# Power Market Reform In the Presence of Flexible Schedulable Distributed Loads. New Bid Rules, Equilibrium and Tractability Issues.

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**Abstract: We investigate the shortcomings of current day-ahead-market designs in eliciting socially optimal demand response and obtaining regulation service reserve offers from flexible loads. More specifically, we show that under the current day-ahead-market rules, individual flexible loads have the perverse incentive to self-schedule based on their estimate of market clearing price trajectories, rather than reveal their true utility. Furthermore, convergence of estimated market clearing prices to the socially optimal equilibrium, although theoretically feasible in a carefully designed iterative approach, is quite impractical for the application at hand. We propose modified market rules that remove the perverse incentives and allow the market to clear and discover the socially optimal equilibrium prices which are stable w.r.t. individual self-dispatch. We prove our claims and verify them with extensive numerical investigation.**

**Keywords: Reserves, multiperiod markets, demand response, power market design, retail locational marginal pricing**.

## I. INTRODUCTION

We follow up on past work [10] on the inadequacy of today's Uniform Price-Quantity Bid (UPQB) power market rules. Indeed, we show that under current market rules, flexible or deferrable loads are unable to express their preferences in the multi-period day-ahead market (DAM). More precisely, we argue that current DAM rules that allow participants to make independent hourly price-quantity bids drive individual flexible load (IFL) participants to engage in a hierarchical game [3] where (*i*) IFLs optimize their hourly quantity transactions on the basis of ex ante DAM hourly clearing price estimates and self-dispatch their hourly transaction quantities by associating them with high price bids; this is followed by (*ii*) clearing of the rest of the market including conventional demand and generation by the Independent System and Distribution Network Operator (ISDO) resulting in ex post clearing prices that determine actual IFL charges. Differences between ex ante estimates of and ex post clearing prices, drive IFLs to revise their estimates and their self-dispatch. We formulate the above hierarchical game and use extensive numerical experiments to show that it converges under a reasonable clearing price estimate revision filter. Moreover, we propose an additional market participation bidding rule which we call inter-temporally coupled or complex bid (ICCB) and show that it enables IFLs to express their true preference and allows the ISDO to clear market prices and quantities which coincide with the hierarchical game's equilibrium.

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Interesting recent work by Huang, Roozbehani, and Dahleh [11] considers deferrable and conventional loads in a sequential decision making context under unknown ex-post real time prices. In this paper we focus on a realistic multi-period DAM model where we consider multiple market participants bidding simultaneously for energy and reserve capacity requirements. Moreover, since flexible loads are connected to the low voltage distribution network, we consider Locational Marginal Prices for energy (LMPs) and for reserves (RLMPs) adjusted by distribution level marginal costs such as line losses and transformer capacity constraints. We refer to these adjusted prices as Distribution Level LMPs (DLMPs) and RLMPs (DRLMPs). The importance of demand side reserve supply [1, 2, 5] is of increasing significance as intermittent/volatile renewable generation is added to the generation mix [4, 7].

The rest of this paper is organized as follows. Section II formulates a DAM co-optimizing energy and reserve capacity transactions expanded to model (*i*) transmission and distribution networks extending radially beyond transmission busses, (*ii*) Congestion of distribution network components, and (*iii*) conventional loads as well as IFLs demanding energy by a known deadline. Section III formulates the behavior of power markets when *uniform price quantity bids* (UPQBs) *constitute* the only bidding rule available to conventional as well as IFL participants. It proceeds to show that UPQBs are unable to represent the inter-temporally coupled preferences of IFLs, leading them to engage in strategic behavior resulting in a hierarchical game between IFLs and the Power Market Independent Transmission System and Distribution Network Operator (ISDO) which converges under conditions to a stable equilibrium. Section IV proposes tractable temporally coupled *complex bid* (TCCB) rules that (*i*) allow flexible load to reveal their true utility, and (*ii*) proves that the new market mechanism enables ISDOs to reach the same equilibrium of the hierarchical game considered in Section III. Section V provides supportive numerical evidence. We conclude in Section VI.

## II. MARKET PARTICIPANT CHARACTERISTICS AND DISTRIBUTION NETWORK LOSSES

Without loss of generality, we model for simplicity (*i*) distribution network losses while disregarding the generally smaller transmission network losses, and (*ii*) only secondary reserves referred to as regulation service reserves (RSR) that represent up and down generation by conventional generation or the equivalent down and up energy consumption by flexible demand.

Starting with the following general definitions we proceed to describe market participant characteristics and distribution network losses.

### *A. General Definitions*

$n$ : A bus/node in the Transmission Network

$n(i)$ : Distribution Network location *i* connected to bus *n*.

$n(\gamma)$ : Generator $\gamma$ connected to the bus $n$ [1].

*t ,j*: DAM hour $t, j \in \{1, 2, ..., 24\}$.

$a, \varsigma, \vartheta, \lambda, \mu, \xi, \rho$ : indicate dual variables as indicated

IFL: Individual Flexible Load,

ISDO: Independent System and Distr. Network Operator

UPQB: Uniform Price Quantity Bid

TCCB: Temporally Coupled Complex Bid

u, x*:* indicate UPQB or TCCB market rule respectively

DLMP: Distribution Locational Marginal Price

DRLMP: Distribution Reserve Locational Marginal Price

### *B. Flexible and Conventional Demand*

Indices

*E, R*: superscripts denoting energy and RSRs respectively

$F_j$: An IFL representing the uncharged battery capacity of EVs with desired departure time at the end of hour *j* [2].

Variables:

${}^{c}d_{n(i)}^{t}$ : Hour $t$ conventional demand at location $n(i)$

${}^{F_j}d_{n(i)}^{t}$ : Hour $t$ demand by $F_j$ at location $n(i)$

${}^{F_j,R}d_{n(i)}^{t}$ : Hour $t$ RSR committed by IFL $F_j$ . ${}^{F_j}x_{n(i)}^{t}$ : The uncharged battery kWh at time $t$ of EVs at $n(i)$ desiring to depart at hour *j,* i.e. belonging to departure class *j*

${}^{F_j}\hat{\omega}_{n(i)}^{t}$ : Estimate of charging demand expected to arrive during hour *t* for EV departure class *j*.

*Parameters:*

${}^{c}\overline{d}_{n(i)}^{t}$ *:* Maximal conventional demand at $n(i)$

${}^{F_j}\underline{d}_{n(i)}^{t}$, ${}^{F_j}\overline{d}_{n(i)}^{t}$ : Minimal and maximal consumption of $F_j$

$\hat{C}_{n(i)}^{t}$ : Distribution capacity at $n(i)$ available to IFL [3].

${}^{F_j}V_{n(i)}$ : Unit Cost per uncharged kWh of EV battery in hour *j*<24.

