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Possible thermodynamic law violations and astrophysical issues for secular acceleration of electrodynamic particles in the vacuum

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A potentially significant consequence of the dynamics due to particles interacting electromagnetically with electromagnetic zero-point plus Planckian radiation is the prediction of a stochastic acceleration mechanism. For particles without constraining forces or other collision mechanisms present, such as due to the walls of a container or to interparticle collisions, this mechanism results in average speeds that continue to increase with time. This secular acceleration effect has been proposed and analyzed in the past by Rueda and others [A. Rueda, Space Sci. Rev. **53**, 223 (1990)]for being a possible source for cosmic-ray production; other possible astrophysical consequences have also been examined. However, the thermodynamics of secular acceleration have presented conceptual problems, such as the apparent source of continual average energy extraction and the possible violation of the second law of thermodynamics. These concerns are examined in detail here, with the conclusion that secular acceleration does not appear to violate these basic laws.

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I. INTRODUCTION

Here the thermodynamic aspects are examined of a rather surprising, but possibly quite significant, theoretically predicted consequence of the interaction of charged particles with electromagnetic zero-point (ZP) radiation. The charged particles are predicted to slowly, but steadily, increase their average kinetic energy over time, unless collisions with other particles, or with macroscopic bodies like the walls of a container, occur to dissipate the rising kinetic energy [1–10]. The same effect is predicted for electrically neutral, but polarizable, particles as well, although the kinetic energy growth is typically much smaller. Moreover, the structure and other properties of a particle, including whether the "particle" is an elementary one, a subatomic one, an atom, or even a molecule, will very strongly dictate and, in most cases, quench the magnitude of the growth. This predicted effect of the ZP fields on particle motion will subsequently be referred to as a secular acceleration mechanism [11].

This acceleration mechanism has its origin in work by Einstein and Hopf [12], which involved classical atomic systems interacting with classical electromagnetic thermal radiation. The atoms were modeled as particles with a mass m that contained a fluctuating classical electric dipole oscillator. Due to interactions with the radiation, Einstein and Hopf recognized that atomic systems must be subject to fluctuating impulses from the radiation, as well as to a velocity-dependent "drag" force. These two mechanisms then compete against each other in terms of increasing and decreasing, respectively, the kinetic energy of particles interacting electromagnetically with thermal radiation.

Much later, Boyer [1] considered the same system as did Einstein and Hopf, but he also investigated the effects of classical electromagnetic ZP, as well as ZP plus Planckian (ZPP) radiation. Due to the Lorentz invariant nature of ZP radiation, the drag force is necessarily absent in the case of ZP radiation alone, thereby leaving the effect of the fluctuating radiation impulses uncompensated. An average continual increase of velocity was then predicted, unless collisions with other matter occur to help dissipate the increased kinetic energy. Further work on this system [2,4,5], including the quantum analog case [9,10,13], has helped strengthen the evidence that this effect is a real one.

This secular acceleration mechanism has been analyzed in some detail by researchers [1–10]. The significance of this mechanism is indicated by a number of astrophysical phenomena that may partially, or perhaps even largely, be explained by it. In intergalactic space, the necessary conditions appear to exist to make the effects of this mechanism most noticeable; namely, the particle densities and the radiation temperature are extremely low, while the size of intergalactic regions is enormous. The result is extremely long relaxation times for particles to come to equilibrium, while allowing sufficiently long times for the radiation to have a slow, but long, accumulation effect on increasing the kinetic energy of the particles.

Consequently, secular acceleration has been proposed [2] and examined as being a viable source for cosmic-ray production by Rueda and others, both classically [2,3,5–10], as well as in the form of its quantum mechanical counterpart [9,10,13]. (See Ref. [10] for a general review on this work.) Indeed, part of the x-ray background observed in space may be due to this secular acceleration mechanism [2,10,14]. Moreover, secular acceleration has been proposed and analyzed for causing the clumping of matter and the associated generation of cosmic voids in very large regions in intergalactic space

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[2,10,11,15].

To date, the details of these predictions agree fairly well with astrophysical observations [16], although certainly considerable further experimental and theoretical work is needed before firmly concluding to what extent secular acceleration is responsible for these phenomena, as other possible causes and theories for these phenomena presently also exist. However, these astrophysical phenomena are mentioned here to emphasize that secular acceleration should have a *number* of observable consequences in nature, with the ones just mentioned being the most significant candidates at this time.

Since the physical dynamic issues about secular acceleration have already been analyzed in considerable detail by past researchers [1–10,12,13], starting from the very early work by Einstein and Hopf [12], it does not seem necessary to reconsider these issues here. However, an important aspect of secular acceleration has largely not been treated to date, namely, basic thermodynamic issues. That task will be undertaken here.

Undoubtedly, secular acceleration seems, at first, to be counterintuitive, if not in direct opposition to the laws of thermodynamics. In particular, the mechanism appears to enable "something to be acquired from nothing," since a source of free energy appears to be available that enables unrestrained matter to continually acquire higher energies, in opposition to the first law of thermodynamics. Moreover, since in a large region of space ZPP radiation should supposedly represent, effectively, a heat bath at temperature T, secular acceleration certainly has the appearance of enabling energy to be extracted from a heat reservoir, thereby possibly violating the second law of thermodynamics.

These questions have been raised in the past and some suggestions have been made to answer them [5,7,9,10]. Unfortunately, as discussed in Sec. V, these suggestions [7,9,10] appear to be incorrect. Instead, here analysis will be carried out by following the lines of some relatively recent work involving the thermodynamics of classical electrodynamic systems [17–22]. In keeping with this previous work, the present analysis will examine the thermodynamics of this secular mechanism within the context of the classical physical theory of stochastic electrodynamics (SED) [23–26,10,21]. The same thermodynamic questions arise in the quantum version of this mechanism. However, as will be seen, one can quickly get to the heart of the apparent paradoxes by discussing the problem classically.

The outline of this article contains, first, a description of the physical system to be analyzed here in Sec. II A, followed by a discussion of the thermodynamic issues in Sec. II B, and then a calculation of the change in caloric entropy for this system in Sec. II C. Section III turns to a numerical calculation on changes in thermodynamic properties, to provide further detailed insight into the thermodynamic behavior of this system. Section IV briefly discusses specific changes that would need to be made to carry out the same detailed analysis for situations where secular acceleration is expected to be a dominant factor. Finally, Sec. V ends with a few concluding remarks.

II. THERMODYNAMIC ANALYSIS

A. System to be analyzed

Our analysis will involve the comparison of two processes that begin and end in thermal equilibrium states; one process will be carried out via a thermodynamic reversible process, while the other will be an irreversible one. The contrast between reversible and irreversible processes will offer detailed insight into the noncontroversial aspect of the second law of thermodynamics [27], namely, where the process in question starts and ends in a thermodynamic equilibrium state.

Let us begin with a qualitative discussion of an example that illustrates the main points of this thermodynamic analysis, then turn to a much more specific system that can be more quantitatively analyzed. Consider a large container with a single, neutral atom bouncing around inside, but also interacting with the infinite degrees of freedom of the *Hohlraum* of thermal radiation in the container via the fluctuating electric dipole of the atom.

