\mathbf{r}-\mathbf{r}_a), \tag{40}$$

where $\psi(\mathbf{r},t)$ is the field operator of the electron. For a two-level atom, $\psi(\mathbf{r},t)$ may be expanded in annihilation operators $c_0(t)$ and $c_1(t)$ for the ground and excited states, respectively,

$$\psi(\mathbf{r},t)=c_0(t)u_0(\mathbf{r})+c_1(t)u_1(\mathbf{r}), \tag{41}$$

where $u_0(\mathbf{r})$ and $u_1(\mathbf{r})$ are the ground- and excited-state wave functions of the atom, respectively, which are assumed to have opposite parities. The annihilation operators $c_0(t)$ and $c_1(t)$ obey the equal-time *anticommutation* relation

$$\{c_i(t),c_j^\dagger(t)\}=\delta_{ij}. \tag{42}$$

The integral over $\mathbf{r}$ in Eq. (38) can be performed if we make the dipole approximation for the atom. This means that the atomic wave functions $u_0(\mathbf{r})$ and $u_1(\mathbf{r})$ are assumed to be localized to within a small neighborhood of the nucleus at $\mathbf{r}_a$. In that case, the vector potential appearing in Eqs. (38) and (39) may be replaced by its value at $\mathbf{r}_a$. Using Eqs. (39) and (41), the first term in Eq. (38), responsible for the radiative decay of the excited atom, can be evaluated:

$$\begin{aligned} H_a^r(t) &\overset{\text{def}}{=} -\int_\infty d^3r\,\mathbf{j}_a\cdot\mathbf{A} \\ &= -\frac{e}{m}[c_0^\dagger(t)c_1(t)\mathbf{A}(\mathbf{r}_a,t)\cdot\mathbf{p}_{01} \\ &\quad +c_1^\dagger(t)c_0(t)\mathbf{A}(\mathbf{r}_a,t)\cdot\mathbf{p}_{10}]+\frac{e^2}{2m}\mathbf{A}^2(\mathbf{r}_a,t) \\ &\quad \times[c_0^\dagger(t)c_0(t)+c_1^\dagger(t)c_1(t)], \end{aligned} \tag{43}$$

where $\mathbf{p}_{01}=\mathbf{p}_{10}^*$ is the matrix element of the operator $(-i\hbar\nabla)$ between the ground- and excited-state wave functions.

For the second term in Eq. (38), we first expand the instantaneous Coulomb potential $U(\mathbf{r},t)$ given by Eq. (8) about $\mathbf{r}_a$,

$$\begin{aligned} U(\mathbf{r},t)=&\int_\infty d^3r'\,\frac{-e\nabla'\cdot[\mathbf{X}(\mathbf{r}',t)\theta(-z')]}{4\pi\epsilon_0} \\ &\times\left[\frac{1}{|\mathbf{r}_a-\mathbf{r}'|}+(x_j-x_{aj})\frac{\partial}{\partial x_{aj}}\frac{1}{|\mathbf{r}_a-\mathbf{r}'|}\right]. \end{aligned} \tag{44}$$

Using Eqs. (44), (40), and (41), the second term in Eq. (38), responsible for the nonradiative decay of the excited atom, can be evaluated,

$$\begin{aligned} H_a^{\text{nr}}(t) &\overset{\text{def}}{=}\int_\infty d^3r\,\rho_a U \\ &=\frac{ie^2[c_0^\dagger(t)c_1(t)p_{01,j}-c_1^\dagger(t)c_0(t)p_{10,j}]}{4\pi\epsilon_0 m\omega_a} \\ &\quad\times\int_{z'<0} d^3r'\,X_i(\mathbf{r}',t)\frac{\partial^2}{\partial x_{aj}\partial x_i'}\frac{1}{|\mathbf{r}_a-\mathbf{r}'|}, \end{aligned} \tag{45}$$

where we have performed an integration by parts and ω_a is the atomic transition frequency.

The radiative and nonradiative perturbation Hamiltonians, Eqs. (43) and (45), may now be used to compute the total decay rate of the excited atom in first-order perturbation theory. This may be accomplished by applying Fermi's golden rule and then expressing the resulting decay rate in terms of Green functions by means of the fluctuation-dissipation theorem. This was the approach taken by Barnett *et al.* [6] in their treatment of the radiative decay rate of an excited atom in an *infinite* homogeneous dielectric medium. Instead, we shall obtain the total decay rate of the excited atom directly in terms of Green functions by considering the self-energy of the excited atom due the perturbations Eqs. (43) and (45).

The self-energy of the excited atom enters into the computation of the Green function of the atom by diagrammatic perturbation technique. Since we are interested in the self-energy of the excited state, we consider the Green function for the excited-state operators

$$g(t_1-t_2)=-\frac{i}{\hbar}\langle T[c_1^{(a)}(t_1)c_1^{(a)\dagger}(t_2)]\rangle^{(a)}, \tag{46}$$

where the superscript (a) denotes *exact* quantities in the combined system of the half space dielectric and the excited atom. Also, the angular brackets denote averaging over the exact ground state of this combined system and T denotes time ordering. Since $c_1^{(a)}(t_1)$ in Eq. (46) is an exact operator for the combined system in the Heisenberg picture, its time dependence is in general unknown. Hence Eq. (46) must be evaluated by diagrammatic perturbation technique. The general result of the diagrammatic technique [7,8] is

$$g(t_1-t_2)=-\frac{i}{\hbar}\langle T[c_1(t_1)S(\infty,-\infty)c_1^\dagger(t_2)]\rangle_{\text{conn}}, \tag{47}$$

where quantities without the superscript (a) are unperturbed quantities, that is, those associated with the Hamiltonian density $\mathcal{H}$ of Eq. (1) without the perturbation H_a. Also, $S(\infty,-\infty)$ is an infinite series of operators

$$\begin{aligned} S(\infty,-\infty)=1+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{-i}{\hbar}\right)^n\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dt_1\cdots dt_n \\ \times T[H_a(t_1)\cdots H_a(t_n)] \end{aligned} \tag{48}$$

and the symbol "conn" in Eq. (47) indicates that only *connected* diagrams are to be included in the calculation.

Since $c_0(t)$ and $c_1(t)$ in Eq. (47) are unperturbed operators, their time dependence is simple harmonic,

$$c_0(t)=c_0e^{-i\omega_0 t}, \tag{49}$$

$$c_1(t)=c_1e^{-i\omega_1 t}, \tag{50}$$

where ω_0 and ω_1 are the energies of the ground and excited states of the atom, respectively, divided by $\hbar$. Thus the zeroth-order Green function obtained by setting $S(\infty,-\infty)=1$ in Eq. (47) is

$$g^{(0)}(\omega)=\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}, \tag{51}$$

where we have taken the Fourier transform with respect to (t_1-t_2).

Next, we compute the first-order correction to the Green function

$$g^{(1)}(t_1-t_2)=\left(-\frac{i}{\hbar}\right)^2\int_{-\infty}^{\infty}dt_3\langle T[c_1(t_1)H_a(t_3)c_1^\dagger(t_2)]\rangle_{\text{conn}} \tag{52}$$

by substituting the $n=1$ term in Eq. (48) into Eq. (47). In order to have a nonvanishing expectation value, there must be the same number of annihilation and creation operators for the ground or excited state in the time-ordered product in Eq. (52). Upon examination of Eqs. (43) and (45), we find that the only term in $H_a(t_3)$ that contributes to the expectation value in Eq. (52) is the term proportional to $\mathbf{A}^2$ in Eq. (43). Substituting this term in Eq. (43) into Eq. (52), expanding the time-ordered product using Wick's theorem and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$g^{(1)}(\omega)=\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}\Sigma^{(1)}\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}, \tag{53}$$

where

$$\Sigma^{(1)}=\frac{i\hbar e^2}{2m}\mathcal{D}_{ii}(\mathbf{r}_a,\mathbf{r}_a;0) \tag{54}$$

is the self-energy of the excited atom in first-order perturbation theory and

$$\mathcal{D}_{ij}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)\overset{\text{def}}{=}-\frac{i}{\hbar}\langle T[A_i(\mathbf{r}_1,t_1)A_j(\mathbf{r}_2,t_2)]\rangle \tag{55}$$

is the transverse photon Green function for the dielectric half space. It should be noted that $\Sigma^{(1)}$ given by Eq. (54) is purely real. Substituting Eq. (55) into Eq. (54), we obtain

$$\Sigma^{(1)}=\frac{e^2}{2m}\langle\mathbf{A}^2(\mathbf{r}_a,t_3)\rangle, \tag{56}$$

which is purely real since $\mathbf{A}$ is Hermitian. Thus $\Sigma^{(1)}$ contributes only to a level-shift of the excited state, but not to its lifetime. Hence, for the purpose of computing the lifetime of the excited atom, we may neglect $\Sigma^{(1)}$.

Next, we consider the second-order contribution to the atom Green function obtained by substituting the $n=2$ term in Eq. (48) into Eq. (47),

$$g^{(2)}(t_1-t_2)=\frac{1}{2!}\left(-\frac{i}{\hbar}\right)^3\int_{-\infty}^{\infty}dt_3\int_{-\infty}^{\infty}dt_4 \times\langle T[c_1(t_1)H_a(t_3)H_a(t_4)c_1^\dagger(t_2)]\rangle_{\text{conn}}. \tag{57}$$

When Eqs. (38), (43), and (45) are substituted into Eq. (57) and the product of the two Hamiltonians expanded, we find that there are terms proportional to e^2, e^3, and e^4. It will be seen below that one of the factors of e in H_a^{nr} given by Eq. (45) is absorbed in the definition of the dielectric function [cf. Eq. (83)], which is of order unity. Hence H_a^{nr} should be counted as a term of order e rather than e^2. Now, since we are only interested in the decay rate of the excited atom to the same approximation as in Fermi's golden rule, we retain only the terms proportional to e^2 in Eq. (57). This amounts to omitting the $\mathbf{A}^2$ term in Eq. (43). Thus we are left with four terms contributing to the self-energy of the excited atom to order e^2: one due to H_a^r acting twice, one to H_a^{nr} acting twice, and the two cross terms. We consider each of these cases separately.