${}^{F_j}v_{n(i)}$ : Unit Cost (typically Smaller than ${}^{F_j}V_{n(i)}$) per uncharged kWh of EV battery incurred in hour 24 for departures scheduled for the next day i.e., *j*>24 [4].

${}^{c}u_{n(i)}^{t}$ : Per kWh utility of conventional demand ${}^{c}d_{n(i)}^{t}$.

Constraints and State Dynamics

For the case of EV IFLs, ${}^{F_j}\underline{d}_{n(i)}^{t} = 0$ [5]. Distribution Network congestion and the up/down nature of RSRs offered by IFLs [2] force ${}^{F_j}d_{n(i)}^{t}$ and ${}^{F_j}d_{n(i)}^{t}$ $\forall t \le j$ to satisfy:

$$ {}^{F_j}d_{n(i)}^{t} \ge {}^{F_j,R}d_{n(i)}^{t} \tag{1.1} $$

$$ {}^{F_j}d_{n(i)}^{t} + {}^{F_jR}d_{n(i)}^{t} \le {}^{F_j}\overline{d}_{n(i)}^{t} \tag{1.2} $$

$$ \sum_j [\,{}^{F_j}d_{n(i)}^{t} + {}^{F_j,R}d_{n(i)}^{t}] \le \hat{C}_{n(i)}^{t} \tag{1.3} $$

For simplicity of exposition we assume here that the equation (1.2) is superseded by (1.3) [6].

The state dynamics can now be written as:

$$ {}^{F_j}x_{n(i)}^{t} = {}^{F_j}x_{n(i)}^{t-1} - {}^{F_j}d_{n(i)}^{t} + {}^{F_j}\hat{\omega}_{n(i)}^{t} \tag{1.4} $$

Lastly, conventional demand must satisfy

$$ {}^{c}d_{n(i)}^{t} < {}^{c}\overline{d}_{n(i)}^{t} \tag{1.5} $$

Utility/Cost

IFLs are deferrable loads that do not derive utility (or incur costs) from energy consumed during a specific hourly period. In fact, their utility is not additively decomposable in terms of hourly consumption levels [2,10,6]. Instead, they derive utility associated with their battery State of Charge, ${}^{F_j}x_{n(i)}^{t}$, representing cumulative past demand arrivals at $n(i)$ of departure class $\tau$ EVs minus cumulative charging. Non-zero state-dependent utility at *t*=*j*, is thus:

$$ {}^{F_j}V_{n(i)}\ {}^{F_j}x_{n(i)}^{t} 1_{t=j} + {}^{F_j}v_{n(i)}\ {}^{F_j}x_{n(i)}^{24} 1_{t=24,j>24} $$

For conventional/inflexible demand, we have the usual time additive hour-specific utility of consumption: ${}^{c}u_{n(i)}^{t}\ {}^{c}d_{n(i)}^{t}$ [7].

[1] We assume that all generators interconnect at the transmission level.

[2] Other examples include HVAC and similar energy demanding loads.

[3] We assume here that the distribution assets serving location *n*(*i*) have a fixed capacity $\hat{C}_{n(i)}^{t} + {}^{c}d_{n(i)}^{t}$

[4] A reasonable value for ${}^{F_j}v_{n(i)}^{24}$ is the Lagrange multiplier of ${}^{F_1}x_{n(i)}^{1}$ in the optimization problems IIIA or IV. The multiplier can be easily estimated in a fast converging iterative process [2]

[5] We do not consider a V2G option

[6] When inequality (1.3) is binding, a situation that occurs when 110V outlets are used for charging, additional state information on the number of vehicles connected at $n(i)$ during hour $t$ is needed [2].

[7] High ${}^{c}u_{n(i)}^{t}$ signals low or non-existing elasticity.

### B. Conventional Generation

Variables

$g^t_{n(\gamma)}$: Hour $t$ output of Conventional generator $n(\gamma)$ .

${}^R g^t_{n(\gamma)}$: Generator $n(\gamma)$ contribution to RSRs.

Parameters

$\underline{g}^t_{n(\gamma)}, \overline{g}^t_{n(\gamma)}$: Hour $t$ min and max generation capacity ${}^c R^t$: Reserve requirement during hour $t$

${}^R ramp^t_{n(\gamma)}$: MWh/minute up/down ramp rate of $n(\gamma)$.

${}^R\tau$: Period length over which generator offers RAS ${}^R g^t_{n(\gamma)}$ should be able to modulate its output from $g^t_{n(\gamma)}$ to $g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)}$ or to $g^t_{n(\gamma)} - {}^R g^t_{n(\gamma)}$ (usually 5 min).

$\overline{c}^t_{n(\gamma)}$: The variable fuel and O&M cost of generator $n(\gamma)$

$\overline{r}^t_{n(\gamma)}$: The unit cost bid in the DAM by generator $n(\gamma)$ to allow the ISDO to manage its generation level in real time in the interval [ $g^t_{n(\gamma)} - {}^R g^t_{n(\gamma)}$, $g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)}$ ] see [2].

Constraints:

$$g^t_{n(\gamma)} - {}^R g^t_{n(\gamma)} \geq \underline{g}^t_{n(\gamma)} \tag{1.6}$$

$$g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)} \leq \overline{g}^t_{n(\gamma)} \tag{1.7}$$

$${}^R g^t_{n(\gamma)} \leq {}^R\tau\,{}^R ramp^t_{n(\gamma)} \tag{1.8}$$

Cost

Disregarding start-up/shut-down costs and minimum up/down times we have variable costs associated with the generation of energy and revenues from responses to regulation service requests made by the power system/whole sale market operator. Net costs for generator $n(\gamma)$ are:

$$c^t_{n(\gamma)}({}^E g^t_{n(\gamma)}, {}^R g^t_{n(\gamma)}) = \overline{c}^t_{n(\gamma)} g^t_{n(\gamma)} + \overline{r}^t_{n(\gamma)}\,{}^R g^t_{n(\gamma)}$$

### C. Distribution Network Losses

Definitions:

$m^t_{n(i)}$: Factor that converts incremental consumption at distribution location $n(i)$ to the quantity that must be made available at the transmission bus $n$ . $\beta_{n(i)}$: Parameter that represents distribution network line characteristics

Losses:

We assume the energy leaving node $n$ to supply total energy consumption in location $n(i)$ equals the energy consumed at $n(i)$ plus the distribution network losses. More specifically, since losses are reasonably approximated by a quadratic function of demand, the conversion factor is

$$m^t_{n(i)} = \frac{\partial}{\partial\,{}^{F_j}d^t_{n(i)}} \{ (\sum_j {}^{F_j}d^t_{n(i)} + {}^c d^t_{n(i)}) + \frac{\beta_{n(i)}}{2} (\sum_j {}^{F_j}d^t_{n(i)} + {}^c d^t_{n(i)})^2 \}$$

$$= \left(1 + \beta_{n(i)}({}^c d^t_{n(i)} + \sum_j {}^{F_j}d^t_{n(i)})\right)$$

## III. STRATEGIC BEVAVIOR OF IFLs UNDER UPQBs

Since IFLs do not have a time additive utility of consumption, they can attain a higher utility in a UPQB market by (*i*) using ex-ante clearing price estimates to optimize their state dependent utility descried above, and (*ii*) using the UPQB rule to self-dispatch their optimized transactions through high price bids for energy and low price offers for RSRs. Such behavior, however, may result in ISDO ex-post clearing prices that differ from the ex-ante estimates, and thus lead to a hierarchical game [3] consisting of cycles of a revised IFL self-dispatch followed by ex-post clearing prices. The hierarchical game is modeled in Section IIIA below and its properties explored in two propositions.

