

---

# Special Theory of Relativity

---

Marc McGuigan

North Quincy High School

---

# Outline

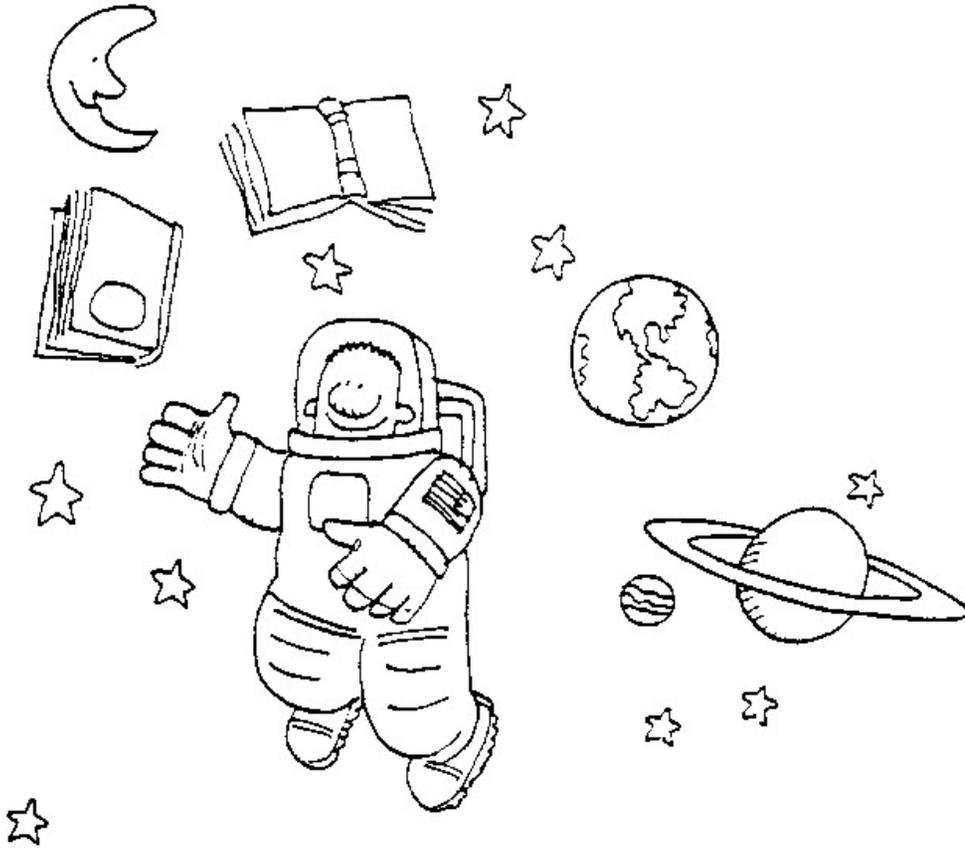
- Introduction
  - Relativity
  - The speed of light
  - Einstein's Solution
  - Time
  - Space
  - Conclusion
-

# Introduction

- The Special Theory of Relativity is the theory put forth by Albert Einstein in his 1905 paper, "On the Electrodynamics of Moving Bodies"
- "Special" because it applies the theory of relativity to inertial reference frames in flat space-time (no gravity)

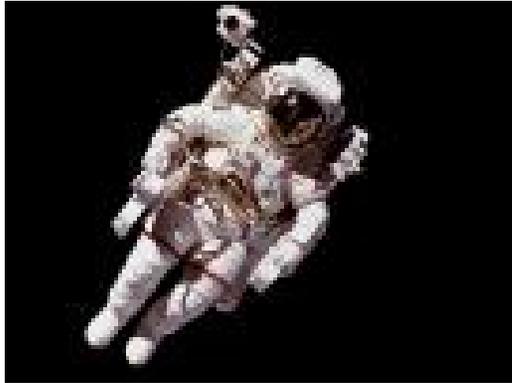


# Relative Motion



- Who is moving?
- How do we know?
- Up until Einstein we used addition and subtraction to calculate the observed speed to other objects

