Discovery of Dynamic Locational Prices on Power Distribution Networks: Efficient and Robust Distributed Algorithms in the Presence of Binding Voltage Constraints

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Abstract- **Distribution network electricity markets can evaluate and consequently integrate Distributed Energy Resources (DERs), including electric vehicles, PV, storage and storage-like flexible loads. Fully distributed algorithms (FDA) proposed so far have been shown to handle full AC load flow modeling and complex DER preferences and capabilities and overcome insurmountable computational limitations of centralized models. Nevertheless, even fully distributed algorithms become impractical and slow under binding voltage constraint situations. This paper addresses Distribution Network Locational Marginal Price (DLMP) discovery under binding voltage constraints by proposing (i) a penalty that replaces hard voltage constraints and (ii) an iterative, partially distributed algorithm (PDA) where DERs adapt to DLMP estimates by solving individual benefit maximization problems in parallel, followed by a centralized AC Load Flow solution. The proposed algorithm is shown to be robust and significantly faster than a fully distributed algorithm (FDA). Further, the PDA algorithm with voltage penalties is implemented on a realistic 800 bus distribution network and is shown to discover robustly and tractably DLMPs for a 24 hour day ahead market clearing problem.**

I. INTRODUCTION

Increasing penetration of Distributed Energy Resources (DER) has sparked interest in the distribution side of the electricity grid. Advances in computation and communication are making it possible to extend wholesale power markets to distribution network connected DERs, as a means of efficient DER integration and valuation.

Much like transmission power markets discover Locational Marginal Prices (LMP) of real power, the proposed distribution power markets may discover Distribution Locational Marginal Prices (DLMPs) of real and reactive power.

Previous work , [1] and [2] amongst others, has used centralized models for distribution power market clearing. Because of the tree structure of typical distribution networks, there is a unique primal solution [3], paired with a unique dual solution. The dual prices of the real and reactive energy balance constraints at the optimal solution are the real and reactive power DLMP respectively. The problem formulation in these papers reveals the huge computational burden of the centralized formulation caused by:

1. Non-convexities in the alternating current (AC) power flow constraints
2. Market participants like electric vehicles and HVAC devices exhibit time coupled constraints
3. Market participants number in the order of hundreds of thousands.

Therefore, relevant literature has moved towards decentralized methods for distribution power market clearing [4], [5]. Our previous work in this direction [6] and [7] extends [4]. In a fully distributed algorithm (FDA) framework, each device (load, generator, DER etc) and each distribution network element (line, transformer) solve an individual problem that optimizes their preferences. This optimization problem takes into account the relevant capabilities as well as messages received from the bus(es) each element is connected to. Generators, loads etc., connected to a single bus, receive such messages from a single bus, while lines and transformers receive messages from the two buses they are connected to. Messages are nothing more and nothing less than tentative DLMP estimates reflecting the imbalance of real power, reactive power and voltage consistency at the relevant bus(es). After the optimization problems of devices and line/transformer have been computed, the results are passed on to the connected bus(es) for the latter to update the prices. This loop repeats until the imbalances are below a threshold.

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Preprint submitted to 2017 American Control Conference.
Received September 20, 2016.

The abovementioned price messages should converge to the DLMPs of real and reactive power, as the iteration count increases and DERs adapt to successive DLMP estimates. [2] and [7] showed the effect that binding voltage magnitude constraints have on the real and reactive power DLMPs. [7] discovered that in cases where voltage levels are persistently binding even in a small subset of busses, convergence to the optimal DLMPs is very slow when fully distributed algorithms are employed. [7] proposes a filter to speed up convergence in both voltage shadow prices as well as real and reactive power prices. In essence, this filter is imposing the system-wide power flow constraints.

Motivated mostly by the shortcomings discussed in [7], this paper proposes a new model that relies on decentralized DER decision making followed by centralized ex-post DLMP calculations. We call this model Partially Distributed Algorithm (PDA). More specifically, DERs solve sub-problems individually aiming at minimizing their costs (or maximizing their benefit) conditional upon DLMP estimates at their connection bus. DLMP estimates and flows on all distribution network resources (lines, transformers, etc.,) are evaluated centrally. This is done by solving an AC load flow and its associated sensitivities that imply ex-post DLMP estimates using the DLMP price decomposition reported in [8]. Equivalently, we can solve a mock centralized market clearing problem with fixed DER behavior whose primal solution are the network flows and dual variables of nodal balances the ex-post DLMPs.