### A. The Individual Flexible Load (IFL) Decision Problem

Each IFL representing flexible demand $F_j$ at $n(i)$, $\forall t \in \{1, 2, ..., 24\}$ will solve consecutive iterations of problem IIIA below as of a hierarchical game:

$$\min_{{}^{F_j}d^t_{n(i)},\, {}^{F_j,R}d^t_{n(i)},\, \forall j,t} \sum_{j,t} \{ \underset{{}^E\lambda^t_n,\, {}^R\lambda^t_n,\, m^t_{n(i)}}{E} [\left(m^t_{n(i)}\,{}^E\lambda^t_n\right) {}^{F_j}d^t_{n(i)} - \left(m^t_{n(i)}\,{}^R\lambda^t_n\right) {}^{F_j,R}d^t_{n(i)}] + {}^{F_j}U^t_{n(i)}({}^{F_j}x^t_{n(i)}) \}$$

Subject to 1.1, 1.3, 1.4 $\forall t, j$ , and increasingly accurate estimates of $m^t_{n(i)}\,{}^E\lambda^t_n$, $m^t_{n(i)}\,{}^R\lambda^t_n$ and $\hat{C}^t_{n(i)}$ based on past ex-post values provided by the ISDO.

After each iteration, IFL solutions to problem IIIA, quantities ${}^{F_j}d^{t*}_{n(i)}$ and ${}^{F_j,R}d^{t*}_{n(i)}$ are conveyed to the ISDO as UPQBs associated with high/low prices[8]. With rare exceptions limited to hours when unmet conventional demand is required for feasibility, the ISDO will schedule these quantities as bid regardless of other participant bids.

Inspecting problem IIIA it is obvious that estimates of $\left(m^t_{n(i)}\,{}^E\lambda^t_n\right)$ and $\left(m^t_{n(i)}\,{}^R\lambda^t_n\right)$ are important. However, assuming that these estimates depend on related information including

$${}^c d^t_{n(i)}, {}^{F_j}d^t_{n(i)}, {}^{F_j,R}d^t_{n(i)}, \sum_{j' \neq j} {}^{F_{j'}}d^t_{n(i)}, \sum_{j' \neq j} {}^{F_{j'},R}d^t_{n(i)}$$

is unreasonable since acquisition of this information is unrealistically expensive. We will simply assume that each IFL uses past ex-post estimates provided by the ISDO, and the dynamics of the hierarchical game will depend on how each IFL uses this information to form a new estimate. Nevertheless, for a given price estimate the following proposition holds.

**Proposition 1**: The solution to problem IIIA will schedule consumption and reserve offers to hours $t < j$ , and will practically do so primarily according to a merit order in

[8] ${}^{F_j}U^j_{n(i)}$ for ${}^{F_j}d^{t*}_{n(i)}$ and 0 for ${}^{F_j,R}d^{t*}_{n(i)}$ .

ascending magnitudes of $m_{n(i)}^{t}({}^{R}\lambda_{n}^{t}-{}^{E}\lambda_{n}^{t})$ modified only by binding local capacity constraints.
*Proof*: By inspection of problem IIIA.

### B. *The ISDO Day-Ahead Market Clearing Problem under strategic IFL behavior induced by the UPQB Rule*

Given IFL selected quantity bids ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$, the ISDO proceeds to schedule conventional generation and demand by solving problem IIIB below:

$$\max_{{}^{c}d_{n(i)}^{t},\, g_{n(\gamma)}^{t},\, {}^{R}g_{n(\gamma)}^{t},\forall t} \sum_{t,i,\gamma}\left({}^{c}u_{n(i)}^{t}\,{}^{c}d_{n(i)}^{t}-\overline{c}_{n(\gamma)}^{t}g_{n(\gamma)}^{t}-\overline{r}_{n(\gamma)}^{t}\,{}^{R}g_{n(\gamma)}^{t}\right)$$

subject to
-Energy Balance Constraints that yield energy clearing prices under the uniform bid rules, ${}^{E}\lambda_{n}^{t}(u)$,

$$\sum_{n(\gamma)}g_{n(\gamma)}^{t}-\sum_{n(i),j}{}^{F_j}d_{n(i)}^{t*}-\sum_{n(i)}{}^{c}d_{n(i)}^{t} -\sum_{n(i)}\frac{\beta_{n(i)}}{2}\left(\sum_{j}{}^{F_j}d_{n(i)}^{t*}+{}^{c}d_{n(i)}^{t}\right)^{2}=0,\ \forall t \tag{2.1u}$$

-Regulation Reserve Requirements Constraints that yield reserve clearing prices under the uniform bid rules ${}^{R}\lambda_{n}^{t}(u)$

$$\sum_{n(\gamma)}{}^{R}g_{n(\gamma)}^{t}+\sum_{j,n(i)}{}^{F_j,R}d_{n(i)}^{t*}+\sum_{n(i)}\Delta Loss_{n(i)}\geq{}^{c}R^{t} \tag{2.2u}$$

-Conventional Demand and Generation capacity and ramp constraints
1.5, 1.6, 1.7, 1.8 $\forall t$ (2.3u)

$\Delta Loss_{n(i)}$ is the increase in RSR provided at location $n(i)$ to a higher RSR delivered at bus $n$. This increase is due to distribution line losses and is a function of $\left(\sum_{j}{}^{F_j,R}d_{n(i)}^{t*},\sum_{j}{}^{F_j}d_{n(i)}^{t*},{}^{c}d_{n(i)}^{t}\right)$. However, since real time RSR provision in response to the ISDO signal represents small variations around the scheduled conventional and IFL hourly consumption, we can reasonably approximate $\Delta Loss_{n(i)}$ as a function of IFL and conventional consumption, namely:

$$\Delta Loss_{n(i)}\approx\sum_{j}{}^{F_j,R}d_{n(i)}^{t}\frac{\partial\beta_{n(i)}\left[\sum_{j}{}^{F_j}d_{n(i)}^{t}+{}^{c}d_{n(i)}^{t}\right]^{2}}{2\partial\,{}^{F_j}d_{n(i)}^{t}}$$