# Relative Motion



- Motion is relative as long as you are not accelerating!
- There is no experiment that you can do in an inertial reference frame that will distinguish it from another inertial reference frame

$$v_{object-ground} = v_{object-train} + v_{train-ground}$$

---

# The Speed of Light

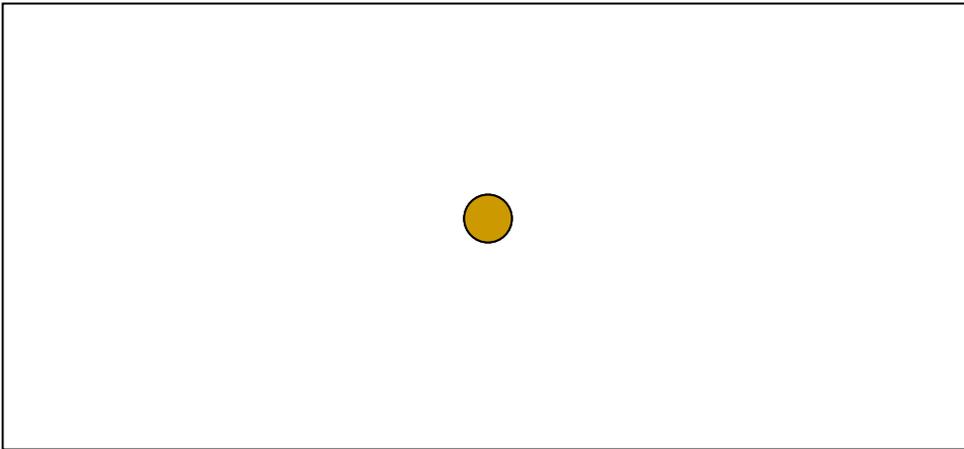
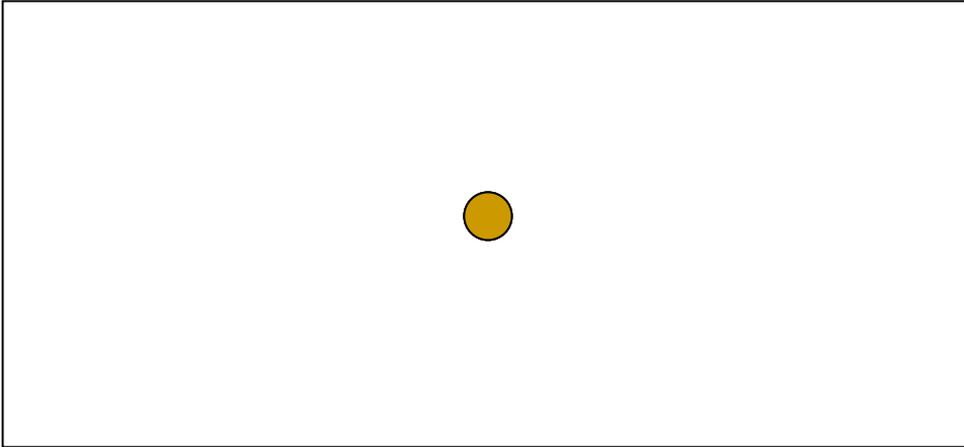
- 670 million miles per hour (in a vacuum)
- 186,000 miles per second
- Light always moves at the same speed
- You can never catch a beam of light!



# Postulates of Special Relativity

1. "The laws of electrodynamics and optics will be valid for all frames of reference in which the laws of mechanics hold good (non-accelerating frames)."
2. "Light is always propagated in empty space with a definite velocity  $c$  that is independent of the state of motion of the emitting body; here the velocity of light  $c$  is defined as the two-way velocity, determined with a single clock."

# Simultaneous Events



Events that are simultaneous for observers in one reference frame are not necessarily simultaneous to observers in another reference frame

