BE 601 / 604: Linear Algebra, statistics, and numerical analysis (Fall 2022)

Instructor: Andy Fan (fana@bu.edu)

TAs: McKayla Vlasity (mvlasity@bu.edu)
Jaehoon Choi (jchoi01@bu.edu)

Class:	MW	10:10 am – 11:55 am	EPC 204
Recitation:	F	2:30 pm – 3:20 pm	EPC 204
Matlab help:	F	3:20 pm – 4:15 pm	EPC 204

Office Hours: By appointment, preferably after lunchtime !
Office: 44 Cummington St, Room 707
TA office hrs: We don't know yet… TBD !

Course documents on Blackboard: https://learn.bu.edu

The three main goals of our BE 601 and 604: "Why should I take another math class – in *grad school?!*" you might ask. We do not intend to make this course as a remedial math tour. Instead, we would like to focus our attention on reinforcing 4 areas of engineering math that you may end up using in your grad school research or in your future job prospects !

a) Linear algebra + basic programming skills

b) Solving physical models involving partial differential equations (PDEs) with a computer

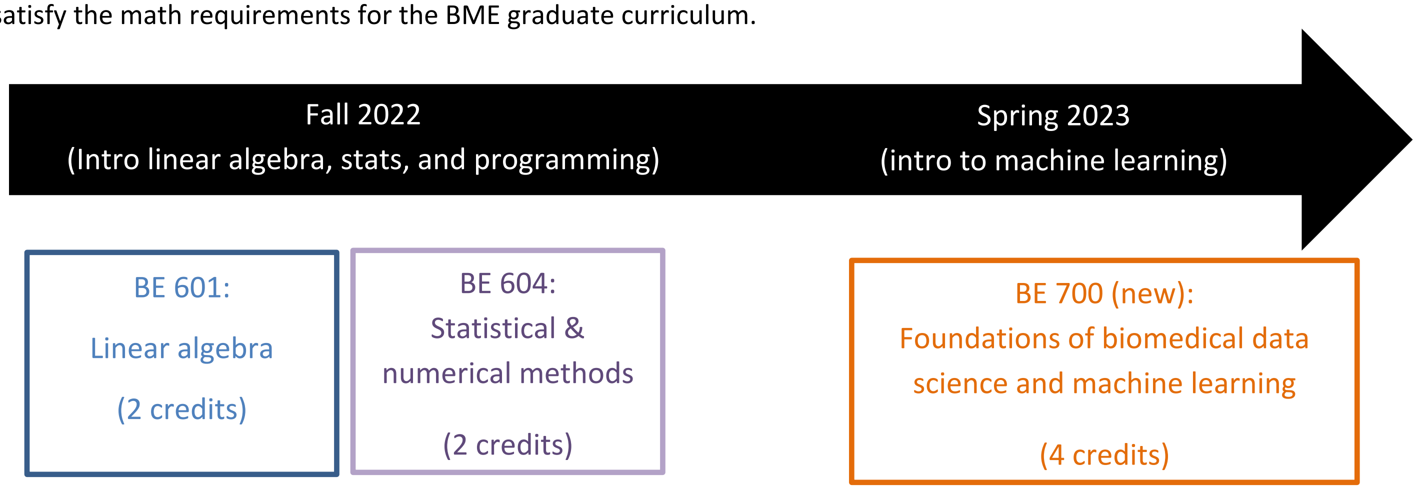
c) Basic digital signal processing and statistics skills

d) 70% of the math you'll need if you want to more about machine learning !

Enrollment: All graduate students or senior undergraduates (need department approval) are welcome !

New "data science" track for the 2022 – 2023 academic year

For the 2022 - 2023 academic year, we are offering a 2-course track that would focus on getting you acquainted with the world of statistical learning (aka. Machine learning) ! The completion either 4-credit "bundle" from the diagram below ill satisfy the math requirements for the BME graduate curriculum.



Intro to graduate math & computer programming; wide range of BME-related topics

Linear algebra & data-driven, programming required

Applying the concepts from BE 601 + 604 to statistical learning problems; programming skills required

Our class in the COVID-19 era:

a) Our Monday / Wednesday lectures

Since BU would like all classes to be in-person, we will not have remote simulcast via Zoom during our normal lecture hours. Having said that, for each lecture, I will take photos of whatever we write on the chalkboard and post them on both our Blackboard and Slack sites.

b) Our Friday recitations (2:30 – 3:20 pm) + the optional matlab help sessions (3:30 – 4:20 pm)

The first part of our Friday afternoon recitations (2:30 – 3:20 pm) will be used to either go over any questions you may have from the lectures, or we may also do 1 or 2 example problems from the materials covered on that particular week. Furthermore, we will briefly introduce the material that you will see on your homework sets.