Using this approximation (2.2u) can be re-written as

$$\sum_{n(\gamma)}{}^{R}g_{n(\gamma)}^{t}+\sum_{j,n(i)}m_{n(i)}^{t}\,{}^{F_j,R}d_{n(i)}^{t*}+\geq{}^{c}R^{t} \tag{2.2u}$$

<u>Discussion of Clearing Price Convergence</u>
Consider the iterative solution of problems IIIA and IIIB described below:

(*i*) sub-problem IIIA solves for ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$, using the DLMP and DRLMP estimates $(\overline{m}_{n(i)}^{t}\,{}^{F_j,E}\overline{\lambda}_{n}^{t}),\ (\overline{m}_{n(i)}^{t}\,{}^{F_j,R}\overline{\lambda}_{n}^{t})$.

(*ii*) IIIB is resolved by the ISDO with ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$ as inputs to obtain new ex-post clearing prices ${}^{E}\lambda_{n}^{t}, {}^{R}\lambda_{n}^{t}$ and marginal loss factors $m_{n(i)}^{t}$.

(*iii*) IFLs revise DLMP and DRLMP estimates and the two steps above are repeated.

Propositions 1 implies that flexible demand loads will be scheduled predominantly in hours $t^{low,k}$ with low $(m_{n(i)}^{t}\,{}^{E}\lambda_{n}^{t,k})$ and high $(m_{n(i)}^{t}\,{}^{R}\lambda_{n}^{t,k})$. If successive DLMP and DRLMP estimates are set equal to the most recent ex-post values provided by the ISDO, iterative solutions of problem IIIB will tend to switch the sets of $\left(t^{low,k},t^{high,k}\right)$ and $\left(t^{low,k+1},t^{high,k+1}\right)$ hours leading to periodic/oscillatory non-converging behavior described in the literature for similar hierarchical games (see for example Zhongjing et al, [9] who also showed that more carefully constructed price estimate updates can lead to converging behavior)

**Proposition 2**. Under mild regularity/continuity conditions, there exist DLMP and DRLMP estimate updates that converge to fixed price vectors, $\mathbf{m}_{n(i)}\,{}^{E}\overline{\boldsymbol{\lambda}}(u)$ and $\mathbf{m}_{n(i)}\,{}^{R}\overline{\boldsymbol{\lambda}}(u)$.

*Proof*: Monotonicity and convexity of consumer and producer surplus guarantee convergence in the absence of step discontinuities. Indeed, numerical experience reported in section V indicates that reducing the singularities caused by the step function nature of demand and supply curves significantly improves convergence. Proposition 3 below proves the existence of equilibrium prices and that these are indeed the socially optimal prices and transactions that clear a market where new bid rules allow IFLs to express their actual utility.

## IV. TCCB BASED MARKET MECHANISM

DAMs expanded by additional bidding rules allowing IFLs to express their inter-temporal battery-state dependent utility via TCCBs reflecting state depended utility, its dynamics and local distribution constraints, can clear markets optimally by solving problem IV below:

$$\max_{{}^{c}d_{n(i)}^{t},g_{n(\gamma)}^{t},{}^{R}g_{n(\gamma)}^{t},{}^{F_j}d_{n(i)}^{t},{}^{F_j,R}d_{n(i)}^{t},{}^{W}g_{n(\gamma)}^{t}\forall t,\gamma,i,\tau}\sum_{t,\gamma,t}\left[{}^{c}u_{n(i)}^{t}\,{}^{c}d_{n(i)}^{t}-\overline{c}_{n(\gamma)}^{t}g_{n(\gamma)}^{t}-\overline{r}_{n(\gamma)}^{t}\,{}^{R}g_{n(\gamma)}^{t}-{}^{F_j}U_{n(i)}^{t}({}^{F_j}x_{n(i)}^{t})\right]$$

Subject to:
-Energy balance constraints yielding complex-bid-based energy clearing prices ${}^{E}\lambda_{n}^{t}(x)$

$$\sum_{n(\gamma)} g^t_{n(\gamma)} - \sum_{n(i),j} {}^{F_j}d^t_{n(i)} - \sum_{n(i)} {}^{c}d^t_{n(i)} - \sum_{n(i)} \frac{\beta_{n(i)}}{2} (\sum_j {}^{F_j}d^t_{n(i)} + {}^{c}d^t_{n(i)})^2 = 0, \quad \forall t \tag{2.1x}$$

-Regulation Reserve Constraints yielding complex-bid-based reserves clearing prices denoted by ${}^{R}\lambda^t_n(x)$

$$\sum_{n(\gamma)} {}^{R}g^t_{n(\gamma)} + \sum_{n(i),j} m^t_{n(i)} \, {}^{F_j,R}d^t_{n(i)} \geq {}^{c}R^t, \quad \forall t \tag{2.2x}$$

-Conventional Demand and Generation capacity and ramp constraints

(1.5), (1.6), (1.7) and (1.8) $\forall t$, (2.3x)

and the IFL optimization constraints of problems IIIA.

1.1, 1.3, 1.4 $\forall t, j$ (2.4x)

TC rules are not unusual. In fact, we encounter them today when ramp constraints coupling conventional generation across hours are included in the ISO problem. Other complex rules, such as examined in [8], are part of European DAMs. Moreover, the introduction of the proposed TCCB rule does not significantly increase the computational complexity of the ISDO problem, primarily because the IFL state dynamics added are linear.

**Proposition 3**. The Complex Bid market mechanism enables ISDOs to reach the same marginal cost equilibrium as the hierarchical game of section III under certain assumptions discussed below, but in a single market clearing step

*Proof*:

To prove the claim, we must show that both formulations have the same first order, feasibility and complementary slackness conditions. We first examine the IFL and ISDO problems under Uniform Bids and analyze the above conditions at the equilibrium prices and quantities to which the hierarchical game described in section III has converged.