We draw attention to an additional difficulty by noting that, despite the existence of a slack bus, upper and lower voltage magnitude constraints can result in the centralized load flow with fixed DER behavior being infeasible. We therefore propose to replace hard voltage constraints by adding convex penalty terms in the objective function. These terms are practically zero in the feasible range and increase steeply as the desirable voltage bounds are exceeded. Under the soft voltage constraint formulation, PDA algorithms are able to overcome infeasibility issues. If we also augment the fully distributed algorithms (FDA) of [7], [6] in this fashion by adding the same cost terms to line sub-problems, we can replace hard voltage constraints in the FDA as well. We compare FDA and PDA algorithms relative to convergence speed. They both speed up when hard voltage constraints are replaced by appropriately designed penalties. The speed-up is more significant when voltage levels are binding in at least some buses. We observe that PDA algorithms converge to the centralized market clearing solution benchmark (i.e., for problem sizes that the centralized problem is tractable) significantly faster than FDA algorithms.

The remainder of this paper is organized as follows: Section II defines notation, Section III defines and compares the different market clearing algorithms. Section IV presents simulation results, while Section V concludes.

II. NOMENCLATURE

Lagrange Multipliers

$\hat{\pi}_b^{P,i}$	Tentative price of real power at bus b at iteration i
$\hat{\pi}_b^{Q,i}$	Tentative price of reactive power at bus b at iteration i
$\hat{\zeta}_{b,b'}^{i}$	Tentative price of voltage magnitude consistency at end b of line (b,b')iteration i
π_b^P	Optimal shadow price of real power balance of bus b
π_b^Q	Optimal shadow price of reactive power balance of bus b
$\rightarrow$	A symbol used to associate a shadow price to an equality or inequality constraint.

Functions

$f(\bullet)$	Convex cost function (or negative utility)
$H(\bullet)$	Heaviside function, whose value is $H(\bullet \geq 0) = \infty$ and $H(\bullet \leq 0) = 0$

Parameters

$\|A_b\|$	Number of devices α connected to bus b
$\|H_b\|$	Number of lines (b,b') connected to bus b, i.e. degree of node b.
π_∞^{OC}	Opportunity cost per kW of substation bus auxiliary generator disabled from producing real power or reserves in order to compensate for reactive power Q_∞
π_∞^{P}	Real Power Locational Marginal Price (LMP) at the substation bus
i	Iteration Count
$k_\bullet$	Constant
$r_{b,b'}, x_{b,b'}$	Resistance and reactance respectively of line or transformer connecting buses b and b'

Subscripts and Sets

(b,b')	Subscript denoting a line or transformer connecting bus b to b'.
α	Subscript denoting a specific device that connects to some network bus b. The notation $\alpha \in G$ means that device α is a generator and that $\alpha \notin D,E,F$
∞	Subscript denoting the substation bus
b,b'	Subscripts denoting a typical distribution bus
G, D, E, F	Set of all network generators, loads, distributed energy resources and capacitors respectively
G_b, D_b, E_b, F_b	Set of generators, loads, DERs and capacitors respectively connected to bus b
H	Set of all network lines or transformers
$H_b, H_b \subset H$	Set of all lines/transformers connected to bus b
$A_b = G_b \cup D_b \cup E_b \cup F_b$	Set of all devices connected to bus b

Distribution Variables

$l_{b,b'}$	Current squared on line or transformer connecting buses b and b'
P_α, Q_α	Real and reactive power respectively of device α. Negative values denote generation, while positive values denote consumption.
$P_{b,b'}$,$Q_{b,b'}$,$S_{b,b'}$	Real, reactive and apparent power flow respectively on line (b,b').
v_b	Voltage magnitude squared of bus b
$v_{b,b'}$	Voltage magnitude squared at end b of line or transformer (b,b')

III. Distribution Market Clearing Formulations

A. *Centralized, Fully Distributed, and Partially Distributed Algorithms*

First, we repeat a higher level version of the centralized formulation of [2], C-OPT with reference to hours omitted for notational convenience. In this formulation, a Distribution System Market Operator with access to all requisite information (network topology, resources, individual participants, etc.,) solves the following problem:

Centralized Formulation C-OPT

$$\underset{P_\alpha, Q_\alpha, v_\infty}{\text{minimize}} \sum_a f(P_\alpha, Q_\alpha) \tag{1}$$

$$\text{subject to } l_{b,b'} = \frac{P_{b,b'}^2 + Q_{b,b'}^2}{v_b} \tag{2}$$

$$v_{b'} = v_b - 2(r_{b,b'} \cdot P_{b,b'} + x_{b,b'} \cdot Q_{b,b'}) + (r_{b,b'}^2 + x_{b,b'}^2) \cdot l_{b,b'} \tag{3}$$

$$\sum_{\alpha, \alpha \in G_b \cup E_b \cup D_b} P_\alpha + \sum_{b'} P_{b,b'} = 0 \rightarrow \pi_b^P \tag{4}$$