For this single-particle system, without a doubt somewhat regular fluctuations in properties will exist, such as the speed of the particle as a function of time or the electromagnetic field values versus time at a fixed point $\mathbf{x}$ in the container. These fluctuations are related to the instantaneous position of the particle within the box and to the size of the box; we expect the particle's average speed to be different near the walls versus at the center of the container [28], and we expect the fluctuating electric and magnetic fields to be correlated with the position of the particle in the box. Nevertheless, the existence of these correlated fluctuations is certainly compatible with the result that a stochastic process that is stationary in time should result after a sufficiently long time of interaction between the particle, radiation, and walls. Assuming the equilibrium state is an ergodic one, then ensemble averages of quantities such as $\langle A(t)B(t+\tau)\rangle$, where A and B might be electromagnetic field components at different points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the box and at times t and $t+\tau$, or the velocity of the particles at these times, or other such combinations, can also be approximated using time averages of the form

$$\frac{1}{N}\sum_{n=0}^{N-1} A(t+n\Delta t)B(t+n\Delta t+\tau) .$$

Here, the time duration $N\Delta t$ should include many, many traversals of the particle across the box. The introduction of Planck's carbon speck [29], a small idealized carbon particle, will ensure that the resulting stochastic process is representative of a thermal equilibrium state.

The irreversible thermodynamic operation that will be discussed here is where the walls of the container are suddenly expanded to make the container many times its original size. The secular acceleration mechanism will then have a much longer time to act between collisions of the atomic system with the walls. Upon impact with a wall, the average kinetic energy of the atom will be larger than it would have been if the walls had not been expanded, resulting in larger amounts of energy to be radiated back into the thermal radiation bathing the system.

We will contrast this process with the following reversible thermodynamic operation, namely, where the walls are very slowly displaced from the original configuration to the same final configuration of the irreversible process just mentioned. The walls must be moved quasistatically, so that the particle has a chance to bounce off the walls many times between infinitesimal displacements of the walls. Thus this process is similar to the reversible thermodynamic operation of slowly expanding a volume of gas, while the above irreversible operation is more closely related to the irreversible operation of a free expansion of the gas.

A key point that we will use here when analyzing the above irreversible operation is to consider only the change in entropy between the initial state of the system in thermal equilibrium, and the final state when the system is again in near thermal equilibrium. This last state will occur well after the expansion has been made and when the particle has had a chance to hit the walls a great number of times, so that the stochastic properties of the system (fields and atom) will have settled down to approximate a stationary stochastic process in time. We will *not* consider, or at least not directly consider, the transient period during which the walls are suddenly expanded, as we wish to investigate the unambiguous and "noncontroversial nature of the second law" [27]. This consideration will enable us to gain insight as to how this secular acceleration mechanism can fit into normal thermodynamic concepts of systems.

The transient period when the walls have been expanded, and before the first impact of the particle with the walls has occurred, is analogous to the situation of particles in cosmic rays. These particles have acquired large kinetic energies and have not yet impacted with other matter; they do not yet exist in thermal equilibrium with their surroundings. Due to the large separation distances of the physical systems involved here, the corresponding relaxation times should also be expected to be quite large. If we were to imagine that the cosmic-ray particles were to impact with the walls of an enormous container of "galactic" size, then bounce back and forth between the container's walls, a near thermodynamic equilibrium between radiation and matter would eventually arise. The full change in entropy of the system could then be calculated; however, intermediate changes in entropy, before thermodynamic equilibrium sets in, are not well defined. Analyzing the thermodynamics of cosmic rays is quite difficult if one restricts attention to this transient period, but the problem becomes much more amenable by placing the process within a larger one that eventually ends in a thermodynamic equilibrium state.

Now let us become more specific so as to enable a more quantitative analysis. Figures 1(a) and 1(b) illustrate an analogous irreversible and reversible thermodynamic operation, respectively, for a more specific system. The left-hand and right-hand sides of the figure represent the equilibrium starting and ending points, respectively, at times t_i and t_f, of both processes. The box shown is intended to represent an idealized perfectly conducting container that contains classical electromagnetic radiation at some temperature T. For such a container, thermal radiation cannot flow out of the box, so operations performed on such a system must be adiabatic ones. To consider other operations that are not adiabatic, let us assume that, for example, stopcocks exist in the walls and can be opened so that radiation energy can flow either in or out, and Planck's idealized carbon speck can be introduced to ensure that thermal equilibrium is always achieved [19,29].

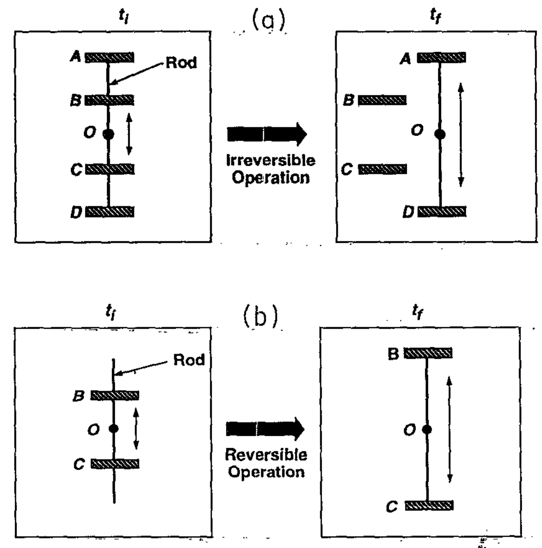

FIG. 1. Examples of an irreversible (a) and a reversible (b) thermodynamic operation on system O, which represents an oscillator that can slide up and down on a rod. (a) The blocks indicated by A, B, C, and D act as barriers to restrict the regions accessible by O. At time t_i, B and C restrain O from moving outside the inner region. Between times t_i and t_f, blocks B and C are rapidly slid to the left, so that O is then free to move between A and D. (b) Here, a closely related reversible thermodynamic operation is performed by infinitely slowly moving blocks B and C apart on the rod while allowing O to repeatedly bounce off them during this operation.

Within the box is a small mechanical system, labeled O, containing a classical, fluctuating electric dipole. Assume that the electric dipole is free to fluctuate in whatever direction is dictated by its interaction with the thermal radiation, but that the mechanical system O housing the dipole must move only vertically up and down along the rod shown. The electric dipole interacts with the radiation bathing the system, resulting in impulses that make O fluctuate in its movements. In turn, accelerations and decelerations of O affect how the electric dipole radiates electromagnetic energy.

In Figs. 1(a) and 1(b), at time t_i, two blocks B and C act as bumpers to initially constrain O to move only within the region of the rod between B and C. In Fig. 1(a), an irreversible operation is carried out by horizontally removing B and C from the rod, thereby suddenly providing O access to the region between bumpers A and D. The system is then no longer in thermal equilibrium; the kinetic energy of the particle, the amount of energy radiated upon colliding with a wall, and the electromagnetic energy density (dependent on position), will all undergo time-dependent changes in their averages. However, upon waiting for enough impacts to occur with bumpers A and D and after introducing Planck's carbon particle, at some time t_f the system again returns to a new (approximately) thermal equilibrium state for $t \geq t_f$. Assuming negligible work is done in displacing B and C off the rod, this example is analogous to a gas undergoing a free expansion with little or no work being done.

In contrast, when passing from time t_i to t_f in Fig. 1(b), the inner blocks B and C are very slowly moved apart on the rod. Essentially the same final mechanical configuration exists at time t_f for both Figs. 1(a) and 1(b); yet, in Fig. 1(b), the final configuration was achieved by allowing O to do work on the slowly moving blocks. This reversible process can be used to calculate the change in entropy in the irreversible operation in Fig. 1(a).