For the Uniform Bid model, the Lagrangian for the IFL problem is given below. Each IFL estimates $\mathbf{E}\left(m^t_{n(i)} \, {}^{E}\lambda^t_n\right)$ and $\mathbf{E}\left(m^t_{n(i)} \, {}^{R}\lambda^t_n\right)$ and treats them as a constant input in solving problem IIIA (which is in fact an LP). We introduce symbols for the dual variables and write the Lagrangian below omitting for simplicity the **E** operator.

$$\begin{aligned}
£_{n(i)} = \sum_{t,j} \; & \Big( m^t_{n(i)} \, {}^{E}\lambda^t_n \, {}^{F_j}d^t_{n(i)} - m^t_{n(i)} \, {}^{R}\lambda^t_{n(i)} \, {}^{F_j,R}d^t_{n(i)} + {}^{F_j}U^t_{n(i)}\left({}^{F_j}x^t_{n(i)}\right)\Big) \\
&+ \sum_{t,j} \alpha^{t,j}_{n(i)} \left({}^{F_j,R}d^t_{n(i)} - {}^{F_j}d^t_{n(i)}\right) \\
&+ \sum_t \mu^t_{n(i)} \Big(\sum_j \left({}^{F_j}d^t_{n(i)} + {}^{F_j,R}d^t_{n(i)}\right) - \hat{C}^t_{n(i)}\Big) + \\
&\sum_{t,\tau} \varsigma^{t,j}_{n(i)} \left({}^{F_j}x^t_{n(i)} - x^{t-1}_{n(i)} + {}^{F_j}d^t_{n(i)} - {}^{F_j}\hat{\omega}^t_{n(i)}\right)
\end{aligned}$$

Where $\alpha, \mu, \zeta$ are the respective dual variables.

The first order optimality conditions are:

$$\nabla_{{}^{F_j}d^t_{n(i)}} £_{n(i)} = m^t_{n(i)} \, {}^{E}\lambda^t_n + \alpha^{t,j}_{n(i)} + \mu^{t,j}_{n(i)} + \varsigma^{t,\tau}_{n(i)} = 0 \tag{3.1u}$$

$$\nabla_{{}^{F_j,R}d^t_{n(i)}} £_{n(i)} = -m^t_{n(i)} \, {}^{R}\lambda^t_{n(i)} + \alpha^{t,j}_{n(i)} + \mu^{t,j}_{n(i)} = 0 \tag{3.2u}$$

$$\nabla_{{}^{F_j}x^t_{n(i)}} £_{n(i)} = {}^{F_j}V_{n(i)} \; 1_{t=j} - {}^{F_j}v_{n(i)} \; 1_{t=24, j>24} + \varsigma^{t,j}_{n(i)} - \varsigma^{t+1,j}_{n(i)} = 0 \tag{3.3u}$$

And the complementary slackness conditions are:

$$\alpha^{t,j}_{n(i)} \left({}^{F_j,R}d^t_{n(i)} - {}^{F_j}d^t_{n(i)}\right) = 0 \quad \forall \, j,t \tag{3.4u}$$

$$\mu^t_{n(i)} \Big(\sum_j \left({}^{F_j}d^t_{n(i)} + {}^{F_j,R}d^t_{n(i)}\right) - \hat{C}^t_{n(i)}\Big) = 0 \quad \forall \, t \tag{3.5u}$$

$$\varsigma^{t,j}_{n(i)} \left({}^{F_j}x^{t+1}_{n(i)} - x^t_{n(i)} + {}^{F_j}d^t_{n(i)} - {}^{F_j}\hat{\omega}^t_{n(i)}\right) = 0 \quad \forall \, j,t \tag{3.6u}$$

The feasibility conditions are described in Section II by the constraints (1.1), (1.3) and (1.4)

The Lagrangian for the ISDO problem is:

$$\begin{aligned}
£_{ISDO(\mathrm{u})} = & \sum_{n,i,\gamma,t} \left(-{}^{c}u^t_{n(i)} \, {}^{c}d^t_{n(i)} + \overline{c}^t_{n(\gamma)} g^t_{n(\gamma)} + \overline{r}^t_{n(\gamma)} \, {}^{R}g^t_{n(\gamma)}\right) \\
& - \sum_{t,n} {}^{E}\lambda^t_n(\mathrm{u}) \Big( \sum_{n(\gamma)} g^t_{n(\gamma)} - \sum_{n(i),j} {}^{F_j}d^{t*}_{n(i)} - \\
& \sum_{n(i)} {}^{c}d^t_{n(i)} - \sum_{n(i)} \frac{\beta_{n(i)}}{2} (\sum_j {}^{F_j}d^{t*}_{n(i)} + {}^{c}d^t_{n(i)})^2 \Big) + \\
& \sum_{t,n} {}^{R}\lambda^t_n(\mathrm{u}) \Big( {}^{c}R^t - \sum_{n(\gamma)} {}^{R}g^t_{n(\gamma)} - \\
& \sum_{j,n(i)} \big(1 + \beta_{n(i)} ({}^{c}d^t_{n(i)} + \sum_j {}^{F_j}d^{t*}_{n(i)})\big) \, {}^{F_j,R}d^{t*}_{n(i)} \Big) + \\
& \sum_{t,n(\gamma)} \xi^t_{n(\gamma)} \left( \underline{g}^t_{n(\gamma)} - g^t_{n(\gamma)} + {}^{R}g^t_{n(\gamma)} \right) + \\
& \sum_{t,n(\gamma)} \rho^t_{n(\gamma)} \left( g^t_{n(\gamma)} + {}^{R}g^t_{n(\gamma)} - \overline{g}^t_{n(\gamma)} \right) + \\
& \sum_{t,n(\gamma)} \vartheta^t_{n(\gamma)} \left( {}^{R}g^t_{n(\gamma)} - {}^{R}\tau \, {}^{R}ramp^t_{n(\gamma)} \right) + \\
& \sum_{t,n(i)} \zeta^t_{n(i)} \left( {}^{c}d^t_{n(i)} - {}^{c}\overline{d}^t_{n(i)} \right)
\end{aligned}$$

Where ${}^{E}\lambda^t_n(\mathrm{u}), {}^{R}\lambda^t_n(\mathrm{u}), \xi^t_{n(\gamma)}, \rho^t_{n(\gamma)}, \vartheta^t_{n(\gamma)}, \zeta^t_{n(i)}$ are the respective dual variables.

The 1st order optimality conditions are:

$$\nabla_{g^t_{n(\gamma)}} £_{ISDO(\mathrm{u})} = \bar{c}^t_{n(\gamma)} - {}^E\lambda^t_n(\mathrm{u}) - \xi^t_{n(\gamma)} + \rho^t_{n(\gamma)} \tag{3.7u}$$

$$\nabla_{g^R_{n(\gamma)}} £_{ISDO(\mathrm{u})} = \bar{r}^t_{n(\gamma)} - {}^R\lambda^t_n(\mathrm{u}) + \xi^t_{n(\gamma)} + \rho^t_{n(\gamma)} + \vartheta^t_{n(\gamma)} \tag{3.8u}$$

$$\nabla_{{}^c d^t_{n(i)}} £_{ISDO(\mathrm{u})} = -{}^c u^t_{n(i)} + {}^E\lambda^t_n(\mathrm{u})\Big(1+ \beta_{n(i)}\big(\sum_j {}^{F_j}d^{t*}_{n(i)} + {}^c d^t_{n(i)}\big)\Big) - {}^R\lambda^t_n(\mathrm{u})\beta_{n(i)} + \zeta^t_{n(i)} \tag{3.9u}$$