$$\sum_{\alpha, \alpha \in G_b \cup E_b \cup D_b \cup F_b} Q_\alpha + \sum_{b'} Q_{b,b'} = 0 \rightarrow \pi_b^Q \tag{5}$$

$$P_{b,b'} + P_{b',b} = r_{b,b'} l_{b,b'} \tag{6}$$

$$Q_{b,b'} + Q_{b',b} = x_{b,b'} l_{b,b'} \tag{7}$$

$$\underline{v}_b \leq v_b \leq \bar{v_b} \rightarrow \mu_b = \bar{\mu}_b - \underline{\mu}_b \tag{8}$$

$$\text{DER capability constraints} \tag{9}$$

The objective function is the sum of the costs (or negative benefit) of all devices, the minimization of which is performed subject to power flow constraints, namely (2)-(8) as well as device capability constraints (generators, loads, capacitors, DERs). The nodal prices of real power (or DLMP) is the dual variable of the real energy balance constraint (4), π_b^P, and similarly the nodal prices of reactive power (i.e. DLMP of reactive power) are the lagrange multipliers of the reactive energy balance constraint (5), π_b^Q.

The fully distributed, iterative algorithm of [6] and [7], FDA-OPT, is described in a high level below.

Preprint submitted to 2017 American Control Conference.
Received September 20, 2016.

Fully Distributed Formulation FDA-OPT

1. Initialize $i \leftarrow 1$.
2. For $\alpha \in D, G, E, F$ solve:

$$\underset{P_\alpha, Q_\alpha}{\text{minimize}} \begin{cases} f(P_\alpha, Q_\alpha) + \hat{\pi}_b^{P,i} \cdot P_\alpha + \hat{\pi}_b^{Q,i} \cdot Q_\alpha \\ +\text{penalty terms} \end{cases} \tag{10}$$

$$\text{subject to DER capacity constraints} \tag{11}$$

3. For $(b, b') \in H$ solve:

$$\underset{\substack{P_{b,b'}, Q_{b,b'}, v_{b,b'} \\ P_{b',b}, Q_{b',b}, v_{b',b}}}{\text{minimize}} \begin{cases} \hat{\pi}_b^{P,i} \cdot P_{b,b'} + \hat{\pi}_b^{Q,i} \cdot Q_{b,b'} + \hat{\zeta}_{b,b'}^{i} \cdot v_{b,b'} \\ +\hat{\pi}_{b'}^{P,i} \cdot P_{b',b} + \hat{\pi}_{b'}^{Q,i} \cdot Q_{b',b} + \hat{\zeta}_{b',b}^{i} \cdot v_{b',b} \\ +\text{penalty terms} \end{cases} \tag{12}$$

$$\text{subject to Power flow constraints (2)-(8)} \tag{13}$$

4. For all buses update:
$v_b = \frac{\sum_{(b,b') \in H_b} v_{b,b'}}{|H_b|}$
$\hat{\pi}_b^{P,i+1} = \hat{\pi}_b^{P,i+1}(\hat{\pi}_b^{P,i}, P_{\alpha,\alpha \in A_b}, P_{b',(b,b') \in H_b})$
$\hat{\pi}_b^{Q,i+1} = \hat{\pi}_b^{Q,i+1}(\hat{\pi}_b^{Q,i}, Q_{\alpha,\alpha \in A_b}, Q_{b,b',(b,b') \in H_b})$
$\hat{\zeta}_{b,b'}^{i+1} = \hat{\zeta}_{b,b'}^{i+1}(\hat{\zeta}_{b,b'}^{i}, v_{b,b',(b,b') \in H_b})$
5. If tolerance criterion satisfied, terminate.
Else, $i \leftarrow i+1$ and go to 2.

Previous work of the authors [7] has revealed the difficulty of algorithm FDA-OPT to reach the optimal solution of C-OPT, when voltage constraints (8) are binding. The same work [7] proposed a convergence enhancement filter that corrects the prices that FDA-OPT approaches, $\hat{\pi}_b^{P,i}, i \to \infty$ and $\hat{\pi}_b^{Q,i}, i \to \infty$, using first order optimality conditions of problem C-OPT. These conditions allow us to relate the optimal dual variables of C-OPT to the sensitivities of the primal, dependent variables of C-OPT at the optimal solution.

The filter imposes in essence system-wide power flow constraints to correct the real and reactive power prices. Motivated by the filter's effectiveness, we propose here a new formulation, where a single agent calculates the system wide power flow and devices $\alpha \in D, G, E, F$ self-schedule based on price signals they receive. We call this formulation partially distributed algorithm (PDA) or PDA-OPT for short and describe the corresponding algorithm.

