Supporting Information

Current-driven terahertz light emission from graphene plasmonic oscillations

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S1. Normalization procedure for the plasmonic emission spectra

In the radiation spectra described in this work, the plasmonic emission peak is superimposed on a broader pedestal arising from greybody radiation [e.g., see red trace of Fig. 2(a) of the main text]. Essentially the same pedestal without the plasmonic peak is obtained by measuring the output light of the same sample held at the same base temperature $T_{\text{base}}$ and driven with the same input electrical power $P_{\text{in}}$, but gated at charge neutrality [e.g., see black trace of Fig. 2(a)]. The plasmonic emission peak can therefore be isolated by taking the difference between these two traces. In fact, the greybody output power can generally be expected to vary with carrier density, and therefore gate voltage, because of two conflicting effects. First, by Kirchhoff’s law of thermal radiation, the long-wavelength graphene emissivity increases with increasing carrier density by an amount equal to the corresponding increase in free-carrier absorption. Second, the larger the carrier density, the smaller the electronic-temperature increment produced by the input power (as discussed in section S4 below). In our measurements we find that, if $T_{\text{base}}$ and $P_{\text{in}}$ are held constant, these two effects tend to balance each other so that the greybody output power only varies by a small amount even for large changes in gate voltage [again as illustrated in Fig. 2(a)]. This observation justifies our normalization procedure of subtracting off the emission spectrum measured at charge neutrality from the spectrum at finite gate voltage to isolate the plasmonic contribution. At the same time, in order to account for the small variations in greybody output power with gate voltage just described, before taking this difference the charge-neutrality spectrum is sometimes rescaled by a constant factor (generally between 1 and 1.1). Specifically, this factor is selected to produce a flat, near-zero baseline in the normalized spectra near the edges of the measured spectral range. Importantly, this small correction does not have any effect on the plasmonic resonance frequencies and output powers extrapolated from the measurement results.

S2. Data fitting procedure for the plasmonic emission frequency

The THz emission peaks measured in this work originate from the fundamental plasmonic resonances of the graphene nanoribbons under study. The frequency of peak emission $\nu_{\text{pl}}$ can therefore be computed from eq. (1) of the main text with $n = 1$, i.e.,

$$
\nu_{\text{pl}} = \frac{\omega_1}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{q^2 v_F (\pi - q_F)}{\sqrt{\hbar \epsilon_0 (\epsilon_{r1} + \epsilon_{r2}) \frac{N^1}{4}}}} \sqrt{\frac{1}{w}},
$$

(S1)
where $N$ is the carrier density, $w$ is the ribbon width, $v_F \approx 1 \times 10^8$ cm/s is the Fermi velocity, $\varphi_r$ is the plasmon reflection phase at the ribbon edges, and $\varepsilon_{r1}$ and $\varepsilon_{r2}$ are the relative permittivities of the surrounding materials immediately below and above the graphene sample. In our device geometry [see Fig. 1(a) of the main text], the medium above is air (i.e., $\varepsilon_{r2} = 1$), while the medium below consists of a thin SiO$_2$ layer of thickness $t_{OX} = 300$ nm on a Si substrate. The permittivity $\varepsilon_{r1}$ is therefore a weighted average of those of SiO$_2$ and Si ($\varepsilon_{OX}$ and $\varepsilon_{Si}$, respectively), with the relative weights determined by the overlap factors of the plasmonic field-intensity distribution with the two materials. Because of the strong sub-wavelength confinement of the plasmonic resonances under study, their field intensity varies with distance $z$ from the graphene plane approximately as $\exp(-2\text{Re}[\beta(\omega)]|z|)$, where $\text{Re}[\beta(\omega)] = (\pi - \varphi_r)/w$ is the in-plane propagation constant described in the main text. Therefore, the effective permittivity of the underlying medium can be evaluated as

$$
\varepsilon_{r1} = \varepsilon_{OX} \left\{ 1 - \exp[-2(\pi - \varphi_r)t_{OX}/w] \right\} + \varepsilon_{Si} \exp[-2(\pi - \varphi_r)t_{OX}/w].
$$

(S2)

with the materials parameters $\varepsilon_{Si}$ and $\varepsilon_{OX}$ obtained as a function of frequency from available experimental data in the literature.$^{S1}$ It should be noted that, when combined with eq. (S1), this expression introduces an additional, weaker $\varphi_r$ dependence to $\nu_{pl}$.

