A Noisy Volunteer's Dilemma

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May 25, 2026

Abstract

The Volunteer's Dilemma asks when individuals will incur a private cost to report a common problem. I alter this setting by adding reporting noise. Strikingly, even if many individuals have arbitrarily small reporting costs, arbitrarily small noise prevents information aggregation in large groups. Rather, successful aggregation requires the existence of individuals with precisely zero reporting costs. Further, reporting noise ensures information aggregation failures extend beyond the Volunteer's Dilemma to more general settings. Finally, I discuss practical solutions for organizations to improve information aggregation.

Keywords: Volunteer's Dilemma, Information Aggregation, Strategic Reporting
JEL Classification Numbers: D82, D71, D83

1 Introduction

The bystander effect has been extensively studied in the social sciences. In this paradigm, a large group observes a common problem and each individual decides whether to speak up to alert a decision-maker. Speaking up incurs a private cost, and thus there exists a diffusion of responsibility: each individual hopes someone else speaks up, making them more likely to remain silent. Research on bystander inaction was motivated by the widely publicized failure to report a violent crime in New York and was later formalized through classic experiments (Darley and Latané, 1968). More recently, the bystander effect has been documented in contexts such as school bullying and sexual harassment reporting (Nickerson et al., 2014) and in organizational misconduct and safety reporting (Bazerman, 2022).

Standard game-theoretic models (cf. Diekmann, 1985) attempt to explain these phenomena through a Volunteer's Dilemma as follows. A problem arises and each individual can report the problem to a decision-maker at a private cost, $\nu_i \geq 0$, or remain silent at no cost.

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If any individual reports the problem, the decision-maker rationally infers a problem arose and investigates the problem, giving every member of the group a benefit of Δ. Proposition 1 proves a standard result that as the group size grows large, the probability that no individual reports the problem converges to the minimal cost-benefit ratio (i.e., $\inf \mathrm{Supp}(\nu_i)/\Delta$), which could be small in the context above.

In contrast to standard models, which assume no reports are received when no problem exists, I assume reporting is noisy: for any state of the world, a report is received from an individual with strictly intermediate probability. I provide several microfoundations for this assumption in the model discussion in Subsection 3.1. These include players who make errors consistent with Quantal Response Equilibrium (McKelvey and Palfrey, 1995), henceforth QRE, and organizations that intentionally introduce noise into reporting (Boudreau et al., 2023). This paper examines how noisy reporting impacts the bystander effect, both by altering individuals' reporting incentives and by influencing how the decision-maker interprets reports.

The first implication of noise is that in large groups the decision-maker may optimally decline to investigate even if reports are made. If, by contradiction, one report is sufficient to trigger an investigation, then in large and noisy groups, a report is always received, implying that an investigation occurs irrespective of the state. In this case, the decision-maker receives no benefit from the reports, contradicting optimality. Further, historical cases align with this prediction: organizations often require multiple warnings before acting. Before both the Boeing 737 MAX crashes and the NASA Challenger disaster, these organizations received employee reports flagging safety concerns, but unfortunately no single report carried enough weight to trigger investigations (Leggett and Burridge, 2020; Vaughan, 1996).[1]

Further, in contrast to the case absent noise, the probability that a report is pivotal goes to zero as the group size grows large irrespective of the distribution of reporting costs. Therefore, in large groups, the decision-maker benefits from reporting if and only if there exist individuals who have precisely zero costs for reporting (Proposition 3). Moreover, the necessity of such individuals extends to more general environments than the Volunteer's Dilemma (Proposition 4). In particular, the action and state spaces can be arbitrary compact subsets of the reals, the decision-maker's preferences can be represented by any continuous utility function, and the decision-maker may commit to any symmetric decision rule.

[1] Others may interpret failures to investigate as intentional cover-ups rather than as outcomes of a Bayesian mechanism-design problem. In my model, however, the qualitative results still apply to organizations whose risk tolerance differs from that of their employees, provided there exists some confidence level at which the organization would investigate (i.e., the decision-maker may benefit from reporting). In support of my interpretation, even Toyota's well-meaning Andon Cord system, which is designed to empower workers to flag problems, effectively requires a second report before management stops the production line, reflecting risk tolerance rather than intentional cover-up (Edmondson, 2023).

These results have implications for a broad range of collective action problems. For instance, one could consider settings where individuals "report" by adding their name to a petition, tweeting about a social movement, or participating in a protest, where the decision-maker corresponds to a politician or potential voter. In these settings, individuals engage in the political movement if and only if they receive personal pleasure from doing so, even when their participation has no bearing on material outcomes. Similarly, one could consider media reporting. In this setting, some partisan outlets act as noise, reporting misconduct when none exists or remaining silent when it does. Unbiased outlets act strategically: they weigh the benefit of exposing malfeasance against strictly positive reporting costs, such as government retaliation or accusations of bias. If individuals are unable to discern which outlets are partisan or not, then as the media market grows large, the diffusion of responsibility drives these unbiased outlets to remain silent. Finally, consider a community enforcement game where individuals must report their partner's action. If (i) there is noise in the reporting, as in Fudenberg and Wolitzky (2024), and (ii) individuals bear a cost of reporting when shirked on due to fear of retribution, then my model predicts that in large populations with long-lived players, players will not report being shirked on, precluding efficient behavior.