Then, for those of yall who haven't touched computer programming in a long time, we will have an ***optional*** matlab programming "after-party" in the same room (EPC 204) from 3:30 – 4:20 pm. For the first 2 weeks, we will go over some basic, remedial programming concepts in matlab. After week 2, we will dive deeper into more advanced topics, such as how to combine loops and ***structs*** to store and organize your data.

Homework and Friday recitations: Depending on the topics covered on a given particular week, problem sets will be handed out usually on Fridays on a weekly (or bi-weekly) basis.

Matlab: Since matrix-dependent elements will be in no short supply in this class, we will adopt Matlab as the standard software from which all course materials, homeworks, and take-home tests will be analyzed with. Programming tips in Matlab will be provided throughout the course during recitations, office hours, or in announcements with template scripts uploaded onto Blackboard Learn.

Textbooks: There are no required textbooks for BE 601 + 604. Please see pages 6 – 7 for more explanations !

Exams: ***Take-home*** exam at the end of BE 604 (given out on the last day of class in December)
Duration = 1 week

Grading: The breakdown per module is:
80% homeworks + participation during recitation
20% from take-home exam

Linear algebra syllabus: **(1st half of Fall 2022)**

Lectures (theme)	Topics	Key concepts	Applications
1-5 **(Ax = b)**	Column multiplication Gaussian elimination / Finite difference eqs	• Exposure to matrix manipulations, "numerical linear algebra" style • Numerical approximations for vector calculus quantities (gradients, divergence)	• Intro to **lumped element modeling** of PDEs
6-7 **(Ax = b)**	1D and 2D convolutions	• The concept of the impulse response • Convolution – multiplication duality	• Signal processing
8 **(Ax = b)**	Least squares ($A^TAx = A^Tb$)	• How to approximate the solution to Ax = b when matrix A is rectangular • Exposure to minimization problems	• Curve fitting • Multi-variable linear regression
9-12 **(Qc = b)**	Orthogonal matrices, inner products and Fourier series Discrete Fourier transforms (DFT)	• **Introduction to orthogonality** and basis sets • Generalized Fourier series • Superposition of discrete sines and cosines (The Fourier matrix)	• Solid deformations (continuum mechanics) • Signals & systems (Fourier series) • Image processing
13 **($Ax = \lambda x$)**	Eigenvalues / eigenvectors	• Symmetric matrices leads to *orthogonal eigenvectors* (important in ODES and PDEs) • **Diagonalization = decoupling** of coordinate systems • Eigenvalues = Describes the geometry of repeated transformations	• Transformations in solid mechanics • Markov chains • Solving ODE systems • Google PageRank (**Markov chains**)
14-16 **($Ax = \lambda x$)**	Quadratic forms, p-norms Change of basis Principal component analysis (PCA)	**Real-life applications of diagonalization and decompositions** • Euclidean vs. statistical "distances" • The links between quadratic forms, ellipses, distances, and the multivariate Gaussian distributions	• The covariance matrix (intro to statistics) • Principal component analysis • K-means clustering
17-18 **(Ax = b)** **($Ax = \lambda x$)**	Eigenvalue and Singular value decompositions (SVD) /	**Relationship between Fourier series, orthogonality, and eigenvectors** • Reducing the complexity of a giant matrix A by only retaining its "dominant" characteristics	• Image processing / data compression using SVD • Principal component analysis revisited

Statistical and Numerical methods: (2nd half of Fall 2022)

Lectures (theme)	Topics	Key concepts	Applications
19-20 **(Ax=b)** **Least-squares revisited**	Gradient descent and the 3D paraboloid Newton's method	***<u>Linear and nonlinear regression fit</u>*** • Intro to iterative algorithms in machine learning • Introduction to finding zeros and local minima for functions and analyzing fixed points for differential equations	• Solving equilibrium problems in chemistry • Prelude to nonlinear optimization algorithms
21-24 **Nonlinear least-squares fit**	Gauss-Newton iterations Levenberg-Marquardt iterations Logistic regression	***<u>Machine learning basics: Gradient descent</u>*** • Explore how linear least squares, the *<u>quadratic form</u>*, the residual, the Jacobian matrix…… are connected to nonlinear least-squares fitting algorithms (Levenberg-Marquardt) • <u>Gradient descent methods</u>: The core mathematics you need for any machine learning class!	• Multi-parameter nonlinear data-fitting for Michaelis-Menton kinetics, sinusoidal data, oxygen saturation curves • Statistics (logistic regression)
25-26 **Basic factor analysis**	1-factor ANOVA 1-factor regression and the ANOVA table 2, 3-factor regression	***<u>Statistics</u>*** ***<u>(Intro to the General Linear Model)</u>*** • Basic statistics review • <u>Interpreting the ANOVA table</u>: Making the connections between the least-squares fit, the residual, and statistics	• Applies to pretty much everything you'll do in grad school !! • General statistical classification and modeling problems
Misc **Heuristic methods** **2-factor ANOVA**	Bootstrap 2-factor ANOVA, with replications	***<u>Statistics</u>*** ***<u>(Model assessment)</u>*** • Combining nonlinear least squares, regression, and elements from statistical learning techniques to validate your model •	• General statistical classification and modeling problems • Intro to machine learning concepts

