Outline

- Overview and Motivation
- Equations for coupled fluid-solid dynamics
- Basic Physics: Localization phenomena
  - non-linear waves
- Shear band formation, reactive flow
- Geodynamics Applications: Mid Ocean Ridges and Subduction Zones
- Open Questions/Future Directions
The Take Away...
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- Magma Dynamics is important for both geodynamics and geochemistry
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• Magma Dynamics is a natural extension of Mantle Convection (just add fluids)
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- The addition of a low-viscosity fluid phase introduces new scales and dynamics.
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- Goals of this lecture
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• Magma Dynamics is important for both geodynamics and geochemistry

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• develop better physical intuition into basic physics of magma dynamics
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• Magma Dynamics is a natural extension of Mantle Convection (just add fluids)
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Goals of this lecture

• develop better physical intuition into basic physics of magma dynamics
• understand the motivation for developing better abstractions for multi-physics solvers
Why Magma Dynamics?

Dynamics of Plate Boundaries

- Mantle convection = Convection with Plates
- Plates are defined by their weak boundaries.
- Convergent and Divergent Boundaries are fundamentally magmatic
- How does magmatism affect the dynamics and structure of plate boundaries and global mantle convection?
Why Magma Dynamics?
Global Geochemical Evolution

Brandenburg et al, EPSL 2008, 2-D Cylindrical High Ra convection calculation

Solid State Convection primarily stirs
Chemical Fractionation, mixing and sampling of the mantle requires a mobile liquid phase
Can we use variation in composition of erupted lavas to infer rate and efficiency of convecting stirring in Earth?
Multi-Scale, Multi-Physics nature of Mantle Convection

- Large Scale Deformation of the Earth is in the solid-state
- Most melting occurs in small scale regions near plate boundaries, but may affect global flow and plate tectonics
- How do we understand the basic physics and interactions across scales and constrain it with chemical data?
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Ingredients for a consistent theory of magma dynamics

images provided courtesy of the NEPTUNE Project (www.neptune.washington.edu) and CEV

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Ingredients for a consistent theory of magma dynamics

- At least two phases (solid & liquid)
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- Significant mass-transfer between phases (melting/crystallization)
- The solid must be *permeable* at some scale
- In the absence of solid flow should look like porous media flow.
- In the absence of liquids, the system must be consistent with mantle convection (viscously deformable)
Governing Equations

Conservation of Mass: Fluid
\[ \frac{\partial (\rho_f \phi)}{\partial t} + \nabla \cdot (\rho_f \phi \mathbf{v}) = \Gamma \]

Conservation of Mass: Solid
\[ \frac{\partial \left[ \rho_s (1 - \phi) \right]}{\partial t} + \nabla \cdot \left[ \rho_s (1 - \phi) \mathbf{v} \right] = -\Gamma \]

Conservation of Momentum for fluid: Darcy’s Law
\[ \phi (\mathbf{v} - \mathbf{V}) = -\frac{K}{\mu} \left[ \nabla P - \rho_f \mathbf{g} \right] \]

Conservation of Momentum for Solid (viscous rheology)
\[ \nabla P = \nabla \cdot \eta \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) + \nabla \left( \zeta - \frac{2\eta}{3} \right) \nabla \cdot \mathbf{v} + \bar{\rho} \mathbf{g} \]

Plus Constitutive Relations/Closures

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A Better (?) Formulation
(McKenzie Tutorial Notes @ CIG, Katz et al, 2007 Pepi)

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Buoyancy
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- **Buoyancy**
- **Volumetric Strain**
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Conservation of Momentum for Solid (viscous rheology)

\[ \nabla P = \bar{\rho} g + \nabla \left( \zeta - \frac{2\eta}{3} \right) \nabla \cdot v + \nabla \cdot \eta \left( \nabla v + \nabla v^T \right) \]

- Buoyancy
- Volumetric Strain
- Shear Strain
A Better (?) Formulation
(McKenzie Tutorial Notes (ClG/bSpace), Katz et al, 2007 Pepi)

Conservation of Momentum for Solid (viscous rheology)

\[ \nabla P = \bar{\rho} g + \nabla \left( \zeta - \frac{2\eta}{3} \right) \nabla \cdot \mathbf{v} + \nabla \cdot \eta \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) \nabla \]

Decompose the pressure into 3 terms

\[ P = P_l + \mathcal{P} + P^* \]

with

- Lithostatic Pressure, \( P_l = \rho_0^s g z \)
- "Compaction Pressure", \( \mathcal{P} = (\zeta - 2\eta/3) \nabla \cdot \mathbf{v} \)
- Dynamic Pressure, \( P^* \)
A Better (?) Formulation