The complementary slackness conditions are:

$${}^E\lambda^t_n(\mathrm{u})\Big(\sum_{n(\gamma)} g^t_{n(\gamma)} - \sum_{n(i),j} {}^{F_j}d^{t*}_{n(i)} - \sum_{n(i)} {}^c d^t_{n(i)} - \sum_{j,n(i)} \frac{\beta_{n(i)}}{2}\big({}^{F_j}d^{t*}_{n(i)} + {}^c d^t_{n(i)}\big)^2\Big) = 0 \quad \forall\, n,t \tag{3.10u}$$

$${}^R\lambda^t_n(\mathrm{u})\Big({}^c R^t - \sum_{n(\gamma)} {}^R g^t_{n(\gamma)} - \sum_{n(i)}\Big(1+\beta_{n(i)}\big({}^c d^t_{n(i)} + \sum_j {}^{F_j}d^{t*}_{n(i)}\big)\Big)\,{}^{F_j,R}d^{t*}_{n(i)}\Big) = 0 \quad \forall\, n,t \tag{3.11u}$$

$$\xi^t_{n(\gamma)}\big(\underline{g}^t_{n(\gamma)} - g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)}\big) = 0 \quad \forall\, n(\gamma),t \tag{3.12u}$$

$$\rho^t_{n(\gamma)}\big(g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)} - \bar{g}^t_{n(\gamma)}\big) = 0 \quad \forall\, n(\gamma),t \tag{3.13u}$$

$$\vartheta^t_{n(\gamma)}\big({}^R g^t_{n(\gamma)} - {}^R\tau\,{}^R ramp^t_{n(\gamma)}\big) = 0 \quad \forall\, n(\gamma),t \tag{3.14u}$$

$$\zeta^t_{n(i)}\big({}^c d^t_{n(i)} - {}^c\bar{d}^t_{n(i)}\big) = 0 \quad \forall\, n(i),t \tag{3.15u}$$

The feasibility conditions are described by the constraints (2.1u), (2.2u), and (2.3u)

The first order, complementary slackness and feasibility conditions between the IFL and ISDO problem have ${}^E\lambda^t_n(\mathrm{u})$, ${}^R\lambda^t_n(\mathrm{u})$, ${}^{F_j,R}d^t_{n(i)}$, ${}^{F_j}d^t_{n(i)}$ in common. The two former are communicated to the IFL by the ISDO (bundled as DLMP and DRLMP) while the two latter are communicated to the ISDO by the IFL. Upon convergence of the iterative approach, however, these variables will be identical in both problems and the conditions for both problems can be solved simultaneously to obtain optimal solutions for the primal and dual variables.

We now turn to the Complex Bid problem. Reformulating as a minimization problem, the Lagrangian is:

$$\begin{aligned}
£_{ISDO(\mathrm{x})} = &\sum_{n,i,\gamma,t} \Big(-{}^c u^t_{n(i)}\,{}^c d^t_{n(i)} + \bar{c}^t_{n(\gamma)} g^t_{n(\gamma)} + \bar{r}^t_{n(\gamma)}\,{}^R g^t_{n(\gamma)} + \sum_j {}^{F_j}U^t_{n(i)}\big({}^{F_j}x^t_{n(i)}\big)\Big) - \\
&\sum_{t,n} {}^E\lambda^t_n(\mathrm{x})\Big(\sum_{n(\gamma)} g^t_{n(\gamma)} - \sum_{n(i),j} {}^{F_j}d^t_{n(i)} - \sum_{n(i)} {}^c d^t_{n(i)} - \sum_{n(i)} \frac{\beta_{n(i)}}{2}\big(\sum_j {}^{F_j}d^t_{n(i)} + {}^c d^t_{n(i)}\big)^2\Big) + \\
&\sum_{t,n} {}^R\lambda^t_n(\mathrm{x})\Big({}^c R^t - \sum_{n(\gamma)} {}^R g^t_{n(\gamma)} - \sum_{j,n(i)}\Big(1+\beta_{n(i)}\big({}^c d^t_{n(i)} + \sum_j {}^{F_j}d^t_{n(i)}\big)\Big)\,{}^{F_j,R}d^t_{n(i)}\Big) + \\
&\sum_{t,n(i),j} \alpha^{t,j}_{n(i)}\big({}^{F_j,R}d^t_{n(i)} - {}^{F_j}d^t_{n(i)}\big) + \\
&\sum_{t,n(i)} \mu^t_{n(i)}\Big(\sum_j\big({}^{F_j}d^t_{n(i)} + {}^{F_j,R}d^t_{n(i)}\big) - \hat{C}^t_{n(i)}\Big) + \\
&\sum_{t,n(i),j} \varsigma^{t,j}_{n(i)}\big({}^{F_j}x^t_{n(i)} - {}^{F_j}x^{t-1}_{n(i)} + {}^{F_j}d^t_{n(i)} - {}^{F_j}\hat{\omega}^t_{n(i)}\big) + \\
&\sum_{t,n(\gamma)} \xi^t_{n(\gamma)}\big(\underline{g}^t_{n(\gamma)} - g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)}\big) + \\
&\sum_{t,n(\gamma)} \rho^t_{n(\gamma)}\big(g^t_{n(\gamma)} + {}^R g^t_{n(\gamma)} - \bar{g}^t_{n(\gamma)}\big) + \\
&\sum_{t,n(\gamma)} \vartheta^t_{n(\gamma)}\big({}^R g^t_{n(\gamma)} - {}^R\tau\,{}^R ramp^t_{n(\gamma)}\big) + \\
&\sum_{t,n(i)} \zeta^t_{n(i)}\big({}^c d^t_{n(i)} - {}^c\bar{d}^t_{n(i)}\big)
\end{aligned}$$

