ME 702 Computational Fluid Mechanics

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Course meetings: MW 2-4 pm, ENG 202

Prerequisite

2^{nd} level fluids course, ME 542 Advanced Fluid Mechanics, or equivalent.

Catalog description

"Numerical techniques for solving the Navier-Stokes and related equations. Topics are selected from the following list, although the emphasis may shift from year to year: boundary integral methods for potential and Stokes flows; free surface flow computations; panel methods; finite difference, finite element and finite volume methods; spectral and pseudo-spectral methods; vortex methods; lattice-gas and lattice- Boltzmann techniques; numerical grid generation."

Textbook

There is no required textbook for this course. Various reading materials will be distributed electronically. Some books that can be recommended are:

- *"Numerical Computation of Internal and External Flows, Volume 1: The Fundamentals of Computational Fluid Dynamics"*, Charles Hirsch, Second Edition: Butterworth-Heinemann/ Elsevier (2007)
- *"Computational Fluid Dynamics"*, John D. Anderson, McGraw-Hill (1995)
- *"Fundamentals of Engineering Numerical Analysis"*, Parviz Moin, Cambridge University Press (2001)

Course aims

This course will prepare students in the fundamentals of the computational approach to study fluid flow problems, and will provide a deeper understanding of the physical models and governing equations of fluid dynamics. It will also present an opportunity to learn the basic skills of programming solutions to differential equations, and present an overview of essential numerical techniques.

Learning objectives

Students will ...

i. deepen their understanding of the governing equations of fluid dynamics, their mathematical nature, and the physical significance of each term thereof.
ii. learn to develop finite difference (FD) discretizations, and implement them in computer code; they will gain understanding of the sources of error in FD approximations.
iii. develop a Navier-Stokes solver step-by-step, and apply it to solve canonical problems in two dimensions.
iv. become familiar with a set of standard CFD techniques (e.g. Lax-Wendroff, McCormack, relaxation, etc.).
v. become acquainted with the finite volume (FV) discretization, spectral methods, and other standard methods of CFD.
vi. appreciate the importance and implications of analytical issues: consistency, stability, convergence, error analysis.

Assessment policies

Assessment is based on student presentations, both oral and written, of assigned work.

Course content

This plan is subject to changes, but approximately, we will cover the following subjects:

1. Review of the Navier-Stokes equations: derivation, physical interpretation, assumptions and applications.
2. Introduction to discretization of partial differential equations with Finite Differences (FDs). Order of accuracy of the FD approximation. Explicit vs. implicit methods. Crank-Nicholson method. Multi-dimensional FD formulas.
3. Simplified model equations, and their mathematical behavior: linear convection equation, inviscid Burgers equation, convection-diffusion equation.
4. Practical Module — "The 12 steps to computing Navier-Stokes"
   This module will take the students through 12 steps, one by one, at the end of which they will have programmed a Navier-Stokes solver, using FDs. The steps are the following:
5. Steps 1–4 are in one dimension: *(i)* linear convection with a step-function initial condition (IC) and appropriate boundary conditions (BC); with the same IC/BCs: *(ii)* nonlinear

convection, and *(iii)* diffusion only; *(iv)* Burgers' equation, with a saw-tooth IC and periodic BCs.

6. Steps 5–10 are in two dimensions: *(v)* linear convection with square function IC and appropriate BCs; with the same IC/BCs: *(vi)* nonlinear convection, and *(vii)* diffusion only; *(viii)* Burgers' equation; *(ix)* Laplace equation, with zero IC and both Neumann and Dirichlet BCs; *(x)* Poisson equation in 2D.
7. Steps 11–12 solve the Navier-Stokes equation in 2D: *(xi)* cavity flow; *(xii)* channel flow.
8. Incompressible Navier-Stokes equations: need for and derivation of the pressure Poisson equation.
9. Analysis of numerical schemes: Consistency, stability, convergence; Lax equivalence theorem. The modified differential equation and truncation errors. Discussion of numerical diffusion, and accuracy issues. Von Neumann stability analysis. Physical interpretation of the CFL condition.
10. Various schemes for convection, and their analysis: Leapfrog, Lax-Friedrichs, Lax-Wendroff, Beam-Warming (second order one-sided differences). Multi-step schemes: Richtmyer/Lax- Wendroff, MacCormack's method.
11. Spectral analysis of numerical errors: numerical dispersion relation, diffusion error and dispersion error. Detailed discussion for hyperbolic problems, using the various schemes for convection; numerical convection speed. Requirements for the number of mesh-points per wavelength.
12. Nonlinear convection: multi-step methods (Richtmyer/Lax-Wendroff, MacCormack.
13. Practical Module — "Inviscid Burgers equation"
    A traveling shock wave, computed with (i) Lax-Friedrichs, (ii) Lax-Wendroff, (iii) MacCormack, (iv) Beam-Warming implicit method, and (v) Beam-Warming with 4th order explicit damping.
14. Numerical solution of the Euler equations: Euler equations in vector-conservation form, the Riemann problem; classic example: the shock-tube problem; discretizing with Lax-Friedrichs, Lax- Wendroff, Richtmyer method and MacCormack method. Sod's test problems.
15. Fundamentals of the finite volume (FV) method.
16. Applications with OpenFOAM
17. Time integration methods for space-discretized equations.
18. Iterative methods for the solution of algebraic systems.