ME 580, Theory of Elasticity

Spring 2012, Mon/Weds, 10 am –12 pm

Course Description

The course content divides essentially into three parts. An emphasis will be placed on modern solution methods, at the partial expense of some of the more classical results in linear elasticity. Thus, we shall not review the stress field around a point source in an infinite solid; on the other hand, we will discuss the theory underlying finite element stress analysis, and mathematical methods such as the method of matched asymptotic expansions.

Prerequisites

Two semesters of undergraduate mechanics of solids and/or fluids. Advanced calculus, and vectors. Familiarity with linear algebra is very useful.

Textbook

- Sadd, Martin H. 2009 *Elasticity: Theory, Applications, and Numerics*, Academic Press, Second Ed.

Grading

The final grade for the course is based on written homework solutions, written and oral reports from small research problems, and participation in homework discussions. Discussions will be held during which students will be expected to present the solutions problems. Collaboration between students in solving homework problems is not recommended!

Topics

Part I: Field Equations (4 weeks)

1. Vector and tensor algebra. Vector and tensor calculus.
2. Analysis of deformation: Definition of strain, linearized strain, principal strains, compatibility equations.
3. Stress and equilibrium: Momentum conservation, definition of stress and traction, analysis of stress tensor.
4. Linear elastic material behavior: Hooke's law, anisotropy, isotropy, thermoelastic constitutive equations. Strain energy, energetic bounds on material constants.

Part II: Basic theorems and elementary $3D$ solutions (2 weeks)

5. Formulation of boundary value problems. Existence and uniqueness.
6. St. Venant's principle.
7. Tension, torsion, and bending of bars by St. Venant's semi-inverse method.
8. Spherically symmetric stress distributions.
9. Reciprocity and integral theorems.

Part III: Plane elastostatics

10. Anti-plane problems (2 weeks):
    (a) Complex variable methods.
    (b) Energetics of a screw dislocation.
    (c) Solutions for hole, crack, crack near surface.
11. Plane stress/strain (3 weeks).
    (a) Airy stress functions.
    (b) Separation of variables: stress concentrations around cirucular hole; crack tip stress fields.
    (c) Interacting holes - using local canonical solutions.
    (d) Dimensional analysis: Wedge solutions; Flamant problem.
    (e) Complex representation of Airy Stress function.

    (f) Elliptical hole and crack.
    (g) Involution: circle theorem in elastostatics.

Part IV: Advanced Topics (as time permits)

12. Stress fields in undeformable solids.
13. Variational Methods:
    (a) Principle of minimum potential energy.
    (b) Principle of minimum complementary energy.
    (c) Principle of virtual work.
    (d) Rayleigh-Ritz and finite element methods.
    (e) Eshelby's tensor, J-integral, applications in fracture mechanics.
14. Effective properties of materials with microstructures.
    (a) Basic variational bounds on material properties.
    (b) Advanced variational bounds on material properties.
    (c) Asymptotic homogenization methods.
15. Multiscale analysis of elastic solids.
    (a) Surface layers in elastostatics: energy of nanoparticle precipitates.
    (b) Asymptotic method of multiple scales.
    (c) Micropolar theory of porous scaffolds.
16. Waves in elastic solids.
    (a) Shear and compressional waves.
    (b) Reflection and mode conversion.
    (c) Surface waves.
    (d) Guided waves.
17. Rods, plates, and shells as asymptotic limit of $3D$ theory.
    (a) Asymptotic derivation of elementary equations.
    (b) Boundary layer edge effects: matched asymptotic expansions.
18. Inverse Elasticity.
    (a) Data requirements to ensure uniqueness.
    (b) Exact $2D$ solution.
    (c) Well-posed variational formulation.

Instructor

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