Partially Distributed Formulation PDA-OPT

1. Initialize $i \leftarrow 1$.
2. For $\alpha \in D, G, E, F$ solve:

$$\underset{P_\alpha, Q_\alpha}{\text{minimize}} f(P_\alpha, Q_\alpha) + \hat{\pi}_b^{P,i} \cdot P_\alpha + \hat{\pi}_b^{Q,i} \cdot Q_\alpha \tag{14}$$

$$\text{subject to DER capacity constraints} \tag{15}$$

3. The Distribution System Operator calculates the power flow:
minimize 0

$$\text{subject to Power flow constraints (2)-(8)} \to \pi_b^P, \pi_b^Q \tag{16}$$

4. Convergence check: if $\max_b(\hat{\pi}_b^{P,i} - \pi_b^P) \leq$ tolerance and $\max_b(\hat{\pi}_b^{Q,i} - \pi_b^Q) \leq$ tolerance, break.
5. DLMP estimate update mindful of oscillation avoidance and convergence:
$\hat{\pi}_b^{P,i+1} = (1 - h(i)) \cdot \hat{\pi}_b^{P,i} + h(i) \cdot \pi_b^P$ and
$\hat{\pi}_b^{Q,i+1} = (1 - h(i)) \cdot \hat{\pi}_b^{Q,i} + h(i) \cdot \pi_b^Q$.
6. $i \leftarrow i+1$ and go to 2.

Aside from the obvious difference of treating lines in parallel or centrally, PDA-OPT and FDA-OPT differ in the device objective functions, with FDA-OPT including penalties for the purpose of avoiding oscillations that are not present in PDA-OPT. Although PDA-OPT requires oscillation avoidance provision as well, this assistance is provided in step 5 of the PDA-OPT where DLMP estimates are updated with decreasing step size adaptation to ex-post DLMPs. In short, oscillation avoidance is provided in different, though equivalent ways.

The following table summarizes a comparison of the differences of the market clearing formulations described above:

Name	Long Name	DER Scheduling	Power Flow Calculation
C-OPT	Fully Centralized	Centalized	Centralized
FDA-OPT	Fully Distributed	Distributed	Distributed
PDA-OPT	Partially Distributed	Distributed	Centralized

Table 1: Comparison of C-OPT, FDA-OPT and PDA-OPT

B. *Voltage Magnitude Constraints replaced by Penalties*

From the formulation of PDA-OPT described above, it is easy to notice that despite the existence of a slack bus with the requisite real and reactive power providing generators attached to it, the existence of the hard voltage constraints (8) could still lead to the power flow problem being infeasible. To avert this, we replace hard voltage constraints with voltage related penalties added to the objective function.

The hard voltage constraints are equivalent to penalty terms like $f(v) = H(\underline{v} - v) + H(v - \bar{v})$, where H is the Heaviside function. Since these constraints are non-differentiable, we use constraints of the type $f(v) = k_1 \cdot (\exp(k_2 \cdot (v - \bar{v})) + \exp(k_3 \cdot (\underline{v} - v)))$. We tune k_1 to control the contribution of this term to the objective function. In fine tuning parameters k_2, k_3, we choose $k_2 >>$ and $k_3 >>$ for the resulting curve to be as close to a square wave as is practical. Such a choice of large enough k_2, k_3 will also ensure that the new optimal solution will be close to the true optimal solution. This modifies PDA-OPT to the following, that we name PDA-SVC, standing for Soft Voltage Constraints.:

Soft Voltage Constrained Partially Distributed Formulation PDA-SVC

1. Initialize $i \leftarrow 1$.
2. For $\alpha \in D, G, E, F$ solve:

$$\underset{P_\alpha, Q_\alpha}{\text{minimize}} f(P_\alpha, Q_\alpha) + \hat{\pi}_b^{P,i} \cdot P_\alpha + \hat{\pi}_b^{Q,i} \cdot Q_\alpha \tag{17}$$

$$\text{subject to DER capacity constraints} \tag{18}$$

3. The Distribution System Operator calculates the power flow:

$$\underset{v_\infty}{\text{minimize}} \sum_b k_{1,b} \cdot (exp(k_{2,b} \cdot (v_b - \bar{v})) + (exp(k_{3,b} \cdot (\underline{v} - v_b)) \tag{19}$$

$$\text{subject to Power flow constraints (2)-(7)} \rightarrow \pi_b^P, \pi_b^Q \tag{20}$$

4. Convergence check: if $\max_b(\hat{\pi}_b^{P,i} - \pi_b^P) \leq$ tolerance and $\max_b(\hat{\pi}_b^{Q,i} - \pi_b^Q) \leq$ tolerance, break.
5. DLMP estimate update mindful of oscillation avoidance and convergence:
$\hat{\pi}_b^{P,i+1} = (1 - h(i)) \cdot \hat{\pi}_b^{P,i} + h(i) \cdot \pi_b^P$ and
$\hat{\pi}_b^{Q,i+1} = (1 - h(i)) \cdot \hat{\pi}_b^{Q,i} + h(i) \cdot \pi_b^Q$.
6. $i \leftarrow i + 1$ and go to 2.