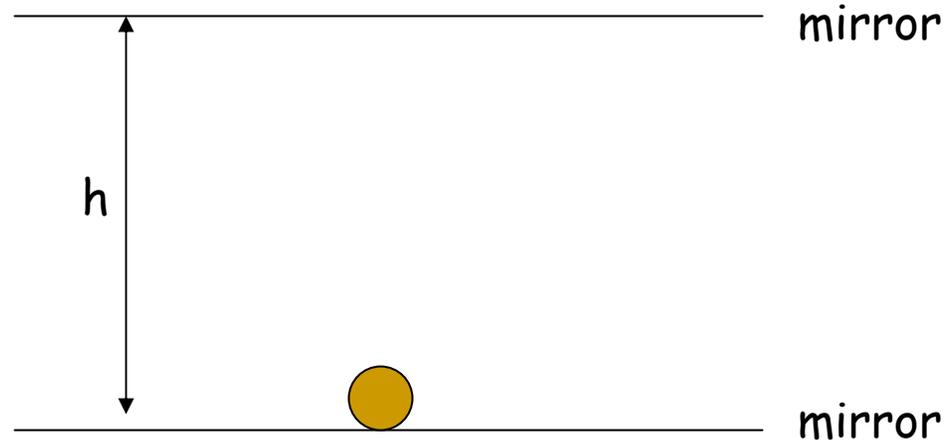
# Time Dilation– Light Clock

- What is time?
- What is a clock?