Recommended "all-purpose" reference textbooks for engineering math: There are numerous "compendium" math texts that span the standard elements of linear algebra, ODE, PDEs, complex variables, and numerical methods. Below is a quick list of common engineering math books you can consult on our Blackboard site (all are available for download under the heading):

/ Resources / Compendium_math_textbooks

	Linear Algebra	ODEs	PDEs	Calculus of variations	Complex Variables	Tensors	Numerical methods / optimization	Prob +Stats	Price (new)	Level of math maturity required	Book focus
Riley	✓	✓	✓✓	✓	✓✓	✓✓	✓	✓	$	♦♦	Chemistry Physics (symmetry)
Boas	✓	✓✓	✓	✓	✓	✓		✓	$$	♦	Physics
Greenberg	✓✓	✓✓	✓✓		✓		✓✓		$$$$	♦	Engineering
Kreyszig	✓	✓	✓		✓		✓✓	✓	$$$	♦	Engineering
Strang (1986)	✓✓✓	✓	✓✓	✓✓	✓		✓✓✓		$	♦♦♦	Computational Physics

** Note: The number of "✓" refers of quality and extent of coverage (I own all 5 books and they're my opinions only !), whereas "♦" denotes the level of mathematical intuition the author expects the reader to have for full comprehension of the material (ie. Not just knowing how to plug + chug basic problems !).

Compendium math textbooks references:

1) G. Strang (1986). Introduction to Applied Mathematics. Wellesley, MA: Wellesley-Cambridge Press

2) K.F. Riley, M.P. Hobson, and S.J. Bence (2006). Mathematical Methods for Physics and Engineering: A Comprehensive Guide (3rd ed.). Cambridge, UK: Cambridge University Press. *Paperback*

3) M. Boas (2006). Mathematical Methods in the Physical Sciences (3rd ed.). Hoboken, NJ: John Wiley & Sons

4) E. Kreyszig (2011). Advanced Engineering Mathematics (10th ed.). Hoboken, NJ: John Wiley & Sons

Recommended references for statistics: Depending on your research interests, there are numerous types of statistics texts you can choose from ! Here are some nice ones (all of them are cited on pages 5 - 7):

Textbook	Focus	Difficulty	Level of math needed
Classical Statistics			
McKillup	1st course in statistics.. *really* nice and easy ! ANOVA, regression, and non-parametric statistics **Audience = biology folks w/ no prior stats knowledge**	♦ (Easiest one out of this list)	College algebra
Zar	Often considered as the "bible of biostatistics" LOTS of example problems, but *very dry reading!* **Audience = biology folks / medical professionals**	♦	College algebra
Box, Hunter & Hunter	Easy intro to statistics Heavy emphasis on *experimental design* Multi-factor ANOVA and regression	♦	College algebra Geometry + logic Linear algebra helps !
Grafen & Hails	Heavy emphasis on *analyzing + interpreting* data Will help you understand ANOVA table outputs from SAS, Excel, minitab, or matlab Multi-factor ANOVA, regression, GLM	♦♦ (ANOVA starts on page 1)	College algebra; some calculus Linear algebra concepts Prior exposure to basic statistics helps!
Johnson & Wichern	Large-scale, hardcore data-crunching statistics (PCA, correlations, classifications)	♦♦♦	Linear algebra is absolutely crucial Prior exposure to random variables
Robeva	"Jack-of-all-trades" raw data analysis Nonlinear ODEs Fourier analysis (DFT) Nonlinear regression	♦♦	Linear algebra ODEs Linear systems concepts Prior exposure to probability
Bayesian Statistics			
Gelman, et al.	Awesome, gentle intro to Bayesian statistics Intro to the "Generalized" Linear Model LOTs of good pedagogical discussions Main focus is on understanding, not on mechanics	♦♦	Requires exposure to prob & stats Linear algebra
Martinez & Martinez	Statistical learning topics, such: Monte Carlo / boostrap Non-parametric regression Data-smoothing, Pattern recognition Easy to read once you get used to the notations	♦♦	Calculus Linear algebra Programming knowledge Prior exposure to probability
Bishop	Exactly like Martinez & Martinez, but more info, explanations, and more formal treatment. Really good intro to statistical learning !	♦♦	Prior knowledge of linear algebra, ANOVA and regression are crucial
Hastie	Exactly like Bishop, but more info, explanations, and more formal treatment. Somewhat more difficult to read than Bishop.	♦♦♦	Same as above, and prior knowledge to Bayesian statistics will help !
Wasserman	Formal & dense treatment of Hastie. Lots of details & proofs that Hastie glosses over	♦♦♦	You better bring your A-game in terms of probability & stats knowledge
Fukunaga	The definitive, most-referenced text on statistical learning + pattern recognition. Less theory than Wasserman & more practical	♦♦♦	Prior knowledge of Bayesian stats and linear algebra are absolutely crucial

BE 601 - 604: Numerical & stats (reading list)

Reading assignments: I will frequently assign readings from this list (especially the red ones) via Blackboard Learn !! Selected PDFs will be available for you to download on the class website.

