

[TP] The rms speed, v_{rms}, of O_2 at 25°C is about ...
("Use units, Luke!" $R = 8.314 \frac{\text{J}}{\text{K mol}} = 8.314 \frac{\text{kg m}^2/\text{s}^2}{\text{K mol}}$)

14%	1.	250 m/s
67%	2.	500 m/s
16%	3.	750 m/s
3%	4.	1000 m/s

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Lecture 10 CH131 Fall 2020

Thursday, October 15, 2020

- Kinetic molecular theory
- Why the gas law does not depend on mass
- Distribution of molecular speeds

Next lecture: Complete: Distribution of molecular speeds; Real gases; Gas law for real gases: van der Waals equation; Ch10: Solids, liquids and phase transitions

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Kinetic molecular theory

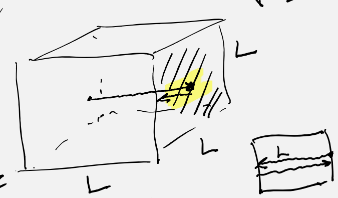
$V = L^3$

$P = \frac{F_{wall}}{A}$

$F = \frac{\Delta p_{wall}}{\Delta t} = \frac{2 m v_i}{\Delta t} = \frac{2 m v_i}{2L/v_i}$

$F_{wall} = \frac{m v_i^2}{L}$

$P_{wall} \approx \frac{F_{wall}}{L^2} = \frac{m v_i^2}{L^3} = \frac{m v_i^2}{V}$



$m v_i \rightarrow$

$-m v_i \leftarrow$

$\Delta p_{part} = p_{fin} - p_{init}$
$= -m v_i - m v_i$
$= -2 m v_i$

elastic

$\boxed{\Delta p_{part} = -\Delta p_{wall}}$

$\Delta p_{wall} = +2 m v_i$

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Kinetic molecular theory

If we assume particles of a gas collide with one another and with the walls of the container elastically, then we can show that the contribution of a single particle i to the pressure of a gas is

$$P_i = \frac{1}{3}\frac{m v_i^2}{V}$$

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Kinetic molecular theory

If we assume particles of a gas collide with one another and with the walls of the container elastically, then we can show that the contribution of a single particle i to the pressure of a gas is

$$P_i = \frac{1}{3}\frac{mv_i^2}{V}$$

Note that the units of this expression, $\frac{\text{energy}}{\text{volume}}$, are correct for pressure.

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Kinetic molecular theory

If we assume particles of a gas collide with one another and with the walls of the container elastically, then we can show that the contribution of a single particle i to the pressure of a gas is

$$P_i = \frac{1}{3}\frac{mv_i^2}{V}$$

Note that the units of this expression, $\frac{\text{energy}}{\text{volume}}$, are correct for pressure.

The factor $\frac{1}{3}$ takes into account that motion takes place in three dimensions.

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Kinetic molecular theory

$PV = nRT$

The total pressure is the sum of the contributions of the $N = nN_A$ particles of the gas,

$$P = P_1 + P_2 + \cdots + P_N = \frac{1}{3}\frac{m}{V}\underbrace{(v_1^2 + v_2^2 + \cdots + v_N^2)}_{N}$$

N

$$\frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}$$

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Kinetic molecular theory

\# of particles in the V

The total pressure is the sum of the contributions of the $N = nN_A$ particles of the gas,

$$P = P_1 + P_2 + \cdots + P_N = \frac{1}{3}\frac{m}{V}(v_1^2 + v_2^2 + \cdots + v_N^2)$$

We can simplify this expression by dividing the sum of squared speeds by the number of particles (N),

$$P = \frac{1}{3}\frac{mN}{V}\left(\frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}\right) = \frac{1}{3}\frac{mN}{V}v_{rms}^2$$

average of squared speeds

mass of one part. moles

$$mN = mN_A n$$

$$= Mn = \text{total mass}$$

$$\boxed{= \frac{1}{3}\frac{Mn}{V} v_{rms}^2}$$

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Kinetic molecular theory

The total pressure is the sum of the contributions of the $N = nN_A$ particles of the gas,

$$P = P_1 + P_2 + \cdots + P_N = \frac{1}{3}\frac{m}{V}(v_1^2 + v_2^2 + \cdots + v_N^2)$$