The experimental data plotted in Fig. 3(c) of the main text ($\nu_{pl}$ versus $N$ for $w = 530$ and 810 nm) have been fitted using these equations, with the reflection phase $\varphi_r$ used as the only fitting parameter. In passing we also note that, for each data point in this figure, the hole density $N$ is computed with the gate capacitor relation from the applied gate voltage $V_{GS}$. For simplicity, the spatial variations in local gate voltage along the nanoribbons (from $V_{GS}$ at the source side to $V_{GS} - V_{DS}$ at the drain side) are not included in this analysis. These variations produce a commensurate gradient in the carrier density $N$ and therefore in the plasmonic resonance frequency $\nu_{pl}$ across the device channel. However, the resulting range of values of $\nu_{pl}$ is sufficiently small compared to the measured linewidths of the emission peaks that it can be neglected in the analysis of Fig. 3(c).

**S3. FDTD simulation results**

Finite difference time domain (FDTD) simulations have been carried out to compute the plasmonic resonance frequencies of both experimental samples EMT1 and EMT2, using the numerical solver FDTD Solutions by Lumerical. In these simulations, the computational window consists of a two-dimensional vertical cross-section of a unit cell of each device, comprising the Si substrate, the
SiO$_2$ gate dielectric, and a nanoribbon. The graphene active material is modeled as a slab of finite thickness (5 nm, limited by the computational mesh size) described by a position-independent Drude permittivity function$^{S2}$ [the same expression used to derive eq. (S1)]. A Drude scattering lifetime $\tau$ of 60 fs is used in the simulations (corresponding to a carrier mobility of $\sim 2500$ cm$^2$/Vs), which produces resonance linewidths comparable to the measured values. With this setup, we compute the transmission spectra through the plane of the nanoribbons using periodic boundary conditions along the lateral direction and perfectly matched layers (PMLs) on the top and bottom boundaries. The input light is a plane wave at normal incidence, with linear polarization perpendicular to the long axis of the nanoribbons. All relevant computational parameters including mesh size, graphene thickness, and vertical extent of the computational window were carefully optimized through extensive convergence tests.

Figure S1. Simulated transmission spectra of graphene nanoribbons. (a, b) Normalized carrier-induced transmission change $1 - T_N(\nu)/T_{\text{CNP}}(\nu)$ computed by FDTD simulations for different values of carrier density $N$. The nanoribbon widths used in these simulations are $w = 530$ nm (a) and 810 nm (b), corresponding to devices EMT1 and EMT2, respectively.

Figure S1 shows simulation results obtained for the nanoribbon width of device EMT1 [panel (a)] and EMT2 [panel (b)], i.e., $w = 530$ and 810 nm, respectively, and for equally spaced carrier-density values in the same range of Figs. 3(a) and 3(b) of the main text. Specifically, each trace in these plots is the normalized transmission change $1 - T_N(\nu)/T_{\text{CNP}}(\nu)$, where $T_{\text{CNP}}(\nu)$ and $T_N(\nu)$ are, respectively, the transmission spectra computed at charge neutrality and for the finite carrier density $N$ listed in the legend. The peaks produced by this normalization are entirely related to optical absorption and scattering by the nanoribbon plasmonic resonances. Therefore, they can be compared to the measured plasmonic emission spectra of Figs. 3(a) and 3(b), and the overall agreement is quite good. In particular, the center frequencies of these FDTD normalized
transmission peaks are in good agreement with the experimental emission frequencies, as illustrated in Fig. 3(c) and discussed in the main text.

S4. Electronic temperature analysis

The elevated electronic temperature $T_{el}$ produced by the injected current in our samples can be computed using eq. (2) of the main text, i.e.,

$$p_{in} = A[T_{el}^3 - T_{latt}^3],$$

(S3)

where $p_{in}$ is the input electrical power density per unit area, $T_{latt}$ is the lattice temperature (related to the substrate temperature as described below), and $A$ is the supercollision electron-phonon coupling constant. This expression is valid if $T_{latt}$ is above a characteristic temperature (the Bloch-Grüneisen limit), where hot electrons predominantly cool down towards the lattice via disorder-assisted acoustic-phonon scattering. For the range of carrier densities considered in this work, the Bloch-Grüneisen temperature is on the order of a few 10 K, so that eq. (S3) is generally applicable. Additionally, this equation neglects diffusive cooling including heat transport into the electrodes, which is a reasonable assumption given the relatively large distance between the source and drain contacts in our samples. As discussed in the main text, all quantities in eq. (S3) vary with position $y$ along the length of the nanoribbons, because of the spatial gradient in $V_G$ (the voltage between the back gate and the graphene channel), leading to commensurate variations in the graphene carrier density $N$. In the following, we discuss how the parameters $p_{in}$, $A$ and $T_{latt}$ can be computed as a function of $y$, so that eq. (S3) can then be solved for $T_{el}(y)$.

