PROBLEMS ON OSCILLATIONS AND PATTERN FORMATION IN MATHEMATICAL BIOLOGY

GEORGIY S. MEDVEDEV

Advisors: Professors T. Kaper and N. Kopell.

Problems of mathematical biology have increasingly attracted attention during the last two decades, in large part due to their scientific significance and potential medical applications. The present dissertation is concerned with two problems of mathematical biology:

a) the formation of spatial patterns in *Proteus mirabilis* bacterial colonies, and

b) synchronization and transient dynamics in the chains of coupled oscillators modeling active dendrites.

Both problems are motivated by biological experiments, and they both require methods of dynamical systems and partial differential equations.

In the first project of this dissertation, we derive and study a model for a *Proteus mirabilis* colony development. The model consists of a degenerate parabolic partial differential equation and an ordinary differential equation. The most interesting feature of this system is that it generates interface dynamics that are time periodic, just as seen in experiments. We show that each period consists of two distinct phases, and we identify the transition mechanisms between them. Also, we demonstrate that a lower-dimensional system for the diffusivity captures well the full system dynamics. The main tools used in the analysis are the method of matched asymptotics and techniques for parabolic partial differential equations.

Modeling calcium dynamics in *dopaminergic* neural cells constitutes the second part of this dissertation. Dopaminergic cells are known to play an important role in several common clinical disorders (*Parkinson's disease, schizophrenia*) and in the mechanism underlying goal oriented behaviors. The objectives of the mathematical modeling are to explain the role of the calcium dynamics in isolated cells and to give clues to the origin of the *in vivo* firing patterns. We study the chains of coupled oscillators modeling cells with excitable dendrites, in which each piece of the dendrite is capable of autonomous oscillations. The analysis focuses on the mechanisms for synchronization and transient dynamics in such chains. It reveals an unintuitive fact about the duration of the transients: they are longer for strong coupling. The main tools used are the geometric theory for singularly perturbed dynamical systems, asymptotic expansions, and a direct Lyapunov method.