The rest of the paper is structured as follows. Section 3 describes the model, where Subsection 3.1 defines and microfounds the noise assumption. Section 4 provides the results in the context of the Volunteer's Dilemma, Section 5 provides generalizations, and Section 6 concludes and discusses policy solutions.

Related Literature This paper is primarily related to three strands of literature: bystander intervention, information aggregation, and pivotality bounds.

First, this paper contributes to the literature on the Volunteer's Dilemma and bystander intervention. My benchmark, without reporting noise, uses a generalization of the bystander intervention model of Diekmann (1985), as described in Bergstrom (2017) and Battaglini and Palfrey (2025), where individuals have heterogeneous reporting costs. When this literature considers other variants of heterogeneity, it is typical to assume immoral types who receive no private value from speaking up (Campos-Mercade, 2022), or different preferences (e.g. Guha, 2020, assumes some types have ambiguity aversion), rather than reporting noise.[2]

The closest related paper is Goeree and Holt (2005), which analyzes a Volunteer's Dilemma assuming QRE. In a QRE, each individual receives a full-support preference shock favoring one of the two decisions. Given these shocks, the probability any individual reports is strictly positive, and thus the probability of no report goes to zero for large groups, implying first-

[2] Other recent papers consider uncertainty over the number of players (Hillenbrand and Winter, 2018) or analyze a dynamic version of the game (Bliss and Nalebuff, 1984; Bergstrom, 2017).

best decision-making in their analysis. However, in their analysis, a problem always occurs and thus a report is always needed. When modeling ex-ante uncertainty about whether a problem exists in these settings, QRE also implies reports are generated with positive probability regardless of the state. Therefore, the key question in my model is whether a Bayesian decision-maker is able to infer the state given the reports. My results imply that upon adding this uncertainty to the framework of Goeree and Holt (2005) the decision-maker would do better than had she observed no reports, but would obtain a payoff bounded away from the first-best for large groups.

Further, a large experimental literature (starting with Diekmann, 1985; see Fischer et al., 2011, for a review) tests the Volunteer's Dilemma. My results are consistent with the behavioral findings from these studies: when a problem exists, reports are received with high probability in large groups.[3] However, to the best of my knowledge, the informational implications — specifically, whether a decision-maker can use noisy reports to infer the presence of a problem and thereby benefit from reporting — have not been directly tested as these papers never consider reporting when a problem does not exist.

Second, this paper connects to the literature on information aggregation in large elections. The Condorcet Jury Theorem demonstrates that majority voting aggregates information perfectly in large groups even though each voter observes only a noisy private signal of the state. Feddersen and Pesendorfer (1996) show that this conclusion is robust to the presence of partisan voters (a form of noise in who participates, as microfounded in Section B.3). Crucially, voting is costless in these models, so the only frictions are informational. In contrast, reporting is costly in my setting, introducing a free-riding friction in which each individual hopes someone else bears the cost. Palfrey and Rosenthal (1983) study a setting with costly voting, however, they preclude aggregate uncertainty about the state of the world, whereas the focus of my analysis is on whether reports are informative about this underlying uncertainty.[4] More recently, several studies have demonstrated that information aggregation can fail in large groups under specific conditions, including uncertainty about signal distributions (Mandler, 2012), ambiguity aversion (Ellis, 2016), and state-dependent election turnout (Ekmekci and Lauermann, 2020). In contrast, I show that information aggregation can fail even when it is common knowledge that every voter perfectly observes the true state of the world.

[3] Such a result is trivially true in my analysis as the noise implies that a report is received with probability bounded away from zero for each individual.

[4] Chemmanur and Fedaseyeu (2018) study information aggregation with costly communication and show that large groups fail to aggregate information when communication costs are homogeneous. However, their learning failures rely on a restriction to pure-strategy equilibria, which typically involves a loss of generality given symmetric costs (cf. Diekmann, 1985).

Finally, on a methodological level, the analysis builds on the theoretical literature bounding pivot probabilities in large games. This literature shows that in large games with noisy types, an individual's pivot probability under any arbitrary mechanism is bounded on the order of $1/\sqrt{n}$ (e.g., Al-Najjar and Smorodinsky, 1996; Fudenberg et al., 1998; Becker, 2012). While the combinatorial properties underpinning these limits are central to my analysis, my economic framework differs substantially. Crucially, applications of these bounds in the prior literature (such as the public goods problem in Al-Najjar and Smorodinsky, 1996) rely on the revelation principle to map individual influence to economic efficiency. In my setting, because reporting costs are incurred directly through the message rather than the mechanism's outcome, the revelation principle breaks down. This precludes using standard mechanism design approaches to evaluate efficiency or predict outcomes, since a vanishing pivot probability does not, a priori, dictate which outcomes will occur. Finally, the decision-maker's challenge in my setting is twofold: the mechanism must induce enough individuals to bear the cost of reporting, and the decision-maker must be able to utilize these reports to properly adapt to the state of the world.[5]

2 Motivating Example

I begin by describing the problem absent noise and what payoffs the decision-maker obtains for large groups.