The fully centralized formulation, with soft voltage constraints, referred to as C-SVC, is similarly modified as follows:

Soft Voltage Constrained Centralized Formulation C-SVC

$$\underset{P_\alpha, Q_\alpha, v_\infty}{\text{minimize}} \begin{cases} \sum_a f(P_\alpha, Q_\alpha) + \\ \sum_b k_{1,b} \cdot (exp(k_{2,b} \cdot (v_b - \bar{v})) + (exp(k_{3,b} \cdot (\underline{v} - v_b)) \end{cases} \tag{21}$$

subject to Power flow constraints (2)-(7)
DER capability constraints

Lastly, we present FDA-SVC, i.e. the fully distributed formulation as augmented in the presence of soft voltage constraints:

Soft Voltage Constrained Fully Distributed Formulation FDA-SVC

1. Initialize $i \leftarrow 1$.
2. For $\alpha \in D, G, E, F$ solve:

$$\underset{P_\alpha, Q_\alpha}{\text{minimize}} \begin{cases} f(P_\alpha, Q_\alpha) + \hat{\pi}_b^{P,i} \cdot P_\alpha + \hat{\pi}_b^{Q,i} \cdot Q_\alpha \\ +\text{penalty terms} \end{cases} \tag{22}$$

$$\text{subject to DER capacity constraints} \tag{23}$$

3. For $(b, b') \in H$ solve [1]:

$$\underset{\substack{P_{b,b'}, Q_{b,b'}, v_{b,b'} \\ P_{b',b}, Q_{b',b}, v_{b',b}}}{\text{minimize}} \begin{cases} \hat{\pi}_b^{P,i} \cdot P_{b,b'} + \hat{\pi}_b^{Q,i} \cdot Q_{b,b'} + \hat{\zeta}_{b,b'}^i \cdot v_{b,b'} \\ + \hat{\pi}_{b'}^{P,i} \cdot P_{b',b} + \hat{\pi}_{b'}^{Q,i} \cdot Q_{b',b} + \hat{\zeta}_{b',b}^i \cdot v_{b',b} \\ + \frac{k_{1,b}}{|H_b|} \cdot (exp(k_{2,b} \cdot (v_{b,b'} - \bar{v})) + \\ (exp(k_{3,b} \cdot (\underline{v} - v_{b,b'}))) \\ + \frac{k_{1,b'}}{|H_{b'}|} \cdot (exp(k_{2,b'} \cdot (v_{b',b} - \bar{v})) + \\ (exp(k_{3,b'} \cdot (\underline{v} - v_{b',b}))) \\ +\text{penalty terms} \end{cases} \tag{24}$$

$$\text{subject to Power flow constraints (2)-(7)} \tag{25}$$

4. For all buses update:
$v_b = \frac{\sum_{(b,b') \in H_b} v_{b,b'}}{|H_b|}$
$\hat{\pi}_b^{P,i+1} = \hat{\pi}_b^{P,i+1}(\hat{\pi}_b^{P,i}, P_{\alpha,\alpha \in A_b}, P_{b,b',(b,b') \in H_b})$
$\hat{\pi}_b^{Q,i+1} = \hat{\pi}_b^{Q,i+1}(\hat{\pi}_b^{Q,i}, Q_{\alpha,\alpha \in A_b}, Q_{b,b',(b,b') \in H_b})$
$\hat{\zeta}_{b,b'}^{i+1} = \hat{\zeta}_{b,b'}^{i+1}(\hat{\zeta}_{b,b'}^i, v_{b,b',(b,b') \in H_b})$
5. If tolerance criterion satisfied, terminate. Else, $i \leftarrow i + 1$ and go to 2.

To conclude this section, we provide a comparison table of all OPT and SVC formulations:

[1] Note the change in the multiplier of the exponential voltage penalty terms. We divide by the degree of each bus b (i.e. the number of lines entering or exiting bus b), since in FDA-SVC this term is used for each line, rather than each bus as in C-SCV and PDA-SVC.