The 1st order optimality conditions -- lining up the equation numbering with the Uniform Problem conditions -- are:

$$\nabla_{{}^{F_j}d^t_{n(i)}} £_{ISDO(\mathrm{x})} = \Big(1+\beta_{n(i)}\big({}^c d^t_{n(i)} + \sum_j {}^{F_j}d^t_{n(i)}\big)\Big)\,{}^E\lambda^t_n(\mathrm{x}) - {}^R\lambda^t_n(\mathrm{x})\beta^t_{n(i)}\,{}^{F_j,R}d^t_{n(i)} + \alpha^{t,j}_{n(i)} + \mu^{t,j}_{n(i)} + \varsigma^{t,j}_{n(i)} = 0 \tag{3.1x}$$

$$\nabla_{{}^{F_j,R}d^t_{n(i)}} £_{ISDO(\mathrm{x})} = -\Big(1+\beta_{n(i)}\big({}^c d^t_{n(i)} + \sum_j {}^{F_j}d^t_{n(i)}\big)\Big)\,{}^R\lambda^t_n(\mathrm{x}) + \alpha^{t,j}_{n(i)} + \mu^{t,j}_{n(i)} = 0 \tag{3.2x}$$

$$\nabla_{{}^{F_j}x^t_{n(i)}} £_{ISDO(\mathrm{x})} = {}^{F_j}V_{n(i)}1_{t=j} - {}^{F_j}v_{n(i)}1_{j>24,t=24} + \varsigma^{t,j} - \varsigma^{t+1,j} = 0 \tag{3.3x}$$

$$\nabla_{g^t_{n(\gamma)}} £_{ISDO(\mathrm{x})} = \bar{c}^t_{n(\gamma)} - {}^E\lambda^t_n(\mathrm{x}) - \xi^t_{n(\gamma)} + \rho^t_{n(\gamma)} \tag{3.7x}$$

$$\nabla_{g^R_{n(\gamma)}} £_{ISDO(\mathrm{x})} = \bar{r}^t_{n(\gamma)} - {}^R\lambda^t_n(\mathrm{x}) + \xi^t_{n(\gamma)} + \rho^t_{n(\gamma)} + \vartheta^t_{n(\gamma)} \tag{3.8x}$$

$$\nabla_{{}^c d^t_{n(i)}} £_{ISDO(\mathrm{x})} = -{}^c u^t_{n(i)} + {}^E\lambda^t_n(\mathrm{x})\Big(1+ \beta_{n(i)}\big(\sum_j {}^{F_j}d^t_{n(i)} + {}^c d^t_{n(i)}\big)\Big) - {}^R\lambda^t_n(\mathrm{x})\beta_{n(i)} + \zeta^t_{n(i)} \tag{3.9x}$$

The complimentary slackness conditions are:

$$\alpha^{t,j}_{n(i)}\big({}^{F_j,R}d^t_{n(i)} - {}^{F_j}d^t_{n(i)}\big) = 0 \quad \forall\, j,n(i),t \tag{3.4x}$$

$$\mu^t_{n(i)}\Big(\sum_j\big({}^{F_j}d^t_{n(i)} + {}^{F_j,R}d^t_{n(i)}\big) - C^t_{n(i)}\Big) = 0 \quad \forall\, n(i),t \tag{3.5x}$$

$$\varsigma_{n(i)}^{t,j}\left({}^{F_j}x_{n(i)}^{t+1} - x_{n(i)}^{t} + {}^{F_j}d_{n(i)}^{t} - {}^{F_j}\hat{\omega}_{n(i)}^{t}\right) = 0 \quad \forall\, j, n(i), t \tag{3.6x}$$

$$ {}^{E}\lambda_n^t(\mathrm{x})\Big(\sum_{n(\gamma)} g_{n(\gamma)}^t - \sum_{n(i),j} {}^{F_j}d_{n(i)}^t - \sum_{n(i)} {}^{c}d_{n(i)}^t - \sum_{j,n(i)} \frac{\beta_{n(i)}}{2}\left({}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t\right)^2\Big) = 0 \quad \forall\, n,t \tag{3.10x}$$

$$ {}^{R}\lambda_n^t(\mathrm{x})\Big({}^{c}R^t - \sum_{n(\gamma)} {}^{R}g_{n(\gamma)}^t - \sum_{n(i)}\big(1+\beta_{n(i)}({}^{c}d_{n(i)}^t + \sum_j {}^{F_j}d_{n(i)}^t)\big){}^{F_j,R}d_{n(i)}^t\Big) = 0 \quad \forall\, n,t \tag{3.11x}$$

$$\xi_{n(\gamma)}^t\left(\underline{g}_{n(\gamma)}^t - g_{n(\gamma)}^t + {}^{R}g_{n(\gamma)}^t\right) = 0 \quad \forall\, n(\gamma), t \tag{3.12x}$$

$$\rho_{n(\gamma)}^t\left(g_{n(\gamma)}^t + {}^{R}g_{n(\gamma)}^t - \overline{g}_{n(\gamma)}^t\right) = 0 \quad \forall\, n(\gamma), t \tag{3.13x}$$

$$\vartheta_{n(\gamma)}^t\left({}^{R}g_{n(\gamma)}^t - {}^{R}\tau\, {}^{R}ramp_{n(\gamma)}^t\right) = 0 \quad \forall\, n(\gamma), t \tag{3.14x}$$

$$\zeta_{n(i)}^t\left({}^{c}d_{n(i)}^t - {}^{c}\overline{d}_{n(i)}^t\right) = 0 \quad \forall\, n(i), t \tag{3.15x}$$

The feasibility conditions are described by the constraints (2.1x), (2.2x), (2.3x) and (2.4x).

Comparing all the optimality conditions between the Complex and Uniform Bid models, we see that with the exception of the first order conditions for ${}^{F_j}d_{n(i)}^t$, equations (3.1x) and (3.1u), all other conditions are equivalent. The Additional term in (3.1x), ${}^{R}\lambda_n^t(\mathrm{x})\beta_{n(i)}^t\, {}^{F_j,R}d_{n(i)}^t$, is in general non-zero, however, we argue that it is negligible and, in fact, goes to 0 as the term

$$\frac{{}^{F_j,R}d_{n(i)}^t}{{}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t} \to 0 \tag{3.16}$$

This argument is based on an assumption regarding the competitiveness of the market, i.e., no single flexible demand type is large relative to the total demand, or equivalently relative to the conventional demand, at a particular location.

Proceeding with this assumption, and making use of the fact that distribution networks are designed so that total losses do not exceed a fraction $K < 1$ (typically about .1 and certainly never above .2) of total consumption, we can write the following relationship

$$\frac{\beta_{n(i)}}{2}\Big(\sum_j {}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t\Big)^2 < K\Big(\sum_j {}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t\Big) \Rightarrow \beta_{n(i)} < K \Big/ \Big(\sum_j {}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t\Big) \tag{3.17}$$

Multiplying both sides of (3.17) by ${}^{R}\lambda_n^t(\mathrm{x})\,{}^{F_j,R}d_{n(i)}^t$ and applying our competitiveness assumption, we have:

$$\beta_{n(i)}\, {}^{R}\lambda_n^t(\mathrm{x})\, {}^{F_j,R}d_{n(i)}^t < \frac{{}^{R}\lambda_n^t(\mathrm{x})\, {}^{F_j,R}d_{n(i)}^t}{\sum_j {}^{F_j}d_{n(i)}^t + {}^{c}d_{n(i)}^t} \to 0$$

Applying condition, (3.16), we see that optimality condition (3.1x) becomes identical to (3.1u) and hence both problems share the same primal-dual optimality conditions and have the same solution, i.e., $\lambda_n^t(\mathrm{x}) = \lambda_n^t(\mathrm{u})$, ${}^{R}\lambda_n^t(\mathrm{x}) = {}^{R}\lambda_n^t(\mathrm{u})$ with small variations in DLMP, DRLMP that tend to 0 as discussed above.