Consider first the effect of H_a^r acting twice. Substituting the first term in Eq. (43) into Eq. (57), expanding the time-ordered product using Wick's theorem and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$g_{AA}^{(2)}(\omega)=\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}\Sigma_{AA}^{(2)}(\omega)\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}, \tag{58}$$

where

$$\Sigma_{AA}^{(2)}(\omega)=\left(\frac{e}{m}\right)^2 p_{10,i}p_{01,j}\int_0^{\infty}d\tau\,\mathcal{D}_{ij}(\mathbf{r}_a,\mathbf{r}_a;\tau)e^{i(\omega-\omega_0)\tau} \tag{59}$$

is the contribution to the self-energy of the excited atom to order e^2 due to H_a^r acting twice.

Next, we consider the contribution to the self-energy of the excited atom due to H_a^{nr} acting twice. Substituting Eq. (45) into Eq. (57), expanding the time-ordered product using Wick's theorem and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$g_{UU}^{(2)}(\omega)=\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}\Sigma_{UU}^{(2)}(\omega)\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}, \tag{60}$$

where

$$\Sigma_{UU}^{(2)}(\omega)=\left(\frac{1}{m\omega_a}\right)^2 p_{10,j}p_{01,n}\int_{z'<0}d^3r'\int_{z''<0}d^3r'' \times\int_0^{\infty}d\tau\, e^{i(\omega-\omega_0)\tau}F_{ji}(\mathbf{r}_a-\mathbf{r}')\mathcal{G}_{il}(\mathbf{r}',\mathbf{r}'';\tau) \times F_{ln}(\mathbf{r}''-\mathbf{r}_a) \tag{61}$$

and

$$\mathcal{G}_{ij}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)\overset{\text{def}}{=}-\frac{i}{\hbar}\langle T[X_i(\mathbf{r}_1,t_1)X_j(\mathbf{r}_2,t_2)]\rangle \tag{62}$$

is the Green function for the matter field $\mathbf{X}$. In deriving Eq. (61), we have used the fact that $\delta(\mathbf{r}_a-\mathbf{r}')$ and $\delta(\mathbf{r}''-\mathbf{r}_a)$ occurring in $F_{ji}(\mathbf{r}_a-\mathbf{r}')$ and $F_{ln}(\mathbf{r}''-\mathbf{r}_a)$, respectively, in this equation are zero for $z_a>0$ and $z',z''<0$.

Finally, we consider the contribution to the self-energy of the excited atom due to H_a^r and H_a^{nr} each acting once. Whether we take $H_a(t_3)$ in Eq. (57) to be H_a^r and $H_a(t_4)$ to be H_a^{nr} or vice versa, we get the same result. Hence we need to consider only one of these two cases and multiply the result by 2. The final result is

$$g_{AU}^{(2)}(\omega)=\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}\Sigma_{AU}^{(2)}(\omega)\frac{1}{\hbar\omega-\hbar\omega_1+i\epsilon}, \tag{63}$$

where

$$\Sigma_{AU}^{(2)}(\omega)=-\frac{e}{m^2\hbar\omega_a}\int_0^\infty d\tau\, e^{i(\omega-\omega_0)\tau}\int_{z'<0}d^3r'\int_{z''<0}d^3r''$$
$$\times F_{jl}(\mathbf{r}_a-\mathbf{r}')\{p_{10,i}p_{01,j}\langle T[A_i(\mathbf{r}_a,\tau)X_l(\mathbf{r}',0)]\rangle$$
$$-p_{01,i}p_{10,j}\langle T[A_i(\mathbf{r}_a,0)X_l(\mathbf{r}',\tau)]\rangle\}. \tag{64}$$

The spontaneous decay rate of the excited atom is proportional to the imaginary part of its self-energy in the excited state

$$W_{\text{spont}}=-2\ \text{Im}\left.\frac{\Sigma(\omega)}{\hbar}\right|_{\omega=\omega_1}, \tag{65}$$

where the self-energy $\Sigma(\omega)$ of the atom in the excited state is given to order e^2 by

$$\Sigma(\omega)=\Sigma^{(1)}(\omega)+\Sigma_{AA}^{(2)}(\omega)+\Sigma_{UU}^{(2)}(\omega)+\Sigma_{AU}^{(2)}(\omega). \tag{66}$$

We now assume that the transition dipole moment of the atom is parallel to one of the coordinate axes, say, the j axis. Its decay rate in free space is given by

$$W_{\text{spont}}^{(0)}=\frac{\omega_a}{3\pi\hbar\epsilon_0c^3}\left(\frac{e}{m}\right)^2p_{10,[j]}p_{01,[j]}, \tag{67}$$

where the square brackets around the index j mean that this index is *not* summed. Then, normalizing the decay rate Eq. (65) to this free-space value and using Eqs. (59), (61), and (64), we obtain

$$\frac{W_{\text{spont},j}}{W_{\text{spont}}^{(0)}}=-\frac{6\pi c^3\epsilon_0}{\omega_a}\,\text{Im}\int_0^\infty d\tau\,e^{i\omega\tau}\left[\mathcal{D}_{[j][j]}(\mathbf{r}_a,\mathbf{r}_a;\tau)+\frac{1}{e^2\omega_a^2}\int_{z<0}\int_{z'<0}d^3r\,d^3r'F_{[j]m}(\mathbf{r}_a-\mathbf{r})\,\mathcal{G}_{mn}(\mathbf{r},\mathbf{r}';\tau)F_{n[j]}(\mathbf{r}'-\mathbf{r}_a)\right.$$
$$\left.\left.+\frac{1}{e\omega_a}\int_{z'<0}d^3r'F_{[j]m}(\mathbf{r}_a-\mathbf{r}')\mathcal{C}_{m[j]}(\mathbf{r}',\mathbf{r}_a;\tau)\right]\right|_{\omega=\omega_a}, \tag{68}$$

where

$$\mathcal{C}_{ij}(\mathbf{r}_1,\mathbf{r}_2;\tau)\overset{\text{def}}{=}\frac{1}{\hbar}\langle T[X_i(\mathbf{r}_1,\tau)A_j(\mathbf{r}_2,0)-X_i(\mathbf{r}_1,0)A_j(\mathbf{r}_2,\tau)]\rangle. \tag{69}$$

The integration over τ in Eq. (68) can be performed explicitly by making use of the analytic properties of the Green functions [7]. The result is

$$\frac{W_{\text{spont},j}}{W_{\text{spont}}^{(0)}}=-\frac{6\pi c^3\epsilon_0}{\omega_a}\,\text{Im}\left[\mathcal{D}_{[j][j]}^{\omega}(\mathbf{r}_a,\mathbf{r}_a)+\frac{1}{e^2\omega_a^2}\int_{z<0}\int_{z'<0}d^3r\,d^3r'F_{[j]m}(\mathbf{r}_a-\mathbf{r})\,\mathcal{G}_{mn}^{\omega}(\mathbf{r},\mathbf{r}')F_{n[j]}(\mathbf{r}'-\mathbf{r}_a)\right.$$
$$\left.\left.+\frac{1}{e\omega_a}\int_{z'<0}d^3r'F_{[j]m}(\mathbf{r}_a-\mathbf{r}')\mathcal{C}_{m[j]}^{\omega}(\mathbf{r}',\mathbf{r}_a)\right]\right|_{\omega=\omega_a}, \tag{70}$$

where the superscript ω denotes Fourier transform with respect to τ.

Equation (70) shows that the spontaneous decay rate of the excited atom near the dielectric surface is, to the same order of approximation as in Fermi's golden rule, given in terms of the Fourier transforms with respect to (t_1-t_2) of the photon Green function Eq. (55), the matter Green function Eq. (62), and the function $\mathcal{C}_{ij}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)$ defined by Eq. (69). The latter three functions are defined with respect to the Hamiltonian density $\mathcal{H}$ for the dielectric half space defined by Eqs. (1)–(7), without the perturbation H_a due to the excited atom. In contrast to the atom Green function Eq. (46), the functions $\mathcal{D}_{ij}^{\omega}$, $\mathcal{G}_{ij}^{\omega}$, and $\mathcal{C}_{ij}^{\omega}$ *cannot* be approximated by the first few terms of their perturbation expansions, since the cumulative effect of the electrons in the dielectric can be large. Instead, these functions are obtained as exact solutions of the Dyson equations that they satisfy. This is discussed in the next section.

V. GREEN FUNCTIONS FOR THE HALF SPACE

The Hamiltonian density $\mathcal{H}$ for the dielectric half space can be rewritten according to Eqs. (1) and (14) as

$$\mathcal{H}=\mathcal{H}_{\text{em}}^{A}+\mathcal{H}_{\text{mat}}^{(0)}+\mathcal{H}_{\text{int}}^{U}+\mathcal{H}_{\text{int}}^{A}. \tag{71}$$

To obtain the exact Green functions for this Hamiltonian density, we first separate the latter into an unperturbed part

$$\mathcal{H}_0=\mathcal{H}_{\text{em}}^{A}+\mathcal{H}_{\text{mat}}^{(0)}+\mathcal{H}_{\text{int}}^{U} \tag{72}$$

and a perturbation $\mathcal{H}_{\text{int}}^{A}$. First, we consider the photon Green function in the subsystem described by the Hamiltonian density of Eq. (72),

$$\mathcal{D}_{ij}^{(0)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[A_i^{(0)}(\mathbf{r}_1,t_1)A_j^{(0)}(\mathbf{r}_2,t_2)]\rangle^{(0)}, \tag{73}$$

where the superscript (0) denotes quantities associated with the Hamiltonian density $\mathcal{H}_0$. From Eqs. (2), (23), (7), and (8) we see that, in the subsystem described by $\mathcal{H}_0$, the transverse radiation field $\mathbf{A}$ is completely decoupled from the matter field $\mathbf{X}$ and the instantaneous Coulomb potential U. Hence, in this subsystem, $\mathbf{A}^{(0)}$ is just the free-space radiation field, which can be expanded in a complete set of transverse plane waves,

$$\mathbf{A}^{(0)}(\mathbf{r},t)=\int\frac{d^3k}{(2\pi)^{3/2}}\sqrt{\frac{\hbar}{2\epsilon_0\omega_k}}\times\sum_{\lambda=1,2}[a_\lambda(\mathbf{k})\mathbf{e}_\lambda(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)}+\text{H.c.}], \tag{74}$$

where $\omega_k=|\mathbf{k}|c$ and $\mathbf{e}_\lambda(\mathbf{k})$, $\lambda=1,2$, are unit vectors such that $[\mathbf{e}_1(\mathbf{k}),\mathbf{e}_2(\mathbf{k}),\mathbf{k}/|\mathbf{k}|]$ form an orthonormal right-handed triad. The commutation relation Eq. (11) implies that

$$[a_\lambda(\mathbf{k}),a_{\lambda'}^\dagger(\mathbf{k}')]=\delta_{\lambda,\lambda'}\delta(\mathbf{k}-\mathbf{k}'), \tag{75}$$

while the commutator between $a_\lambda(\mathbf{k})$ and $a_{\lambda'}(\mathbf{k}')$, or between $a_\lambda^\dagger(\mathbf{k})$ and $a_{\lambda'}^\dagger(\mathbf{k}')$, vanishes. Equation (75) shows that $a_\lambda^\dagger(\mathbf{k})$ and $a_\lambda(\mathbf{k})$ are the creation and annihilation operators of a transverse photon of wave vector $\mathbf{k}$ and polarization λ.

