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There are many links between mathematics and the humanities, including literature and art. I'm excited to visit Kilachand and to explore some of these links with you. There's plenty more to discover, too. To whet your appetite, I've suggested below a few preparatory explorations you can look at if you wish, before we meet. I hope they will enrich your experience, but you'll still be able to follow everything even if you don't get the chance to delve into any these resources beforehand, so there's absolutely no pressure.

## My Background

I'm a Professor of Mathematics at Birkbeck College (one of the colleges of the University of London), and concurrently Professor of Geometry at Gresham College, an institution founded in 1597 to provide free public lectures for all (happily nowadays only in English, after the requirement to lecture in both English and Latin was removed in the $18^{\text {th }}$ century). My mathematics research is in an area of abstract algebra known as group theory, but alongside that l've always been fascinated by the place of mathematics as a creative art within culture and society more broadly. Here are some links if you want to find out more.

- The New York Times profiled me and my work in an article by Siobhan Roberts https://www.nytimes.com/2021/03/06/science/math-gresham-sarah-hart.html.
- I enjoyed being a guest last year Steven Levitt's podcast People I (Mostly) Admire. Steven is an economist best known for his smash hit book Freakonomics. You can listen at https://freakonomics.com/podcast/mathematician-sarah-hart-on-why-numbers-are-music-to-our-ears/ (or wherever you get your podcasts).
- As Gresham Professor of Geometry I give six free public lectures each year. They are all broadcast live, as well as recorded and posted on YouTube for anyone to watch free, forever. The most watched in the last year was "Where do Mathematical Symbols Come From?".


## The Mathematics of Perspective in Art

One of the great achievements of Renaissance art is the discovery of the rules of perspective by Filippo Brunelleschi. But where do these rules come from, and how do they work? In this lecture we will explore the mathematics of perspective, and see how it has been used to create some of our greatest works of art. Along the way we'll encounter effects like "anamorphic" perspective used to create the apparently distorted skull in Holbein's famous painting "The Ambassadors". You can also see this technique used in televised sports, where the sponsor's logo is painted onto the pitch so as to be seen correctly from the angle of the cameras. We'll also talk about how perspective painting is linked to the strange world of projective geometry, in which the famous parallel postulate of Euclid does not hold.

We'll explain the math behind the rules of perspective when we meet, but one of the rules is that parallel lines that aren't parallel to the picture plane, meet in what we call a vanishing point. We can test
this out nowadays by looking at photographs. For example, here's a picture of a London street, on the left the original picture and on the right with the receding parallel lines extended - they do indeed meet.


Sometimes we have more than one vanishing point, if there are several sets of parallel lines, as in this painting Paris Street, Rainy Day, by Gustave Caillebotte.


Once we understand the rules of perspective, we can play with them to create fun effects, like pictures hidden within pictures. The most famous classical example of this is in Holbein's Ambassadors, where that weird smear is, if you look at it from exactly the right vantage point, actually a skull. These effects are known as anamorphic perspective.


When we meet we'll discuss the math required to produce this kind of effect, but ahead of that I'd like you to have a go at producing your own anamorphic perspective pictures.

## Things to try

1. Extend the parallel lines in this lovely photograph, taken of Manhattan Bridge in 1935, by the photographer Berenice Abbott, to find the vanishing point.
2. Take your own photo of a street or building (the more parallel lines, the better), and find the vanishing point.
3. Become a perspective detective: I've told you that the rules of perspective were first discovered in Italy during the Renaissance. Before the discovery, if we try and find vanishing points in art we would expect to find that parallel lines aren't meeting. Visit the Web gallery of art https://www.wga.hu/index.html and see if you can guess just from the art when the rules were discovered. (Obviously you could google it but where's the fun in that?) Over the page are some suggestions to get you started.

- Giotto, Legend of St Francis:


Apparition at Arles (1300) https://www.wga.hu/html/g/giotto/assisi/upper/legend/franc18.html

- Agnolo Gaddi, The Last Supper (1395) https://www.wga.hu/html/g/gaddi/agnolo/various/lastsup.html
- Sassetta, The Last Supper (1423) https://www.wga.hu/html/s/sassetta/eucharis/4lastsup.html
- Andrea del Castagno, The Last Supper (1454) https://www.wga.hu/html/a/andrea/castagno/1 1440s/08lasts1.html
- Raphael, The School of Athens (1509) https://www.wga.hu/html/r/raphael/4stanze/1segnatu/1/athens.html

4. Over the page l've created an anamorphic design - to view it, hold the page sideways and put your eye as close as you can manage above the page, a little way to the right of the image. It might help to close the other eye. You can draw your own anamorphic designs too. To try this, you'll first need an image that fits in a square grid. On the right is an $8 \times 8$ grid with a simple design that you can use if you like. Alternatively, you can draw your own picture in the empty grid below. If you are a good artist (unlike me!) you can draw anything you like, but you could also look online for $8 \times 8$ pixel art, for example here: http://www.pixelart.name/pixel-art-8x8/ (this was where I got the simple bird design from). Then you need to
 transfer the design to the distorted grid over the page. Each square "pixel" becomes a trapezium. If you do have pixels, just colour the corresponding trapezium pixels in the same way. If you have drawn a more complex design, you'll need to use your artistic prowess to distort the image, with the grid as a guide. When you're done, view it from just above the page an inch or so to the right of the finished design, and you should see the undistorted image.

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