Unfortunately, even the simple system described here is still extremely complicated, particularly when considering the collisions with the bumpers and in view of the resulting radiated energy and the spectral radiation density. Consequently, let us now consider an even simpler system. Let system O be replaced with a single classical charged particle of charge e and mass m. Let the particle be constrained to move up and down in the vertical direction while being bound to the center point on the rod by a simple harmonic oscillator (SHO) potential. For simplicity of presentation, let us assume the particle motion to be subrelativistic; this problem can then be solved in some detail. A reversible operation analogous to the one in Fig. 1(b) can be performed by very slowly reducing the "spring constant" that binds the charged particle, thereby reducing the binding force on the particle and allowing it to traverse a greater distance from the center of the rod. Likewise, an irreversible operation analogous to the one in Fig. 1(a) can be accomplished by changing the spring constant very quickly. We can now calculate the changes in internal energy, work done, and changes in entropy for this system.

Taking the ensemble average of the internal energy in Eq. (31) in Ref. [21], following the nonrelativistic, resonant harmonic oscillator limit in Refs. [17,20], and using the thermal radiation internal energy expression in Refs. [18,19] for a large volume, $\mathcal{V}$, one can show that

$$U_{\text{int}} = \tfrac{1}{2} m \langle \dot{x}^2 \rangle + \sigma' T^4 \mathcal{V} + C$$
$$\approx \tfrac{1}{2}\pi^2 h^2(\omega_o, T) + \sigma' T^4 \mathcal{V} + C \,, \quad (1)$$

where C is a constant and $\sigma' = \pi^2 k_B^4/(c^3 \hbar^3 15)$. Here, the thermal spectral energy density is $\rho(\omega, T) = \omega^2 h^2(\omega, T)/c^3$, while ω_o is the resonant frequency of the oscillator. The above internal energy only considers the kinetic and electromagnetic energy as the contributors, since the force due to the harmonic oscillator potential is treated as an externally applied force, as is typically done for the pressure on the walls of a gas container [21]. The walls surrounding the volume $\mathcal{V}$ of the system are assumed to be perfectly reflecting to the electromagnetic radiation at all frequencies, so the system of particle plus radiation may be considered to be closed, unless stopcocks are opened in the walls to allow nonadiabatic thermodynamic operations to be performed.

The ensemble average of the work done on the particle when very slowly changing the spring constant $\kappa = m\omega_0^2$, between times t_{I} and t_{II}, is

$$W = \left\langle \int_{t_{\text{I}}}^{t_{\text{II}}} \mathbf{F}(t)\cdot\mathbf{v}(t)\,dt \right\rangle = \int_{t_{\text{I}}}^{t_{\text{II}}} \langle [-m\omega_0^2(t) x(t)] \dot{x}(t) \rangle\, dt$$
$$= -\frac{m}{2}\int_{t_{\text{I}}}^{t_{\text{II}}} \omega_0^2(t) \frac{d}{dt}\langle x^2(t)\rangle\, dt = -\tfrac{1}{2}\pi^2 h^2(\omega_{o\text{II}}, T_{\text{II}}) + \tfrac{1}{2}\pi^2 h^2(\omega_{o\text{I}}, T_{\text{I}}) + \pi^2 \int_{t_{\text{I}}}^{t_{\text{II}}} dt \frac{h^2(\omega_o, T)}{\omega_o}\frac{d\omega_o}{dt} \,, \quad (2)$$

where $\mathbf{F}(t)$ is the harmonic restoring force and $\mathbf{v}(t)$ is the velocity of the particle. Only $x(t)$ varies stochastically —hence the third equality. In the last equality, an integration by parts was done and the resonant result of $\langle x^2(t) \rangle = \pi^2 h^2(\omega_0, T)/m\omega_0^2$ was used. The last term in the last equality depends on how ω_o and T are slowly changed with time, rather than just their values at t_{I} and t_{II}; thus this integral is path dependent, as should be expected for the thermodynamic work function. If the spring constant is not changed, but only the temperature is altered, then this last term equals zero. Nevertheless, work is still done, just as happens when a balloon of gas is heated and the balloon expands, in opposition to the outside atmospheric pressure.

Before proceeding with computing the change in entropy of the above system, we should note that a disadvantage of carrying out the above analysis is that nonrelativistic and dipole approximations are being invoked when evaluating the ensemble averages of the internal energy and work done. These approximations make the calculations much more tractable; however, the results of the secular acceleration mechanism do then not directly appear. Only if we went beyond these approximations, so that the particle traversed larger distances from the center of the restoring force, would this effect be evident. The dipole approximation, in particular, would then be invalid.

Nevertheless, the mechanism of secular acceleration, and the increased kinetic energy that results, have been analyzed in some detail in the past [1–10], so it is not critically important that we further investigate these points here. Instead, what is much more important is that some key physical concepts be clarified so as to explain how the analytical and quantitative predictions already made about this mechanism fit within our traditional concepts of physics and thermodynamics. The simplified system described here will aid this understanding.

B. Discussion on thermodynamic concerns

Consequently, a brief discussion is warranted on where the additional kinetic energy caused by the ZPP radiation must come from, including the amount above what we just calculated if the small oscillator assumption is removed. For the irreversible process of Fig. 1(a), the net energy within the perfectly conducting container must be a constant, provided that the atomic system does not hit or exit through the walls [30]. The constant energy is a singular quantity, since it includes ZPP radiation; however, changes in the total energy are the physically important quantities. For physically realizable processes, the changes must be finite and, in the present case, must equal zero. Hence the increased kinetic energy between impacts with the bumpers or, for example, between subsequent midpoints of the particle's motion, over the example we just calculated, must come from the total electromagnetic radiation energy within the container. This energy must include that of the radiation bathing the particle, the energy radiated by the particle, and the electromagnetic energy cross terms [21]. Just as the shape, size, and type of a material boundary can alter the resulting radiation density within a closed cavity, as occurs when analyzing Casimir forces [19], having a chaotically moving electrodynamic particle, or many such particles as occurs in a gas, must result in the radiation density being modified.

Thus the first objection to ZPP radiation giving rise to secular acceleration is easily overcome. Something is not acquired from nothing. As in other cases involving ZP radiation, such as with Casimir forces [19] or van der Waals forces [17,18], energy is still conserved despite the singular nature of the energies and forces involved. If secular acceleration is indeed the main contributor to cosmic rays, then the apparent free energy that has been acquired by these very high velocity particles is due to a finite change in the enormous amount of electromagnetic energy available in space.

To further emphasize this point, Ref. [21] showed in detail how the change in energy within any volume $\mathcal{V}$ of space must equal the net flow of energy into $\mathcal{V}$ plus the work done by external forces on the particles within $\mathcal{V}$, where $\mathcal{V}$ contains N classical charged particles. Mass renormalization, as well as the existence of the singular ZPP radiation, was taken into account. When applying these results to specific systems of dipole oscillators [17,18,20] and blackbody radiation [19], finite and measurable physical properties were predicted for pressures, forces, and specific heats, despite the singular energy of the ZPP radiation. In the case of movable conducting walls, the change in the blackbody radiation energy as a function of position within the container walls is due to the walls imposing boundary conditions on the radiation at points in space that change as the walls are displaced [19]. Rapidly fluctuating currents exist in the walls to impose these boundary conditions. Except at $T=0$, the temperature of the radiation as well as the cavity walls will change as the cavity is slowly expanded or contracted, thereby emphasizing the interplay of energy that must result between particles and radiation, despite the net singular energy possessed by the radiation. A very similar situation must hold for the case of Fig. 1.