A state $\omega \in \{0,1\}$ is drawn, where $\omega = 1$ represents a problem. Each of n individuals observes ω and can report the problem ($r_i = 1$) or remain silent ($r_i = 0$). Silence incurs no cost, but reporting incurs a cost ν_i which is i.i.d. across individuals with $\nu_i \sim F(\cdot)$, where F admits a strictly positive, continuous density on $[\underline{\nu}, \bar{\nu}]$ with $0 < \underline{\nu}$. A decision-maker observes $\sum r_i$ and chooses whether to investigate ($a \in \{0,1\}$). All players receive a benefit $\Delta > \underline{\nu}$ if an investigation occurs when a problem exists and a loss $L < 0$ if an investigation occurs when it does not. Finally, I assume the decision-maker would not investigate at her prior (i.e., $\Pr(\omega = 1)\Delta + \Pr(\omega = 0)L < 0$) and I restrict attention to symmetric equilibria.

Proposition 1 (**Benchmark: No Noise**). *Fix a distribution of ν_i as described above and assume $a = 1$ if and only if $\sum r_i > 0$. In the unique symmetric equilibrium, there is never an investigation when $\omega = 0$ and the probability of no investigation when $\omega = 1$ converges to $\underline{\nu}/\Delta < 1$ as $n \to \infty$.*

Without noise, the decision rule $a = 1 \iff \sum r_i > 0$ is ex-post optimal and is the decision rule typically considered in the literature for the decision-maker. The proof

[5] Additionally, unlike standard mechanism design settings (cf. Holmström, 1979) where a principal can use transfers to achieve first-best outcomes despite noise, my setting precludes transfers.

below follows directly from standard arguments (Battaglini and Palfrey, 2025), with the only difference being that these papers abstract from uncertainty about ω.

Proof of Proposition 1. When $\omega = 0$, $r_i = 0$ is a dominant action. Focusing on $\omega = 1$, as the cost of $r_i = 1$ is increasing in ν_i, there exists a threshold, $\nu^*(n)$, for choosing $r_i = 1$. At $\nu^*(n)$, the individual is indifferent between $r_i = 0$ and 1 as stated below:

$$\nu^*(n) = \Delta\big(1 - F(\nu^*(n))\big)^{n-1}. \tag{1}$$

Therefore $\nu^*(n)$ is decreasing, and as it is bounded below by $\underline{\nu}$, $\lim_{n\to\infty} \nu^*(n)$ exists. If, by contradiction, $\lim_{n\to\infty} \nu^*(n) > \underline{\nu}$, then the right-hand side of Equation (1) is equal to zero, deriving a contradiction. Therefore, $\lim_{n\to\infty} \nu^*(n) = \underline{\nu}$. Finally,

$$\Pr(\sum r_i = 0|\omega = 1) = (1 - F(\nu^*(n)))^n = \Big(\frac{\nu^*(n)}{\Delta}\Big)^{\frac{n}{n-1}},$$

and taking the limit of this expression gives the condition in the proposition. □

I introduce noise as follows: suppose a fraction $\epsilon > 0$ of individuals are mechanical, each choosing $r_i \in \{0, 1\}$ uniformly at random, while the remaining $1 - \epsilon$ are rational as above. Alternatively, Appendix B formalizes how one can interpret this assumption as stating a proportion $\epsilon/2$ of individuals have preferences misaligned with those of the decision-maker and always want investigations and another $\epsilon/2$ never want investigations.

Proposition 2 (Benchmark: With Noise).
Fix any $\epsilon > 0$. There exists n^* such that for all $n \geq n^*$ and any threshold rule where $a = 1 \iff \sum r_i \geq t_n$, the decision-maker's payoff is bounded above by the payoff she would obtain absent any reports.

The main analysis shows that this learning failure continues to hold for more general utility functions and arbitrary symmetric decision rules for the decision-maker. Further, the analysis provides conditions on the reporting costs under which the decision-maker can benefit from reporting in large groups, unlike in the benchmark case.

Proof of Proposition 2. Each mechanical individual reports $r_i = 1$ with probability 1/2 regardless of ω, so in any symmetric equilibrium $\Pr(r_i = 1 \mid \omega) \in [\epsilon/2, 1 - \epsilon/2]$ for all ω. An individual is pivotal if and only if $\sum_{j\neq i} r_j = t_n - 1$. As $\sum_{j\neq i} r_j \sim \text{Bin}(n-1, p)$ for some $p \in [\epsilon/2, 1 - \epsilon/2]$,

$$\Pr\Big(\sum_{j\neq i} r_j = t_n - 1\Big) \leq \max_k \Pr\big(\text{Bin}(n-1, \epsilon/2) = k\big) \leq \frac{C_\epsilon}{\sqrt{n}}, \tag{2}$$

where C_ϵ is a constant depending only on ϵ. This constant is derived from the Berry-Esseen theorem, as formalized in Appendix C. Further, the benefit of a pivotal report is at most Δ, so the expected benefit of reporting is bounded above by $\Delta C_\epsilon/\sqrt{n}$. Therefore, there exists an n^* above which this falls below $\underline{\nu}$, so no rational individual reports. As only mechanical types report, their reports are uninformative about ω, implying the decision-maker's payoff is weakly below the payoff she would obtain absent any reports. □

