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C S 329 E
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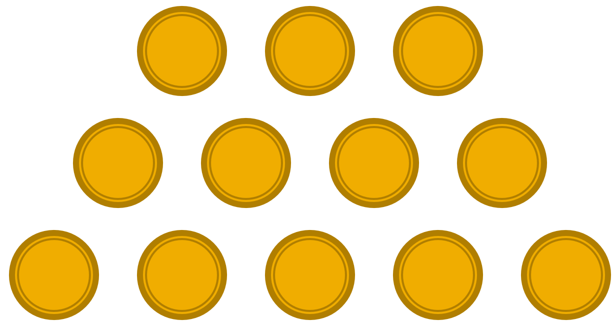
The Game of Nim

How to Play

The setup:

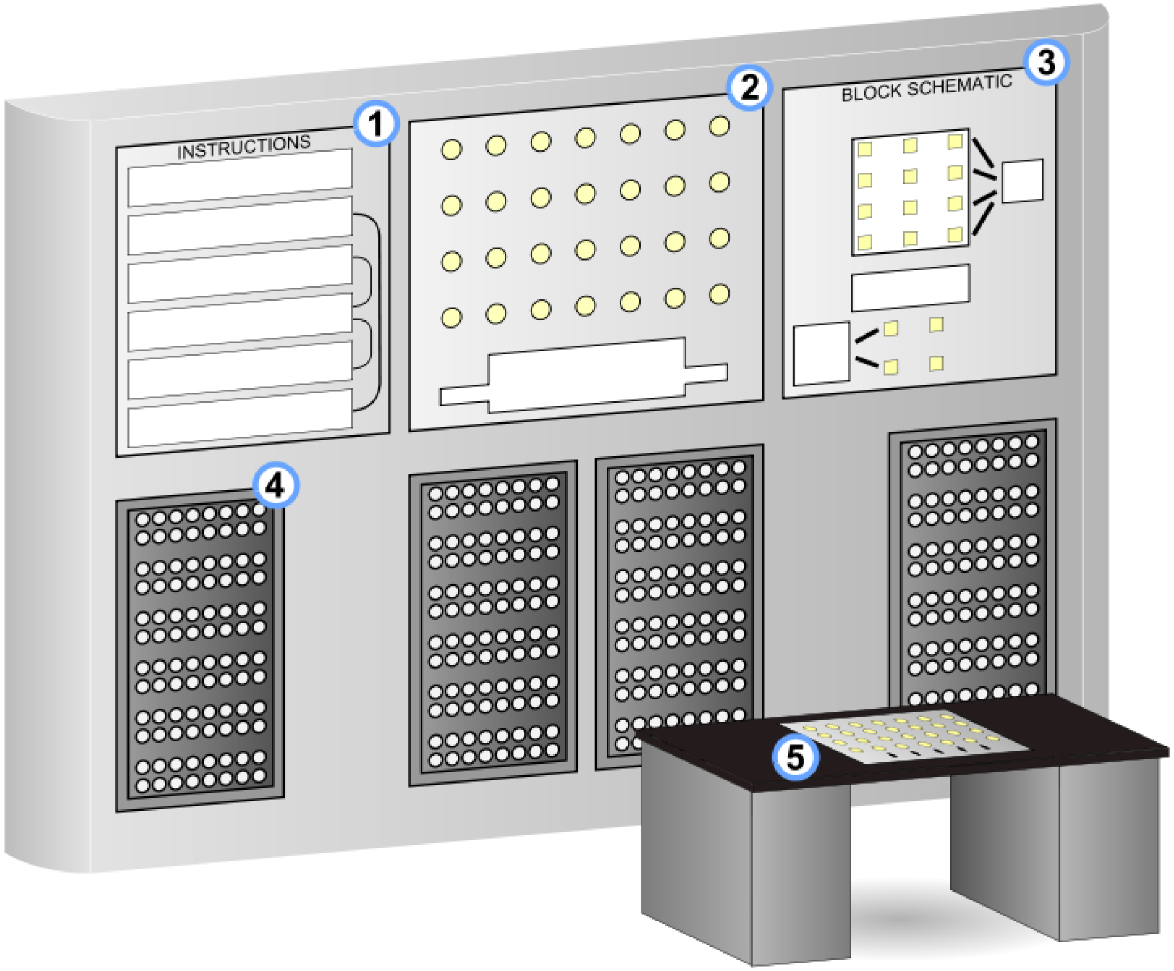


- 2 players take turns picking circles from each row (we call the rows “heaps”).
- At each turn, at least 1 circle has to be picked.
- A player cannot pick from more than 1 row.

Background

- Variants played since ancient times
  - resemblance to Chinese “picking stones”
- Current name and theory developed by C. Bouton of Harvard in 1901
  - name taken from German *nimm* meaning “take”

Nimrod

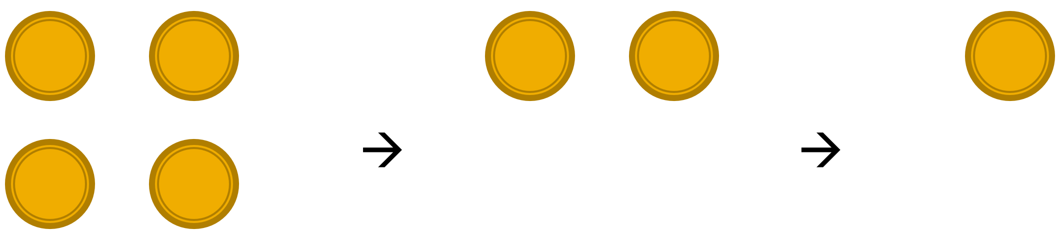


http://en.wikipedia.org/wiki/Nimrod_(computing)

Simple Example

Player 1 takes 2 from heap 2

Player 1 is forced to take the last one



Player 2 takes 1 from heap 1

Player 2 wins!

Theory

- Theory completely solved for any number of heaps/objects by C. Bouton
- Based upon *binary digital sum* of heap sizes
  - also known as “nim-sum”

Algorithm!

- Write the size of each heap in binary
- Add the sizes without carrying
  - Simple rule of thumb:
    - Column w/ even # of 1's = 0
    - Column w/ odd # of 1's = 1

Binary Digital Sum

1 0

1 1

= 0 1

1 0

1 0

= 0 0

Strategy

- Winning strategy: finish each move such that the nim-sum is zero
  - If your partner gives you a non-zero nim-sum, it is **always** possible for you to make it into a zero nim-sum.
  - If your partner gives you a zero nim-sum, it is **never** possible for you to keep it at a zero nim-sum. You will have to change it into a non-zero nim-sum.

Example

010

011

100

= 101

→

010

011

001

= 000

Endgame

- When the next move will result in heaps of size 1.
  - <u>Normal play</u>: Move such that an *even* number of heaps of size 1 remain. Here, you will lose with Normal play!
  - <u>Misère play</u>: Move such that an *odd* number of heaps of size 1 remain.

Any Questions?