Half life simulation using coin flips
Notes on General Chemistry

In lecture we illustrated how half life decay works using coin flips to mark each half life. Here are the details of what we did and how they relate to calculations using half life decay data.

We began with everyone in the class standing.

The first half life consisted of the results of a coin flip: Students whose flipped coin was head down sat down. Those students whose flipped coin was head up remained standing. Students recorded the results of the half life using their CPS response pads to select "Still standing" or "Sitting down."

The second and subsequent half lives consisted of the results of a coin flip by *only those students who were still standing after the previous half life*: Students whose flipped coin was head down sat down, and students whose flipped coin was head up remained standing. *Only the students who participated in the coin flip* (that is, only the students who were standing before the coin flip) recorded the results of the half life using their CPS response pads to select "Still standing" or "Sitting down."

Half lives elapsed until only several students remained standing.

Simulated results

Here are example results for an initial "population" N_0 of 256 students, in tabular form and plotted as points together with the idealized population decay curve.

coin flip	still standing
0	256
1	131
2	76
3	29
4	19
5	12
6	5
7	1

Example results for an initial "population" N_0 of 256 students.
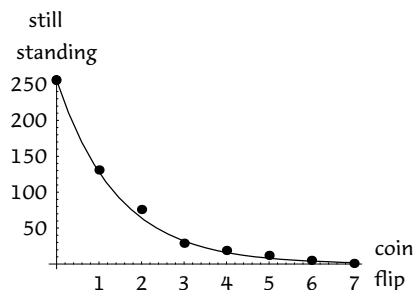
Example results for an initial "population" N_0 of 256 students plotted as points together with the idealized population decay curve.

The results at each half life differ from the idealized curve, because with such a small sample, it is quite unlikely that exactly half of the coin flips will be head up or head down.

Idealized decay curve

The idealized decay curve assumes that exactly half of the population decays during each half life. This means that after n half lives the remaining population is

$$N_n = (1/2)^n N_0.$$

Rearranging this expression and then taking the natural logarithm of both sides, we get

$$\ln(N_n / N_0) = \ln[(1/2)^n] = -n \ln(2),$$

since $\ln(1/2) = \ln(1) - \ln(2) = -\ln(2)$. Taking the antilogarithm of both sides,

$$N_n / N_0 = e^{-n \ln(2)},$$

we can express the population after n half lives as

$$N_n = N_0 \, e^{-n \ln(2)}.$$

The right hand side of this equation is the idealized decay curve. It is important because it allows the calculation of the population not just for integer values of n but fractional half lives too.

Counting class attendance by counting half lives

In lecture we found that after $n = 6$ coin flips $N_6 = 3$ students remained. If we "round" this to $N_6 = 2$, then the initial number of students—the class attendance—evaluates to

$$N_0 = N_6 \, 2^6 = 2^7 = 128.$$

If we "round" this to $N_6 = 4$, then the initial number of students instead evaluates to

$$N_0 = N_6 \, 2^6 = 2^8 = 256.$$

In fact, there were 141 students. This shows that to within a power of 2 we were able to use "half life decay" to "count" the initial population size.

For such a small sample, this is definitely the hard way to count. But if we had a very large sample, say the population of a city, it would be much quicker than counting each person individually.

Estimating how many half lives have elapsed

Sometimes we want to know how much time is required for a substance with a known half life to decay to a certain fraction of its original amount. A way to answer this kind of question is to write the fraction remaining as

$$N_n / N_0 = (1/2)^n$$

in terms of the number n of half lives that have elapsed. By taking the logarithm of both sides, it is then easy to solve for n.

For example, how much time is required for a sample with a half life of 21 minutes to decay to 40% of its original amount. We have

$$\log(N_n / N_0) = \log(0.40) = \log[(1/2)^n] = -n \log(2)$$

Since $\log(0.40) = \log(4.0 \times 10^{-1}) = \log(4.0) - 1 = 0.60 - 1 = -0.40$ and since $\log(2) = 0.30$, we have $-0.40 = -n\, 0.30$ and so $n = 4/3$. This means $4/3 \times 21 = 28$ minutes us required.