ME 580, Theory of Elasticity

Spring 2014, Mon/Weds, 10 am –12 pm

Course Description

The course content divides essentially into three parts. An emphasis will be placed on modern solution methods, at the partial expense of some of the more classical results in linear elasticity. Thus, we shall not review complex variable methods; on the other hand, we will discuss materials with microstructure, and mathematical methods such as matched asymptotic expansions.

Prerequisites

Two semesters of undergraduate mechanics of solids and/or fluids. Advanced calculus, and vectors. Familiarity with linear algebra is very useful.

Textbook

- Sadd, Martin H. 2011 *Elasticity: Theory, Applications, and Numerics*, Academic Press, Third Ed.
- $\mathcal{NB}$: This book is available electronically through the BU Library.

Grading

The final grade for the course is based on written homework solutions, written and oral reports from small research problems, and participation in homework discussions. Discussions will be held during which students will be expected to present the solutions of homework problems.

Topics

Part I: Field Equations (4 weeks)

1. Vector and tensor algebra. Vector and tensor calculus.
2. Analysis of deformation: Definition of strain, linearized strain, principal strains, compatibility equations.
3. Stress and equilibrium: Momentum conservation, definition of stress and traction, analysis of stress tensor.
4. Linear elastic material behavior: Hooke's law, anisotropy, isotropy, thermoelastic constitutive equations. Strain energy, energetic bounds on material constants.

Part II: Basic theorems and elementary $3D$ solutions (2 weeks)

5. Formulation of boundary value problems. Existence and uniqueness.
6. St. Venant's principle.
7. Tension, torsion, and bending of bars by St. Venant's semi-inverse method.
8. Spherically symmetric stress distributions.
9. Reciprocity and integral theorems.

Part III: Basic elastostatics (3 weeks)

10. Plane stress/strain (2 weeks).
    (a) Plane stress & strain as special cases of $3D$ problems.
    (b) Airy stress functions and separation of variables:
        - Wedge and Flamant problem (dimensional analysis).
        - Stress concentrations around a circular hole (topology).
        - Crack tip stress fields (asymptotic behavior of stress fields).
11. Three-dimensional Problems
    - Displacement potential functions.
    - Kelvin's problem.

Part IV: Advanced Topics (4 weeks)

12. Waves in elastic solids.
    (a) Shear and compressional waves; Helmholtz decomposition.
    (b) Reflection, transmission, and mode conversion at an interface.
    (c) Surface waves (Rayleigh waves).
    (d) Guided waves (SH waves in a layer); end effects.
13. Rods, plates, and shells as asymptotic limit of $3D$ theory.
    (a) Asymptotic derivation of elementary equations.
    (b) Boundary layer edge effects: matched asymptotic expansions.
14. Multiscale analysis of elastic materials with microstructure.
    (a) Asymptotic method of multiple scales; derivation of effective properties.
    (b) Exotic continuum behavior of materials with extreme microstructures.

Instructor

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