In the field-effect-transistor device geometry under study, $V_G$ varies linearly from $V_{GS}$ to $V_{GS} - V_{DS}$ in going from the source to the drain contact, with $V_{GS} < 0$ and $V_{DS} > 0$ for nearly all measurements described in this work. The corresponding local carrier density can then be computed as a function of $y$ from the modified gate capacitor relation

$$N(y) = \left(\pm N_{CV}(y) + \sqrt{N_{CV}^2(y) + 4N_0^2}\right)/2,$$

(S4)

where the lower (upper) sign corresponds to holes (electrons), and

$$N_{CV}(y) = \varepsilon_0 \varepsilon_{OX} [V_G(y) - V_{CNP}] / (q t_{OX}).$$

(S5)

In eq. (S4) the residual carrier density $N_0$ accounts for the finite conductance measured at the charge neutrality point. A value of $4 \times 10^{11}$ cm$^{-2}$ is used for this parameter, following ref. S5 and
consistent with resistivity measurements on our samples. Inclusion of a nonzero residual carrier density in the present analysis is important when computing the electronic temperature at charge neutrality, whereas its effect becomes increasingly negligible with increasing \(|V_{GS} - V_{CNP}|\). For completeness, this correction has also been included in the plasmonic-resonance-frequency analysis of Fig. 3(c) of the main text, although its impact in that context is also quite minimal.

Given \(N(y)\), the input power density \(p_{in}(y)\) can be obtained together with the corresponding value of the carrier mobility \(\mu\) by solving the following two equations:\(^{\text{S7}}\)

\[
p_{in}(y) = J_{DS}^2/|q\mu N(y)|, \tag{S6}
\]

\[
P_{in} = W \int_0^L dy p_{in}(y). \tag{S7}
\]

Here, \(L\) and \(W\) are the length and combined width of all the nanoribbons, respectively; \(J_{DS} = V_{DS}/(WR)\) is the current density flowing through the device, determined from the experimental values of the bias voltage \(V_{DS}\) and resistance \(R\); and \(p_{in} = V_{DS}^2/R\) is the total electrical power fed into the device. This model neglects the dependence of scattering lifetime on carrier density and temperature (so that \(\mu\) can be taken to be constant with \(y\)), which is a reasonable assumption for the range of values of \(N\) and \(T_{\text{base}}\) used in this work.\(^{\text{S7}}\) For each measured emission spectrum, \(\mu\) is computed from the relevant experimental parameters \((V_{GS} - V_{CNP}, V_{DS}, R)\) by substituting eqs. (S4)-(S6) into (S7) and solving the resulting equation. Values close to 1000 \(\text{cm}^2/\text{Vs}\) are consistently obtained, in agreement with the results of electrical measurements [resistivity versus gate voltage, as in Fig. 1(b) of the main text]. Given \(\mu\), \(p_{in}\) can then be evaluated as a function of \(y\) from eq. (S6).

Next, the electron-phonon coupling coefficient of eq. (S3) is estimated using the expression

\[A \approx 7.5 \times 10^{-4} (D^2 N/k_{\text{fL}}) \left[ W/(m^2K^3) \right],\tag{S5}\]

where \(N\) is given in units of \(10^{12} \text{ cm}^2\), \(D\) is the graphene deformation potential in eV, and \(k_{\text{fL}}\) is the normalized disorder-limited carrier mean free path (which does not depend on carrier density\(^{\text{S3}}\)). The parameter \(D\) provides the main source of uncertainty in this temperature analysis, since a wide range of values (from about 10 eV to 70 eV) has been quoted in the literature.\(^{\text{S4,55}}\) In the following we use the intermediate choice \(D = 30\) eV.\(^{\text{S8,59}}\) The product \(k_{\text{fL}}\) is also not precisely known, and a value of 2 is selected, consistent with the analysis of ref. S5 (rescaled to account for the lower mobility of our samples). In passing it should be noted that, because of its linear dependence on \(N\), the coupling constant \(A\) also exhibits significant variations with position \(y\) along the length of the nanoribbons.