Name	DER Scheduling	Power Flow Calculation	Voltage Constrain
C-OPT	Centralized	Centralized	Hard
C-SVC	Centralized	Centralized	Soft
FDA-OPT	Distributed	Distributed	Hard
FDA-SVC	Distributed	Distributed	Soft
PDA-OPT	Distributed	Centralized	Hard
PDA-SVC	Distributed	Centralized	Soft

Table 2: Comparison of OPT vs SVC formulations

IV. Numerical Results

This section includes numerical results on a realistic 47 bus distribution feeder from Southern California Edison, that was first presented in [9]. That is the same network we used in our past paper [7]. To be able to draw comparisons, we use the same parameters. Specifically, we are focusing on an edge case where distributed real and reactive power provision is disabled, and there are curtailable loads with utility loss functions $-A_\alpha \cdot P_\alpha + B_\alpha \cdot P_\alpha^2, A_\alpha \geq 0, B_\alpha \geq 0$. (This is referred to as case B2 in [7]).

The centralized formulation with hard voltage constraints C-OPT is used as a benchmark. This result will be the true optimal solution of the market clearing problem.

The following two figures illustrate the closeness of the solution obtained by the relaxation of the centralized problem, C-SVC, as compared to C-OPT. Figure 1 does so by means of illustrating the total cost for real power service paid by the curtailable loads, while Figure 2 shows the voltage magnitudes at all load buses. In C-SVC voltages are at most 0.04% away from their true optimal values in C-OPT.

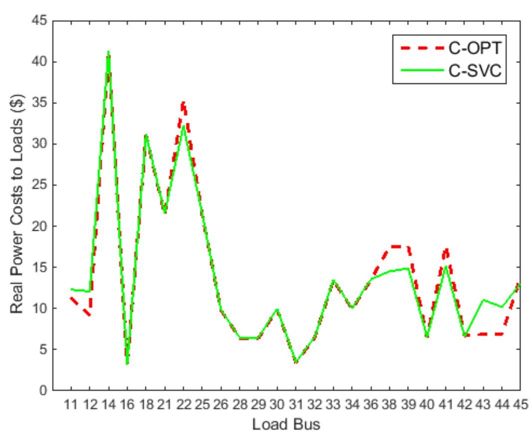

Figure 1: Real Power Costs to Curtailable Loads ($)

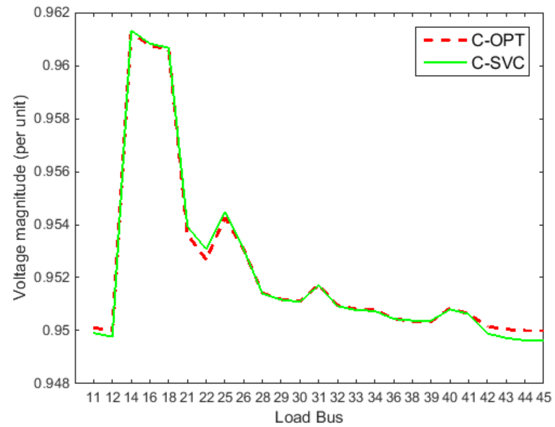

Figure 2: Voltage magnitude of load buses (per unit)

We proceed by concentrating now on the fully distributed algorithms, FDA-OPT and FDA-SVC. We remind the interested reader that we have individually shown convergence results of FDA-OPT in our past work [7], but we will be repeating them here to easily contrast them to the improvements achieved by FDA-SVC.

With the proposed relaxation regarding the voltage magnitude constraints, the fully distributed algorithm is able to converge fully (accuracy of 0.1% error in the prices) to the centralized benchmark after 7500 iterations. The previous version proposed in [7] is unable to reach this level of accuracy even after 50000 iterations. Therefore, the computational effort improvement of the proposed realxation is at least 6 times.

Figures 3 and 4 show the convergence of the two variations of the fully distributed algorithm via the average and maximum deviation of the prices from the optimal benchmark respectively.

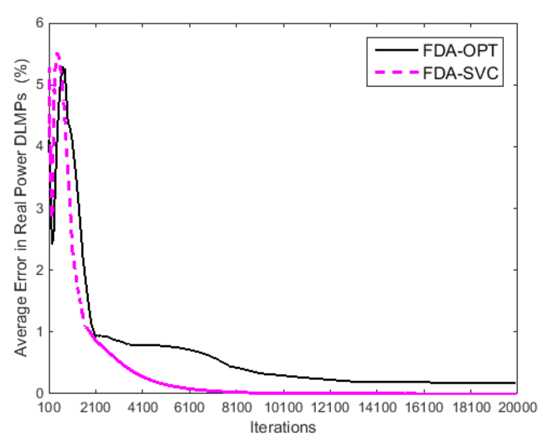

Figure 3: Fully Distributed Algorithms, Average Error in Real power DLMPs per iteration (%)

