Qualifying Exam MA 582 problems April 2016

Problem 1.

Consider the $N(0, \theta)$ family, where $\theta > 0$; note that $var(X) = \theta$ if $X \sim N(0, \theta)$.

a. Does this family satisfy all the regularity conditions for MLE?
b. Find the maximum likelihood estimator (MLE) of θ, call it Y_n.
c. Show whether or not Y_n is unbiased for θ.
d. Show whether or not Y_n is a consistent estimator for θ.
e. Show whether or not Y_n is asymptotically normal, and if it is, identify its asymptotic normal variance.
f. Find the MLE of θ^4. Show whether or not it is biased.
g. Find a function g so that $n^{1/2}(g(Y_n) – g(\theta))$ is asymptotically standard normal for all values of $\theta > 0$.

Problem 2.

We say the rv X has the W distribution with parameter $\theta > 0$ (written $X \sim W(\theta)$) if X has pdf

$$f(x, \theta) = 3x^2/\theta^3, \text{ for } 0 < x < \theta, \text{ and } f(x) = 0, \text{ elsewhere.}$$

Consider the parameterized W family $\{W(\theta) : \theta > 0 \}$.

a. Show that the MLE of θ is the sample maximum.
b. Let Y_n be the maximum of the random sample of size n. Show that Y_n is a consistent estimator of θ.
c. Find the pdf of Y_n. (*Hint:* Find the cdf first.)
d. Show that Y_n is NOT an unbiased estimator of θ.
e. Show that $n(\theta - Y_n)$ converges in distribution, and find its asymptotic distribution explicitly.
f. Find an unbiased estimator of θ, call it T_n. Show that T_n is a consistent estimator of θ.
g. Show that $n(\theta - T_n)$ converges in distribution, and find its asymptotic distribution explicitly. (Hint: Use parts (e) and (f) here.)