The free-space photon Green function can be calculated by substituting Eq. (74) into Eq. (73). After taking the Fourier transform with respect to (t_1-t_2), we obtain

$$\mathcal{D}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\epsilon_0}\int\frac{d^3k}{(2\pi)^3}\frac{\left(\delta_{ij}-\dfrac{k_ik_j}{k^2}\right)}{\omega^2-k^2c^2+i\epsilon}e^{i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)}. \tag{76}$$

Next, we consider the Green function for the matter field $\mathbf{X}$ for the unperturbed Hamiltonian density of Eq. (72). To do so, it is necessary to partition $\mathcal{H}_0$ further into a part without the Coulomb interaction

$$\mathcal{H}_{00}=\mathcal{H}_{\text{em}}^{A}+\mathcal{H}_{\text{mat}}^{(0)} \tag{77}$$

and the Coulomb interaction term $\mathcal{H}_{\text{int}}^{U}$. We first obtain the Green function for $\mathbf{X}$ in the subsystem described by $\mathcal{H}_{00}$,

$$\mathcal{G}_{ij}^{(00)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[X_i^{(00)}(\mathbf{r}_1,t_1)X_j^{(00)}(\mathbf{r}_2,t_2)]\rangle^{(00)}, \tag{78}$$

where the superscript (00) denotes quantities associated with the Hamiltonian density of Eq. (77). For this purpose, the first term in Eq. (77) has no effect, since there is no coupling between the radiation field and matter in this subsystem. Next, since the term $\mathcal{H}_{\text{mat}}^{(0)}$ given by Eq. (25) is diagonal in the dressed matter operators $\mathbf{B}_\omega$, the time dependence of the latter operators in this subsystem is simple harmonic. Substituting Eq. (34) into Eq. (78), using the commutation relation Eq. (26) and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{i}{2\rho\bar{\omega}_0}\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\int_0^\infty d\omega'|h(\omega')|^2\times\left[\frac{i}{\omega-\omega'+i\epsilon}-\frac{i}{\omega+\omega'-i\epsilon}\right]. \tag{79}$$

Using Eqs. (36), (27), and (28), we obtain

$$|h(\omega')|^2=\frac{\bar{\omega}_0}{\omega'}\xi(\omega'), \tag{80}$$

where

$$\xi(\omega')\overset{\text{def}}{=}\frac{\bar{\omega}_0\omega'V^2(\omega')}{|\omega'^2-\bar{\omega}_0^2z(\omega')|^2}. \tag{81}$$

Equation (81) shows that $\xi(\omega)$ is an even function of ω, since $V^2(\omega)$ is odd and $z(-\omega)=z^*(\omega)$. Furthermore, $\xi(\omega)$ is analytic on the real ω axis [5].

The integral over ω' in Eq. (79) can be rewritten as

$$\begin{aligned}J(\omega)&\overset{\text{def}}{=}\int_0^\infty d\omega'|h(\omega')|^2\left[\frac{1}{\omega-\omega'+i\epsilon}-\frac{1}{\omega+\omega'-i\epsilon}\right]\\&=\int_0^\infty d\omega'\frac{\bar{\omega}_0}{\omega'}\xi(\omega')\frac{2\omega'}{\omega^2-(\omega'-i\epsilon)^2}\\&=\frac{\bar{\omega}_0}{\omega}\int_0^\infty d\omega'\xi(\omega')\left[\frac{1}{\omega-\omega'+i\epsilon}+\frac{1}{\omega+\omega'-i\epsilon}\right].\end{aligned} \tag{82}$$

Following Huttner and Barnett [5], we define the dielectric function of the dielectric as

$$\epsilon(\omega)=1-\frac{e^2}{2\rho\epsilon_0\omega}\int_{-\infty}^\infty d\omega'\frac{\xi(\omega')}{\omega-\omega'+i\epsilon}. \tag{83}$$

This function satisfies the Kramers-Krönig relations, since Eq. (83) shows that $\epsilon(\omega)$ is analytic in the upper half of the complex plane. Comparing the RHS of Eqs. (82) and (83) and using the fact that $\xi(\omega)$ is an even function of ω, we find that

$$J(\omega)=-\frac{2\rho\epsilon_0\bar{\omega}_0}{e^2}[\epsilon(|\omega|)-1]. \tag{84}$$

Substituting Eq. (84) into Eq. (79), we obtain

$$\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{\epsilon_0[\epsilon(|\omega|)-1]}{e^2}\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{85}$$

When the Coulomb interaction Eq. (9) is added, the matter Green function $\mathcal{G}_{ij}^{(0)}$ can be expressed in terms of quantities without the Coulomb interaction using the general result of the diagrammatic technique,

$$\mathcal{G}_{ij}^{(0)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[X_i^{(00)}(\mathbf{r}_1,t_1)S^U(\infty,-\infty)$$

$$\times X_j^{(00)}(\mathbf{r}_2,t_2)]\rangle_{\text{conn}}^{(00)}, \qquad (86)$$

where

$$S^U(\infty,-\infty) = 1+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{-i}{\hbar}\right)^n\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dt_1\cdots dt_n$$

$$\times T[H_{\text{int}}^{(00)U}(t_1)\cdots H_{\text{int}}^{(00)U}(t_n)]. \qquad (87)$$

Substituting Eq. (87) into Eq. (86), expanding the time-ordered products using Wick's theorem and taking the Fourier transform with respect to (t_1-t_2), we obtain an infinite series that can be summed in closed form

$$\mathcal{G}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)$$

$$=\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)+\int_{z'<0}d^3r'\int_{z''<0}d^3r''$$

$$\times\mathcal{G}_{il}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}')F_{lm}(\mathbf{r}'-\mathbf{r}'')\mathcal{G}_{mj}^{(00)\omega}(\mathbf{r}'',\mathbf{r}_2)+\cdots,$$

$$=\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)+\int_{z'<0}d^3r'\int_{z''<0}d^3r''$$

$$\times\mathcal{G}_{il}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}')F_{lm}(\mathbf{r}'-\mathbf{r}'')\mathcal{G}_{mj}^{(0)\omega}(\mathbf{r}'',\mathbf{r}_2). \qquad (88)$$

Then, substituting Eq. (85) into Eq. (88), we obtain

$$\mathcal{G}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{\epsilon_0[\epsilon(|\omega|)-1]}{e^2}\left[\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\right.$$

$$\left.+\int_{z_3<0}d^3r_3F_{im}(\mathbf{r}_1-\mathbf{r}_3)\ \mathcal{G}_{mj}^{(0)\omega}(\mathbf{r}_3,\mathbf{r}_2)\right]. \qquad (89)$$

Equation (89) is the Dyson equation for the matter Green function $\mathcal{G}_{ij}^{(0)\omega}$ for the system described by the Hamiltonian density $\mathcal{H}_0$ of Eq. (72). This equation can be solved exactly for $\mathcal{G}_{ij}^{(0)\omega}$ using an extension of the Wiener-Hopf technique. The solution is [9]

$$\mathcal{G}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{\epsilon_0[\epsilon_L(|\omega|)-1]}{e^2}\ \delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)$$

$$+\frac{\epsilon_0[\epsilon_L(|\omega|)-1]^2}{4\pi\epsilon_L(|\omega|)e^2}\left[\frac{\partial^2}{\partial x_{1i}\partial x_{2j}}\left(\frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}\right)\right.$$

$$\left.+\frac{\epsilon_L(|\omega|)-1}{\epsilon_L(|\omega|)+1}\ \frac{\partial^2}{\partial x_{1i}\partial x_{2j}}\left(\frac{1}{|\mathbf{r}_1-\tilde{\mathbf{r}}_2|}\right)\right], \qquad (90)$$

where $\tilde{\mathbf{r}}_2$ is the image of $\mathbf{r}_2$ in the plane $z=0$ and

$$\epsilon_L(|\omega|)=1+\frac{\epsilon(|\omega|)-1}{1-\frac{1}{3}[\epsilon(|\omega|)-1]} \qquad (91)$$

is the dielectric function including local-field effects.

So far, we have obtained the Green functions for photons and matter, Eqs. (76) and (90), for the system described by the Hamiltonian density $\mathcal{H}_0$ of Eq. (72). Next, we calculate the Green functions for the complete Hamiltonian density $\mathcal{H}$ given by Eq. (71). First, we consider the photon Green function Eq. (55), which, by the diagrammatic technique, can be written as

$$\mathcal{D}_{ij}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[A_i^{(0)}(\mathbf{r}_1,t_1)S^A(\infty,-\infty)$$

$$\times A_j^{(0)}(\mathbf{r}_2,t_2)]\rangle_{\text{conn}}^{(0)}, \qquad (92)$$

where

$$S^A(\infty,-\infty)=1+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{-i}{\hbar}\right)^n\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dt_1\cdots dt_n$$

$$\times T[H_{\text{int}}^{(0)A}(t_1)\cdots H_{\text{int}}^{(0)A}(t_n)]. \qquad (93)$$

Here $H_{\text{int}}^{(0)A}(t)$ is the Hamiltonian corresponding to the Hamiltonian density $\mathcal{H}_{\text{int}}^{A}(\mathbf{r},t)$ of Eq. (6),

$$H_{\text{int}}^{(0)A}(t)=\int_{z<0}d^3r\left[-\frac{e}{\rho}\mathbf{A}^{(0)}(\mathbf{r},t)\cdot\mathbf{P}_X^{(0)}(\mathbf{r},t)\right.$$

$$\left.+\frac{e^2}{2\rho}\mathbf{A}^{(0)}(\mathbf{r},t)\cdot\mathbf{A}^{(0)}(\mathbf{r},t)\right]. \qquad (94)$$

As we have mentioned, $\mathcal{D}_{ij}$ cannot be approximated by the first few terms of its perturbation expansion. Instead, the entire infinite series in Eq. (93) must be substituted into Eq. (92). However, we shall see shortly that the resulting infinite series can be summed exactly to give a closed-form Dyson equation for the photon Green function.