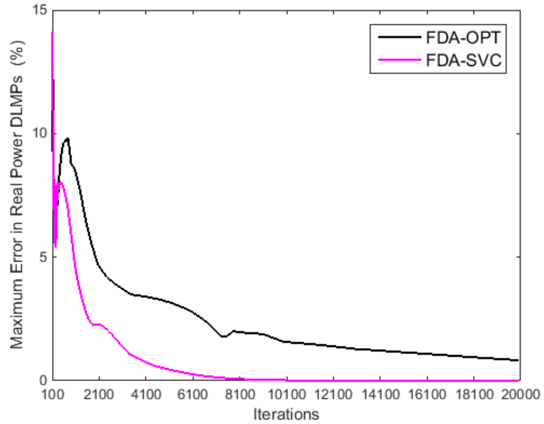

Figure 4: Fully Distributed Algorithms, Maximum Error in Real power DLMPs per iteration (%)

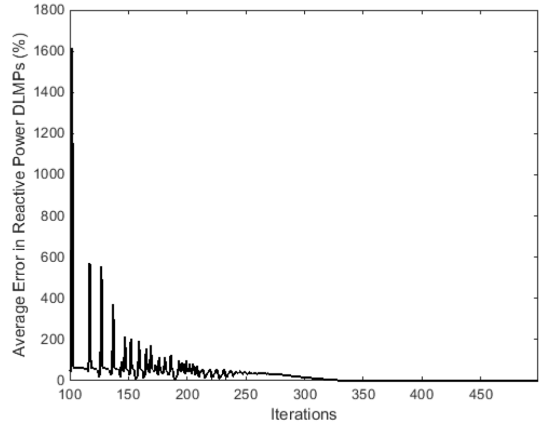

Figure 6: Partially Distributed Algorithm PDA-SVC, Average Error in the reactive DLMPs per iteration (%)

We note that the errors in prices depicted above are not relative to the same value. Errors of FDA-OPT are based on the benchmark values of C-OPT and errors of FDA-SVC are based on the benchmark vales of C-SVC.

The results section continues with the partially distributed (PDA) models. First, we note the inability of PDA-OPT to solve this instance, as the power flow problem returns infeasibility. We continue by showing the convergence of our newly proposed model, PDA-SVC. The figures that follow show the convergence of the average error in the real and reactive power prices, as well as voltage magnitudes. Exact converge is observed after about 400 iterations only.

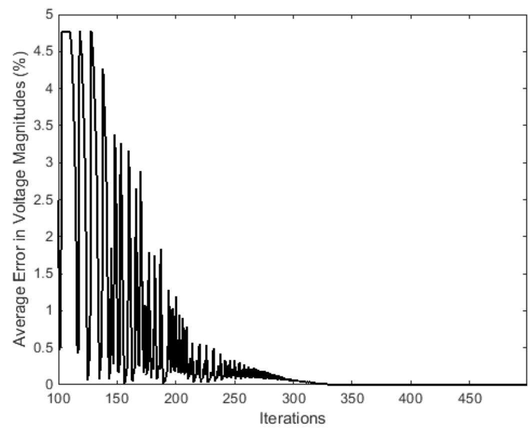

Figure 7: Partially Distributed Algorithm PDA-SVC, Average Error in voltage magnitudes per iteration (%)

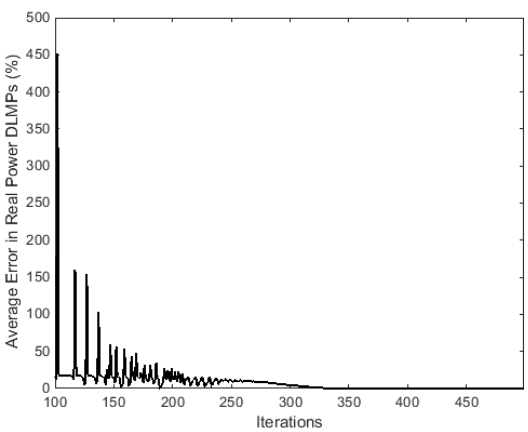

Figure 5: Partially Distributed Algorithm PDA-SVC, Average Error in the real DLMPs per iteration (%)

The behavior of the convergence curves of the real and reactive prices is a result of the DLMP estimate update that we perform after the solution of the power flow problem (namely step 5 of PDA-SVC in section III above). The decline in prices is steeper in the first iterations since we use a decreasing stepsize per iteration, specifically $h(i) = 10/i$.

Last but not least, we conclude this section by evaluating the overall improvement achieved in this work compared to our previous work in [7]. This is done graphically, by showing the first 500 iterations of the proposed PDA-SVC, that are actually adequate for absolute convergence to the benchmark, together with the first 500 iterations of the fully distributed algorithms FDA-OPT and FDA-SVC. It can be seen that there is an overall benefit of more than 100 times.

Preprint submitted to 2017 American Control Conference.
Received September 20, 2016.