3 Model

There is a collective action problem with n individuals, $i \in \{1, \dots, n\}$, an unknown state of the world $\omega \in \Omega \subseteq \mathbf{R}$, and a decision-maker who chooses an action $a \in A \subseteq \mathbf{R}$. The timing of the game is as follows. The decision-maker commits to a decision rule that maps the group's reports to a distribution over actions $\Delta(A)$. Next, all individuals observe ω and the decision rule, and simultaneously make a binary decision of whether to report ($r_i = 1$) or not ($r_i = 0$).[6]

The decision-maker's utility given $a \in A$ and $\omega \in \Omega$ is denoted by $u^p(a, \omega)$. I assume that A, Ω are compact and $u^p(\cdot)$ is continuous. Each individual has an i.i.d. private type $\nu_i \in \mathbf{R}$ distributed according to $F(\cdot)$. Further, each individual chooses $r_i \in \{0, 1\}$ to maximize:

$$u^i(a, \omega, \nu_i, r_i) := u^p(a, \omega) - r_i \cdot \nu_i, \tag{3}$$

given the chosen decision rule. The only assumption imposed on the decision rule is symmetry: the decision rule can only condition on the total number of reports. Formally, $d : \{0, 1, \dots, n\} \to \Delta(A)$. I note that the decision rule need not be monotone, and need not be ex-post efficient.[7]

Throughout, I utilize the following definitions.

Definition 1. *The decision-maker's <u>first-best payoff</u>, Π^{FB}, and <u>status-quo payoff</u>, Π^{SQ}, are*

$$\Pi^{FB} = \mathbb{E}_\omega(\max_a u^p(a, \omega)) \text{ and } \Pi^{SQ} = \max_a \mathbb{E}_\omega(u^p(a, \omega)). \tag{4}$$

Therefore, her first-best payoff is the payoff she obtains when she can observe ω, and the status-quo payoff is what she obtains absent information from the individuals. To avoid a

[6]The restriction to binary messages is without loss of generality when applied to the volunteer's dilemma. With more general state spaces, this restriction is with loss of generality. However, I show in Section 5 that in various contexts, despite this restriction, the decision-maker can still obtain her first-best payoff (defined below) with binary messages.

[7]As the qualitative message of the paper is that noise results in poor payoffs for the decision-maker, restricting to monotone or ex-post efficient decision rules would only strengthen the message of the paper.

trivial analysis, I assume $\Pi^{FB} > \Pi^{SQ}$.

Finally, to avoid discussions of equilibrium multiplicity, I focus on the decision-maker's preferred symmetric equilibrium.[8] Formally, denote by $D(n)$ the space of all symmetric decision rules with n individuals, with prototypical element d. I denote the set of all symmetric Nash equilibria given decision rule d as $\mathcal{E}(d)$ and define,

$$\Pi(n) := \max_{d \in D(n), e \in \mathcal{E}(d)} \mathbb{E}(u^p(a, \omega) | e, d). \tag{5}$$

The focus of the analysis will be on the limit of $\Pi(n)$ as n grows large.

3.1 Defining Noise

The simplest way to incorporate noise into the model is to assume that the distribution of ν_i, the costs of reporting, incorporates individuals who (i) have large enough costs of reporting such that they never prefer to report and (ii) have large enough benefits from reporting (i.e., ν_i is negative and large) such that they always report.

Assumption 1. *Define* $M := \max_{\omega, a, a'} |u^p(a, \omega) - u^p(a', \omega)|$. *The distribution of* ν_i *satisfies* $1 - F(M) > 0$ *and* $F(-M) > 0$.

Additionally, this assumption is implied by the modeling of QRE whereby individuals receive full-support preference shocks (cf. Goeree and Holt, 2005). Rather than modeling both the preference shock for choosing a specific action and the cost of reporting, I interpret ν_i as the net cost of reporting. Given this assumption, we have the following result.

Lemma 1. *Given Assumption 1, there exists an* $\epsilon > 0$ *such that in any equilibrium and for any decision rule,* $\Pr(r_i = 1 | \omega) \in [\epsilon, 1 - \epsilon]$.

Any microfoundation of "noise" that results in the same conclusion as this lemma will be sufficient for the subsequent results. Appendix B discusses how behavioral types (as in the motivating example in Section 2) or noisy communication channels that sometimes flip messages (as discussed in Section 4.3 and Appendix B) would also imply Lemma 1. Further, upon restricting attention to monotone decision rules, Appendix B shows that biased

[8] The restriction to symmetric equilibria is standard in the Volunteer's Dilemma literature. Additionally, given the noise assumption, the results in Battaglini and Palfrey (2025) imply that if $F(\cdot)$ has full support, then asymmetric equilibria do not exist for large groups, which is the focus of the analysis. Finally, to gain intuition behind the multiplicity, as is standard in communication games there will also exist "babbling" equilibria where the reports are independent of the state, implying that the decision-maker's action is independent of the reports, thus the agents have no incentive to condition the reports on the state.

individuals (those who always prefer higher actions) would result in similar aggregation failures.

Finally, I note that the standard Volunteer's Dilemma already includes situations in which $1 - F(M) > 0$, namely where there are individuals who never report. These individuals always result in false negatives and such individuals on their own will not result in information aggregation failures, as one report remains sufficient to trigger an investigation. The introduction of noise implies both false negatives (i.e., $1-F(M) > 0$) and false positives which are instrumental for the results.

