

**Boston University College of Engineering,
Department of Electrical and Computer Engineering**

BU ENG EK 501 Mathematical Methods I

Spring 2025

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Course Description: Mathematical methods I and II (501 & 502) present mathematical methods and concepts which are widely used in science and engineering. Unifying and/or geometric concepts are emphasized while stressing representative applications. The first course emphasizes calculus, linear algebra and ordinary differential equations (see content below). The second emphasizes partial differential equations and numerical methods. For official descriptions, see <http://www.bu.edu/academics/eng/courses/eng-ek-501/>.

Where and when: IEC B13, 888 Comm Ave., M, W, 12:20 -2:05 PM.

See: <https://www.bu.edu/classrooms/classroom/iec-b13/>

Office hours: Given the small size of the class, we have a lot of flexibility. From the class matrix, and the instructor's meeting schedule, it seems that late Friday morning seems to work well, as do Mondays and Wednesdays after class. Let me know what works for you.

Prerequisites: Previous exposure to univariable calculus, linear algebra, vector calculus, ordinary differential equations, complex variable methods, and transform methods. (Don't panic!) The course will review all of these topics. However, we won't develop them from scratch; the emphasis will be on unifying themes that appear through application.

Goals:

- Present linear algebra in a manner consistent with practices in scientific computing.
- Develop vector calculus and complex variables in a conceptually coherent manner.
- Develop an appreciation of the distinction between (i) how constructive proofs lead to algorithms, and (ii) how "fundamental theorems" (of arithmetic, algebra, calculus, ...) lead to constraints on algorithms and seemingly constructive procedures, but not actual algorithms. (This is a bit heady but will become clearer as we go along.)
- Develop an appreciation of the research that goes on in our college and university.

Textbook (I will not follow it closely, but it sets the approach to some key concepts)

"All the Mathematics You Missed- But need to Know for Grad School", Thomas A. Garrity, Cambridge University Press, (1st Ed. 2001, 2nd Ed.2021)- Available Barnes & Noble at BU. More on books below.)

Evaluation: The table below shows two evaluation schemes; we will follow Scheme 1. I presented scheme 2 to contrast the course's evaluation scheme with a more conventional

approach. Suggestions for modifying the evaluation scheme are welcome but will have no impact on the current semester if they do not come in the first week or two of class.

	Scheme 1	Scheme 2
Homework	10	10
First Midterm exam	20	25
Second Midterm exam	20	25
Final Exam	-	40
Final project written report	40	-
Final project class presentation	5	-
Attendance during class presentations	5	-
Total	100 %	100 %

Currently, in order to develop basic mathematical communication skills, an effort is being made to add a Latex component to the course and the evaluation scheme.

A Reminder and an Expectation:

Students are expected to attend class regularly and develop their own interests in the context of the course. If this seems difficult or unreasonable, be sure to engage the instructor.

The Bottom line: If you're having difficulty relating the course material or your interests to this course, it is time to talk to, and engage the instructor! Be sure to ask:

Am I having fun?

Will the course material help change the world?

Course Content:

At least two weeks will be spent on each of the first four sections below. Emphasis on remaining three sections will be dictated by the needs of the class:

- A development of the notions of numbers and polynomials which sets the stage for many involved results and applications of the course material. Some algorithmic themes will be developed in this context.
- Univariable calculus: review of basic techniques with an emphasis on algebraic and computational aspects such as rational functions and Pade approximations.
- Linear algebra: basic properties of linear systems, solution of linear systems by both direct and iterative methods, sparse matrices, inner product spaces, condition numbers, Krylov subspaces, eigenvalue and singular value decompositions, normal matrices, functions of matrices, structured (circulant, Hankel and Toeplitz) matrices.
- Multivariable calculus: multivariable Taylor approximations and critical point theory, implicit and inverse function theorems, fundamental theorem of multivariable (vector) calculus.
- Complex variable methods: Cauchy-Riemann equations, analytic functions, conformal mapping, positive real functions, analytic extension, Cauchy's theorem, the principle of the argument, special functions.

If I get really inspired and depending on the interests of the class, I might emphasize the following topics:

- Ordinary differential equations: Emphasis will be on aspects which reinforce concepts from linear algebra; systems of linear differential equations and Sturm-Liouville theory.
- Fourier, Hilbert and Laplace transform methods: Emphasis will be on complex inversion formulas and applications to systems theory (causality, passivity, Nyquist stability), Poisson resummation (sampling theorem), and integral equations.
- Emerging topics motivated by students' research and paper topics.

In general, there's insufficient time to explore all technical topics that might relate to this course; we have to make choices! If there is a technical topic that is particularly useful for developing your paper or pursuing your professional aspirations, let me know and I'll see if I can work it into the broader themes covered in class.