We can simplify this expression by dividing the sum of squared speeds by the number of particles (N),

$$P = \frac{1}{3}\frac{mN}{V}\left(\frac{v_1^2+v_2^2+\cdots+v_N^2}{N}\right) = \frac{1}{3}\frac{mN}{V}v_{rms}^2$$

The quantity $v_{\text{rms}}^2 = \frac{v_1^2+v_2^2+\cdots+v_N^2}{N}$ is the mean (average) of the squared speeds of the individual particles.

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Kinetic molecular theory

Comparing the expression for pressure from the kinetic molecular theory

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\frac{m\mathrm{N_A}n}{V}v_{rms}^2 = \frac{1}{3}\frac{Mn}{V}v_{rms}^2$$

to that from the ideal gas law,

$$P = \frac{nRT}{V},$$

we get the fundamental connection between temperature and the mean of the squared speeds,

$$v_{rms}^2 = \frac{3RT}{M}$$

average energy / mole

$$= \frac{1}{2} M v_{rms}^2 = \frac{1}{2}\cancel{M}\frac{3RT}{\cancel{M}}$$

$$= \frac{3}{2}RT$$

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Kinetic molecular theory

The square root of the mean of the squared speeds is known the root-mean-squared speed,

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

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[TP] The rms speed, v_{rms}, of O_2 at 25°C is about …

("Use units, Luke!") $R = 8.314 \frac{\text{J}}{\text{K mol}} = 8.314 \left(\frac{\text{kg m}^2/\text{s}^2}{\text{K mol}}\right)$

1%	1.	250 m/s
95%	2.	500 m/s
3%	3.	750 m/s
1%	4.	1000 m/s

25
273.15
298

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3 * 8.314 \,\cancel{\text{kg}}\, \text{m}^2/\text{s}^2 * 298\,\cancel{\text{K}}}{\cancel{\text{K}}\,\cancel{\text{mol}} * \frac{32\,\cancel{\text{g}}}{\cancel{\text{mol}}} * \frac{1\,\cancel{\text{kg}}}{10^3\,\cancel{\text{g}}}}} = 482 \text{ m/s}$$

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Kinetic molecular theory

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$R = 8.314\,\frac{\mathrm{J}}{\mathrm{K\,mol}} = 8.314\,\frac{\mathrm{kg\,m^2/s^2}}{\mathrm{K\,mol}}$$

Answer: 482 m/s ≈ 500 m/s

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[TP] The rms speed of O_2 at 25°C is about 500 m/s. Without using a calculator, use the rms speed of O_2 to estimate that the rms speed of H_2 at the same temperature is about …

12%	1.	4000 m/s
72%	2.	2000 m/s
13%	3.	1000 m/s
3%	4.	500 m/s
1%	5.	250 m/s

$$\sqrt{\frac{3RT}{M}} = v_{rms}$$

$$\frac{v_{H_2}}{v_{O_2}} = \frac{\sqrt{\frac{\cancel{3RT}}{M_{H_2}}}}{\sqrt{\frac{\cancel{3RT}}{M_{O_2}}}} = 4$$

$$= \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32\ \cancel{g/mol}}{2\ \cancel{g/mol}}} = \sqrt{16} = 4$$

$$v_{H_2} = 4 v_{O_2} = 4 * 500\ m/s = 2000\ m/s$$

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Why the gas law does not depend on mass

We have seen that the microscopic model of gas pressure is that

$$P = \frac{1}{3}\frac{Mn}{V}v_{rms}^2 \text{ where } v_{rms} = \sqrt{\frac{3RT}{M}}$$

Why doesn't P depend on molar mass (M)?