The zeroth-order term in Eq. (93) gives just the free-space photon Green function $\mathcal{D}_{ij}^{(0)}$ of Eq. (73), whose Fourier transform is given by Eq. (76). The $n=1$ term in Eq. (93) gives a first-order correction to the free-space photon Green function. Since there must be an even number of **A** operators in the time-ordered product to give a nonvanishing expectation value, we see that only the second term in Eq. (94) contributes in this order. Thus we obtain

$$\mathcal{D}_{ij}^{(1)\omega}(\mathbf{r}_1,\mathbf{r}_2)=\frac{e^2}{\rho}\int_{z_3<0}d^3r_3\mathcal{D}_{im}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_3)\mathcal{D}_{mj}^{(0)\omega}(\mathbf{r}_3,\mathbf{r}_2), \qquad (95)$$

where we have taken the Fourier transform with respect to (t_1-t_2).

Next, we consider the contribution from the $n=2$ term in Eq. (93),

$$\mathcal{D}_{ij}^{(2)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=\frac{1}{2!}\left(\frac{-i}{\hbar}\right)^3\int_{-\infty}^{\infty}dt_3\int_{-\infty}^{\infty}dt_4$$

$$\times\langle T[A_i^{(0)}(\mathbf{r}_1,t_1)$$

$$\times H_{\text{int}}^{(0)A}(t_3)H_{\text{int}}^{(0)A}(t_4)A_j^{(0)}(\mathbf{r}_2,t_2)]\rangle_{\text{conn}}^{(0)}. \qquad (96)$$

When Eq. (94) is substituted into Eq. (96) and the product of the two Hamiltonians expanded, there is a total of four terms: one due to the first term in Eq. (94) acting twice, one to the second term in Eq. (94) acting twice, and the two cross terms. The latter cross terms contribute nothing, since they each contain an odd number of **A** operators.

Consider first the contribution due to the first term in Eq. (94) acting twice. Substituting the first term in Eq. (94) into Eq. (96), expanding the time-ordered products using Wick's theorem and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$\mathcal{D}_{ij}^{(2)\omega}(\mathbf{r}_1,\mathbf{r}_2)=\left(\frac{-e}{\rho}\right)^2\int_{z_3<0}d^3r_3\int_{z_4<0}d^3r_4 \times\mathcal{D}_{il}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_3)\,\mathcal{Q}_{lm}^{(0)\omega}(\mathbf{r}_3,\mathbf{r}_4)\mathcal{D}_{mj}^{(0)\omega}(\mathbf{r}_4,\mathbf{r}_2), \tag{97}$$

where $\mathcal{Q}_{lm}^{(0)\omega}$ is the Fourier transform of the quantity $\mathcal{Q}_{ij}^{(0)}$ defined by

$$\mathcal{Q}_{ij}^{(0)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)\overset{\text{def}}{=}-\frac{i}{\hbar}\langle T[P_{X,i}^{(0)}(\mathbf{r}_1,t_1)P_{X,j}^{(0)}(\mathbf{r}_2,t_2)]\rangle^{(0)}. \tag{98}$$

The quantity $\mathcal{Q}_{ij}^{(0)}$ is the Green function for the operator $\mathbf{P}_X$ in the subsystem described by $\mathcal{H}_0$ of Eq. (72). To compute this quantity, we follow the same procedure as used in computing $\mathcal{G}_{ij}^{(0)}$ by partitioning $\mathcal{H}_0$ into a sum of $\mathcal{H}_{00}$ and $\mathcal{H}_{\text{int}}^U$ where $\mathcal{H}_{00}$ is given by Eq. (77). The Green function for $\mathbf{P}_X$ in the system described by $\mathcal{H}_{00}$ is given by an expression similar to Eq. (78), but with X replaced by P_X. Comparing Eqs. (34) and (35), we see that P_X is obtained from X by replacing $h(\omega)$ by $\rho\tilde{\omega}_0 g(\omega)$. Hence, following the steps leading from Eq. (78) to Eq. (79), we obtain for $\mathcal{Q}_{ij}^{(00)\omega}$ a result similar to Eq. (79) but with $h(\omega)$ replaced by $\rho\tilde{\omega}_0 g(\omega)$,

$$\mathcal{Q}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{i\rho\tilde{\omega}_0}{2}\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\int_0^\infty d\omega'|g(\omega')|^2 \times\left[\frac{i}{\omega-\omega'+i\epsilon}-\frac{i}{\omega+\omega'-i\epsilon}\right]. \tag{99}$$

Using Eqs. (37), (27), and (28), we obtain

$$|g(\omega')|^2=\frac{\omega'}{\tilde{\omega}_0}\xi(\omega'), \tag{100}$$

where $\xi(\omega')$ is given by Eq. (81). Following the steps leading from Eq. (79) to Eq. (85), we obtain

$$\mathcal{Q}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\rho\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\left\{1+\frac{\rho\epsilon_0\omega^2}{e^2}[\epsilon(|\omega|)-1]\right\}, \tag{101}$$

where we have used the fact that [5]

$$\int_0^\infty d\omega'\xi(\omega')=1. \tag{102}$$

When the Coulomb interaction Eq. (9) is added, the Green function for $\mathbf{P}_X$ is given by an infinite series similar to Eq. (86),

$$\mathcal{Q}_{ij}^{(0)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[P_{X,i}^{(00)}(\mathbf{r}_1,t_1)S^U(\infty,-\infty) \times P_{X,j}^{(00)}(\mathbf{r}_2,t_2)]\rangle_{\text{conn}}^{(00)}, \tag{103}$$

where $S^U(\infty,-\infty)$ is given by Eq. (87). When the latter equation is substituted into Eq. (103) and the resulting time-ordered products expanded using Wick's theorem, we obtain an infinite series

$$\begin{aligned}\mathcal{Q}_{ij}^{(0)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)={}&\mathcal{Q}_{ij}^{(00)}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)+\left(\frac{-i}{\hbar}\right)^2\int_{-\infty}^{\infty}dt_3\\&\times\int_{z'<0}d^3r'\int_{z''<0}d^3r''\langle T[P_{X,i}^{(00)}(\mathbf{r}_1,t_1)\\&\times X_l^{(00)}(\mathbf{r}',t_3)]\rangle^{(00)}F_{lm}(\mathbf{r}'-\mathbf{r}'')\\&\times\langle T[X_m^{(00)}(\mathbf{r}'',t_3)P_{X,j}^{(00)}(\mathbf{r}_2,t_2)]\rangle^{(00)}\\&+\cdots.\end{aligned} \tag{104}$$

We now have to compute the quantities $\langle T[P_{X,i}^{(00)}X_j^{(00)}]\rangle^{(00)}$ and $\langle T[X_i^{(00)}P_{X,j}^{(00)}]\rangle^{(00)}$. Using Eqs. (34) and (35) and following the steps leading from Eq. (78) to Eq. (79), we obtain

$$\begin{aligned}&-\frac{i}{\hbar}\langle T[P_{X,i}^{(00)}(\mathbf{r}_1,t_1)X_j^{(00)}(\mathbf{r}_2,t_2)]\rangle^{(00)}|^{\omega}\\&=\frac{1}{2}\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\int_0^\infty d\omega'\left[\frac{g^*(\omega')h(\omega')}{\omega-\omega'+i\epsilon}-\frac{g(\omega')h^*(\omega')}{\omega+\omega'-i\epsilon}\right],\end{aligned} \tag{105}$$

where the vertical bar with the superscript ω indicates Fourier transform with respect to (t_1-t_2). Using Eqs. (36) and (37), we calculate

$$g^*(\omega')h(\omega')=-i\{|\alpha_0(\omega')|^2-|\beta_0(\omega')|^2 +2i\ \text{Im}[\alpha_0(\omega')\beta_0^*(\omega')]\}. \tag{106}$$

From Eqs. (27) and (28), we see that $[\alpha_0(\omega')\beta_0^*(\omega')]$ is purely real. Hence using Eqs. (27), (28), and (81), we obtain

$$g^*(\omega')h(\omega')=-i\xi(\omega'). \tag{107}$$

Substituting Eq. (107) into Eq. (105), we obtain

$$\begin{aligned}&-\frac{i}{\hbar}\langle T[P_{X,i}^{(00)}(\mathbf{r}_1,t_1)X_j^{(00)}(\mathbf{r}_2,t_2)]\rangle^{(00)}|^{\omega}\\&=-i\rho\omega\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2),\end{aligned} \tag{108}$$

where we have used Eqs. (82), (84), and (85).

For $\langle T[X_i^{(00)}P_{X,j}^{(00)}]\rangle^{(00)}$, we obtain an expression similar to Eq. (105) but with $g(\omega')$ and $h(\omega')$ interchanged. According to Eq. (107), this amounts to an extra minus sign. Hence

$$-\frac{i}{\hbar}\langle T[X_i^{(00)}(\mathbf{r}_1,t_1)P_{X,j}^{(00)}(\mathbf{r}_2,t_2)]\rangle^{(00)}|^{\omega}=i\rho\omega\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2). \tag{109}$$

We now take the Fourier transform of Eq. (104) with respect to (t_1-t_2) and then use Eqs. (108) and (109),

$$\mathcal{Q}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{Q}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)+\rho^2\omega^2\int_{z'<0}d^3r'\int_{z''<0}d^3r'' \times\mathcal{G}_{il}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}')F_{lm}(\mathbf{r}'-\mathbf{r}'')\mathcal{G}_{mj}^{(00)\omega}(\mathbf{r}'',\mathbf{r}_2) +\cdots. \tag{110}$$

Comparing Eqs. (88) and (110), we see that the infinite series in these two equations, from the second term on the RHS of each of the two equations onward, are identical except for a factor $\rho^2\omega^2$. Hence we conclude that

$$\mathcal{Q}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)-\mathcal{Q}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2) =\rho^2\omega^2[\mathcal{G}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)-\mathcal{G}_{ij}^{(00)\omega}(\mathbf{r}_1,\mathbf{r}_2)]. \tag{111}$$

Substituting Eqs. (85) and (101) into Eq. (111), we obtain

$$\mathcal{Q}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)=-\rho\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)+\rho^2\omega^2\mathcal{G}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2), \tag{112}$$

where $\mathcal{G}_{ij}^{(0)\omega}$ is given by Eq. (90). Equation (112) can now be used in Eq. (97) to calculate $\mathcal{D}_{ij}^{(2)\omega}$.