(McKenzie Tutorial Notes (CIG), Katz et al, 2007 Pepi)

Compressible Flow

\[ \frac{D\phi}{Dt} = (1 - \phi) \frac{P}{\xi} + \Gamma / \rho_s \]

\[ -\nabla \cdot \frac{K}{\mu} \nabla P + \frac{P}{\xi} = \nabla \cdot \frac{K}{\mu} [\nabla P^* + \Delta \rho \mathbf{g}] + \Gamma \frac{\Delta \rho}{\rho_f \rho_s} \]

“Incompressible” Flow

\[ \nabla \cdot \mathbf{v} = \frac{P}{\xi} \]

\[ \nabla P^* = \nabla \cdot \eta (\nabla \mathbf{v} + \nabla \mathbf{v}^T) - \phi \Delta \rho \mathbf{g} \]

with

- \[ \xi = (\zeta - 2\eta/3) = \eta \left( \frac{1}{\phi} - \frac{2}{3} \right) \approx \eta / \phi \]
- \[ \Delta \rho = \rho_s - \rho_f \]
Comparison to Thermal Convection
(McKenzie Tutorial Notes (CIG/bSpace), Katz et al, 2007 Pepi)

\[
\frac{D\phi}{Dt} = (1 - \phi) \frac{P}{\xi} + \Gamma / \rho_s \quad \text{“Magma”}
\]

\[-\nabla \cdot \left( \frac{K}{\mu} \nabla P \right) + \frac{P}{\xi} = \nabla \cdot \left( \frac{K}{\mu} \left[ \nabla P^* + \Delta \rho g \right] \right) + \Gamma \frac{\Delta \rho}{\rho_f \rho_s} \]

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\]

\[
\nabla P^* = \nabla \cdot \eta \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) - \phi \Delta \rho g
\]

\[
\frac{DT}{Dt} = \nabla^2 T
\]

\[
\nabla \cdot \mathbf{v} = 0
\]

\[
\nabla P = \nabla \cdot \eta \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) - \text{Ra} T g
\]
Non-linear wave equations for porosity


\[ \frac{D\phi}{Dt} = (1 - \phi) \frac{P}{\xi} + \frac{\Gamma}{\rho_s} \]

\[ - \nabla \cdot \left( \frac{K}{\mu} \nabla P \right) + \frac{P}{\xi} = \nabla \cdot \left( \frac{K}{\mu} \left[ \nabla P^* + \Delta \rho g \right] \right) + \frac{\Delta \rho}{\rho_f \rho_s} \]

with

- \( \xi = (\zeta - 2\eta/3) = \eta \left( \frac{1}{\phi} - \frac{2}{3} \right) \approx \eta/\phi \)
- \( \Delta \rho = \rho_s - \rho_f \)
**Intrinsic length Scale: The compaction length**


The Compaction Length

\[ \delta = \sqrt{\frac{K(\phi)\zeta(\phi)}{\mu}} \]

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\[ \delta = \sqrt{\frac{K(\phi)\zeta(\phi)}{\mu}} \]

Permeability \hspace{1cm} K(\phi) \propto \phi^n

Solid Bulk Viscosity \hspace{1cm} \zeta(\phi) \propto \eta/\phi^m

Length scale of pressure variations due to a change in flux.

Length scale of pressure variations due to a change in flux.

Graph showing the relationship between porosity, flux, pressure, and compaction lengths.
Non-linear porosity waves

Variations in melt flux propagate as *non-linear* porosity waves
Non-linear porosity waves

- Variations in melt flux propagate as non-linear porosity waves
- Speed and structure of porosity waves depends on permeability and solid rheology

Monday, January 10, 2011
Non-linear porosity waves

Collision of 2, 2D-porosity waves. P2-P2 FEM with Semi-Lagrangian 2nd-order time stepping. Hybrid FEniCS/PETSc codes.
Non-linear porosity waves
Non-linear porosity waves

\[ \frac{\partial w}{\partial t} \cdot u + \frac{\partial w}{\partial x} \cdot \sigma + M \cdot \nabla w(x) \cdot S(t) = 0, \quad s = 0 \text{ at } t = 0 \]

Note: Free stress boundary conditions \( n = 0 \) are automatically included as natural boundary conditions.

\[ \text{Issue is simply choice of Discrete function space } V \text{ for test functions } w \text{ and } u. \]

• Instability of 1D-3D waves. 3-D mixed finite elements. Hybrid FEniCS/PETSc codes. (CIG)
Non-linear porosity waves
Non-linear porosity waves


- Instability of 1D-3-D waves. Spiegelman and Wiggins, 1994, GRL.
- FV geometric multi-grid code.

\begin{equation}
\mathbf{\Omega} \mathbf{\rho} \mathbf{\nabla} \mathbf{\cdot} \mathbf{u} \nabla \mathbf{=} \mathbf{\Omega} \mathbf{\nabla} \mathbf{\cdot} \mathbf{\sigma} + \mathbf{M} \mathbf{\nabla} \mathbf{w}(x_s) S(t) \nabla \mathbf{s} \nabla \mathbf{= 0}
\end{equation}

Note: Free stress boundary conditions are automatically included as natural boundary conditions.