The reason is that root-mean-square speed offsets the explicit mass dependence,

$$P = \frac{1}{3}\frac{Mn}{V}v_{rms}^2 = \frac{1}{3}\frac{Mn}{V} \times \frac{3RT}{M} = \frac{nRT}{V}$$

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Distribution of molecular speeds

Distribution of speeds is a plot of the number of particles (y axis) at each speed (x axis).

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Distribution of molecular speeds

Say we 20,000 particles, all with the same speed v = 500 m/s.

Make a sketch of the corresponding distribution of speeds.

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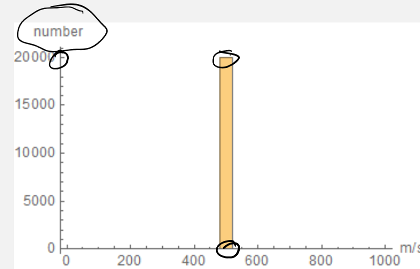
Distribution of molecular speeds

Say we 20,000 particles, all with the same speed v = 500 m/s.

Make a sketch of the corresponding distribution of speeds.



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Distribution of molecular speeds

Collisions between gas particles result in the particles having different speeds.

Make a sketch of a possible distribution of just three speeds, 300, 500, and 700 m/s, of the 20,000 particles.

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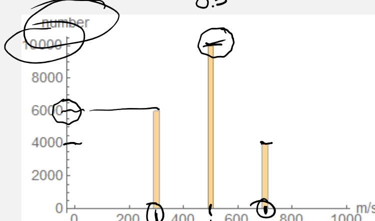
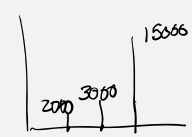
Distribution of molecular speeds

We have said that collisions between gas particles results in the particles having different speeds.

Make a sketch of a possible distribution of just three speeds, 300, 500, and 700 m/s, of the 20,000 particles.

0.5





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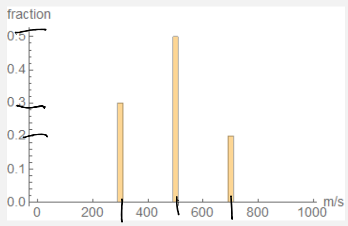
Distribution of molecular speeds

We have said that collisions between gas particles results in the particles having different speeds.

It is convenient to express the distribution in terms of fractional number of particles rather than number of particles.



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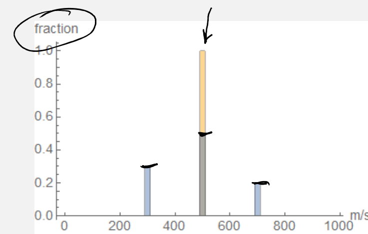
Distribution of molecular speeds

We have said that collisions between gas particles results in the particles having different speeds.

It is convenient to express the distribution in terms of fractional number of particles rather than number of particles.



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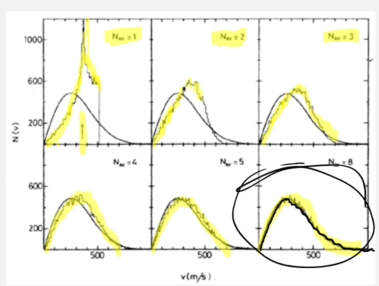
Distribution of molecular speeds

Here is what happens to the speeds of 20,000 particles, all initially at the same speed, after they each have undergone successive numbers of collisions.

Bonomo & Riggi,
Am. J. Phys., Vol 52, p 54 (1984)
http://dx.doi.org/10.1119/1.13809



Maxwell-Boltzmann

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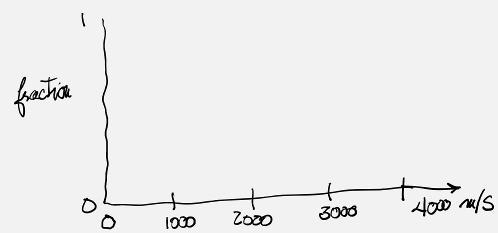
Distribution of molecular speeds

At 300 K, the rms speed of O_2 is $v_{rms} = 500$ m/s.