In section V we examine the effect of assumption (3.16) on the stability of our convergence result, which should give some intuition as to how much the market can deviate from perfect competition while still allowing flexible loads to reveal their true utility in a Complex Bid setting.

## V. NUMERICAL RESULTS

We employ a three bus system. Each bus feeds three distinct distribution network locations –either predominantly commercial/evening IFLs or residential/morning IFLs -- each with conventional and flexible loads. We measure convergence by the distance of LMP and RLMP prices, **λ**(u) and **λ**(x), i.e., the iterative estimates in the hierarchical game and the ISDO Problem allowing TCCBs for IFLs. We do not report explicitly on DLMPs and DRLMPs which also converge, albeit asymptotically as the extra term goes to 0. To demonstrate the effect of regularization/smoothing of the conventional generation supply we simulate a base case with 15 conventional generating units (Base Case) and a regularized case with 60 generating units (Smooth Case). Variable generating costs range from \$20/MWh to \$100/MWh in both cases but the step sizes in the supply function are smaller in the later, thus coming closer to a smooth supply function. Figure 1 exhibits **λ**(u) and **λ**(x) vector distance with increasing hierarchical game iteration number for both the Base and Smooth cases. Clearly, smoothing the supply function results in significant improvement in convergence, which is in fact exact for both the LMP and RLMP.

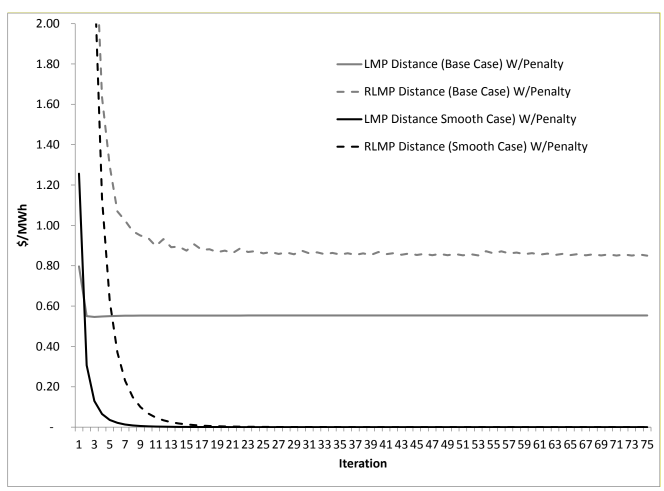


Figure 1 Hierarchical Game Convergence to TCCB Clearing Prices.

Minor price discrepancies occur at the distribution level and are due to the additional term ${}^{R}\lambda_n^t(\mathrm{x})\beta_{n(i)}^{t}\,{}^{F_j,R}d_{n(i)}^t$ discussed in Proposition 3. While this term is small and for most cases relatively insignificant compared to the rest of the terms in (3.1x), it contributes to small, yet observable discrepancies at the distribution level. To demonstrate the role of the extra term, we show the difference in distribution level prices for a specific location in Figure 2, where the extra term is relatively large.

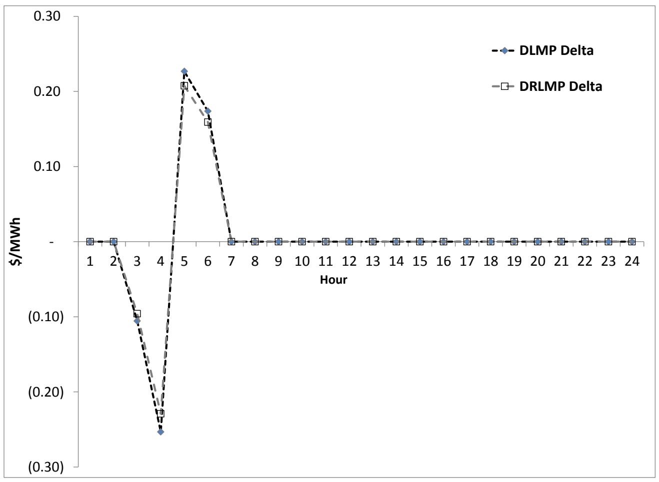

Figure 2: Specific hour price differences between the Hierarchical Game Equilibrium Prices and the TCCB ISDO solution,

In the hierarchical game equilibrium, that same location's IFL schedules Flexible demand and regulation in hours 3 and 4, while the TCCB ISDO market clears the same quantities in hours 5 and 6. In hours 5 and 6, the DLMP price difference of hierarchical game equilibrium and the TCCB ISDN solution is of the order of .44% and .33 %. Using (3.1x) we can write the DLMP at location $n(i)$:

$$DLMP_{n(i)}^t = \left(1+\beta_{n(i)}\left({}^{c}d_{n(i)}^t + \sum_j {}^{F_j}d_{n(i)}^t\right)\right){}^{E}\lambda_n^t(\mathrm{x}) =$$

$$-{}^{R}\lambda_n^t(\mathrm{x})\beta_{n(i)}^t\,{}^{F_j,R}d_{n(i)}^t + \alpha_{n(i)}^{t,j} + \mu_{n(i)}^{t,j} + \varsigma_{n(i)}^{t,j}$$

The additional term ${}^{R}\lambda_n^t(\mathrm{x})\beta_{n(i)}^{t}\,{}^{F_j,R}d_{n(i)}^t$ in (3.1x) but not in (3.1u) accounts for .45 and .34 % of the right hand side of the above equation (for the particular case depicted in Figure 2), which is extremely close to DLMP, DRLMP price differences observed. Additional support for this observation on the role of the extra term is provided by the fact that reducing the importance of distribution network losses, specifically setting $\beta_{n(i)}^t$=0, eliminates the small discrepancy between the hierarchical game equilibrium and TCCB ISDO Distribution location specific prices. In the case of $\beta_{n(i)}^t$=0, we observe exact convergence in prices. This poses an interesting research topic on competitiveness conditions that needs to be investigated in the near future.

## VI. CONCLUSION

We first showed that UPQBs characterizing today's Power Market mechanism provide incentives for flexible loads and renewable intermittent generation to engage in strategic behavior. We then proposed an additional temporally coupled complex bid (TCCB) whose addition to the power market rules (*i*) allows flexible loads and intermittent generators to convey their preferences accurately and (*ii*) reflect the rich spatio-temporally varying marginal distribution network costs. We finally prove that the resulting clearing prices are socially optimal under mild competitiveness conditions. We intend to investigate this further in future research.

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