Issue is simply choice of Discrete function space V for test functions w and u.
Non-linear porosity waves

• Wave behavior is the natural consequence of non-linearity of flux with porosity and viscous deformation of the solid.
• Waves are generated by obstructions in the flux.
• Implies that magma dynamics is highly time dependent
• Solitary waves provide an excellent non-linear benchmark for space-time codes.
• Simpson and Spiegelman, JSC, 2010 provides sinc-codes for calculating spectrally accurate wave profiles in 1, 2 and 3-D.
Other Localization instabilities
Mechanical shear band formation, experiments

Kohlstedt and Holtzmann,
Ann Rev Geophys., 2009
Mechanical shear band instability
(Katz et al, 2006 Nature)

- Full equations with porosity weakening shear viscosity $\eta(\phi, V)$
- Neglect gravity (at lab scale)
- PETSc codes with segregated SNES
- Spontaneously develops shear band instability

**Figure a**
Olivine + chromite (4:1) + 4 vol. % MORB $\gamma = 3.4$

**Figure b**
Simulated porosity (volume fraction) $\gamma = 2.79$

**Figure c**
Simulated perturbation vorticity (%) $\gamma = 2.79$

**Figure d**
FFT bin amplitude vs. Band count

Katz, Spiegelman & Holtzman, 2006 Nature

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Mechanical shear band instability
(Katz et al, 2006 Nature)

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d FFT bin amplitude

Katz, Spiegelman & Holtzman, 2006 Nature

a Cartoon

melt flow

opening

solid flow

opening
Mechanical shear band instability
(Katz et al, 2006 Nature)

a. Olivine + chromite (4:1) + 4 vol. % MORB

\[ \gamma = 3.4 \]

b. Simulated porosity (volume fraction)

\[ \gamma = 2.79 \]

c. Simulated perturbation vorticity (%)

\[ \gamma = 2.79 \]

d. FFT bin amplitude

\[
\begin{array}{c|c|c}
\text{Band angle, } \theta & \text{Simulation} & \text{Experiment} \\
\hline
10^\circ & 0.5 & 1 \\
20^\circ & 0.5 & 0.5 \\
30^\circ & 0.5 & 0.5 \\
40^\circ & 0.5 & 0.5 \\
50^\circ & 0.5 & 0.5 \\
60^\circ & 0.5 & 0.5 \\
70^\circ & 0.5 & 0.5 \\
80^\circ & 0.5 & 0.5 \\
90^\circ & 0.5 & 0.5 \\
\end{array}
\]

Katz, Spiegelman & Holtzman, 2006 Nature

Linear Analysis

\[ \gamma = 1 \]

\[ \gamma = 3.4 \]

\[ \gamma = 2.79 \]
Other sources of melt channelization

Reactive infiltration instability

Chemistry

Figure 5. Photomosaic of a mountainside in the Muscat Massif. As at all scales, dunite orientations measured across the image area are used to correct dunite widths. The lighter rocks are dunite, the darker are harzburgites. The two geologists in the center of the image are standing ~ 50 m apart.

(Dunite)

Hartzburgite

(Oman Ophiolite)

Concentration/Cl

La Ce Nd Sm Dy Er

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Reactive Infiltration Instability

(Spiegelman et al. 2001)

(a) Reactive Zone
$c^s = 0.95 + \epsilon$

(b) Unreactive Zone
$\phi = 1, w = 1, W = 0$

Periodic (wrap around)

Melt outflow

Melt inflow

Height (z/δ)

$\Gamma_0 \sim 0.025$

$C^{eq}_f$
Reactive Infiltration Instability

(Spiegelman, Kelemen and Aharonov, JGR 2001)
Reactive Infiltration Instability

Porosity

Solid Concentration
Chemical Consequences of Melt Channeling
(Spiegelman & Kelemen, 2003, G3)

Calculations

Melts

Center melts
Depleted edges

Mean concentration
13.61% Batch melt

Residues

La Ce Nd Sm Eu Gd Tb Dy Ho Er Tm Yb


t=70.00 [ 0.306849 3.1543 ]
Chemical Consequences of Melt Channeling
(Spiegelman & Kelemen, 2003, G3)