We can now turn to the second thermodynamic concern involving secular acceleration, which, on the surface, undoubtedly appears more troublesome. This acceleration of classical charged particles, which is predicted to occur even at $T=0$, appears to be extracting energy from a heat reservoir, thereby possibly violating the second law of thermodynamics [31]. In particular, the Clausius statement of the second law is [32], "No process is possible whose sole result is the transfer of heat from a cooler to a hotter body." This law appears to invalidate the prediction that the acceleration mechanism will enable a charged particle to increase its kinetic energy at the expense of the ZPP radiation energy. After all, Refs. [1,2,4] predict that, as a result of the secular acceleration mechanism, the particle's kinetic energy will continue to grow, on the average, until the particle hits a wall; upon impact, some energy will be radiated off and the process will begin anew. Until the impact with the wall, this scenario has the appearance of heat being transferred from a cooler body (the ZPP radiation) to a hotter body (the charged particle). Hence the dilemma.

This initial dilemma (we will propose another one shortly) can be addressed fairly quickly. The above process does *not* violate Clausius's statement of the second law, because the statement implicitly assumes that the initial and final states being compared are both states of thermal equilibrium. Indeed, the unambiguous nature of the second law of thermodynamics requires this stipulation [27]. Examining the period of time after a collision with a wall and before the next collision, or, from an ensemble point of view, taking an ensemble of particles bouncing back and forth between walls, and comparing only the subensemble of particles shortly after a collision with the subensemble of particles that exist right before a collision, is, strictly speaking, *not* what the second law involves. Neither of these subensembles represents a thermal equilibrium state.

Of course, we can readily understand why one might naturally think of comparing these two situations. In particular, when the walls are very far apart, as in the related case of charged particles traveling through space and between collisions with celestial bodies, one might naturally suppose that a quasithermal state should exist some time after a collision, but well before the next collision. After all, for a large colloidal particle in a liquid at some temperature T, the particle *will* quickly reach an equilibrium state after hitting a wall or after being agitated by, for example, a spoon. After interaction with the spoon, the kinetic energy of the colloidal particle again settles down to a steady, average value.

Although the same idea might intuitively seem reasonable to hold for charged or polarizable particles in large regions of space, the theoretical prediction of the secular acceleration mechanism [1,2,4] and the observance of phenomena that appear to fit this description (e.g., cosmic rays [10]) indicate that a portion of the motion of charged particles in "free flight," or between collisions with walls or other particles, is *not* appropriate to consider when discussing thermal equilibrium; rather, the entire motion is essential. Although a charged particle may be in thermal equilibrium with the container walls and with thermal radiation, its kinetic energy is predicted to be quite different before, after, and partway between collisions with the walls. Only the full motion of the particle, as averaged over trajectories involving *many* collisions, is truly representative of the thermal equilibrium state.

Hence, this initial dilemma is readily resolved. However, a more complicated dilemma exists that does involve the full motion. Consider the average kinetic energy of the particle over many traversals of the box by the particle, both before and well after the box has been (greatly) expanded. Now we are safe because we are indeed considering states of thermal equilibrium. Roughly speaking, additional kinetic energy that is picked up by a particle upon traversing the box, due to interaction with the radiation, will be radiated off upon impact with the walls. Consequently, the average kinetic energy of a particle, over many traversals of the box, should be an approximately constant, steady value, so here this is nothing mysterious happening. However, provided the box is large enough, the average kinetic energy should be larger after the expansion of the box due to the additional time for the acceleration mechanism to act between wall impacts. Again, the appearance is that of heat being transferred from the ZPP radiation to the particle; hence the new dilemma.

Nevertheless, this problem can also be readily resolved. The process of enlarging the volume does *not* violate Clausius's statement because this statement specifies a process "...whose *sole* result is the transfer of heat ... ." However, "heat" transfer is not the sole result in the process we are presently discussing because an important parameter specifying the state of the system is also changed, namely, the volume containing the particle. Consequently, a charged particle acquiring kinetic energy from the vacuum does not represent a violation of the second law of thermodynamics any more than the following situation: namely, two neutral conducting plates held apart, then released. The two plates will move together and acquire kinetic energy as a result of the Casimir force, due to the vacuum, acting upon them [22]. In both this case, as well as the former one, a macroscopic parameter was changed, i.e., the volume in the first case and the constraint on the plate separation in the second case.

Another example that illustrates the same point perhaps even more clearly is when a relatively large colloidal particle is in a hot liquid bath, but the particle is at first attached to the side of the container. If one releases the particle, it will then acquire an average kinetic energy as a result of the Brownian motion induced by the bombardment of the rapidly moving smaller molecules in the liquid. The acquired kinetic energy comes from the internal energy of the heat reservoir of the liquid, just as the cosmic-ray particles may acquire their energy from the heat reservoir of ZPP radiation. The change in the state of the colloidal particle, i.e., the removal of the confinement of the particle, and thereby the excitation of previously quenched translational degrees of freedom, enables the particle to absorb energy from the heat bath. Likewise, expanding the volume of the box containing a particle bouncing around inside should enable the particle to acquire from the ZPP radiation a higher average kinetic energy, due to the relaxation of the confinement of the constraining walls. In return, a finite change in the average radiation density must result.

C. Calculation of change in entropy

Returning to the earlier computational problem, let us now calculate the change in caloric entropy for the irreversible process of Fig. 1(a). Of particular interest here will be whether our detailed calculations yield that the caloric entropy increases for this irreversible process, as predicted by the second law of thermodynamics [33]. From Eqs. (1) and (2), the heat, $Q=\Delta U_{\rm int}-W$, in the form of electromagnetic radiation that flows into the box due to a small change δT in temperature and $\delta\omega_o$ in resonant frequency equals

$$Q=\delta\omega_o\pi^2\left[\frac{\partial h^2}{\partial\omega_o}-\frac{h^2}{\omega_o}\right]+\delta T\left[4\sigma' T^3\mathcal{V}+\pi^2\frac{\partial h^2}{\partial T}\right] . \quad (3)$$

We note that for an isothermal reversible process, with $\delta T=0$, we again find that only one spectrum will yield no heat flow, namely, the zero-point spectrum described by $h^2=\kappa\omega$, in agreement with the thermodynamic definition of $T=0$ [17–20]. A connection can also be made for this spectrum to the adiabatic invariants in classical mechanics [34]. Just as no heat is predicted to flow for this simple system at $T=0$, so also must no heat flow even for a system of galactic size, should it hypothetically be in thermal equilibrium at $T=0$, and should we, also hypothetically, be able to change its state via a quasistatic, isothermal operation. As demanded by the thermodynamic definition of $T=0$ [17], this result should occur even though the kinetic energies of particles may change enormously during their motion from one wall of the container to the next one [35].