S6
Finally, the lattice temperature is estimated from the following expression

\[ T_{\text{latt}}(y) = T_{\text{base}} + \left(1/\alpha_{\text{vdW}} + t_{\text{OX}}/\kappa_{\text{OX}}\right)p_{\text{in}}(y). \]  

(S8)

where \( T_{\text{base}} \) is the substrate base temperature controlled by the cryostat. The second term in eq. (S8) accounts for heat transport across the van der Waals interface between the graphene active layer and the underlying SiO\(_2\), described by the transfer coefficient \( \alpha_{\text{vdW}} = 2.5 \times 10^7 \text{ W/m}^2/\text{K} \).\(^{10} \)

The last term accounts for heat transport across the SiO\(_2\) gate dielectric, which has thickness \( t_{\text{OX}} = 300 \text{ nm} \) and thermal conductivity \( \kappa_{\text{OX}} = 1.3 \text{ W/m/K} \).\(^{11} \)

**Figure S2.** Temperature analysis of graphene nanoribbons under bias. (a) Calculated temperatures of the 2DEG (red trace) and the lattice (blue trace) plotted as a function of position from the source to the drain contact in the nanoribbons of device EMT1, for \( T_{\text{base}} = 80 \text{ K}, P_{\text{in}} = 0.2 \text{ W}, \) and \( V_{\text{GS}} = V_{\text{CNP}} \). (b) Same as (a) for \( V_{\text{GS}} = V_{\text{CNP}} - 75 \text{ V} \). In both panels, the dotted black line shows the base temperature.

Given \( p_{\text{in}}, A, \) and \( T_{\text{latt}} \), the electronic temperature \( T_{\text{el}} \) can then be computed as a function of \( y \) from eq. (S3). Figure S2 shows the results of this analysis for the experimental conditions of Fig. 2 of the main text, i.e., \( T_{\text{base}} = 80 \text{ K}, P_{\text{in}} = 0.2 \text{ W}, \) with \( V_{\text{GS}} = V_{\text{CNP}} \) [Fig. S2(a)] and \( V_{\text{CNP}} - 75 \text{ V} \) [Fig. S2(b)]. As illustrated by these plots, the graphene lattice temperature remains quite close to the base value across the entire device, by virtue of the relatively high thermal conductance of both the graphene/SiO\(_2\) interface and the SiO\(_2\) gate dielectric. In contrast, the electronic temperature is increased significantly by the injected current, with a maximum value of over 450 K near the source contact at charge neutrality. Additionally, large variations in \( T_{\text{el}} \) with position along the nanoribbons are observed in Fig. S2, inversely related to the corresponding variations in carrier density. This behavior originates from two distinct mechanisms. First, the graphene electrical conductivity increases linearly with increasing \( N \), so that a larger fraction of the input electrical power is absorbed in the regions of lower carrier density near the source contact [see eq. (S6)]. Second, the supercollision electron-phonon coupling coefficient \( A \) of eq. (S3) is also
proportional to N, and as a result the thermal relaxation of the 2DEG to the lattice is less efficient in regions of lower carrier densities (and vice versa).

**S5. Plasmonic-emission power analysis**

In our measurements, the voltage $V_{DS}$ across the drain and source contacts, and therefore the current $I_{DS}$ flowing through the nanoribbons, are switched on and off periodically with a slow repetition rate of 200 Hz. In the process, the electronic temperature is modulated from its initial equilibrium value $T_{\text{base}}$ to the position-dependent elevated value $T_{\text{el}}$ described in the previous section. A lock-in amplifier is then used to measure the corresponding change in THz output power. When the device is held at charge neutrality (where no significant plasmonic oscillations can be excited due to the lack of sufficient free carriers), the output light predominantly consists of greybody radiation. Its power spectral density per unit bandwidth can then be computed using Planck’s law,

$$\rho_{gb}(\nu, T) = \frac{2\pi h eWL\nu^3}{c^2} \frac{1}{\exp(h\nu/k_BT)-1},$$  (S9)

where $\nu$ is the optical frequency and $\varepsilon$ is the greybody emissivity. For the latter parameter, a value of $1.6\pm0.8\%$ has been measured in electrically biased unpatterned samples, in reasonable agreement with the $2.3\%$ absorptivity of graphene in thermal equilibrium. The signal detected in our measurements at charge neutrality can therefore be written as

$$S_{\text{CNP}}(\nu) = C(\nu)\Delta\rho_{gb}(\nu)$$  \hspace{1cm} (S10)

$$\Delta\rho_{gb}(\nu) = \langle \rho_{gb}(\nu, T_{el}) \rangle - \rho_{gb}(\nu, T_{\text{base}}),$$  \hspace{1cm} (S11)

where the brackets $\langle \rangle$ indicate averaging over the length of the nanoribbons, and the prefactor $C(\nu)$ is the frequency-dependent transfer function of the measurement setup, including factors such as collection efficiency, transmission through the cryostat windows and bolometer filter, atmospheric losses, and bolometer responsivity.

