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[TP] For steam \rightarrow water, which of the following must always be true?

14% 1. $\Delta S_{\text{sys}} < 0$ and $\Delta S_{\text{sur}} < 0$

14% 2. $\Delta S_{\text{sys}} < 0$ and $\Delta S_{\text{sur}} = 0$

14% 3. $\Delta S_{\text{sys}} < 0$ and $\Delta S_{\text{sur}} > 0$

14% 4. $\Delta S_{\text{tot}} < 0$

14% 5. $\Delta S_{\text{tot}} = 0$

14% 6. $\Delta S_{\text{tot}} > 0$

14% 7. None of the above

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Lecture 30 CH102 A1 (MWF 9:05 am)
Friday, April 12, 2019

- Counting energy dispersal
- Heat (energy) flow \rightarrow entropy change
- Spontaneity of phase transitions: water \rightleftharpoons steam

Next lecture: Continue ch17

Notes: Spontaneity: Second law of thermodynamics
<http://quantum.bu.edu/courses/ch102-spring-2018/handouts.html>

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Counting energy dispersal

The goal: Entropy change is
proportional to enthalpy change and
inversely proportional to absolute temperature ...

$$\Delta S = \frac{\Delta H}{T}$$

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Counting energy dispersal

Unique (distinguishable) arrangements of
 q identical quanta distributed among
 m identical molecules

For example, four quanta among three molecules ...
 $q|q|qq, q|qqq, |qqq|, q|qq|q, \text{ etc.}$

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Counting energy dispersal

Four quanta among three molecules ...
 $q|q|qq, q||qqq, |qqqq|, q|qq|q,$ etc.

How many unique such arrangements, $W_e(4,3)$?

$$W_e(4,3) = \frac{6!}{(4! 2!)} = 15$$

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Counting energy dispersal

Unique (**distinguishable**) arrangements of
 q identical quanta among
 m identical molecules ...

$$W_e(q, m) = \frac{(q + m - 1)!}{q! (m - 1)!}$$

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Heat (energy) flow \rightarrow entropy change

Adding energy increases energy dispersal, $W_e(q, m)$.
 This means that adding energy increases S .
 How does ΔS depend on the energy (number of energy units q) **initially present**?
 How does ΔS depend on the **energy added** ($\Delta q = q_f - q_i$)?

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ΔS versus energy initially present (q)

Which energy change $(q, m) \rightarrow (q + 1, m)$ has **larger ΔS** ...
 adding 1 quantum to 4: $(4, m) \rightarrow (5, m)$ or ...
 adding 1 quantum to 9: $(9, m) \rightarrow (10, m)$?

We can answer these questions using by analyzing **energy dispersal** and the corresponding **entropy change**.

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[TP] The change $W_e(9, 4) \rightarrow W_e(10, 4)$ is $286 - 220 = 66$,
and the change $W_e(4, 4) \rightarrow W_e(5, 4)$ is $56 - 35 = 21$.

For which change do you expect the entropy increase to be greater?

- 50% 1. $W_e(9, 4) \rightarrow W_e(10, 4)$
50% 2. $W_e(4, 4) \rightarrow W_e(5, 4)$



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[TP] The general expression for the entropy change of a 4-molecule system with q quanta gaining 1 quantum is ...

- 25% 1. $W_e(q + 1, 4) - W_e(q, 4)$
25% 2. $\ln[W_e(q + 1, 4)] - \ln[W_e(q, 4)]$
25% 3. $\ln[W_e(q + 1, 4)/W_e(q, 4)]$
25% 4. Both 2 and 3



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[TP] Which is larger? (Speeds the evaluations by simplifying ratios.)

- 0% 1. $W_e(10, 4)/W_e(9, 4)$
0% 2. $W_e(5, 4)/W_e(4, 4)$
0% 3. They are equal
0% 4. More information is needed



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ΔS versus energy initially present (q)

Which change $(q, m) \rightarrow (q + 1, m)$ has larger ΔS ,

adding 1 to 5: $(4, m) \rightarrow (5, m)$ or ...

adding 1 to 9: $(9, m) \rightarrow (10, m)$?

$(4, m) \rightarrow (5, m)$ has larger ΔS than $(9, m) \rightarrow (10, m)$

For a given energy change, ΔS is larger, the less energy (q) initially present.

ΔS is larger, the lower T



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Heat (energy) flow \rightarrow entropy change

For a given energy change, ΔS is larger, the less energy (q) initially present.

How does ΔS depend on the amount of energy added, $\Delta q = q_f - q_i$, for a given amount of energy present initially, q_i ?

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[TP] Which is larger? (Simplifying ratios speeds the evaluations.)

25% 1. $W_e(6,4)/W_e(4,4)$
 25% 2. $W_e(5,4)/W_e(4,4)$
 25% 3. They are equal
 25% 4. More information is needed

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0 of 0

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ΔS versus the energy added (Δq)

Which change $(q, m) \rightarrow (q + 1, m)$ or $(q + 2, m)$ has larger ΔS ,

adding 1 to 4: $(4, m) \rightarrow (5, m)$ or ...
 adding 2 to 4: $(4, m) \rightarrow (6, m)$?

$(4, m) \rightarrow (6, m)$ has larger ΔS than $(4, m) \rightarrow (5, m)$

For a given energy initially present, q_i , ΔS is larger the larger the energy added, $\Delta q = q_f - q_i$.

ΔS is larger, the greater ΔH

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Heat (energy) flow \rightarrow entropy change

Which changes have larger ΔS ?

$(4, m) \rightarrow (5, m)$ has larger ΔS than $(9, m) \rightarrow (10, m)$; $\Delta S \propto 1/T$

$(4, m) \rightarrow (6, m)$ has larger ΔS than $(4, m) \rightarrow (5, m)$; $\Delta S \propto \Delta H$

Taken together, these results illustrate that ...

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Heat (energy) flow → entropy change

ΔS is larger the lower T

ΔS larger the greater ΔH

$\Delta S = \Delta H/T$

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Spontaneity of phase transitions

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Taking stock

Spontaneity **means** that ...

$$\Delta S_{\text{tot}} = \Delta S_{\text{sys}} + \Delta S_{\text{sur}} > 0$$

Spontaneity **does not** require that ...

$$\Delta S_{\text{sys}} > 0 \text{ or } \Delta S_{\text{sur}} > 0$$

A neat illustration of the **separate roles** of ΔS_{sys} and ΔS_{sur} is understanding why **steam condenses** and **water boils**.

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steam → water at 94 °C

Super-cooled steam at 94 °C condenses **spontaneously** to water.

Spontaneity means $\Delta S_{\text{tot}} > 0$

Since condensation releases ΔH_{vap} to the surroundings, $\Delta S_{\text{sur}} > 0$

But “gas → liquid” means $\Delta S_{\text{sys}} < 0$

This means it must be ΔS_{sur} that makes $\Delta S_{\text{tot}} > 0$



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