4 Volunteer's Dilemma

This section presents results on a special case of the model in Section 3: the standard Volunteer's Dilemma (cf. Diekmann, 1985). Here, $\omega \in \{0,1\}$, $a \in \{0,1\}$ and the decision-maker's utility satisfies $u^p(1,1) > u^p(0,1)$ and $u^p(0,0) > u^p(1,0)$.[9] One can interpret $r_i = 1$ (respectively, 0) as reporting (respectively, remaining silent), $a = 1$ (respectively, 0) as whether an investigation occurs (respectively, does not), and $\omega = 1$ (respectively, 0) as whether an investigation is needed (respectively, not needed). Finally, as in Proposition 1, assume that $\mathbb{E}(u^p(1,\omega)) < \mathbb{E}(u^p(0,\omega))$, implying that at her prior, the decision-maker would not investigate.

This section provides asymptotic results, presents a numerical example to showcase which group sizes in practice fail to aggregate information, and concludes with a discussion.

4.1 Information Aggregation

I now analyze the environment described at the beginning of this section, hereafter imposing Assumption 1. Assumption 1 states that due to noisy reporting, $\Pr(r_i = 1|\omega)$ is strictly intermediate for all ω. Without Assumption 1, the probability of being pivotal need not go to zero as the group size grows large; for example, when $\underline{\nu} > 0$ in Equation (1). As shown below, this is false given Assumption 1.

Lemma 2. *There exists a constant $\tilde{M} > 0$, independent of n, such that (i) if $\nu_i > \frac{\tilde{M}}{\sqrt{n}}$, then $r_i = 0$ and (ii) if $\nu_i < \frac{-\tilde{M}}{\sqrt{n}}$, then $r_i = 1$.*

The intuition is that an individual's report only changes the decision-maker's action when the remaining $n-1$ reports land exactly at a threshold, so the expected gain from

[9] If these inequalities did not hold, then, up to symmetry, the decision-maker's optimal decision is independent of the state.

reporting equals the gain from being pivotal multiplied by the probability of being pivotal. By Assumption 1, the reports of others are i.i.d. with variance bounded away from zero, so by arguments in the spirit of the Central Limit Theorem, the probability of any particular count is at most of order $1/\sqrt{n}$. Finally, since the maximal utility gain from being pivotal is bounded by some constant, the expected gain from reporting is therefore at most $\tilde{M}/\sqrt{n}$.

Lemma 2 implies that information about ω can only come from $\nu_i \in \left[\frac{-\tilde{M}}{\sqrt{n}}, \frac{\tilde{M}}{\sqrt{n}}\right]$. Outside this set, all other types choose their action based on ν_i alone, rather than information about ω. The proof of this lemma appears below and, in fact, applies to the general environment described in Section 3.

To build intuition for the subsequent results on information transmission, note that the decision-maker faces two interrelated problems. First, the probability that any individual's report contains information about ω shrinks as n grows large. Second, the standard deviation of the noise in $\sum r_i$ is of order $\sqrt{n}$ asymptotically.[10] Therefore, as formalized in the Appendix, for the decision-maker to obtain her first-best payoff (respectively, exceed her status-quo payoff) the difference in the number of reports when $\omega = 1$ compared to $\omega = 0$ must be larger than $\sqrt{n}$ (respectively, proportional to $\sqrt{n}$) as stated below,

$$\begin{aligned}
\lim_{n\to\infty} \Pi(n) = \Pi^{FB} \iff \lim_{n\to\infty} \frac{\sum \Pr(r_i = 1|\omega = 1) - \sum \Pr(r_i = 1|\omega = 0)}{\sqrt{n}} = \infty. \\
\lim_{n\to\infty} \Pi(n) > \Pi^{SQ} \iff \lim_{n\to\infty} \frac{\sum \Pr(r_i = 1|\omega = 1) - \sum \Pr(r_i = 1|\omega = 0)}{\sqrt{n}} > 0. \qquad (6)
\end{aligned}$$

In contrast, absent noise, the condition for the decision-maker to obtain her first-best payoff is far more lenient: one more report must be sent when $\omega = 1$ compared to $\omega = 0$. Finally, Lemma 2 implies the following upper bound for the limit in Equation (6), where, recall, $F(\cdot)$ is the distribution of ν_i:

$$\lim_{n\to\infty} \frac{n(F(\frac{\tilde{M}}{\sqrt{n}}) - F(\frac{-\tilde{M}}{\sqrt{n}}))}{\sqrt{n}} = \tilde{M} \cdot \lim_{\delta\to 0^+} \frac{F(\delta) - F(-\delta)}{\delta} = 2 \cdot \tilde{M} \cdot F'(0), \qquad (7)$$

when such a derivative exists. Here, the first equality follows from a change of variables, and the second equality is the definition of a derivative. The following proposition formalizes these intuitions and provides additional bounds for $\Pi(n)$ in terms of Π^{SQ}, Π^{FB}, as defined in Definition 1.

Proposition 3. *The following hold given a distribution of reporting costs, $F(\cdot)$:*

1. *If there exists a $\delta > 0$ such that $F(\delta) - F(-\delta) = 0$, then there exists $n^* \in \mathbf{N}$ such that*

[10]$\mathrm{Var}(\sum r_i) = n \cdot \mathrm{Var}(r_i)$ and $\mathrm{Var}(r_i)$ has an exogenous lower bound by Assumption 1.

for all $n > n^*$, $\Pi(n) = \Pi^{SQ}$.