The change in the caloric entropy is given by $\delta S_{\rm cal}=Q/T$. Requiring that $S_{\rm cal}$ be a function of state results in equating two expressions from Eq. (3) for $\partial^2 S_{\rm cal}/\partial T\,\partial\omega_0$, thereby enabling one to deduce that the frequency-temperature version of the generalized Wien's displacement law must hold [19,36], or $h^2(\omega,T)=c^3\omega f(\omega/T)$. The caloric entropy can then be found by integrating $\delta S_{\rm cal}=Q/T$. One obtains, with $\Theta=\omega/T$,

$$S_{\rm cal}(\omega_{o\rm II},T_{\rm II})-S_{\rm cal}(\omega_{o\rm I},T_{\rm I})$$
$$=c^3\pi^2\int_{\omega_{o\rm I}/T_{\rm I}}^{\omega_{o\rm II}/T_{\rm II}}d\Theta\,\Theta\frac{df}{d\Theta}+\tfrac{4}{3}\sigma'(T_{\rm II}^3-T_{\rm I}^3)\mathcal{V} . \quad (4)$$

Upon substituting the ZPP spectrum [1] of

$$\rho(\omega,T)=\frac{\omega^3\hbar}{2\pi^2c^3}\coth\left[\frac{\hbar\omega}{2k_BT}\right], \tag{5}$$

into Eq. (4), with $\rho=\omega^2h^2/c^3=\omega^3 f(\omega/T)$, one obtains

$$\Delta S_{\rm cal}=\left[\frac{\hbar\omega_0}{2T}\coth\left[\frac{\hbar\omega_o}{2k_BT}\right]-k_B\ln\sinh\left[\frac{\hbar\omega_o}{2k_BT}\right]\right]\Bigg|_{\rm I}^{\rm II}+\tfrac{4}{3}\sigma'(T_{\rm II}^3-T_{\rm I}^3)\mathcal{V}\ . \tag{6}$$

Equation (6) is explicitly expressed in terms of the initial and final values of T and ω_o of the thermal equilibrium states connected by the thermodynamic process of interest. In Fig. 1(a), we are interested in a rapid "free relaxation" of the spring constant from ω_{oi} to ω_{of}, thereby mimicking the free expansion of a gas in a container where the volume is suddenly increased. Let us *define* an irreversible free relaxation of the oscillator system by whatever irreversible change to the spring constant, from ω_{oi} to ω_{of}, results in $\Delta U_{\rm int}$ equaling zero upon carrying out this operation and then waiting for the system to again relax to thermal equilibrium, when the entire process is carried out adiabatically so that no stopcocks are open in the walls of the perfectly conducting container. Then $Q=\Delta U_{\rm int}=0$, so $W=0$ as well, in close analogy with the situation for a free expansion of a gas. We can deduce T_f from the constraint that $\Delta U_{\rm int}=0$; Eqs. (1) and (5) then enable T_f to be found in terms of T_i, ω_{oi}, and ω_{of}.

For this irreversible process, we can now show that $\Delta S_{\rm cal}>0$, as well as that $T_f>T_i$. (The first point is certainly the more critical one, as $\Delta S_{\rm cal}>0$ should always occur for an irreversible process, while the temperature need not always increase for such an operation.) To prove $\Delta S_{\rm cal}>0$, one can first readily show that $\partial U_{\rm int}/\partial\omega_o>0$ and $\partial U_{\rm int}/\partial T\geq 0\ \forall\ T\geq 0$ and $\omega_o\geq 0$, where $\partial U_{\rm int}/\partial T=0$ only when $T\to 0$. Hence, $U_{\rm int}$ monotonically increases (decreases) when T is held fixed and ω_o is increased (decreased), or when ω_o is held fixed and T is increased (decreased). Since in our example $\omega_{of}<\omega_{oi}$, to have $\Delta U_{\rm int}=0$ we must have that $T_f>T_i$; hence, the temperature increases for this irreversible process.

Using this result to show that $\Delta S_{\rm cal}>0$, first let

$$S'\equiv\frac{\hbar}{2}\vartheta\coth\left[\frac{\hbar\vartheta}{2k_B}\right]-k_B\ln\sinh\left[\frac{\hbar\vartheta}{2k_B}\right], \tag{7}$$

where $\vartheta=\omega_o/T$. Then,

$$\frac{\partial S'}{\partial\vartheta}=-\frac{\hbar^2\vartheta}{4k_B}\left[\sinh\left[\frac{\hbar\vartheta}{2k_B}\right]\right]^{-2}. \tag{8}$$

Hence, $\partial S'/\partial\vartheta\leq 0\ \forall\ \vartheta\geq 0$, and $S'=0$ only when $\vartheta\to\infty$. Hence, when ω_o/T decreases, as in our example, S' increases. Incorporating this information with the additional term of $(\frac{4}{3}\sigma'T^3\mathcal{V})$ in Eq. (6), we see that when $\omega_{of}<\omega_{oi}$ and $T_f>T_i$ then $\Delta S_{\rm cal}>0$. Since $\Delta S_{\rm cal}$ represents the net change in caloric entropy during this adiabatic irreversible process for the radiation and oscillator system within $\mathcal{V}$, as well as the surrounding universe, we have verified that the second law of thermodynamics does hold here.

III. EXAMPLE OF AN EXPLICIT NUMERICAL CALCULATION

To gain more insight into the typical thermodynamic behavior that results in T and $S_{\rm cal}$ both increasing for this irreversible process, we now turn to a numerical example worked out in detail. Let $T_i=100$ K, $\omega_{oi}=2.0\times10^{14}$ sec^{-1}, and $\omega_{of}=2.0\times10^{13}$ sec^{-1}. Hence, ω_o is assumed here to have been rapidly decreased by a factor of 10, to correspond with the idea of rapidly expanding the region accessible to the oscillator. Let the conducting box enclosing the one-dimensional oscillator be a cube with sides of length 188 μm. [This size was chosen to allow the use of the continuum thermal radiation energy expression of $\Delta(\sigma'T^4\mathcal{V})$.] As a guide for selecting a sufficiently large size to allow this approximation, the criterion of $d\approx\lambda/2$, where $\lambda=2\pi c/\omega_o$ is the wavelength at the resonant frequency of the oscillator and d is the length of the side of the cavity, was used as a gauge for the point at which taking discrete radiation modes into account in the analysis is known to become important [37,38]. If d is much larger than $\lambda/2$, then the continuum approximation should be adequate. In the present case, with $\omega_{oi}=2.0\times10^{14}$ sec^{-1} and $\omega_{of}=2.0\times10^{13}$ sec^{-1}, $\lambda/2$ equals 4.7 and 47.0 μm, respectively. Since the cutoff for needing discrete mode analysis is fairly abrupt [37,38], choosing the side of the cavity to be four times the largest of these quantities (i.e., we let $d=4\times47$ μm) should be quite reasonable.

Due to the relatively large volume chosen, the predicted temperature variation will only be a fraction of a Kelvin, which is a small effect to be sure, but still large enough to adequately show significant thermodynamic changes. One should note that a much smaller volume will yield *dramatically* larger temperature variations. For example, if $d\approx5$ μm, thereby clearly placing this problem within the regime of what has become known as cavity quantum electrodynamics [37] (SED has been used to study phenomena in this domain [38]), then a rough estimate indicates that the temperature variation over this process would be several hundred Kelvin. Since our numerical example here is only for illustrative purposes, we merely note this fact and continue to proceed with the less dramatic, but also much simpler to analyze, larger volume domain.