When the device is gated with a voltage $V_{GS}$ away from charge neutrality, the plasmonic emission peak emerges over the greybody background, and the detected signal becomes

$$S_{V_{\text{gs}}}(\nu) = C(\nu)[\Delta\rho_{gb}(\nu) + \Delta\rho_{pl}(\nu)],$$  \hspace{1cm} (S12)
where $\Delta \rho_{pl}(v)$ is the increase in plasmonic output-power spectral density caused by the current-driven temperature increment. The corresponding increase in total plasmonic output power, integrated over the entire emission peak, can be approximated as $\Delta P_{pl} = \Delta \rho_{pl}(v_{pl})\delta v$, where $v_{pl}$ and $\delta v$ are the peak center frequency and linewidth (full width at half maximum), respectively. From the ratio of eq. (S12) to eq. (S10) we finally obtain

$$\Delta P_{pl} = \left[ \frac{S_{VGS}(v_{pl})}{S_{CNP}(v_{pl})} \right] - 1 \Delta \rho_{gb}(v_{pl})\delta v.$$  

(S13)

For any given set of operating conditions (base temperature, gate voltage, and input electrical power), the frequencies $v_{pl}$ and $\delta v$ and the ratio $S_{VGS}(v_{pl})/S_{CNP}(v_{pl})$ can be obtained directly from the FTIR measurement results, such as the data shown in Fig. 2 of the main text. The greybody power density increment $\Delta \rho_{gb}(v_{pl})$ can be computed by combining eqs. (S9) and (S11) and the results of the temperature analysis described above. Equation (S13) can then be used to evaluate $\Delta P_{pl}$.

All values of the plasmonic output power presented in the main text were obtained using this procedure. For example, Fig. 2(a) shows the spectra $S_{CNP}(v)$ (black trace) and $S_{VGS=V_{CNP}-75V}(v)$ (red trace) measured with device EMT1 at $T_{\text{base}} = 80$ K and $P_{in} = 0.2$ W. From these data we find $v_{pl} = 7.9$ THz, $\delta v = 2.0$ THz, and $S_{VGS=V_{CNP}-75V}(v_{pl})/S_{CNP}(v_{pl}) = 1.9$. Using eqs. (S9) and (S11) with the electronic temperature distribution $T_{el}(x)$ plotted in Fig. S2(a), we compute a greybody power density increment $\Delta \rho_{gb}(v_{pl})$ of 1.9 nW/THz. By combining these values in eq. (S13), we finally estimate $\Delta P_{pl} = 3.4$ nW.

**S6. Additional measurement results**

Figure S3 shows several normalized emission spectra of device EMT2, measured at different temperatures for $P_{in} = 0.2$ W in panel (a), and for different input powers at $T_{\text{base}} = 80$ K in panel (b). The corresponding increments in 2DEG temperature over the substrate $\Delta T = T_{el} - T_{\text{base}}$ (at the source side of the nanoribbons) and in plasmonic output power $\Delta P_{pl}$ are plotted versus $T_{\text{base}}$ in Fig. S3(c) and versus $P_{in}$ Fig. S3(d). The behavior observed in these plots is qualitatively analogous to that of Fig. 4 of the main text, which presents similar data measured with device EMT1.
Figure S3. Temperature and input-power dependence of plasmonic emission from graphene nanoribbons. (a) Normalized plasmonic emission peaks measured with device EMT2 at different base temperatures for $P_{in} = 0.2$ W. (b) Normalized emission peaks of the same device at $T_{base} = 80$ K for different values of the input electrical power. (c, d) Plasmonic output power $\Delta P_{pl}$ and electronic temperature increment $\Delta T = T_{el} - T_{base}$ (near the source contact) versus base temperature (c) and versus input power (d). In all these measurements, the applied gate voltage is $V_{GS} = V_{CNP} - 60$ V.

REFERENCES


