Review of Vector Concepts

1. Coordinates:

   Cartesian Coordinates = (x,y) coordinates

   A point: (x,y)

   A vector: $< x, y >$

   Polar Coordinates = (r, θ) coordinates

   A point: (r, θ)

   A vector: $< r, \theta >$

2. Converting between the two coordinate systems:

   r = length of the vector = $\sqrt{x^2 + y^2}$ by Pythagorean Theorem

   $\theta = \tan^{-1}(\frac{y}{x})$ by the properties of trigonometric functions.

   $x = r\cos\theta$

   $y = r\sin\theta$ since r is the hypotenuse of the right triangle with sides

   x, y, r

3. Trigonometry:

   SOHCAHTOA tells us that for RIGHT triangles, we have:

   $\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{O}{H}$

   $\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{A}{H}$

   $\tan(\theta) = \frac{opposite}{adjacent} = \frac{O}{A}$

   !! $\tan(\theta) = \frac{O}{A} = \frac{O/H}{A/H} = \frac{\sin\theta}{\cos\theta}$

   Using INVERSE trigonometric functions to find angle θ

   $\theta = \sin^{-1}(\sin(\theta)) = \sin^{-1}(\frac{O}{H})$

   $\theta = \cos^{-1}(\cos(\theta)) = \cos^{-1}(\frac{A}{H})$

   $\theta = \tan^{-1}(\tan(\theta)) = \tan^{-1}(\frac{O}{H})$

4. Vector Addition and Multiplication by Scalar:

   Given vectors: $\vec{v} =< a, b >$ and $\vec{w} =< \alpha, \beta >$ and scalar: c

   Algebra:

   $\vec{v} + \vec{w}$

   $=< a, b > + < \alpha, \beta >$

   $=< a + \alpha, b + \beta >$

   $c * \vec{v}$

   $= c* < a, b >$

   $=< c * a, c * b > \leftarrow$ this just makes the vector (arrow) longer.

   Graphically:

   Head - to - Tail Method

   Place the TAIL of one vector ONTO the HEAD of the second vector and draw the RESULTANT vector that connects the two ends .

   Parallelogram Method:

   Use the two vectors to draw and PARALLELOGRAM and the DIAGONAL from (0,0) is the RESULTANT vector.

5. Independence of Vector Components:

   Question:

   Does changing the x - component of a vector affect the y - component?

   Test:

   Pretend we are walking in the woods all day and we move 4 miles to the northeast, 2 miles north and 6 miles south. What is our displacement?

   $< 4, NE > + < 2, N > + < 6, S >$ in polar coordinates.

   Convert to Cartesian coordinates.

   $=< 4 \cos 45, 4 \sin 45 > + < 2 \cos 90, 2 \sin 90 > + < 6 \cos 270, 6 \sin 270 >$

   $=< 4\frac{1}{\sqrt{2}}, 4\frac{1}{\sqrt{2}} > + < 0, 2 > + < 0, -6 >$

   $=< 2\sqrt{2}, 2\sqrt{2} > + < 0, 2 > + < 0, -6 >$

   $=< 2\sqrt{2}, 2\sqrt{2} + 2 + (-6) >$

   $=< 2\sqrt{2}, 2\sqrt{2} - 4 >$

   So even though we moved North and South, our x-component remained the same!!

   $\Rightarrow$ x and y component do NOT affect each other - They are INDEPENDENT.

6. Projectile:

Projectiles are objects that are moving with only the force of gravity acting on them, like a rocket, falling monkey or thrown potato.

Recall: $d_f = d_0 + v_0 t + \frac{1}{2}at^2$ was used for constant acceleration and falling objects. This equation is ONLY for 1-Dimension. Projectiles are in 2-Dimensions. Therefore, we need to use two of the above equations, one for each direction.

$d_{yf} = d_{y0} + v_{y0}t + \frac{1}{2}a_y t^2$

$d_{xf} = d_{x0} + v_{x0}t + \frac{1}{2}a_x t^2$

where $v_0 =< v_{x0}, v_{y0} >$, $d_0 =< d_{x0}, d_{y0} >$ and $a =< a_x, a_y >$.

Since acceleration due to gravity is ONLY in the y-direction,

then $a =< 0, -g >=< 0, -9.8\, m/s^2 >$ and

$d_{yf} = d_{y0} + v_{y0}t + \frac{1}{2}gt^2$

$d_{xf} = d_x 0 + v_{x0}t$

For VELOCITY:

$v_{xf} = v_{x0} + a_x t = v_{x0}$ since $a_x = 0$

$v_{yf} = v_{y0} + a_y t = v_{y0} - 9.8t$

1) A tomato falls off a building that is 20 meters high with initial velocity of 20 m/s. How long does it take to fall to the ground? How far from the building does the tomato land? What is the tomato's final velocity? Draw the tomato's path through the air.

2) Xiaoyu kicks a soccer ball at 30 degrees with initial velocity of 10 m/s. Draw the trajectory of the ball? What are the components of the initial velocity? Where does the ball land? How high did it go? What is the ball's velocity when it lands?

Homework:

What if Xiaoyu was on the moon (moon's gravity is about $\frac{1}{6}$ g), then what would these answers be?

7. Force:

   Remember, Acceleration is vector $\longrightarrow$ FORCE is a vector.

   1) A gymnast is hanging from the rings with her arms spread out at 45 degrees off the horizontal. She is not accelerating. Which of Newton's Law do we use? What forces are acting on the block? Determine these forces.
   What if the gymnast has mass of m = 85 kg and is on earth? What is the tension in each arm?

   2) A block sitting on a table with mass m and is not accelerating. Which of Newton's Law do we use? What forces are acting on the block?

   3) A block on an "inclined plane" of mass 75 kg and it has a frictional coefficient of 0.25. Which of Newton's Law do we use? What forces are acting on the block? Determine these forces and the net force.

   4) CeiWei is pushing a block of mass 5 kg up a hill at 10 degrees and the block is accelerating at $2m/s^2$. Draw the "Free Body Diagram." Which of Newton's Law do we use? What forces are acting on the block? What is Net Force?