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Concentration/C0

La Ce Nd Sm Dy Er

Cpx in Oman harzburgite

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Primitive Mantle Normalized

Ba Th U La Ce Nd Sm Eu Gd Tb Dy Ho Er Tm Yb

Melts

Niu&Batlz Seaamounts PetDB Offaxis PetDB On–Axis

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Cpx in Oman harzburgite
Summary
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- Basic Magma Dynamics is Stokes coupled to a dispersive non-linear wave equation.
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- Basic Magma Dynamics is Stokes coupled to a dispersive non-linear wave equation
- Coupling non-linear permeability with a deformable matrix introduce a wide range of behavior and leads to development of small-scale structures
Basic Magma Dynamics is Stokes coupled to a dispersive non-linear wave equation

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- length scale of features controlled by the compaction length (0-10km)
Open Questions
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- What are the observable consequences of these mechanisms.

- How/do small scale physics influence large scale mantle dynamics?
Petascale AMR FEM/Rhea

Global Convection code with parallel adaptive mesh refinement

- Minimum mesh spacing \( \sim 1 \text{km} \) resolves weak boundaries
- Adaptive refinement in weak/plastic regions
- Full refinement at \( h = 1 \text{km} \sim 10^{12} \) elements (exascale?)
- Can accomplish, goal oriented adaptation to convergence with 150-300 million elements \( (10^3-10^4) \) savings
Geo problems: mid-ocean ridge models
(Courtesy Richard Katz)

Melt and solid flow field for a heterogeneous melting mantle beneath a mid-ocean ridge

Full solution of magma dynamics using the "enthalpy method"
Katz, J. Pet, 2008

PETSc parallel, structured FV code on staggered mesh.
Location of Volcanoes in Subduction Zones

Global Slab Contours and Volcanoes
(Syracuse and Abers, 2006)
Geo problems: Subduction Zone models
(Spiegelman, van Keken, Hacker, 2009)
Nicaragua Model

(Syracuse et al, 2009)

Nicaragua:
Thermal/Flow Model
van Keken
Unstructured FEM
Kinematic Slab
Olivine, Dislocation creep
rheology for wedge

Monday, January 10, 2011
Nicaragua:
Slab Water model
Hacker
Perple_X
Wt % water bound in slab minerals
Nicaragua:
Slab Water model
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Monday, January 10, 2011
Permeable Flow model on subdomain
Spiegelman (MADDs-FP -- CIG)

Nicaragua:
Fluid Flow on SubDomain
Unstructured FEM/P2P2
Hybrid FEniCS/PETsc Codes
Full Magma Dynamics

Monday, January 10, 2011
Fluid Flow Trajectories given dehydration rates

Time: 0.0

w0 = 100 cm/yr
eta0 = 1.0e21
phi_0 = 0.005
L = 15 km
w0/U0 = 13.9
delta = 56.3 km
Comparison to TUCAN Data

low permeability

132.9 ± 4.1 km
(Syracuse & Abers)

high permeability

$w_0 = 10 \text{ cm/yr}$
$\phi_0 = 1.21$
$L = 15 \text{ km}$
$u_0 = 13.9$
$\Delta = 56.3 \text{ km}$
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- Magma Dynamics is a natural extension of Mantle Dynamics (Stokes + Darcy)
- addition of a melt phase introduces new dynamics and new length & time scales
- Many different mechanisms suggest some form of mesoscale organization into melt channels in the mantle which may have significant observational consequences.
- Small changes in couplings can significantly change the physics and computational requirements of these problems
Open Questions

- What are the interactions/dominant mechanism for localization at the meso-scale?

- What are the interaction between meso-scale and plate-boundary scale flow? Plate boundary dynamics and global mantle convection?

- What are the observational consequences of these processes and can important inferences be made from existing data on the structure and processes of partially molten regions?
Computational & Software issues

- Magma Dynamics is fundamentally a coupled multi-scale, multi-physics problem.

- How do we develop flexible, high-performance tools for more readily exploring the space of models and behavior?

- This is a completely different issue than finding/tuning a well understood problem (e.g. Navier-Stokes, Seismic Wave tomography).

- Much of the essential software already exists (e.g. PETSc, FEniCS). Next time will detail how we can use it to develop some flexible and general approach to solving multi-physics models.
Philosophy of multi-physics PDE based models

- Overall Structure and Choices
- Software design for managing choices (PETSc, FEniCS)
- General abstractions of Non-linear multi-physics problem
- Examples in Hybrid FEniCS/PETSc codes.
- HPC issues...