Energetics of nuclear reactions

Notes on General Chemistry

http://quantum.bu.edu/notes/GeneralChemistry/NuclearEnergetics.pdf
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The energy change in nuclear reactions is due to the gain or loss of mass in the reaction. According to the Einstein mass-energy equivalence principle, if mass changes by Δm, then there is an associated energy change $\Delta E = \Delta m\, c^2$, where c = 299792458 m/s is the speed of light (in a vacuum).

■ Nuclear binding energy per nucleon

A neutral atom of mass number A and atomic number Z is composed of Z electrons, Z protons, and $A - Z$ neutrons. If we compare the mass of these components to the mass of the atom, *the atom always has less mass*! This means that the atom is energetically more stable than its separated components.

To see how to confirm the mass loss on formation of an atom from its components, let's calculate the mass loss for the formation of an atom of argon-40, the most stable isotope of argon. Web Elements,

> http://www.webelements.com/webelements/elements/text/periodic-table/isot.html

lists the molar masses of every isotope of every element in the periodic table. The entries for argon at

> http://www.webelements.com/webelements/elements/text/Ar/isot.html

list argon-40 as the most abundant naturally occurring isotope and as having the molar mass 39.9623837 g/mol. Argon-40 has $A = 40$ and $Z = 18$, and so its components are 18 electrons, 18 protons, and $40 - 18 = 22$ neutrons. To compute the mass of these components, we can use the electron mass, 9.10938×10^{-31} kg, the proton mass, 1.67262×10^{-27} kg, and the neutron mass, 1.67493×10^{-27} kg, to calculate the corresponding molar masses.

> Verify that the molar mass of the electron, proton, and neutron is 0.00054858 g/mol, 1.00728 g/mol, and 1.00866 g/mol, respectively.

Taking into account how many moles of electrons, protons, and neutrons compose a mole of argon-40, we can calculate the total molar mass of these components.

> Verify that the total molar mass of the components is 40.3314790 g/mol.

Since this is more than the molar mass of argon-40, 39.9623837 g/mol, there is a mass loss of

$$\Delta m = m_f - m_i = 39.9623837\, g/\text{mol} - 40.3314790\, g/\text{mol} = -0.369095\, g/\text{mol}$$

and a corresponding energy loss of

$$\Delta E = \Delta m\, c^2 = -0.369095\, g\,/\,\text{mol} \times (299792458\, m\,/\,s)^2 = -3.31726 \times 10^{10}\ \text{kJ}\,/\,\text{mol}.$$

> Use the fact that $1\ \text{kJ} = 10^3\ \text{J} = 10^3\ \text{kg m}^2\,/\,\text{s}^2$ to verify this energy change.

The energy loss is known as the molar nuclear binding energy of the isotope.

> Will a different isotope of argon have a different molar nuclear bindng energy?

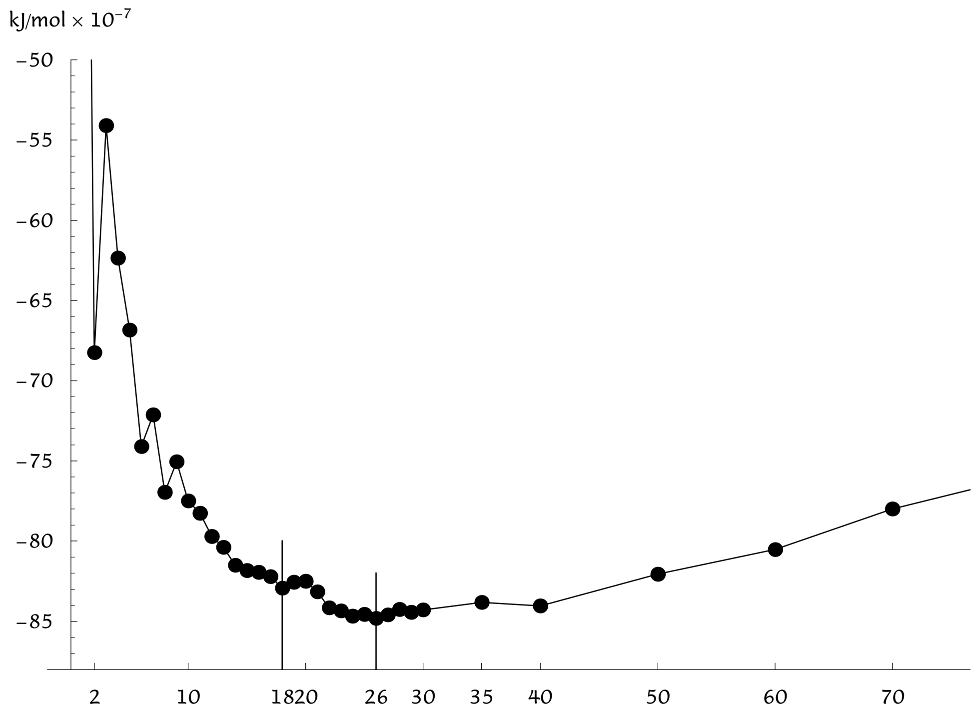
■ Trend of nuclear binding energy per nucleon

Since different atoms have different numbers of nucleons, it is helpful to compare binding energies in terms of the binding energy per nucleon, by dividing by the number of nucleons in the nucleus.

> Show that the binding energy energy per nucleon of argon-40 is 82.9316×10^7 kJ/mol.

This is the value shown for $Z = 18$ (argon) in Figure 3.18 on page 187 of the text *Chemistry/A Project of the American Chemical Society*.

Using the data from Web Elements we can compute the nuclear binding energy per nucleon for all of the elements. Here is a display of the results for the most stable isotopes of each element up to $Z = 30$ (zinc) and representative elements with higher Z.



Binding energy per nucleon versus atomic number Z for the most stable isotope of each element. The thin vertical line marks the value -82.9316×10^7 kJ/mol. for Ar-40 ($Z = 18$) and the value -84.8135×10^7 kJ/mol for Fe-56 ($Z = 26$). The value of H-1, $-0.0000893456 \times 10^7$ kJ/mol (not shown), is essentially zero on this scale.

The figure shows that the most stable element is iron-56. This stability valley is the reason that there is a peak at iron in the cosmic abundance of elements. Formation of elements lighter than iron by fusion of still lighter elements releases energy whereas energy is required to form elements heavier than iron by fusing lighter elements.