Table I lists seven different thermodynamic operations performed on the oscillator system. Plots involving temperature, resonant frequency, internal energy, and caloric entropy during these thermodynamic operations are shown in Fig. 2. Figure 2(a) shows all the operations versus resonant frequency and temperature, while Figs. 2(b), 2(d), and 2(f) show how $U_{\rm int}$ and $S_{\rm cal}$ vary with temperature, and Figs. 2(c) and 2(e) show how $U_{\rm int}$ and $S_{\rm cal}$ vary with ω_o, for these paths.

Path No. 1 in Table I represents the irreversible thermodynamic operation corresponding to Fig. 1(a); here the spring constant of our one-dimensional oscillator has

TABLE I. Thermodynamic operations performed on the oscillator system.

Path No.	Thermodynamic path	Description of process
1	$a \to c$	Irreversibly "free relaxation" of ω_o ($U_{\rm int}$ fixed)
2	$a \to b$	Reversibly and adiabatically decrease ω_o ($S_{\rm cal}$ fixed)
3	$b \to c$	Reversibly heat while holding ω_o fixed (ω_o fixed)
4	$a \to e$	Reversibly and isothermally decrease ω_o (T fixed)
5	$e \to c$	Reversibly and adiabatically increase ω_o ($S_{\rm cal}$ fixed)
6	$c \to d$	Reversibly and adiabatically increase ω_o ($S_{\rm cal}$ fixed)
7	$d \to a$	Reversibly cool while holding ω_o fixed (ω_o fixed)

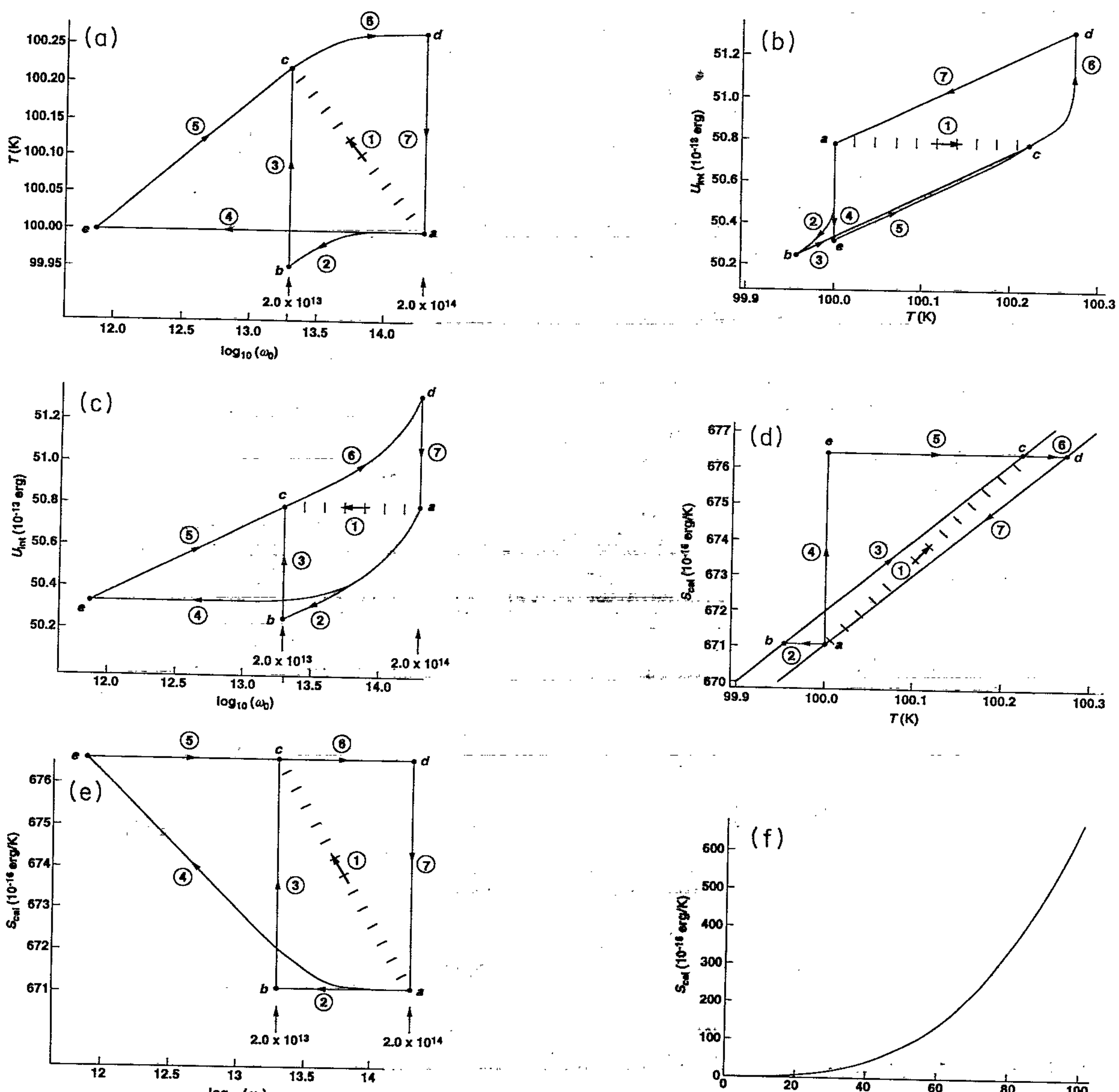

FIG. 2. A set of plots showing thermodynamic properties of the one-dimensional oscillator system while undergoing the processes listed in Table I. The hatched lines represent the irreversible process corresponding to Fig. 1(a), while all other paths are reversible. (a) T vs ω_o; (b) $U_{\rm int}$ vs T; (c) $U_{\rm int}$ vs ω_o; (d) $S_{\rm cal}$ vs T; (e) $S_{\rm cal}$ vs ω_o; (f) $S_{\rm cal}$ vs T.

been rapidly relaxed from ω_{oi} to ω_{of}. The hatched lines in each of the plots in Fig. 2 indicate this irreversible process. Point c in these plots was deduced by requiring that U_{int} at point c equaled U_{int} at point a, resulting in $T_c = 100.221$ K and $\Delta T = +0.221$ K.

To correspond with Fig. 1(b), several reversible thermodynamic processes are indicated in Table I and Fig. 2 that connect states a and c. For example, during path No. 2, the resonant frequency of the oscillator was slowly decreased from ω_{oi} to ω_{of}, while preventing any heat from flowing out of the containing box [the constant S_{cal} can be seen in Figs. 2(d) and 2(e) for path No. 2]. Hence this path corresponds directly with Fig. 1(b). Since work was essentially being done *by* the system during this adiabatic resonant frequency relaxation, U_{int} decreased [Figs. 2(b) and 2(c)], and the temperature decreased to $T_b = 99.954$ K.

To reversibly change the resulting state at b to the same final state of the irreversible process at c, a second path was chosen: path No. 3. Here, the resonant frequency was held fixed, stopcocks were opened, and the box and its contents were slowly heated until point c was reached. As can be seen by path No. 3 in Figs. 2(a), 2(b), and 2(d), the amount of heat needed was sufficiently high to raise the final temperature of T_c above T_a, as predicted. Furthermore, S_{cal} clearly rose so that $S_{\text{cal},c} > S_{\text{cal},a} = S_{\text{cal},b}$ [Figs. 2(d) and 2(e)].

