

12 steps to a Fast Multipole Method on GPUs

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Why would I want to learn FMM?

The combination of algorithmic acceleration and hardware acceleration can have tremendous impact. Therefore, we are interested in studying and developing GPU implementations of modern and efficient algorithms.

The FMM is a fast algorithm for calculating matrix vector multiplications in $O(N)$ time, and it runs very fast on GPUs. Its combination of high degree of parallelism and $O(N)$ complexity make it an attractive solver for the Peta-scale and Exa-scale era. It has a wide range of applications, e.g. quantum mechanics, molecular dynamics, electrostatics, acoustics, structural mechanics, fluid mechanics, and astrophysics.

What are the prerequisites?

- Basic understanding of linear algebra and calculus
- Some experience with C or C++

The aim of this course

This course will provide a hands-on tutorial with simple exercises that will lead to a full FMM code that runs on GPUs. Each step is carefully planned so that there is no sudden jump in the difficulty. During the tutorial, each attendee will have remote access to the GPU cluster in Nagasaki Japan. There will be Teaching Assistants walking around to assist you, so that no one is left behind.

Taking small steps is the key to programming complicated algorithms.

Saving these incremental steps helps the people who want to follow your code

Simple examples that incrementally lead to the final code are much more helpful than a manual

The 12 steps

The following topics will be covered:

- step 1 : direct N-body
- step 2 : multipole expansion
- step 3 : tree structure
- step 4 : link lists, interaction lists
- step 5 : treecode
- step 6 : M2L translation
- step 7 : single level FMM
- step 8 : M2M,L2L translation
- step 9 : multi-level FMM
- step 10 : direct N-body on GPUs
- step 11 : P2P on GPUs
- step 12 : M2L on GPUs

Supplemental material

http://www.ics.uci.edu/~ihler/papers/ihler_area.pdf

http://math.nyu.edu/faculty/greengar/shortcourse_fmm.pdf

<http://www.umiacs.umd.edu/~ramani/cmssc878R/index.html>

Flow of the FMM
 The FMM calculates the interaction between the source particles (shown in red) and the target particles (shown in blue) in the following steps:

- Step 1. particle to multipole expansion
- Step 2. multipole to multipole expansion
- Step 3. multipole to local expansion
- Step 4. local to local expansion
- Step 5. local expansion to particle
- Step 6. particle to particle