2. *If $F'(0) = 0$, then $\lim_{n\to\infty} \Pi(n) = \Pi^{SQ}$.*

3. *If $F'(0) > 0$, then $\Pi^{SQ} < \liminf_{n\to\infty} \Pi(n) \leq \limsup_{n\to\infty} \Pi(n) < \Pi^{FB}$.*

4. *If $\lim_{\delta\to 0^+} F(\delta) - F(-\delta) > 0$, then $\lim_{n\to\infty} \Pi(n) = \Pi^{FB}$.*

This proposition states that the decision-maker's payoff is determined by the prevalence of individuals who have precisely zero reporting costs. If no such individuals exist (i.e., Statements 1 and 2), then she obtains her status quo payoff, which was defined as her maximal payoff absent reporting. If these individuals exist, but not with a discrete probability, then she obtains a payoff exceeding her status quo payoff, but less than her first best payoff, defined as her payoff when she can directly observe the state (Statement 3). Finally, if there exists a discrete probability that an individual has exactly zero reporting costs, then she obtains her first best payoff (Statement 4).

The proof of Statement 1 is that if such a δ exists, then by Lemma 2, there exists an n^* above which an individual's action is a function of their private type, ν_i, alone. Therefore, the decision-maker optimally disregards the reports and obtains her status-quo payoff. Statement 2 and the upper bound in Statement 3 follow from formalizations of Equations (6) and (7).

The lower bound in Statement 3 requires the construction of an equilibrium where the decision-maker's payoff exceeds her status-quo payoff in the limit. By Equation (6), this necessitates that the difference in reporting probabilities when ω is one compared to zero is proportional to $\frac{1}{\sqrt{n}}$. I show that the ex-post efficient decision rule accomplishes this goal and allows the decision-maker to use this information efficiently.[11] Finally, Statement 4 states that if there is a mass-point at $\nu_i = 0$, the decision-maker's payoff converges to her first-best payoff. To see why, any individual with $\nu_i = 0$ can simply choose $r_i = \omega$. Therefore the difference in the probabilities of reporting $r_i = 1$ does not converge to zero as n grows large, thus satisfying the condition for $\lim_{n\to\infty} \Pi(n) = \Pi^{FB}$ in Equation (6).

4.2 Example

The preceding results show that information aggregation fails in large and noisy groups whenever reporting costs are bounded away from zero. Figure 1 illustrates this finding

[11]The actual details of the proof are more involved, as naive decision rules result in a probability of being pivotal proportional to $\frac{1}{n}$. In contrast, the ex-post efficient decision rule exhibits this property only when the decision-maker's threshold for choosing $a = 1$ lies near the median of the distribution of reports. However, of course, this threshold is endogenous, complicating the proof.

numerically and shows that the group sizes needed for these asymptotics to apply are reasonable given the motivating examples. Here, I fix a utility function for the decision-maker and parameterize the amount of noise by assuming that with probability ϵ an individual has a sufficiently large (or negative) ν_i such that they have a dominant strategy to report (or not report).[12] With probability $1 - \epsilon$, an individual is as in the standard Volunteer's Dilemma. Rather than solving for the decision-maker's globally optimal decision rule, which is computationally intractable for large n, the figure reports her payoff under an optimally chosen threshold rule T_n whereby $a = 1 \iff \sum r_i \geq T_n$. One can see that even a small amount of noise (such as $\epsilon = .01$) can cause aggregation failures for groups of size 200. Given that the motivating applications involve engineering teams that may number in the thousands or students witnessing misconduct in settings with hundreds of bystanders, these thresholds for aggregation failure appear well within the range encountered in practice. Finally, an important comparative static to note is that the decision-maker's payoff is increasing in the group size when $\epsilon = 0$, but is decreasing in the group size for $\epsilon > 0$. This is because when $\epsilon = 0$, a larger group size increases the probability that an individual has a low cost realization and thus reports. Further, when $\epsilon = 0$, the optimal threshold for large groups remains $T_n = 1$. In contrast, for $\epsilon > 0$, the decision-maker must choose larger thresholds for larger groups. These higher thresholds decrease the probability an individual is pivotal and thus decrease both the number of reports and the utility of the decision-maker.

[12] As shown in the Appendix, for the decision rules considered here, it is equivalent to assume that $\epsilon/2$ individuals always want a higher action and $\epsilon/2$ individuals always want a lower action.