We still have to consider the contribution from the second term in Eq. (94) acting twice, as well as the contributions of the higher-order terms in Eq. (93). These contributions can be taken into account by using diagrammatic analysis.

The perturbation series Eq. (92) can be represented by an infinite series of diagrams as shown in Fig. 1. Here the complete photon Green function is represented by a heavy dashed line and the free-space photon Green function by a light dashed line. The contribution Eq. (95) due to the second term in Eq. (94) acting once is represented by the second diagram on the RHS of Fig. 1(a), the contribution Eq. (97) due the the first term in Eq. (94) acting twice by the third diagram, the contribution due to the second term in Eq. (94) acting twice by the fourth diagram, etc. It can be seen that any one of the higher-order diagrams in Fig. 1(a) is constructed from just two types of building blocks, or self-energy parts, represented by a cross and a vertical dumbell, which correspond to the second term in Eq. (94) acting once and the first term in Eq. (94) acting twice, respectively. We can separate the higher-order diagrams into two groups: those containing a self-energy part of the first type at the bottom and those containing a self-energy part of the second type at the bottom, as shown in Fig. 1(b). The infinite series connected to either one of these factors consists of all possible diagrams constructed from an arbitrary number of self-energy parts of either type strung together in any order by free-space photon Green functions. Such a series is just the complete photon Green function. Hence we obtain the equivalent representation shown in Fig. 1(c). This means that the higher-order diagrams are all included in just two diagrams obtained from the second and third diagrams on the RHS of Fig. 1(a) by replacing the light dashed lines at the top of the latter diagrams by heavy dashed lines. Mathematically, the complete photon Green function is given by the sum of the free-space photon Green function and the two terms derived from Eqs. (95) and (97) by replacing the factor $\mathcal{D}_{mj}^{(0)\omega}$ in these equations by $\mathcal{D}_{mj}^{\omega}$,

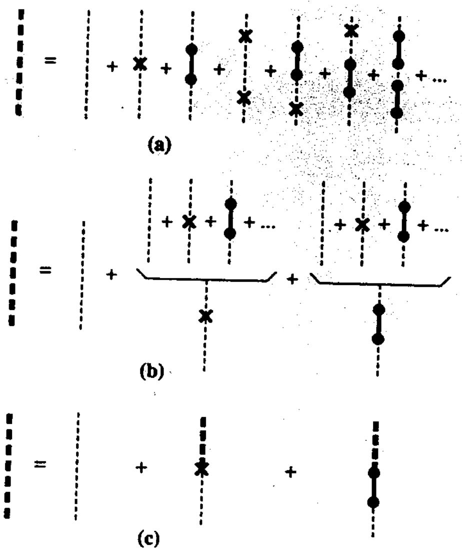

FIG. 1. Perturbation series for the photon Green function. A heavy dashed line represents the complete photon Green function. A light dashed line represents the free-space photon Green function. A cross represents an interaction vertex due to the **A**·**A** term in Eq. (94) acting once. A vertical dumbell consists of two interaction vertices due to the **A**·$\mathbf{P}_X$ term in Eq. (94) acting twice.

$$\mathcal{D}_{ij}^{\omega}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{D}_{ij}^{(0)\omega}(\mathbf{r}_1-\mathbf{r}_2)+\frac{e^2}{\rho}\int_{z_3<0}d^3r_3 \times\mathcal{D}_{im}^{(0)\omega}(\mathbf{r}_1-\mathbf{r}_3)\mathcal{D}_{mj}^{\omega}(\mathbf{r}_3,\mathbf{r}_2) +\left(\frac{-e}{\rho}\right)^2\int_{z_3<0}d^3r_3\int_{z_4<0}d^3r_4 \times\mathcal{D}_{il}^{(0)\omega}(\mathbf{r}_1-\mathbf{r}_3)\mathcal{Q}_{lm}^{(0)\omega}(\mathbf{r}_3,\mathbf{r}_4)\mathcal{D}_{mj}^{\omega}(\mathbf{r}_4,\mathbf{r}_2). \tag{113}$$

Substituting Eq. (112) into Eq. (113), we obtain

$$\mathcal{D}_{ij}^{\omega}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{D}_{ij}^{(0)\omega}(\mathbf{r}_1-\mathbf{r}_2)+e^2\omega^2\int_{z_3<0}d^3r_3\int_{z_4<0}d^3r_4 \times\mathcal{D}_{il}^{(0)\omega}(\mathbf{r}_1-\mathbf{r}_3)\mathcal{G}_{lm}^{(0)\omega}(\mathbf{r}_3,\mathbf{r}_4)\mathcal{D}_{mj}^{\omega}(\mathbf{r}_4,\mathbf{r}_2). \tag{114}$$

We may now substitute Eq. (90) into Eq. (114). For the second term in Eq. (90), we perform integration by parts twice and make use of the transversality of the photon Green function,

$$\frac{\partial}{\partial x_i}\mathcal{D}_{ij}^{\omega}(\mathbf{r},\mathbf{r}')=\frac{\partial}{\partial x_j'}\mathcal{D}_{ij}^{\omega}(\mathbf{r},\mathbf{r}')=0. \tag{115}$$

The result is

$$\mathcal{D}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{D}^{(0)\omega}_{ij}(\mathbf{r}_1-\mathbf{r}_2)-\omega^2\epsilon_0[\epsilon_L(|\omega|)-1]$$
$$\times\int_{z_3<0}d^3r_3\mathcal{D}^{(0)\omega}_{im}(\mathbf{r}_1-\mathbf{r}_3)\mathcal{D}^{\omega}_{mj}(\mathbf{r}_3,\mathbf{r}_2)$$
$$+\frac{\omega^2\epsilon_0[\epsilon_L(|\omega|)-1]^2}{2\pi[\epsilon_L(|\omega|)+1]}\int\int d^2s_3d^2s_4$$
$$\times\mathcal{D}^{(0)\omega}_{i3}(\mathbf{r}_1-\mathbf{s}_3)\frac{1}{|\mathbf{s}_3-\mathbf{s}_4|}\mathcal{D}^{\omega}_{3j}(\mathbf{s}_4,\mathbf{r}_2)\ , \quad (116)$$

where $\mathbf{s}_3$ and $\mathbf{s}_4$ are integration points on the plane $z=0$.

Equation (116) is the Dyson equation for the transverse photon Green function for the complete Hamiltonian density $\mathcal{H}$ of Eq. (1). This equation can be solved exactly for $\mathcal{D}^{\omega}_{ij}$ using an extension of the Wiener-Hopf technique, as discussed in the Appendix.

Next, we calculate the matter Green function $\mathcal{G}^{\omega}_{ij}$ for the complete Hamiltonian density $\mathcal{H}$. This is given according to the diagrammatic technique by

$$\mathcal{G}_{ij}(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)=-\frac{i}{\hbar}\langle T[X^{(0)}_i(\mathbf{r}_1,t_1)S^A(\infty,-\infty)$$
$$\times X^{(0)}_j(\mathbf{r}_2,t_2)]\rangle^{(0)}_{\text{conn}}, \quad (117)$$

where $S^A(\infty,-\infty)$ is given by Eq. (93). Substituting the latter equation into Eq. (117), expanding the time-ordered products using Wick's theorem, and taking the Fourier transform with respect to (t_1-t_2), we obtain

$$\mathcal{G}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{G}^{(0)\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)+\left(-\frac{i}{\hbar}\right)^2\left(\frac{-e}{\rho}\right)^2\int_{z_3<0}d^3r_3$$
$$\times\int_{z_4<0}d^3r_4\langle T[X^{(0)}_i(\mathbf{r}_1,t_1)$$
$$\times P^{(0)}_{X.l}(\mathbf{r}_3,t_3)]\rangle^{(0)}|^{\omega}\mathcal{D}^{\omega}_{lm}(\mathbf{r}_3,\mathbf{r}_4)$$
$$\times\langle T[P^{(0)}_{X.m}(\mathbf{r}_4,t_4)X^{(0)}_j(\mathbf{r}_2,t_2)]\rangle^{(0)}|\omega. \quad (118)$$

We now have to compute the quantities $\langle T[P^{(0)}_{X,i}X^{(0)}_j]\rangle^{(0)}$ and $\langle T[X^{(0)}_iP^{(0)}_{X,j}]\rangle^{(0)}$. These are obtained in terms of the quantities $\langle T[P^{(00)}_{X,i}X^{(00)}_j]\rangle^{(00)}$ and $\langle T[X^{(00)}_iP^{(00)}_{X.j}]\rangle^{(00)}$ by the diagrammatic technique with the Coulomb interaction $H^{(00)U}_{\text{int}}$ as perturbation,

$$-\frac{i}{\hbar}\langle T[P^{(0)}_{X,i}(\mathbf{r}_1,t_1)X^{(0)}_j(\mathbf{r}_2,t_2)]\rangle^{(0)}$$
$$=-\frac{i}{\hbar}\langle T[P^{(00)}_{X.i}(\mathbf{r}_1,t_1)S^U(\infty,-\infty)X^{(00)}_j(\mathbf{r}_2,t_2)]\rangle^{(00)}_{\text{conn}}, \quad (119)$$

where $S^U(\infty,-\infty)$ is given by Eq. (87). Comparing the two infinite series Eqs. (86) and (119), we see that each term of the latter series differs from the corresponding term of the former series only in replacing, in the former series, a factor $(-i/\hbar)\langle T[X^{(00)}_iX^{(00)}_l]\rangle^{(00)}=\mathcal{G}^{(00)}_{il}$ by $(-i/\hbar)\langle T[P^{(00)}_{X,i}X^{(00)}_l]\rangle^{(00)}$. By Eq. (108), this amounts to multiplying each term of the series in Eq. (88) by $-i\rho\omega$. Thus we conclude

$$-\frac{i}{\hbar}\langle T[P^{(0)}_{X,i}(\mathbf{r}_1,t_1)X^{(0)}_j(\mathbf{r}_2,t_2)]\rangle^{(0)}|^{\omega}=-i\rho\omega\mathcal{G}^{(0)\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2). \quad (120)$$