$c = 3.0 \times 10^8 \text{ m/s}$

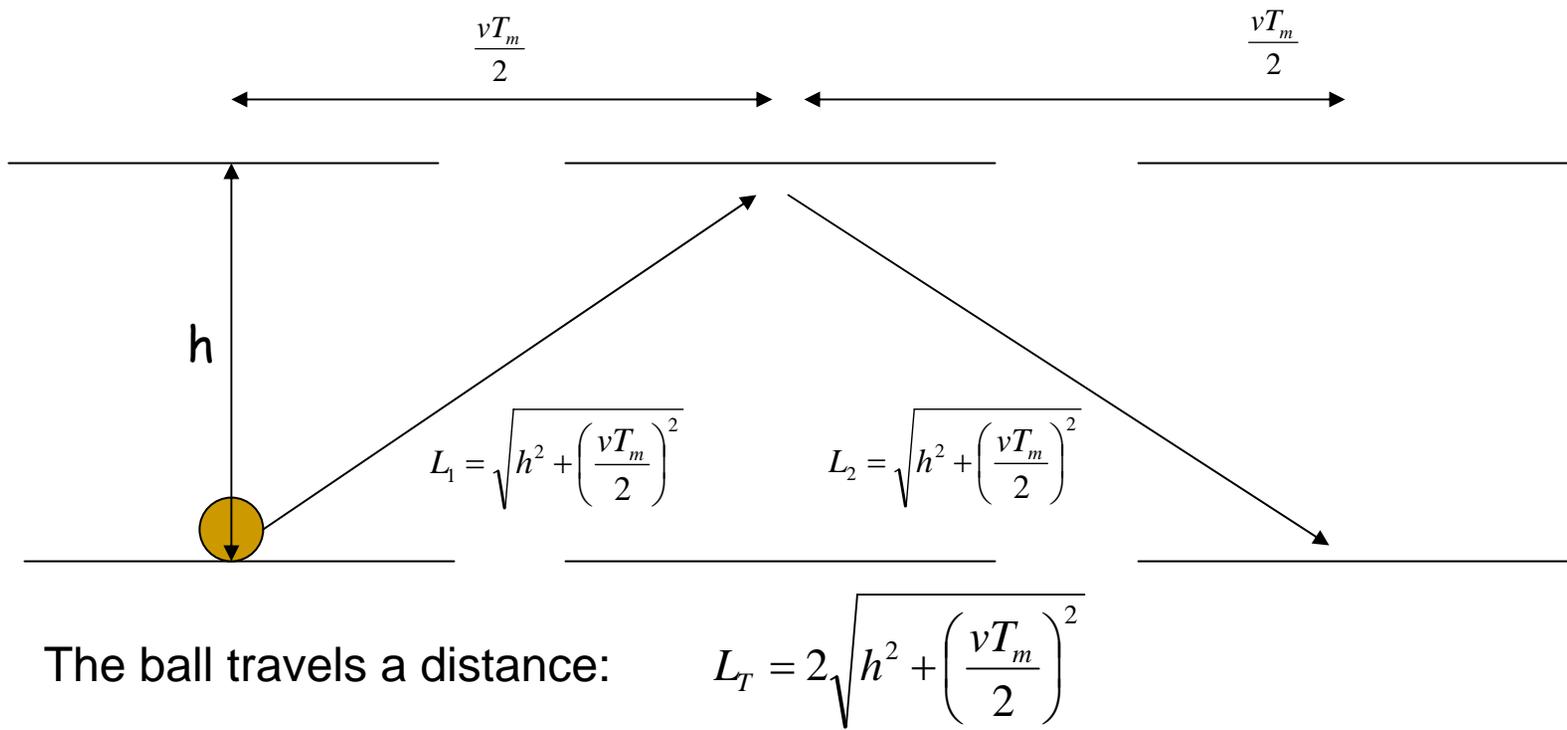
Separation  $h = 15 \text{ cm}$

Period =  $T_s = 1$  billionth of a second



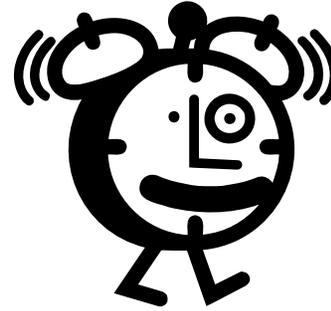
# Moving Light Clock

- What happens if the light clock moves with a velocity  $v$ ?



# Time Dilation

$$T_m = \frac{L_T}{c} = \frac{2\sqrt{h^2 + \left(\frac{vT_m}{2}\right)^2}}{c}$$



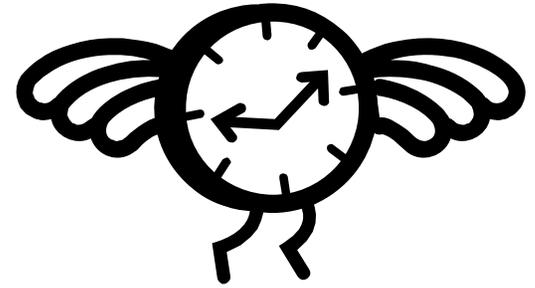
$$T_m = \frac{2h}{\sqrt{c^2 - v^2}}$$

$$T_s = \frac{2h}{c}$$

# Time Dilation



$$T_m = \frac{T_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$T_m = \gamma T_s$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From one inertial reference frame it seems as though a clock moving in another frame ticks slower!

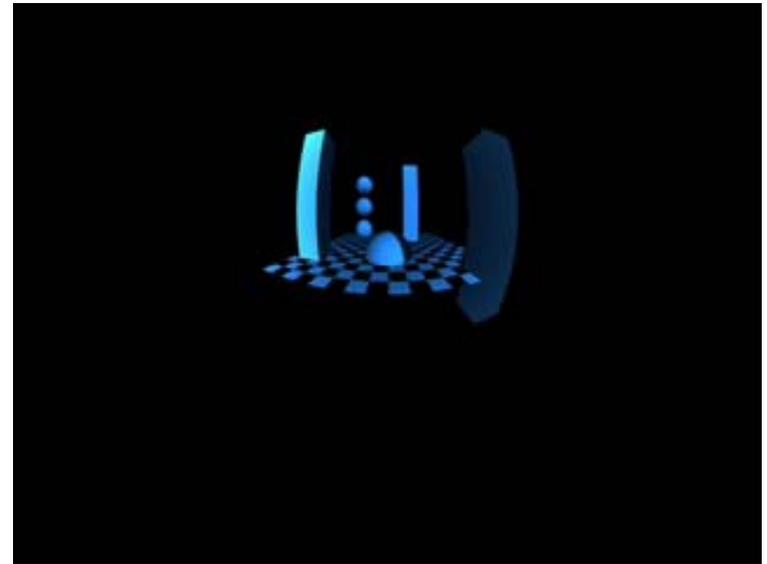
# What about Length?

$$L_m = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$



# Space-Time

- We can think of 4 dimensions, three space and time
- Objects move through space-time at the speed of light!
- If an object is “stationary” its motion is used to travel through time
- If the object “moves” in space, it travels more slowly through the time dimension



# Relative Motion

$$v_{object-ground} = \frac{v_{object-train} + v_{train-ground}}{1 + \frac{v_{object-train} v_{train-ground}}{c^2}}$$