Other reversible thermodynamic processes that could be chosen to connect states a and c were also plotted in Fig. 2. For example, paths No. 4 and No. 5 represent two sections of a Carnot cycle consisting of, respectively, a reversible isothermal and a subsequent reversible adiabatic process. Point e between these two paths resulted in a significantly lower resonant frequency of 7.27×10^{11} sec^{-1} to connect the two curves. Consequently, we should emphasize that point e, and the part of the paths near it, are calculated here with an inaccurate physical description, since ω_o is certainly small enough here to require the discrete mode analysis mentioned above, rather than continuing to treat the thermal radiation spectrum as being continuous. Despite this deficiency, paths No. 4 and No. 5 are still included, simply to indicate the basic path structure and to show what would happen if a continuum spectrum was used throughout.

The other paths in Fig. 2 are accurately calculated. Paths No. 6 and No. 7 represent the reverse procedure of paths No. 2 and No. 3. They connect states $c \rightarrow d$ and $d \rightarrow a$, respectively, thereby forming a thermodynamic cycle of alternating operations with paths No. 2 and No. 3.

The two diagonal curves in Fig. 2(d) represent curves of constant ω_o for S_{cal} vs T, when $\omega_{oi} = 2 \times 10^{14}$ sec^{-1} and $\omega_{of} = 2 \times 10^{13}$ sec^{-1}. Figure 2(f) attempts to place 2(d) in proper perspective by showing S_{cal} vs T for a much larger range of T. The diagonal curves in Fig. 2(d) are too close together to be seen separately in Fig. 2(f); however, they obey the structure of being wider apart for $T > 0$ than at $T = 0$, and joining together as $T \rightarrow 0$, thereby enabling the system to satisfy the third law of thermodynamics (see, for example, Ref. [32], Fig. 19–12(b)).

An interesting feature to note is what happens to the plots in Fig. 2 as $T_i \rightarrow 0$. As $T_i \rightarrow 0$, the adiabatic curves merge with the isothermal curves. Thus, paths No. 2, No. 5, and No. 6, which are all adiabatic ones, would become increasingly horizontal in Fig. 2(a) as $T_i \rightarrow 0$, and vertical in Fig. 2(b) (i.e., they become more like isothermal curves as $T_i \rightarrow 0$). Although these paths would remain horizontal in Figs. 2(d) and 2(e), since $S_{\text{cal}} = \text{const}$ for these paths, the ΔT width of these curves in Fig. 2(d) would decrease, in correspondence with the same curves in Fig. 2(b) becoming more vertical. In the limit of $T_i \rightarrow 0$, these curves in Fig. 2(d) would shrink to a point; namely, the point at $T = 0$ where all constant ω_o curves in Fig. 2(f) must converge.

Regarding Fig. 2(a), if $T_i \rightarrow 0$, then as mentioned above path No. 2 would become a horizontal line, with $T_b = 0$. Path No. 3 (as well as No. 7) would remain a vertical line, since this path simply involves heating the system. Moreover, the structure of path No. 1 would not be affected, since irreversible processes can certainly occur at $T = 0$ and raise the temperature to T_c. However, as $T_i \rightarrow 0$, point e would be pushed to the left in the direction of $\omega_o = 0$ because path No. 5 would become increasingly more horizontal. Eventually, as T_i is lowered, there exists some value T_i that can in principle be readily calculated, where there may not exist a point e that will enable an adiabatic path No. 5 to run between e and c, with $T_c > 0$.

IV. SECULAR ACCELERATION CONSIDERATIONS

The example of a SHO in a box served as a relatively simple vehicle to illustrate thermodynamic effects involving finite changes in singular energies due to ZP and ZPP radiation. We note, however, that this numerical example did not directly involve secular acceleration. Nevertheless, the same basic procedures for analyzing the SHO's thermodynamic behavior should still hold as the effects of this mechanism become more dominant.

For example, suppose we significantly lowered the spring constant, or resonant frequency, of the oscillator. Now the particle would be able to oscillate over much larger regions of space, thereby increasing the effects due to secular acceleration. To adequately analyze the resulting physical behavior, we would need to go beyond the approximations made here, such as the dipole, the subrelativistic, and (probably) the resonant approximations. Moreover, since ω_o would be reduced to a much smaller value than in the previous numerical example, we would need to consider a much larger box size to remain outside the region of cavity quantum electrodynamics $(d > \lambda/2 = \pi c/\omega_o)$ and thus allow the continued use of the continuum expression for the thermal radiation spectrum.

In conjunction with secular acceleration becoming a very significant factor in the behavior of the oscillator, the average kinetic energy change of $\Delta\{mc^2\langle[1-(\dot{x}/c)^2]^{-1/2}\rangle\}$, rather than $\Delta[\frac{1}{2}m\langle\dot{x}^2\rangle]$, would need to be calculated, since we should expect a greater range in speed between the turning points of the oscillating particle. The result should still be a function of ω_o and T, as in Eq. (1), because these are the only controlling parameters of the motion. The compensating change in the average thermal radiation energy would still be $\sigma'\mathcal{V}\Delta(T^4)$, assuming of course that we are still investigating the same irreversible process of path No. 1 in Fig. 2, where $U_{\rm int}=$const.

If the box is not made sufficiently large so that quantum electrodynamic cavity effects can be ignored, then the spectral continuum approximation cannot be made and image charge effects, plus spatially dependent electromagnetic energy density within the box, would need to be taken into account. Also, one would need to check the importance of the change in the electromagnetic energy cross-term contributions, as discussed in Refs. [21,17,20].

Despite these complicating features, there seems little here to indicate that the laws of thermodynamics would be violated via the effects of this mechanism. We would again analyze the effects of irreversible processes via following idealistic, reversible ones. As in the SHO example shown here, we should still have no net heat flow at $T=0$ during reversible, isothermal processes, due to the definition of $T=0$, yet, as also shown here, we should still be able to have irreversible processes leading to nonzero temperature cases starting initially from $T=0$. Moreover, as illustrated in the SHO example, we should still see (1) conservation of energy [21], (2) $\Delta S_{\rm cal}>0$ for irreversible processes, and (3) $\Delta S_{\rm cal}\to 0$ as $T\to 0$ during reversible isothermal processes. The exact, specific shapes of the curves in Fig. 2 would certainly change as secular acceleration becomes more of a factor; however, general features, such as the reversible, adiabatic paths beginning to look more like reversible, isothermal paths as $T\to 0$, and $S_{{\rm cal},c}>S_{{\rm cal},a}$, as seen clearly by following paths No. 2 and No. 3 in Fig. 2(e), should remain the same.

V. CONCLUDING REMARKS

This article has concentrated on the conceptual thermodynamic aspects of secular acceleration, since previous work by others, starting with the early work of Einstein and Hopf [12], has already studied the dynamics of this mechanism in some detail. The thermodynamics appear to readily fit within conventional ideas, provided one adequately considers the subtleties involved with very large systems coming to equilibrium and with the interplay between electromagnetic particles and electromagnetic thermal radiation.

Moreover, we find nothing obvious to suggest that the laws of thermodynamics are violated if an electromagnetic particle in a very large container acquires, on the average, a higher speed as it approaches the walls of the container, versus its speed at the center of the container. The particle may pick up energy from the radiation in the box, just as particles can acquire kinetic energy via much different and more familiar mechanisms, such as that due to image charge effects. The total energy of the entire system will certainly be conserved in either case.

