ME 580, Theory of Elasticity

Spring 2019, Mon/Weds, 10:10 am –11:55 pm

Course Description

The course content divides essentially into four parts. An emphasis will be placed on concepts (e.g. modeling, linearity, superposition, scaling) and fundamental phenomena (e.g. boundary layers, timescales, energy) exhibited through a sampling of model problems. The approaches to solve the model problems provide exposure to a variety of solution methods (complex variable, perturbation, variational methods). This choice comes at the expense of a comprehensive review of some of the more classical results in linear elasticity.

Prerequisites

Two semesters of undergraduate mechanics of solids and/or fluids. Advanced calculus, and vectors. Familiarity with linear algebra is very useful.

Reference Materials

- Sadd, Martin H. 2011 *Elasticity: Theory, Applications, and Numerics*, Academic Press, Third Ed.

  $\mathcal{NB}$: This book is available electronically through the BU Library. It'll be our primary reference for the course.

- Lee A. Segel and G. H. Handelman, *Mathematics Applied to Continuum Mechanics*, SIAM, 2007.

  $\mathcal{NB}$: This book is available electronically through the BU Library. It's a classic and shows the connection between elasticity theory and the more general continuum mechanics. The introductory material on vectors and tensors is especially good.

- Love, A.E.H. 1892 *A Treatise on the Mathematical Theory of Elasticity*, Cambridge University Press.

  $\mathcal{NB}$: The year of publication of this book is not a misprint. It provides excellent historical content and context, and it covers many of the classical solutions. It also has a lot of material that is not generally covered in modern texts (e.g. the elastica), and so remains the only and best comprehensive coverage of those early topics in the subject. The notation, however, is difficult for a modern reader.

Grading

The final grade for the course is based on written homework solutions, participation in homework discussions, and written and oral reports from small research problems. Discussions will be held weekly during which students will be expected to present the solutions of homework problems.

Topics

Part I: Field Equations (3 weeks)

1. Vector and tensor algebra. Vector and tensor calculus.
2. Analysis of deformation: Definition of strain, linearized strain, compatibility equations.
3. Stress and equilibrium: Momentum conservation, definition of stress and traction, analysis of stress tensor, principal stresses.
4. Linear elastic material behavior: Hooke's law, anisotropy, isotropy, thermoelastic constitutive equations. Strain energy, energetic bounds on material constants.

Part II: Basic theorems and elementary $3D$ solutions (3 weeks)

5. Formulation of boundary value problems. Existence and uniqueness.
6. St. Venant's principle.
7. Tension, torsion, and bending of bars by St. Venant's semi-inverse method.
8. Spherically symmetric stress distributions.
9. Reciprocity and integral theorems.

Part III: Basic elastostatics (3 weeks)

10. Plane stress/strain (2 weeks).
    (a) Plane stress & strain as special cases of $3D$ problems.
    (b) Airy stress functions and separation of variables:
        - Wedge and Flamant problem (dimensional analysis).
        - Stress concentrations around a circular hole (topology).
        - Crack tip stress fields (asymptotic behavior of stress fields).
11. Three-dimensional Problems
    - Displacement potential functions.
    - Kelvin's problem.

Part IV: Advanced Topics (4 weeks) Topics will be selected from the list below based on class preferences.

12. Variational Methods: (2-3 lectures)
    (a) Principle of minimum potential energy.
    (b) Principle of minimum complementary energy.
    (c) Principle of virtual work.
    (d) Rayleigh-Ritz and finite element methods.
13. Effective properties of materials with microstructures.
    (a) Basic (Hashin-Shtrikman) variational bounds on material properties.
    (b) Advanced (Willis) variational bounds on material properties.
    (c) Asymptotic homogenization methods.
14. Multiscale analysis of elastic materials with microstructure.
    (a) Surface layers in elastodynamics.
    (b) Asymptotic method of multiple scales; derivation of effective properties.
    (c) Exotic continuum behavior of materials with extreme microstructures.
15. Rods, plates, and shells as asymptotic limit of $3D$ theory.
    (a) Asymptotic derivation of elementary equations.
    (b) Boundary layer edge effects: matched asymptotic expansions.
16. Waves in elastic solids.
    (a) Shear and compressional waves; Helmholtz decomposition.
    (b) Reflection, transmission, and mode conversion at an interface.
    (c) Surface waves (Rayleigh waves).
    (d) Guided waves (SH waves in a layer); end effects.
17. Poroelasticity.
    (a) Biot poroelasticity: confined vs. unconfined compression
    (b) Multiscale poroelasticity: "vascularized" poroelastic solids.

Instructor

Paul E. Barbone, EMA 221, (617)353-6063, barbone@bu.edu
Office hours: Wednesdays 4-5pm, or by appointment (email me).