More on books:

This course dwells on "Fundamental Theorems" (of arithmetic, algebra, calculus,...), and their technological implications, but the lack of a "Fundamental theorem of linear algebra" points to the fact that "matrix theory" frames much of computational mathematics, and it isn't emphasized in the text. So here are two recommended books to broaden our perspective:

- Golub & VanLoan, Matrix Computations, Johns Hopkins, 3rd ed., 1996, ISBN: 0801854148
- D. Serre, Matrices; Theory and Applications, 2nd Edition. Graduate Texts in Mathematics No. 216, Springer, New York, 2010.

The following books were once required for the course, and now serve as references. Note that the first five books are well known books in their respective fields, yet the price of all five is less than some textbooks. Besides being a great value, each is small enough to carry around, to read, and to refer to!

- M. R. Spiegel, Schaum's Outline of Complex Variables, McGraw-Hill Book Companies, (ISBN 0070602301).
- C.H. Edwards, Advanced Calculus of Several Variables, Dover, 1973, ISBN: 0-486-68336-2
- K. Yoshida, Lectures on Differential and Integral Equations, Dover, 1991, ISBN: 0-486-66679-4
- N. Young, An Introduction to Hilbert Space, Camb. U. P., 1998, ISBN: 0-521-337178
- The Mathworks, MATLAB-Student ed., documentation.

Resources for exploration and project ideas:

Eventually, I would like to compile a list of books which students found inspiring in the course of their project work. What you find below is a feeble start based on feedback from students.

David Benson, "Music: A Mathematical Offering". Cambridge University Press 2007.
Info. and an online .pdf version can be found at:

<http://www.maths.abdn.ac.uk/~bensondj/html/maths-music.html>

Do not be intimidated by this book; file it. Although many of the chapters contain mathematical material, most chapters can be read independently and the author has a wonderful way of introducing mathematical concepts in an easily accessible context. ECE students should note that the author maintains the web page for Yamaha synthesizers, and that the Book is a real gem for Musical EEs.

Manfred Schroeder, "Number Theory in Science and Communication", Springer-Verlag (various editions).

Again, don't try read this book in a night. The math is nontrivial but the author is quite a master of introducing seemingly advanced topics in a very practical context. If you get into concert hall or recording studio acoustics, this book is a must. The author is a very well known engineer who, amongst other things, has fixed the acoustics of various concert halls (after they were built!).

Jeffrey Hoffstein, Jill Pipher and Joseph H. Silverman, "An Introduction to Mathematical Cryptography". Undergraduate texts in mathematics, Springer, 2008. This book is remarkably accessible given the nature of the material. Joe Silverman is a local area mathematician who grew his cryptography start-up into a respectable company.

[Google's PageRank and Beyond: The Science of Search Engine Rankings](#) by **Amy N. Langville and Carl D. Meyer**, Princeton University Press, 2006. This book is a self-contained account of Google's page rank algorithm and an account of all the underlying numerical linear algebra. A related book is: [A Course on the Web Graph \(Graduate Studies in Mathematics\)](#) by **Anthony Bonato**, American Mathematical Society, 2008.

Christiane Rousseau and Yuan Saint-Aubin, "Mathematics and Technology" (Springer Texts in Mathematics and Technology) 2008 edition (translated from the French original.) ISBN-13: 978-0387692159 ; ISBN-10: 0387692150 . This book is in no way encyclopedic, but it does point to project ideas.

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The following "timeless classics" were written by exceptional people and intended for the non-specialist. I include them because of their intention and expository style:

Mark Kac and Stanislaw M. Ulam, "Mathematics and Logic", Dover Publications Inc., Mineola, N.Y., 1992. This book was originally published in 1968 as part of the celebration of the 200th anniversary of the Encyclopedia Britannica. It takes a very concrete approach to getting the reader to think about the nature of mathematics, how it is created and practiced, how it developed alongside science.

The following wonderful book caught my eye because its attitude. I was tempted to adopt it on account of its orientation, but it would have been overwhelming in terms of detail:

Z. A. Melzak, Companion to Concrete Mathematics, Dover Publications Inc., 2007. (Two volumes bound as one).ISBN-13: 978-0-486-45781-9.

References to other books and web resources will be given as the need arises, or as particular interests are identified; especially if examinations are replaced with projects.

The following two books were written in the first half of the last century, are intended for people with little more than a high school education and intended to be relatively elementary:

Richard Courant and Herbert Robbins, “What is Mathematics”.

The latest edition is edited by Ian Stewart: What Is Mathematics? An Elementary Approach to Ideas and Methods **by Courant and Robbins; edited by Ian Stewart.**

Rademacher and Otto Toeplitz, “The Enjoyment of Mathematics”, Published at various times by both Princeton University Press and Dover Publications, Inc..