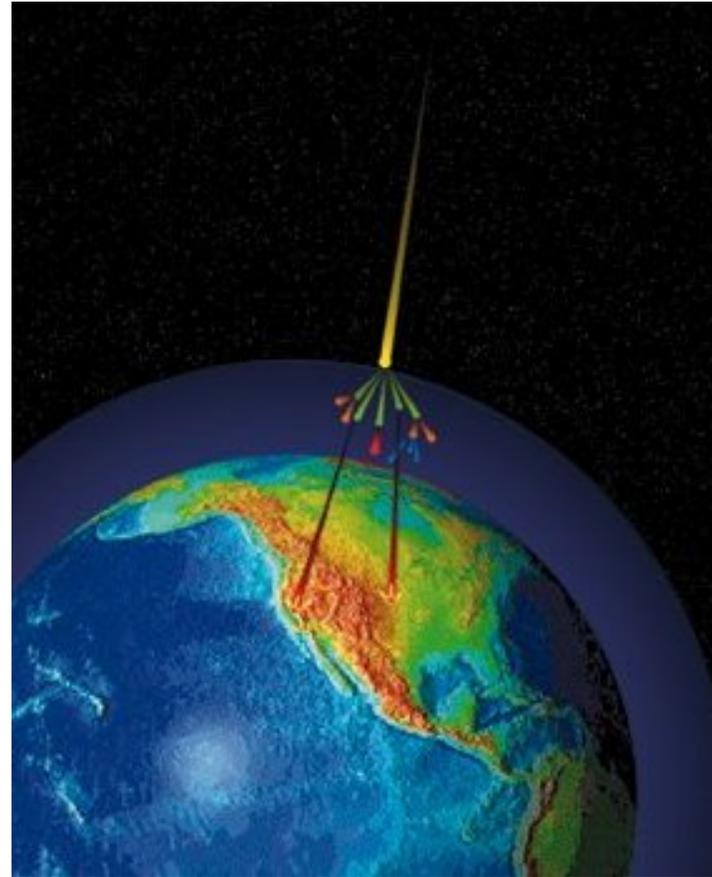
What happens if the velocity of the object with respect to the train is the speed of light?



$$v_{object-ground} = c$$

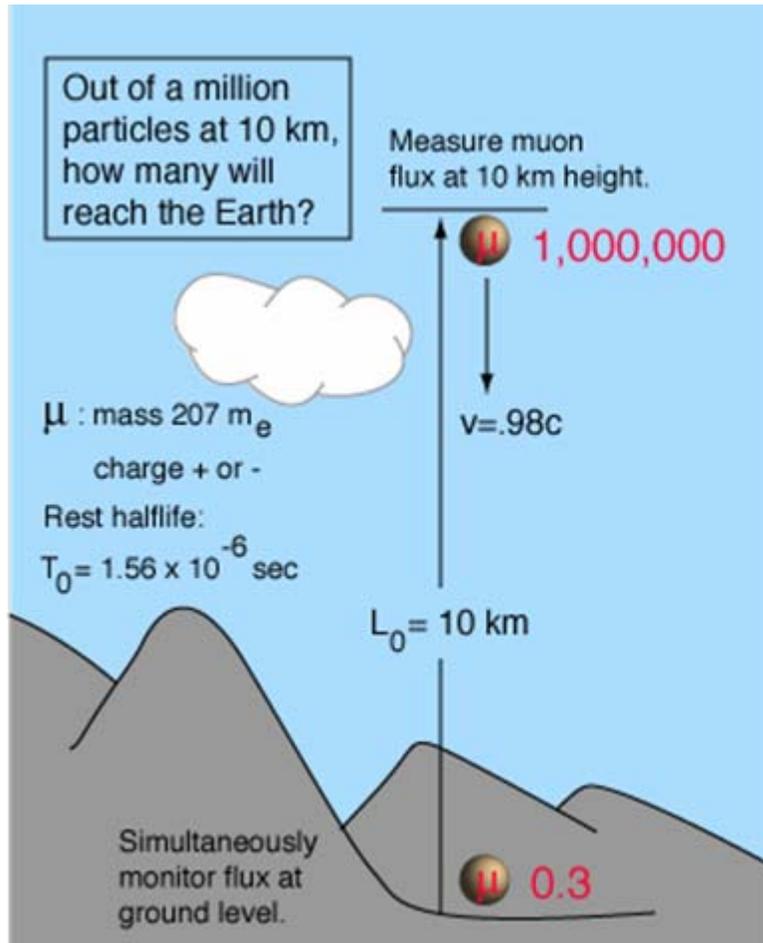
# What is a Muon?