Name	Iterations to Convergence	Computation Reduction
FDA-OPT	>50000	-
FDA-SVC	7500	≥6
PDA-SVC	400	≥100

Table 3: Comparison of FDA-OPT, FDA-SVC and PDA-SVC

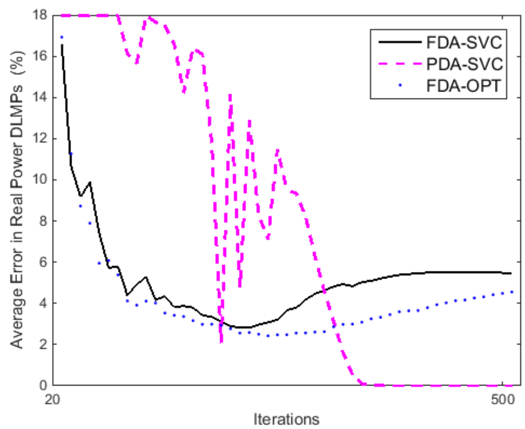

Figure 8: Comparison of Average Error in Real power DLMPs during 500 first iterations (%)

We refer the reader to [10] for a discussion on communication costs and associated security issues of distributed approaches.

Finally, we apply our proposed algorithm, PDA-SVC, to solve a 24 hour day ahead market for an 800 bus realistic distribution feeder. The feeder, adapted from PNNL data, includes PVs, electric vehicles, HVAC loads as well as voltage-sensitive loads. Complete information on the DERs and the network can be found in [8]. Although we use a cold starting point, with careful fine tuning of the DLMP estimate updates, we are able to achieve reasonable convergence in a much smaller number of iterations, namely about 1% deviation in the prices after 20 iterations. The convergence speed in the results reported for the smaller network above is much slower because our intention was to analyze the relative convergence of various algorithms. As such, the DLMP update step was not as fine tuned.

V. CONCLUSIONS

This paper is motivated by the shortcomings of existing distribution electricity market clearing models: the intractability of centralized models and the inability of fully distributed algorithms to handle hard voltage constraints efficiently. We propose (*a*) the replacement of hard voltage magnitude constraints with soft penalty constraints, introduced as cost components in the objective function, to speed up convergence of the fully distributed algorithm models, and (*b*) a Partially Distributed Algorithm (PDA) model that distributes DER scheduling but estimates ex-post locational marginal costs centrally. This new formulation exhibits robustness and superior convergence performance. We finally present results from the PDA with Soft Voltage Constraints on a 24 hour Day Ahead, 800 bus distribution network that demonstrate the applicability of the new algorithm to real size markets.

References

[1] E. Ntakou and M. Caramanis, "Price discovery in dynamic power markets with low-voltage distribution-network participants," in *2014 IEEE PES T D Conference and Exposition*, April 2014, pp. 1–5.

[2] ——, "Distribution network spatiotemporal marginal cost of reactive power," in *2015 IEEE Power Energy Society General Meeting*, July 2015, pp. 1–5.

[3] M. E. Baran and F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 725–734, Jan 1989.

[4] M. Kraning, E. Chu, J. Lavaei, and S. Boyd, "Dynamic network energy management via proximal message passing," *Found. Trends Optim.*, vol. 1, no. 2, pp. 73–126, Jan. 2014. [Online]. Available: http://dx.doi.org/10.1561/2400000002

[5] Q. Peng and S. H. Low, "Distributed algorithm for optimal power flow on a radial network," in *53rd IEEE Conference on Decision and Control*, Dec 2014, pp. 167–172.

[6] E. Ntakou and M. Caramanis, "Distribution network electricity market clearing: Parallelized pmp algorithms with minimal coordination," in *53rd IEEE Conference on Decision and Control*, Dec 2014, pp. 1687–1694.

[7] ——, "Enhanced convergence rate of inequality constraint shadow prices in pmp algorithm cleared distribution power markets," in *2016 American Control Conference (ACC)*, July 2016, pp. 1433–1439.

[8] E. Ntakou, "Spatiotemporal marginal-cost-based retail electricity markets: Efficiency, structure and feasibility," http://bu.edu/pcms/caramanis/ElliDistr.pdf, 2015.

[9] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy, "Inverter var control for distribution systems with renewables," in *Smart Grid Communications (SmartGridComm), 2011 IEEE International Conference on*, Oct 2011, pp. 457–462.

[10] M. Caramanis, E. Ntakou, W. W. Hogan, A. Chakrabortty, and J. Schoene, "Co-optimization of power and reserves in dynamic t amp;d power markets with nondispatchable renewable generation and distributed energy resources," *Proceedings of the IEEE*, vol. 104, no. 4, pp. 807–836, April 2016.

Preprint submitted to 2017 American Control Conference.
Received September 20, 2016.