Similarly, using Eq. (109),

$$-\frac{i}{\hbar}\langle T[X^{(0)}_i(\mathbf{r}_1,t_1)P^{(0)}_{X,j}(\mathbf{r}_2,t_2)]\rangle^{(0)}|^{\omega}=i\rho\omega\mathcal{G}^{(0)\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2). \quad (121)$$

Substituting Eqs. (120) and (121) into Eq. (118), we obtain

$$\mathcal{G}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\mathcal{G}^{(0)\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$$
$$+e^2\omega^2\int_{z_3<0}d^3r_3\int_{z_4<0}d^3r_4\mathcal{G}^{(0)\omega}_{il}(\mathbf{r}_1,\mathbf{r}_3)$$
$$\times\mathcal{D}^{\omega}_{lm}(\mathbf{r}_3,\mathbf{r}_4)\mathcal{G}^{(0)\omega}_{mj}(\mathbf{r}_3,\mathbf{r}_2). \quad (122)$$

Equation (122) is an explicit expression for the complete matter Green function $\mathcal{G}^{\omega}_{ij}$ since $\mathcal{G}^{(0)\omega}_{ij}$ is known from Eq. (90) and the complete photon Green function $\mathcal{D}^{\omega}_{ij}$ is known from the Appendix.

Finally, we calculate the function C_{ij} given by Eq. (69). We consider each of the two terms in the latter equation separately. By the diagrammatic technique, we have

$$\frac{1}{\hbar}\langle T[X_i(\mathbf{r}_1,\tau)A_j(\mathbf{r}_2,0)]\rangle=\frac{1}{\hbar}\langle T[X^{(0)}_i(\mathbf{r}_1,\tau)S^A(\infty,-\infty)$$
$$\times A^{(0)}_j(\mathbf{r}_2,0)]\rangle^{(0)}, \quad (123)$$

where $S^A(\infty,-\infty)$ is given by Eq. (93). Following the steps leading from Eq. (117) to Eq. (122), we obtain

$$\frac{1}{\hbar}\langle T[X_i(\mathbf{r}_1,\tau)A_j(\mathbf{r}_2,0)]\rangle|^{\omega}$$
$$=e\omega\int_{z_3<0}d^3r_3\mathcal{G}^{(0)\omega}_{il}(\mathbf{r}_1,\mathbf{r}_3)\mathcal{D}^{\omega}_{lj}(\mathbf{r}_3,\mathbf{r}_2). \quad (124)$$

Similarly, we obtain

$$\frac{1}{\hbar}\langle T[X_i(\mathbf{r}_1,0)A_j(\mathbf{r}_2,\tau)]\rangle|^{\omega}$$
$$=\frac{1}{\hbar}\langle T[X_i(\mathbf{r}_1,\tau)A_j(\mathbf{r}_2,0)]\rangle|^{-\omega}$$
$$=-e\omega\int_{z_3<0}d^3r_3\mathcal{G}^{(0)-\omega}_{il}(\mathbf{r}_1,\mathbf{r}_3)\mathcal{D}^{-\omega}_{lj}(\mathbf{r}_3,\mathbf{r}_2). \quad (125)$$

It can be seen from Eq. (90) and the Appendix that the Green functions $\mathcal{G}^{(0)\omega}_{ij}$ and $\mathcal{D}^{\omega}_{ij}$ are even functions of ω, since ω enters into these Green functions only in the form $|\omega|$ or ω^2. Hence the RHS of Eq. (125) is equal and opposite to the RHS of Eq. (124). Taking the Fourier transform of Eq. (69) with respect to τ and using Eqs. (124) and (125), we obtain

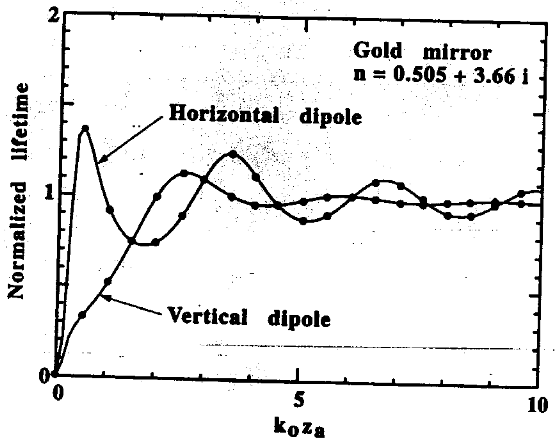

FIG. 2. Lifetime of an excited atom near a gold mirror. Solid lines are the results of classical electromagnetic theory. Dots are the results of our quantum theory. k_0 is the wave vector in air. z_a is the distance of the atom above the mirror.

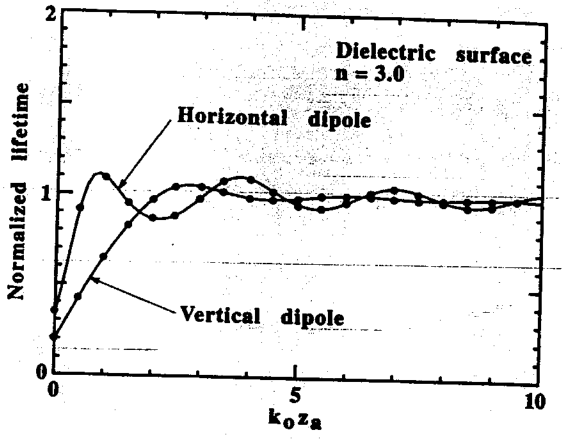

FIG. 3. Lifetime of an excited atom near a lossless dielectric surface. Solid lines are the results of the spatial-mode quantization theory. Dots are the results of our quantum theory.

$$\mathcal{C}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)=2e\omega\int_{z_3<0}d^3r_3\mathcal{G}^{(0)\omega}_{il}(\mathbf{r}_1,\mathbf{r}_3)\mathcal{D}^{\omega}_{lj}(\mathbf{r}_3,\mathbf{r}_2). \quad (126)$$

We now have all the quantities we need, namely, $\mathcal{D}^{\omega}_{ij}$, $\mathcal{G}^{\omega}_{ij}$, and $\mathcal{C}^{\omega}_{ij}$ to compute the spontaneous decay rate of the excited atom using Eq. (70).

VI. COMPARISON WITH THE CLASSICAL THEORY

Results for the total decay rate calculated from Eq. (70) using the exact functions $\mathcal{D}^{\omega}_{ij}$, $\mathcal{G}^{\omega}_{ij}$, and $\mathcal{C}^{\omega}_{ij}$ are in excellent agreement with those of classical electromagnetic theory [2]. These are shown in Fig. 2 for a gold mirror with refractive index $n=0.505+3.66i$. The total decay rate contains a nonradiative component due to energy transfer from the excited atom to the absorbing mirror via the near-field Coulomb interaction. The contribution due to the Coulomb potential U alone may be obtained from Eq. (70) by setting $\mathcal{D}^{\omega}_{ij}$ equal to zero,

$$\frac{W^{U}_{\text{spont},j}}{W^{(0)}_{\text{spont}}}=-\frac{6\pi c^3\epsilon_0}{\omega_a}\,\mathrm{Im}\frac{1}{e^2\omega_a^2}\int_{z<0}\int_{z'<0}d^3r\;d^3r'\times F_{[j]m}(\mathbf{r}_a-\mathbf{r})\mathcal{G}^{(0)\omega_a}_{mn}(\mathbf{r},\mathbf{r}')F_{n[j]}(\mathbf{r}'-\mathbf{r}_a). \quad (127)$$

Using Eqs. (10) and (90), the integrals over $\mathbf{r}$ and $\mathbf{r}'$ can be evaluated to give

$$\frac{W^{U}_{\text{spont},j}}{W^{(0)}_{\text{spont}}}=\frac{3\,\theta_j}{8\omega_a^3z_a^3}\,\mathrm{Im}\left[\frac{-1}{\epsilon_L(\omega_a)+1}\right], \quad (128)$$

where $\theta_j=2$ for $j=z$ and $\theta_j=1$ for $j=x$ or y. Equation (128) agrees with the classical result [2] for the rate of nonradiative energy transfer in the limit $z_a\rightarrow 0$.

Nonradiative decay is absent for a perfect dielectric. In this case, our results for the decay rate W_j are in excellent agreement with the results of classical electromagnetic theory and with those based on quantization of macroscopic spatial modes [4]. These are shown in Fig. 3 for a dielectric half space with refractive index $n=3$.

VII. CONCLUSION

In this paper, we have presented an exact solution of a microscopic Hamiltonian model of an absorbing dielectric half space and used it to calculate the spontaneous emission rate to order e^2 of an excited atom near the surface. Because our calculation is based on a fully canonical quantization scheme, it provides a fundamental demonstration of the validity of the classical electromagnetic theory of the rate of spontaneous emission near an absorbing dielectric surface. This serves to increase our confidence in the results of recent work on spontaneous lifetime based on classical electromagnetic theory [10]. Also, the exact photon Green function for the half space given in the Appendix can be used to treat other quantum-mechanical interaction phenomena between charged particles and the electromagnetic field near an absorbing plane surface, such as the level shift of an electron undergoing cyclotron motion near such a surface [11]. In the above discussion, we have only considered the case for which the excited atom is on the air side of the surface. However, our approach can be extended to treat the other case also.

APPENDIX: GREEN FUNCTION FOR THE TRANSVERSE PHOTONS

In this appendix, the steps involved in solving the Dyson equation Eq. (116) exactly for the photon Green function $\mathcal{D}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$ are outlined and the complete solution is given. The details of the calculations can be found in [9].