Linear algebra :

1) G. Strang (2009). Introduction to Linear Algebra (4th ed.). Wellesley, MA: Wellesley-Cambridge Press.
2) G. Strang (2005). Linear algebra and its Applications (4th ed.). Boston, MA: Cengage Learning
3) C.D. Meyer (2000). Matrix Analysis and Applied Linear Algebra. Philadelphia, PA: Society for Industrial and Applied Mathematics (Siam)

PDE + Heat transfer:

1) S.J. Farlow (1993). Partial Differential Equations for Scientists and Engineers. New York, NY: John Wiley & Sons
2) T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P.DeWitt (2011). Fundamentals of Heat and Mass Transfer. Hoboken, NJ: John Wiley & Sons

Numerical methods:

1) L.N. Trefethen and D. Bau III (1997). Numerical Linear Algebra. Philadelphia, PA: Society for Industrial and Applied Mathematics (Siam)
2) J.W. Demmel (1997). Applied Numerical Linear Algebra. Philadelphia, PA: Society for Industrial and Applied Mathematics (Siam)
3) R. LeVeque (2007). Finite Difference methods for Ordinary and Partial Differential Equations: Steady-State and Time-dependent Problems Philadelphia, PA: Society for Industrial and Applied Mathematics (Siam)
4) J. Nocedal, S. Wright (2000). Numerical Optimization (2nd ed). New York, NY: Springer-Verlag
5) Y. Saad (2003). Iterative Methods for Sparse Linear Systems (2nd ed.). Philadelphia, PA: Society for Industrial and Applied Mathematics (Siam)
6) R.S. Robeva, J.R. Kirkwood, R.L. Davies, L.S. Farhy, M.L. Johnson, B.P. Kovatchev, and M. Straume (2008). An Invitation to Biomathematics. Burlington, MA: Elsevier

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Statistical methods (for life scientists); statistical analysis

1) S. McKillup (2012). Statistics Explained: An Introductory Guide for LIfe Scientists (2nd ed.). Cambridge, UK: Cambridge University Press.
2) A. Grafen and R. Hails (2002). Modern Statistics for the Life Sciences. New York, NY: Oxford University Press, Inc.
3) R. A. Johnson and D. W. Wichern (2007): Applied Multivariate Statistical Analysis. Upper Saddle River, NJ: Prentice Hall
4) L. Wasserman (2004). All of Statistics: A Concise Course in Statistical Inference. New York, NY: Springer

5) W.L. Martinez and A.R. Martinez (2002). Computational Statistics Handbook with Matlab. New York, NY: Chapman & Hall / CRC Press

6) T. Hastie, R. Tibshirani, and J. Friedman (2009). The Elements of Statistical Learning: Data Mining, Inference, and Prediction (2^{nd} ed). New York, NY: Springer

7) J.H. Zar (2009). Biostatistical Analysis (5th ed.). Upper Saddle River, NJ: Prentice-Hall

8) G. E. P. Box, J. S. Hunter, and W. G. Hunter (2005): Statistics for Experiments: Design, Innovation, and Discovery (2^{nd} ed). Hoboken, NJ: John Wiley & Sons

9) A. C. Rencher, G. B. Schaalje (2008): Linear Models in Statistics. Hoboken, NJ: John Wiley & Sons

10) A. Gelman, J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari, and D.B. Rubin (2014). Bayesian Data Analysis (3^{rd} ed). Boca Raton, FL: CRC Press

11) A. Dobson (2008). An Introduction to Generalized Linear Models (3rd ed). New York, NY: Chapman & Hall / CRC Press

12) K. Fukunaga (1990). Introduction to Statistical Pattern Recognition (2^{nd} ed). San Diego, CA: Academic Press

13) C. Bishop (1990). Pattern recognition and machine learning. New York, NY: Springer

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Probability theory

1) E.T. Jaynes (2003). Probability Theory: The Logic of Science. Cambridge, UK: Cambridge University Press

2) S. Kay (2006). Intuitive Probability and Random Processes using Matlab. New York, NY: Springer

3) A. Papoulis and S.U. Pillai (2002). Probability, Random Variables, and Stochastic Processes (4^{th} ed). New York, NY: McGraw-Hill