Sketch the Maxwell-Boltzmann distribution of speeds of O_2 at 300 K, from 0 m/s to 4000 m/s, marking the position of v_{rms} on the x axis.



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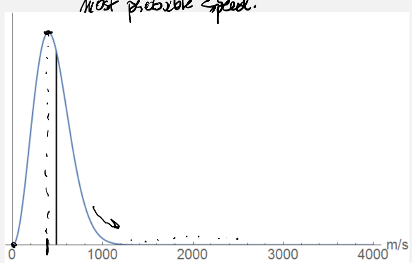
Distribution of molecular speeds

At 300 K, the rms speed of O_2 is $v_{rms} = 500$ m/s.

Sketch the Maxwell-Boltzmann distribution of speeds of O_2 at 300 K, from 0 m/s to 4000 m/s, marking the position of v_{rms} on the x axis,

most probable speed.



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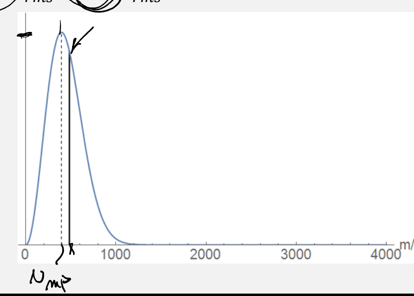
Distribution of molecular speeds

The rms speed, u_{rms}, is slightly higher than the most probable speed, v_{mp}, because of the exponential tail extending to very high speeds.

$$v_{mp} = \sqrt{2/3}\, v_{rms} \approx 0.82\, v_{rms}$$

$v_{mp} < v_{rms}$



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[TP] Use the rms speed of O_2 at 300 K, $v_{rms} = 500$ m/s. Without a calculator, use this value to find the rms speed of O_2 at 1200 K.

4%	1.	4000 m/s
13%	2.	2000 m/s
82%	3.	1000 m/s
2%	4.	500 m/s
0%	5.	250 m/s

algebra

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{v_{1200K}}{v_{300K}} = \frac{\sqrt{\frac{3R\,1200K}{M}}}{\sqrt{\frac{3R\,300K}{M}}} = \sqrt{\frac{1200K}{300K}} = \sqrt{4}$$

$$v_{1200K} = v_{300K}\sqrt{4}$$

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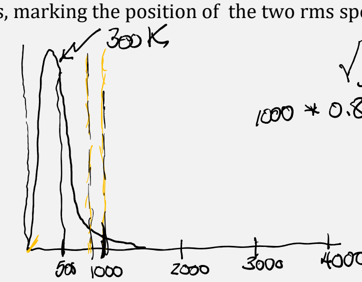
u_{rms} versus temperature

The rms speed of O_2 at 300 K is 500 m/s and the rms speed of O_2 at 1200 K is 1000 m/s. Sketch the distribution of speeds for O_2 at these two temperatures on the same axes, marking the position of the two rms speeds on the x axis.



$\sqrt{\frac{2}{3}}\, v_{rms}$

$1000 * 0.8 = 800$ m/s

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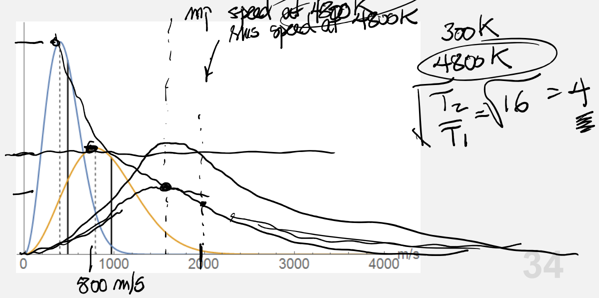
u_{rms} versus temperature

The rms speed of O_2 at 300 K is 500 m/s and the rms speed of O_2 at 1200 K is 1000 m/s. Sketch the distribution of speeds for O_2 at these two temperatures on the same axes, marking the position of the two rms speeds on the x axis.



300 K

4800 K

$$\sqrt{\frac{T_2}{T_1}} = \sqrt{16} = 4$$

34

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