Following the standard Wiener-Hopf technique [12], we decompose the unknown function $\mathcal{D}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$ into a sum of two quantities $\mathcal{D}^{(+)\,\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$ and $\mathcal{D}^{(-)\,\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$, the first of which vanishes for $z_1>0$ and the second of which vanishes for $z_1<0$. Then, the Fourier transform of Eq. (116) with respect to (x_1-x_2), (y_1-y_2), and z_1 is taken. After rearrangement of terms, we obtain

$$\left\{[\omega^2-\Pi(\omega)-k^2c^2+i\varepsilon]\ \delta_{im}+\Pi(\omega)\frac{k_ik_m}{k^2}\right\}$$

$$\times\frac{D_{mj}^{(+)}(k_z)}{\omega^2-k^2c^2+i\varepsilon}+D_{ij}^{(-)}(k_z)$$

$$=D_{ij}^{(0)}(k_z)+\Sigma(\omega,K)\frac{\left(\delta_{i3}-\dfrac{k_ik_z}{k^2}\right)}{\omega^2-k^2c^2+i\varepsilon}\tilde{D}_{3j}^{\omega}(0), \qquad \text{(A1)}$$

where $\Pi(\omega)$ and $\Sigma(\omega,K)$ are the bulk and surface photon self-energies divided by ϵ_0,

$$\Pi(\omega)=-\omega^2[\epsilon_L(|\omega|)-1], \qquad \text{(A2)}$$

$$\Sigma(\omega,K)=\frac{\omega^2[\epsilon_L(|\omega|)-1]^2}{K[\epsilon_L(|\omega|)+1]}, \qquad \text{(A3)}$$

and $K=\sqrt{k_x^2+k_y^2}$. In Eq. (A1), we have suppressed the dependence of the Fourier transforms $D_{ij}^{(\pm)}(k_z)$ and $D_{ij}^{(0)}(k_z)$ on z_2,ω, and the transverse momentum (k_x,k_y). Also, $\tilde{D}_{ij}^{\omega}(0)$ is defined to be the Fourier transform of $\mathcal{D}_{ij}^{\omega}(\mathbf{r}_1,\mathbf{r}_2)$ with respect to (x_1-x_2) and (y_1-y_2) *only* and evaluated at $z_1=0$, while $D_{ij}^{(0)}(k_z)$ is the Fourier transform of $\mathcal{D}_{ij}^{(0)\omega}(\mathbf{r}_1,\mathbf{r}_2)$ with respect to (x_1-x_2), (y_1-y_2), and z_1,

$$D_{ij}^{(0)}(k_z)=\frac{1}{\epsilon_0}\frac{\left(\delta_{ij}-\dfrac{k_ik_j}{k^2}\right)}{\omega^2-k^2c^2+i\epsilon}e^{-ik_zz_2}. \qquad \text{(A4)}$$

The 3×3 matrix in curly brackets on the LHS of Eq. (A1) can be factorized into a product $\mathcal{M}_{il}^{(-)}\mathcal{M}_{lm}^{(+)}$ where

$$\mathcal{M}_{ij}^{(\pm)}=[iL(K)\pm k_zc]\delta_{ij}+i[S(K)-L(K)]\frac{k_ik_j}{k^2} \qquad \text{(A5)}$$

and

$$L(K)=\sqrt{K^2c^2+\Pi(\omega)-\omega^2-i\varepsilon}, \qquad \text{(A6)}$$

$$S(K)=\sqrt{K^2c^2-\omega^2-i\varepsilon}. \qquad \text{(A7)}$$

The branches of the square roots in Eqs. (A6) and (A7) are chosen so that the real parts of $L(K)$ and $S(K)$ are always positive. Notice that $\mathcal{M}_{ij}^{(\pm)}$ have singularities at $k_z=\pm iK$ due to the factor $1/k^2$. As a result, the standard Wiener-Hopf technique must be extended to deal with these singularities. This consists of subtracting appropriate poles from both sides of Eq. (A1). After multiplying this equation throughout by $(iS-k_zc)$ times the inverse of $\mathcal{M}^{(-)}$ and rearranging terms, we obtain

$$\mathcal{M}_{im}^{(+)}\left[\frac{D_{mj}^{(+)}(k_z)}{iS+k_zc}-A_{mj}^{(+)}\right]+\mathcal{M}_{im}^{(-)-1}[D_{mj}^{(-)}(k_z)(iS-k_zc)-A_{mj}^{(-)}]$$

$$=\ \mathcal{M}_{im}^{(-)-1}\left[D_{mj}^{(0)}(k_z)+\Sigma(\omega,K)\frac{\left(\delta_{m3}-\dfrac{k_mk_z}{k^2}\right)}{\omega^2-k^2c^2+i\varepsilon}\tilde{D}_{3j}^{\omega}(0)\right](iS-k_zc)-\mathcal{M}_{im}^{(+)}A_{mj}^{(+)}-\mathcal{M}_{im}^{(-)-1}A_{mj}^{(-)}\ , \qquad \text{(A8)}$$

where we have subtracted quantities $\mathcal{M}_{im}^{(+)}A_{mj}^{(+)}$ and $\mathcal{M}_{im}^{(-)-1}A_{mj}^{(-)}$ from both sides of the equation so as to make the residues of the poles at $k_z=iK$ and $k_z=-iK$ due to $\mathcal{M}_{im}^{(+)}$ and $\mathcal{M}_{im}^{(-)-1}$ in the first and second terms, respectively, on the left-handside (LHS) of Eq. (A8) vanish. As a result, the first and second terms on the LHS of this equation are analytic in $\Sigma^{(+)}$ and $\Sigma^{(-)}$, respectively, where $\Sigma^{(+)}$ contains the upper half of the k_z plane, $\Sigma^{(-)}$ contains the lower half, and $\Sigma^{(+)}$ and $\Sigma^{(-)}$ overlap in an open region.

The RHS of Eq. (A8) can be decomposed [9] into a sum of two terms $P_{ij}^{(+)}(k_z)$ and $P_{ij}^{(-)}(k_z)$ analytic in $\Sigma^{(+)}$ and $\Sigma^{(-)}$, respectively. By equating the first and second terms on the LHS of Eq. (A8) to $P_{ij}^{(+)}(k_z)$ and $P_{ij}^{(-)}(k_z)$, respectively, we obtain the formal solutions

$$D_{ij}^{(+)}(k_z)=[iS(K)+k_zc][A_{ij}^{(+)}+\mathcal{M}_{im}^{(+)-1}P_{mj}^{(+)}(k_z)], \qquad \text{(A9)}$$

$$D_{ij}^{(-)}(k_z)=\frac{1}{iS(K)-k_zc}[A_{ij}^{(-)}+\mathcal{M}_{im}^{(-)}P_{mj}^{(-)}(k_z)]\ . \qquad \text{(A10)}$$

The solutions Eqs. (A9) and (A10) still contain the unknown quantities $\tilde{D}_{3j}^{\omega}(0)$, $A_{ij}^{(+)}$, and $A_{ij}^{(-)}$ the latter two of which occur only in the combinations $r_mA_{mj}^{(+)}$ and $r'_mA_{mj}^{(-)}$, where $\mathbf{r}$ and $\mathbf{r}'$ are vectors with components

$$r_i=(k_x,k_y,iK), \qquad \text{(A11)}$$

$$r'_i=(k_x,k_y,-iK). \qquad \text{(A12)}$$

To solve for these quantities, we first use the fact that $D_{ij}^{(+)}(k_z)$ and $D_{ij}^{(-)}(k_z)$ must be analytic in $\Sigma^{(+)}$ and $\Sigma^{(-)}$, respectively. Thus the residues of the poles at $k_z=iK$ and $k_z=-iK$ due to $\mathcal{M}_{im}^{(+)-1}$ and $\mathcal{M}_{im}^{(-)}$ in Eqs. (A9) and (A10), respectively, must vanish. This gives two simultaneous algebraic equations

$$\lim_{k_z\to iK}k_mP_{mj}^{(+)}(k_z)=0, \qquad \text{(A13)}$$

$$\lim_{k_z\to -iK}k_mP_{mj}^{(-)}(k_z)=0, \qquad \text{(A14)}$$

which can be solved for the unknowns $r_mA_{mj}^{(+)}$ and $r'_mA_{mj}^{(-)}$. The results for $z_2\geqslant0$ are

$$r_m A_{mj}^{(+)} = -\frac{2i}{\Delta_0}\left\{\frac{1}{(S-L)(S-Kc)} r_i' - \frac{L+S+2Kc}{(S-L)^2(S+Kc)} r_i\right\} \frac{\left(\delta_{im} - \frac{q_i q_m}{q^2}\right)}{L+S} \left[\frac{1}{\epsilon_0} e^{-Sz_2/c} \delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right]$$
$$-\frac{2i}{\Delta_0(S-L)^2(S-Kc)} r_m' \left[\frac{1}{\epsilon_0} e^{-Kz_2}\delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right], \tag{A15}$$

$$r_m' A_{mj}^{(-)} = \frac{2i}{\Delta_0}\left\{\frac{L+Kc}{S-L} r_i + \frac{(S+Kc)(L+Kc)(L+S+2Kc)}{(S-L)^2(S-Kc)} r_i'\right\} \frac{\left(\delta_{im} - \frac{q_i q_m}{q^2}\right)}{L+S} \left[\frac{1}{\epsilon_0} e^{-Sz_2/c}\delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right]$$
$$-\frac{2i(S+Kc)^2}{\Delta_0(S-L)^2(S-Kc)} r_m' \left[\frac{1}{\epsilon_0} e^{-Kz_2}\delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right], \tag{A16}$$

where

$$q_i = \left(k_x, k_y, -\frac{iS}{c}\right), \tag{A17}$$

$$q_i' = \left(k_x, k_y, \frac{iS}{c}\right), \tag{A18}$$

$$\Delta_0 = 1 + \left[\frac{L+S+2Kc}{S-L}\right]^2. \tag{A19}$$

Next, the inverse Fourier transform of $D_{zj}^{(-)}(k_z)$ with respect to k_z for $z_1 \geqslant 0$ is calculated from Eq. (A10) by contour integration. The result for $z_2 \geqslant 0$ is