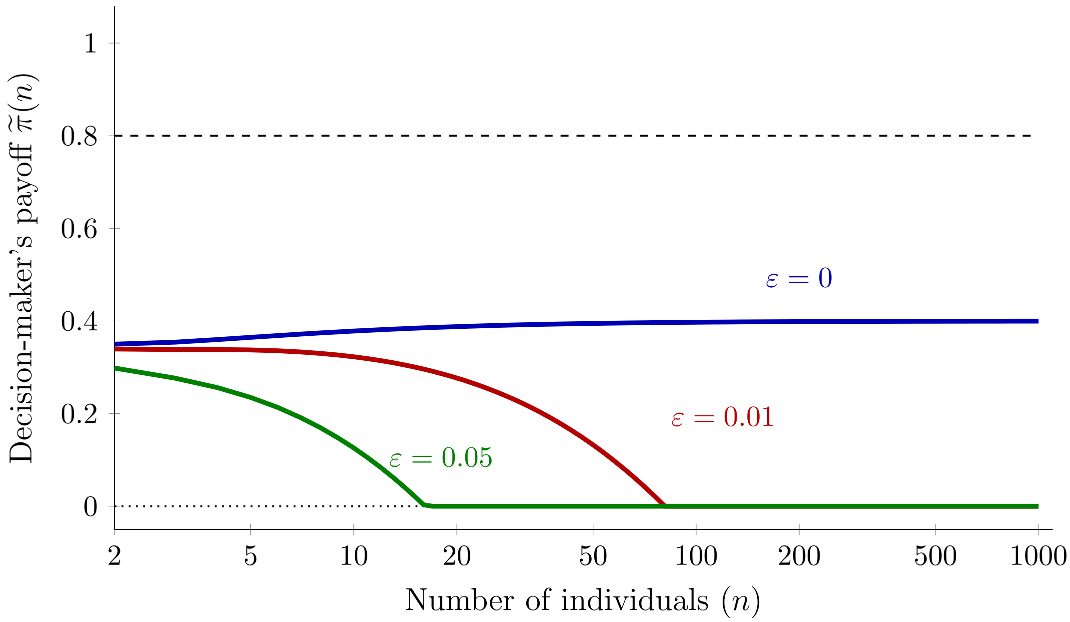

Figure 1: The decision-maker's utility is $u^p(a, \omega) = a(5.5\omega - 1.5)$, where $\omega \in \{0, 1\}$ with $\Pr(\omega = 1) = 0.2$. With probability $\epsilon/2$, ν_i is sufficiently negative that an individual always reports, with probability $\epsilon/2$, ν_i is sufficiently positive that an individual never reports, and with probability $1 - \epsilon$, $\nu_i \sim U[2, 6]$. In blue is the case where $\epsilon = 0$, as in the standard Volunteer's Dilemma, and thus the decision-maker uses a threshold of 1 to choose $a = 1$. In red is the case where $\epsilon = 0.01$, and in green is the case where $\epsilon = 0.05$. In both these cases, the decision-maker's payoff is calculated as the maximum over all possible thresholds, as maximizing over all possible decision rules for large n is computationally infeasible. Finally, the dashed line at 1 corresponds to the first-best payoff and the dashed line at 0 corresponds to the status-quo payoff.

4.3 Discussion

The results in the previous subsections show the deleterious effects of reporting noise (i.e., Assumption 1). For instance, absent Assumption 1, if the support of ν_i contains zero, then asymptotically the decision-maker obtains her first-best payoff, as can be shown with an analogous argument to Proposition 1. I then show in Subsection 4.1 that this is not the case with Assumption 1. Additionally, absent Assumption 1, if the support of ν_i does not contain zero, but contains points arbitrarily close to zero, then asymptotically the decision-maker's payoff approaches the first-best payoff arbitrarily closely. In contrast, Subsection 4.1 shows that with Assumption 1, the decision-maker's payoff never approaches her first-best payoff and equals her status-quo payoff whenever the distribution of ν_i, $F(\cdot)$, satisfies $F'(0) = 0$. These results imply that noisy reporting in collective action games leads to significantly less efficient information aggregation.

Let us now apply these results to the motivating examples (such as reporting harassment, bullying, or safety concerns within organizations). In these contexts, Assumption 1 states that with positive probability (i) a decision-maker may observe reports of a problem even if no problem exists and (ii) the decision-maker may observe no reports of a problem even if a problem exists. When this assumption holds (for any of the reasons discussed in Subsection 3.1), then information aggregation succeeds if and only if there exist individuals who have precisely zero reporting costs.

One of the microfoundations for Assumption 1 is organizations that intentionally add noise to reporting, as in Boudreau et al. (2023). In their setting, adding noise increases plausible deniability for individuals reporting their managers, and thus decreases the probability of retaliation from a disgruntled manager. Through the lens of my model, one could view the cost of reporting (i.e., ν_i) as including potential retaliation from a manager. Therefore, while not formally modeled in my analysis, garbling could be viewed as shifting the distribution of ν_i downward, which on its own encourages more reporting. Boudreau et al. (2023) empirically show that doing so can induce reporting, but they do not consider differential effects by team size. Further, such an analysis would be difficult to interpret causally because team size is not exogenous in their context. Consistent with their findings, my analysis implies that garbling may be helpful for smaller teams by encouraging reports, as it may decrease the cost of reporting. By contrast, for large groups, my analysis implies that the noise requires the decision-maker to increase her threshold for investigation (due to the potential for false positives, which Boudreau et al. (2023) abstract from), which decreases the incentives for individuals to report truthfully.

5 Generalized Model

This section expands on the information aggregation failures described in the previous section. I show that these failures persist with more general preferences, action spaces, and state spaces, as described in Section 3. Throughout, I note that the restriction to binary messages was without loss of generality given the assumption that ω was binary, implying that an individual's private information about ω was also binary. However, in this more general analysis it entails a loss of generality. Despite this, even with binary messages, if there exists a positive fraction of individuals with zero reporting cost, the decision-maker achieves her first-best payoff in the limit.