- Semi-stable fundamental particle
- Muon mass is 207 times the mass of the electron
- Decays into a positron and two neutrinos



# Evidence of Time Dilation - Muons

Non relativistic



Distance:  $L_0 = 10^4 \text{ meters}$

Time:  $T = \frac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 34 \times 10^{-6} \text{ s} = 21.8 \text{ halflives}$

Survival rate:

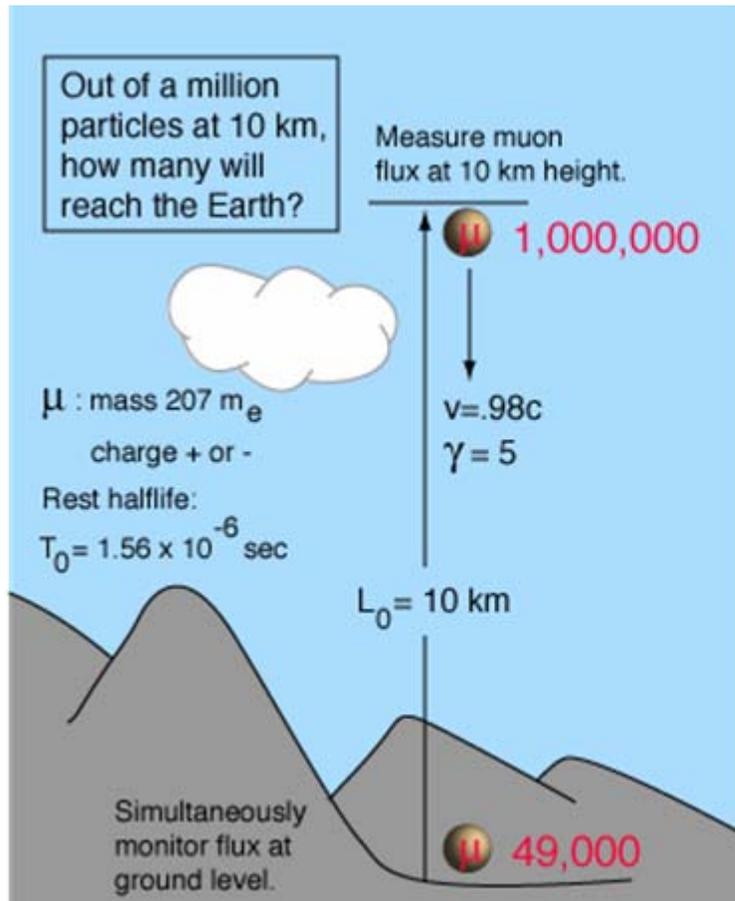
$$\frac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$$

Or only about 0.3 out of a million.

# Evidence of Time Dilation - Muons

Relativistic

Earth Frame



Distance:  $L_0 = 10^4$  meters

Time:  $T = \frac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 34 \times 10^{-6} \text{ s} = 4.36$  halfives

Survival rate:

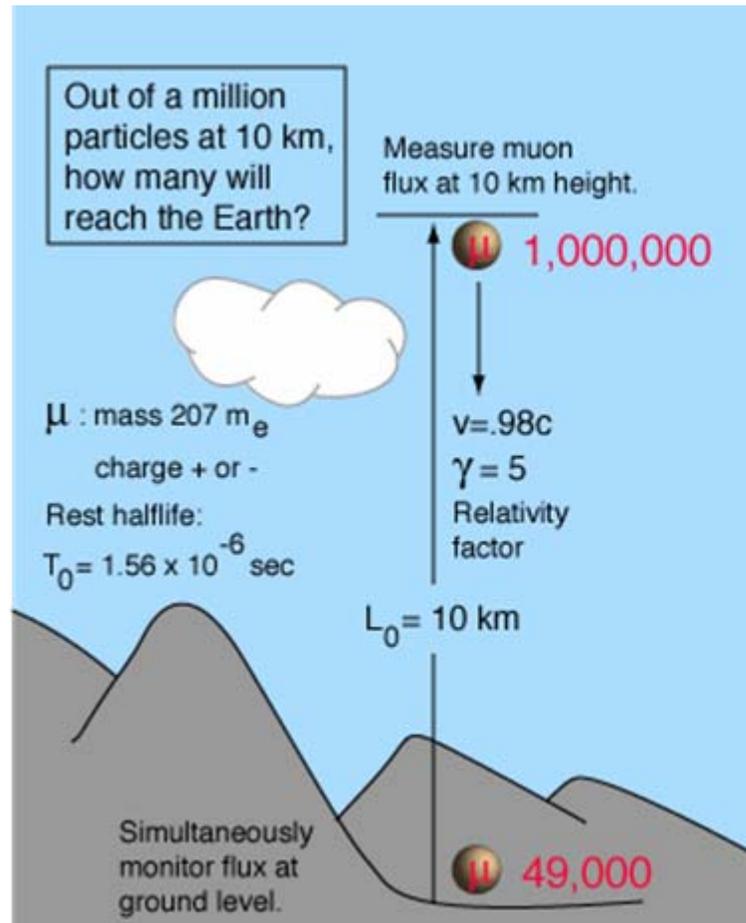
$$\frac{I}{I_0} = 2^{-4.36} = 0.049$$

Or about 49,000 out of a million.

The muon's clock is time-dilated, or running slow by the factor  $T = \gamma T_0$ , so its measured half-life is  $5 \times 1.56 \mu\text{s} = 7.8 \mu\text{s}$ .

# Evidence of Time Dilation - Muon

## Relativistic Muon Frame



Distance:  $L_0 = 10^4$  meters

Time:  $T = \frac{2000 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 6.8 \times 10^{-6} \text{ s} = 4.36$  half-lives

Survival rate:

$$\frac{I}{I_0} = 2^{-4.36} = 0.049$$

Or about 49,000 out of a million.

The muon sees distance as length-contracted so that  $L = L_0 / \gamma = 0.2L_0 = 2$  km.

# Twin Paradox

Twins - Earth Man and Rocket Man

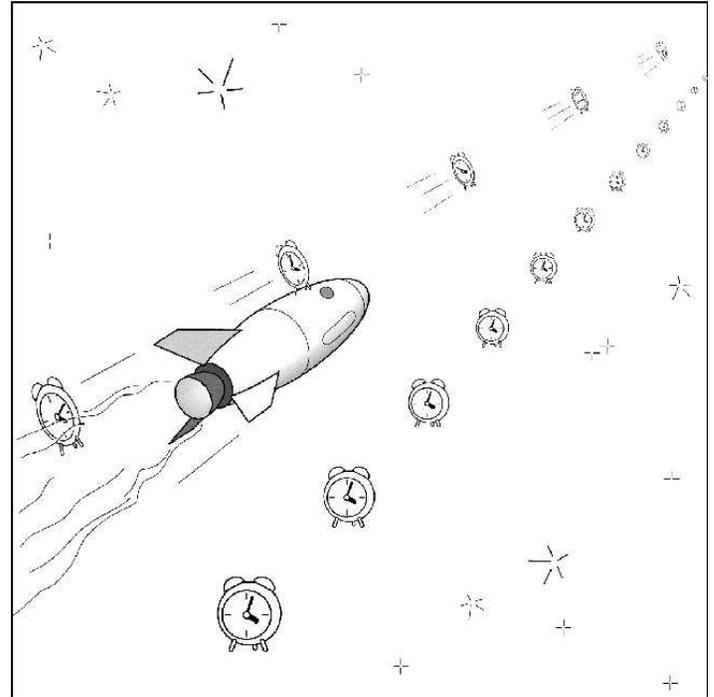
Rocket Man takes off at a velocity  
60 % of the speed of light

Rocket Man travel out 12 hours (by  
his watch) and then turns around and  
returns to Earth at the same speed  
that he left with.

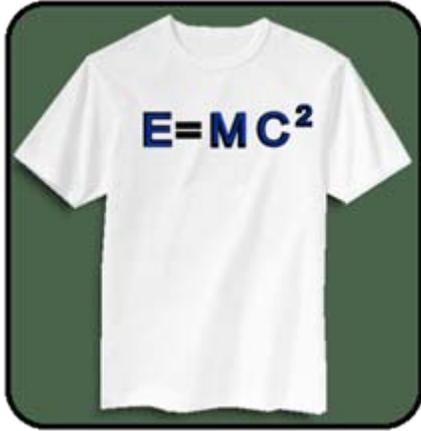
When Rocket Man returns to Earth  
he has aged 24 hours.

**How much has Earth Man Aged?**

**Answer: Earth Man has aged 30  
hours!**



# $E=mc^2$



$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \gamma m_0 v$$

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E_{rest} = m_0 c^2$$

$$E = \sqrt{p^2 c^2 + m^2 c^4}$$

# The End