$$\tilde{D}_{ij}^{\omega}(z_1) = \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} e^{ik_z z_1} D_{ij}^{(-)}(k_z)$$
$$= \tilde{D}_{ij}^{(0)\omega}(z_1) - \left[\frac{1}{2Sc}\left(\delta_{i3} - \frac{q_i' q_3'}{q^2}\right) e^{-Sz_1/c} + \frac{r_i r_3}{2K\omega^2} e^{-Kz_1}\right] \Sigma(\omega,K)\tilde{D}_{3j}^{\omega}(0)$$
$$-\frac{S-L}{2(L+S)}\left[\frac{1}{Sc}\left(\delta_{il} - \frac{q_i' q_l'}{q^2}\right) e^{-Sz_1/c} + \frac{r_i r_l}{K\omega^2} e^{-Kz_1}\right]\left(\delta_{lm} - \frac{q_l q_m}{q^2}\right)\left[\frac{1}{\epsilon_0} e^{-Sz_2/c}\delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right]$$
$$-\frac{S-L}{2K(L+Kc)}\left[\frac{1}{\omega^2}\left(\delta_{il} - \frac{q_i' q_l'}{q^2}\right) r_l' e^{-Sz_1} + \frac{r_i}{2(S-Kc)^2} e^{-Kz_1}\right] r_m' \left[\frac{1}{\epsilon_0} e^{-Kz_2}\delta_{mj} + \Sigma(\omega,K)\delta_{m3}\tilde{D}_{3j}^{\omega}(0)\right]$$
$$+\frac{i(S-L)^2}{2K(L+Kc)(S+Kc)^2}\left\{\frac{S+Kc}{2(S-Kc)} e^{-Kz_1} r_i - \left[r_i' - \frac{q_i' K\left(K + \frac{S}{c}\right)}{q^2}\right] e^{-Sz_1/c}\right\} r_m' A_{mj}^{(-)}$$
$$-\frac{i(S-L)^2}{2K(S-Kc)}\left\{\frac{S+L-2Kc}{2(S-L)} e^{-Kz_1} r_i + \left[r_i - \frac{q_i' K\left(K - \frac{S}{c}\right)}{q^2}\right] e^{-Sz_1/c}\right\} r_m A_{mj}^{(+)}, \tag{A20}$$

where

$$\widetilde{D}_{ij}^{(0)\omega}(z_1)=\frac{1}{\epsilon_0}\begin{cases} -\dfrac{\left(\delta_{ij}-\dfrac{q_i'q_j'}{q^2}\right)}{2Sc}e^{-S(z_1-z_2)/c}-\dfrac{r_ir_j}{2K(\omega^2+i\epsilon)}e^{-K(z_1-z_2)}, & z_1>z_2 \\ -\dfrac{\left(\delta_{ij}-\dfrac{q_iq_j}{q^2}\right)}{2Sc}e^{S(z_1-z_2)/c}-\dfrac{r_i'r_j'}{2K(\omega^2+i\epsilon)}e^{K(z_1-z_2)}, & z_1<z_2 . \end{cases} \tag{A21}$$

Since the RHS of Eq. (A20) contains the unknown quantity $\widetilde{D}_{3j}^{\omega}(0)$, we obtain an algebraic equation for this unknown quantity by setting $i=3$ and $z_1=0$ on both sides of Eq. (A20). The resulting solution for $\widetilde{D}_{3j}^{\omega}(0)$ can be written in the form

$$\widetilde{D}_{3j}^{\omega}(0)=\gamma_1\delta_{3j}+\gamma_2r_j'+\gamma_3r_j+\gamma_4q_j, \tag{A22}$$

where

$$\gamma_1=\frac{1}{\Delta_1}\left[-\frac{1}{2Sc}-\frac{S-L}{2Sc(L+S)}\left(1+\frac{2S^2}{q^2c^2}\right)\right]\frac{1}{\epsilon_0}e^{-Sz_2/c}, \tag{A23}$$

$$\gamma_2=\frac{1}{\Delta_1}\left(\left\{\frac{i}{2\omega^2}+\frac{i(S-L)}{2(L+Kc)\ \omega^2}\left[1+\frac{S\left(K+\frac{S}{c}\right)}{q^2c}\right]-\frac{i(S-L)}{4(L+Kc)(S-Kc)^2}\right\}\frac{1}{\epsilon_0}e^{-Kz_2}-\sigma_1(\alpha_1+\alpha_3)-\sigma_2(\beta_2+\beta_3)\right), \tag{A24}$$

$$\gamma_3=\frac{1}{\Delta_1}\left[-\frac{i(S-L)}{2(L+S)\ \omega^2}\frac{1}{\epsilon_0}e^{-Sz_2/c}-\sigma_1\alpha_2-\sigma_2\beta_1\right], \tag{A25}$$

$$\begin{aligned}\gamma_4=\frac{1}{\Delta_1}\Bigg\{&\left[-\frac{i}{2q^2c^2}-\frac{i(S-L)\left(K^2+\frac{S^2}{c^2}\right)}{2(L+S)q^4c^2}+\frac{i(S-L)K\left(K+\frac{S}{c}\right)}{2(L+S)\ \omega^2q^2}\right]\frac{1}{\epsilon_0}e^{-Sz_2/c}+\sigma_1\frac{K}{q^2}\left[\alpha_1\left(K-\frac{S}{c}\right)+\alpha_2\left(K+\frac{S}{c}\right)\right]\\ &+\sigma_2\frac{K}{q^2}\left[\beta_1\left(K+\frac{S}{c}\right)+\beta_2\left(K-\frac{S}{c}\right)\right]\Bigg\},\end{aligned} \tag{A26}$$

and $\Delta_1=(1+\sigma_1W_1+\sigma_2V_1-U_1)$.

Equations (A23) to (A26) contain the quantities α_i, β_i, and σ_i, as well as the quantities U_1, V_1, and W_1 through Δ_1. These quantities are given for $z_2\geqslant 0$ by

$$\alpha_1=-\frac{2i}{\Delta_0(S-L)(S-Kc)(L+S)}\frac{1}{\epsilon_0}e^{-Sz_2/c}, \tag{A27}$$

$$\alpha_2=\frac{2i(L+S+2Kc)}{\Delta_0(S-L)^2(S+Kc)(L+S)}\frac{1}{\epsilon_0}e^{-Sz_2/c}, \tag{A28}$$

$$\alpha_3=-\frac{2i}{\Delta_0(S-L)^2(S-Kc)}\frac{1}{\epsilon_0}e^{-Kz_2}, \tag{A29}$$

$$\beta_1=\frac{2i(L+Kc)}{\Delta_0(S-L)(L+S)}\frac{1}{\epsilon_0}e^{-Sz_2/c}, \tag{A30}$$

$$\beta_2=\frac{2i(S+Kc)(L+Kc)(L+S+2Kc)}{\Delta_0(S-L)^2(S-Kc)(L+S)}\frac{1}{\epsilon_0}e^{-Sz_2/c}, \tag{A31}$$

$$\beta_3=-\frac{2i(S+Kc)^2}{\Delta_0(S-L)^2(S-Kc)}\frac{1}{\epsilon_0}e^{-Kz_2}, \tag{A32}$$

and

$$\sigma_1=-\frac{(S-L)^2}{2(S-Kc)}\left\{\frac{S+L-2Kc}{2(S-L)}+\left[1-\frac{S\left(K-\frac{S}{c}\right)}{q^2c}\right]\right\}, \tag{A33}$$

$$\sigma_2=\frac{(S-L)^2}{2(L+Kc)(S+Kc)^2}\left\{\frac{S+Kc}{2(S-Kc)}+\left[1+\frac{S\left(K+\frac{S}{c}\right)}{q^2c}\right]\right\}, \tag{A34}$$

$$\begin{aligned}U_1={}&-\frac{1}{2Sc}\left(1+\frac{S^2}{q^2c^2}\right)+\frac{K}{2\,\omega^2}-\frac{S-L}{2Sc(L+S)}\left[1+\frac{2S^2}{q^2c^2}+\frac{S^2\left(K^2+\frac{S^2}{c^2}\right)}{q^4c^2}\right]+\frac{(S-L)K}{2(L+Kc)\,\omega^2}\left[1+\frac{S\left(K+\frac{S}{c}\right)}{q^2c}\right]\\&+\frac{(S-L)K}{2(L+S)\,\omega^2}\left[1+\frac{S\left(K+\frac{S}{c}\right)}{q^2c}\right]-\frac{(S-L)K}{4(L+Kc)(S-Kc)^2},\end{aligned} \tag{A35}$$

$$V_1=\frac{2i}{\Delta_0}\left\{\frac{L+Kc}{S-L}r_i+\frac{(S+Kc)(L+Kc)(L+S+2Kc)}{(S-L)^2(S-Kc)}r_i'\right\}\frac{\left(\delta_{i3}-\frac{q_iq_3}{q^2}\right)}{L+S}\Sigma(\omega,K)-\frac{2K(S+Kc)^2}{\Delta_0(S-L)^2(S-Kc)}\Sigma(\omega,K), \tag{A36}$$

$$W_1=-\frac{2i}{\Delta_0}\left\{\frac{1}{(S-L)(S-Kc)}r_i'-\frac{L+S+2Kc}{(S-L)^2(S+Kc)}r_i\right\}\frac{\left(\delta_{i3}-\frac{q_iq_3}{q^2}\right)}{L+S}\Sigma(\omega,K)-\frac{2K}{\Delta_0(S-L)^2(S-Kc)}\Sigma(\omega,K). \tag{A37}$$

This completes our determination of all the unknown quantities on the RHS of Eq. (A20). To compute the spontaneous decay rate by Eq. (70), the photon Green function $\mathcal{D}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)$ appearing in the first term on the RHS of this equation is first computed by evaluating the inverse Fourier transform of $\tilde{D}^{\omega}_{ij}(z_1)$ given by Eq. (A20) with respect to k_x and k_y numerically,

$$\mathcal{D}^{\omega}_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\int\int\frac{dk_xdk_y}{(2\pi)^2}e^{ik_x(x_1-x_2)+ik_y(y_1-y_2)}\tilde{D}^{\omega}_{ij}(z_1). \tag{A38}$$

Next, by using Eqs. (10), (122), and (126), the integrals over $\mathbf{r}$ and $\mathbf{r}'$ in the second and third terms on the RHS of Eq. (70) can be reduced to integrals over the transverse momentum (k_x,k_y) in which the photon Green function occurs only through the quantity $\tilde{D}^{\omega}_{3j}(0)$ given by Eq. (A22).

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