Furthermore, there appears nothing obvious to suggest that such an increase in kinetic energy acts to violate the noncontroversial view of the second law of thermodynamics involving the differences between two thermal equilibrium states. A key point here is that comparing a subensemble of particles just before colliding with the walls, with a subensemble of particles in the center of the container, does *not* represent two thermal equilibrium states. A second key point is that increasing the space within which an electromagnetic particle is free to move *can* enable the particle to pick up additional energy from thermal radiation, just as releasing a colloidal particle adhered to the walls of a container will enable that particle to gain in kinetic energy due to the induced Brownian motion from the heat bath of the liquid in which it is immersed. In neither case are thermodynamic laws violated by the particle in question gaining kinetic energy from the corresponding heat bath.

The increase of kinetic energy of a charged particle, due to ZP plus thermal radiation, when the particle passes from one wall to the next, then bounces off and starts the same process over again, is not so drastically different from the behavior of a charged particle in a SHO potential with ZPP radiation present. A somewhat analogous behavior occurs in both cases although, in the former, the random-walk aspects that lead to increased kinetic energy between wall bounces are the dominant feature. For the SHO example, discussed classically in some detail here, the higher kinetic energy that occurs on the average at the equilibrium point of the SHO potential is largely due to the energy exchanged between the kinetic and potential energies of the system, with the random-like behavior due to the radiation playing a less dominant, but still important role. In both cases, reversible thermodynamic processes can, in principle, be performed on the system to analyze the changes, such as in entropy, that occur in the actual irreversible processes in nature.

To conclude this article, some previously published comments in the literature on this subject [7,9,10] appear to need correction, as they are in conflict with the work presented here. First, Ref. [7] stated, "A free particle is seen to accelerate spontaneously. This seems to violate the first law, but indeed it does not, as the ZPF, when taken as real, has an infinite amount of energy." Actually, the huge amount of energy in the ZPF (zero-point field) does not explain why the first law is not violated by secular acceleration. As with Casimir forces, *changes* in the total energy are the important entities, versus the total amount. As discussed here, the change in kinetic plus electromagnetic energy in any given region of space is certainly predicted to be in exact correspondence with the energy that flows into or out of the region, in agreement with the first law of thermodynamics [21].

Second, some ideas in Refs. [7] and [10] on the possible nonviolation of the second law of thermodynamics need correction. Quoting from p. 302 in Ref. [10]: "As the ZPF of SED is the result of the accelerated motion of all charges in the Universe, it follows that its nature is very different from that of thermal radiation. From this and the arguments above we conclude that no specific temperature can be assigned in a proper manner to the ZPF. There is then no violation of the second law by the acceleration phenomenon as the ZPF is definitely not a reservoir at zero temperature but rather in some sense it behaves as a reservoir at all temperatures."

References [7] and [10] offered a number of ideas as to why one might argue that no specific temperature can be assigned to the ZPF. However, from the more recent work of Refs. [17–22], we see that the ZPF *can* be assigned the temperature $T=0$, in correspondence with the thermodynamic definition of $T=0$ being that heat does not flow at $T=0$ during isothermal, reversible thermodynamic operations [17,19]. Moreover, whether the entire universe could, hypothetically, be held at a temperature of $T=0$, as opposed to a nonzero, finite temperature, the phenomena of secular acceleration would still be an issue, due to the high velocity tail that is predicted to exist for particles situated in ZPP radiation as well as in ZP radiation [4]. Indeed, one need not think of the ZPF as a separate, special entity, but as having the same basic properties of electromagnetic radiation at a nonzero temperature. Our definition of $T=0$, as well as any other nonzero temperature, can be based on the definition of heat flow during reversible, isothermal thermodynamic operations on a particular system in question [32,18,21].

In closing, the secular acceleration mechanism becomes increasingly important as the region of space that an electromagnetic particle can access without colliding with other particles becomes larger. The detailed dynamical mechanism for this behavior seems fairly clear [1–10,12]. However, in the past, there have been concerns that this detailed physical process, involving impulses from ZPP radiation creating a random-walk-like behavior in velocity space for a free particle, somehow appears to violate one or more of the thermodynamic laws of nature. Our investigation of this phenomenon does not reveal any such violation.

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[30] For this reason we considered internal "bumpers" or other constraining force mechanisms in the above thought experiment, to keep the particle away from the perfectly conducting walls [i.e., the outer box drawn in Figs. 1(a) and 1(b)]. This means of devising the thought experiment is certainly not essential; rather, it just seemed to simplify the concerns. One can certainly also have an adiabatic reversible process without having perfectly conducting walls. To do so, one needs to adjust the temperature of the medium bathing the container as the reversible process is slowly carried out, such that no heat flows into (out of) the container, from (into) the surrounding medium.

[31] Indeed, this apparent behavior led others to suspect at one time that the ZP spectrum does not represent a true heat reservoir at $T=0$. See Ref. [10], p. 302. However, as discussed here, we see that ZPP radiation is acceptable as a heat reservoir at temperature T, including the case of ZP radiation at $T=0$.

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[33] See, for example, Sec. 8.8 in Ref. [32].

[34] The total kinetic plus potential energy of the oscillator equals, in the resonant limit,

$$\mathcal{E}=\frac{m}{2}\langle \dot{x}^2\rangle+\frac{m\omega_0^2}{2}\langle x^2\rangle=\pi^2h^2(\omega_o,T) .$$

Rewriting Eq. (3) in terms of $\mathcal{E}$, we see that for $\delta T=0$,

$$Q=\delta\omega_o\omega_o\frac{\partial}{\partial\omega_o}\left[\frac{\mathcal{E}}{\omega_o}\right] .$$

For the SHO, the quantity of $\mathcal{E}/\omega_o$ is significant since it is equal to the adiabatic invariant of an uncharged SHO in classical mechanics, as defined by the action variable $J=\oint p\,dq$. In order for $\mathcal{E}/\omega_o$ to remain equal to a constant value of $\pi^2\kappa$ for a charged SHO, while slowly changing the resonant frequency of the oscillator and while bathing it in thermal radiation, then $h^2=\mathcal{E}/\pi^2=\kappa\omega$. Thus the following two features are seen here to be closely connected, namely, the spectrum that yields "no heat flow" during reversible processes at $T=0$, and the spectrum that is invariant under slow (adiabatic) changes of external parameters, like spring constants and applied fields. This connection can be made even tighter, but we leave this analysis for future work. The second property above, involving adiabatic invariants, was deduced by T. H. Boyer, Phys. Rev. A **18**, 1238 (1978).

[35] For a sufficiently large and massive hypothetical container, gravitational effects involving the curvature of space and time would certainly need to enter into our discussion, thereby complicating the required analysis. We do not address such points here, but simply note that they exist. The basic, conceptual, physical points discussed in this article should still remain the same, despite such complicating issues.

[36] The qualification of "generalized" Wien's displacement law means that the usual form of Wien's displacement law is shown to hold here, but, as in Refs. [18,19], the law is shown to hold even in the case when the thermal equilibrium radiation spectrum does not vanish when $T\rightarrow0$. As discussed in Refs. [18,19], the original derivations for Wien's displacement law implicitly made the assumption that $\rho(\omega,T)\rightarrow0$ as $T\rightarrow0$.

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