Proposition 4. *The following hold given a distribution of reporting costs, $F(\cdot)$:*

1. *If there exists a $\delta > 0$ such that $F(\delta) - F(-\delta) = 0$, then there exists a finite cutoff n^* such that for all $n > n^*$, $\Pi(n) = \Pi^{SQ}$.*
2. *If $F'(0) = 0$, then $\lim_{n\to\infty} \Pi(n) = \Pi^{SQ}$.*

3. *If* $F'(0) > 0$*, then* $\Pi^{SQ} < \liminf_{n\to\infty} \Pi(n) \leq \limsup_{n\to\infty} \Pi(n) < \Pi^{FB}$.

4. *If* $\lim_{\delta\to 0^+} F(\delta) - F(-\delta) > 0$ *and* $u^p(a,\omega)$ *satisfies increasing differences, then* $\lim_{n\to\infty} \Pi(n) = \Pi^{FB}$.

This proposition states that the communication breakdowns described in Proposition 3 are not unique to the payoff environment of the Volunteer's Dilemma and occur much more generally. The reason is that Lemma 2 (detailing a necessary condition on ν_i for r_i to be informative about ω) does not rely on any details of the environment and, in fact, holds whenever $u^p(a,\omega)$ is bounded. Recall that Lemma 2 implies that reports that convey information about ω may only arise from individuals with types in $\left[-\frac{\tilde{M}}{\sqrt{n}}, \frac{\tilde{M}}{\sqrt{n}}\right]$. The proof of Proposition 4 proceeds by deriving an upper bound on the decision-maker's payoff by ignoring feasibility constraints, and assuming these individuals fully convey information about ω. As before, the relative proportion of such individuals will determine whether information transmission completely fails for finite n (Statement 1), completely fails asymptotically (Statement 2), or is inefficient asymptotically (Statement 3). Finally, if the utility function satisfies increasing differences, I can show that despite the binary reporting technology the decision-maker's payoff still converges to the first-best. The intuition is that the individuals with zero cost can randomize whether to report with a probability that is increasing in ω. Further, the decision-maker then responds to a higher number of reports with a higher action. There is thus a fixed point for the individuals with zero cost: reporting with too high (respectively, low) a probability results in too high (respectively, low) an action.

6 Conclusion

This paper considers a collective action problem among n individuals who commonly observe a state of the world and then choose whether to submit a report to a decision-maker. The decision-maker then chooses an action based on her beliefs about the state from the reports. I assume that there is noise in the reporting such that reports are made with strictly intermediate probability regardless of the state. With this assumption, even with infinitely many individuals with arbitrarily low reporting costs, I show that the decision-maker's payoff need not exceed her payoff absent any information. Finally, I show that a necessary condition for information transmission is the existence of individuals who have precisely zero reporting costs.

Additionally, the impossibility results continue to hold in settings even more general than those analyzed in this paper. Upon maintaining the noise assumption (Assumption 1), I could relax the following assumptions in the analysis and continue to derive information-transmission failures. First, I assumed that the preference shocks were drawn identically across individuals. One could allow asymmetrically distributed shocks that are correlated across individuals. Second, I assumed there was common knowledge of the number of individuals, which could instead be randomly drawn. Finally, I assumed that, ignoring the preference shocks, the individuals share the decision-maker's utility function. One could instead assume individuals had preferences that differ from one another and from the decision-maker's, provided such preferences are bounded.

Therefore, the preceding analysis yields a clear policy implication: organizations facing collective reporting problems subject to noise should either reduce the effective group size or break the symmetry of the decision rule. In contrast, absent noise, the decision-maker's payoff is increasing in group size (Figure 1 shows the payoff is increasing for any $n > 2$), and Battaglini and Palfrey (2025) show that each individual's utility is weakly increasing in the group size because reports increasingly derive from lower-cost individuals. With noise, I have shown that larger groups can perform worse than smaller ones, and may result in a complete breakdown in information transmission. Both policy levers (reducing group size and introducing asymmetry) can be viewed as concentrating the decision-maker's reliance on a smaller set of identifiable individuals, thereby circumventing the information aggregation failure. Below, I discuss practical implementations of such policies in specific settings.

The Volunteer's Dilemma applies naturally to students reporting bullying in hallways or cafeterias. A common institutional response is to appoint a hallway monitor: a designated individual who is publicly tasked with reporting problems. Through the lens of this model, the decision-maker observes the sum of the other reports but also can individually observe the monitor's report. The monitor, aware of this asymmetry, faces a higher probability of being pivotal and thus has a stronger incentive to report, mitigating the bystander effect.

Analogous practices are widespread in organizational settings. Firms routinely establish dedicated compliance or safety units: small teams that are explicitly charged with identifying and escalating problems and whose reports are treated asymmetrically from those of the broader workforce. This effectively reduces the relevant group size to the unit itself and assigns it a privileged role in the decision rule. A related mechanism is the appointment of a devil's advocate: an individual whose designated role is to voice dissent. If the costs of reporting stem in part from deviating from organizational norms, the devil's advocate understands that a report of wrongdoing must originate from them, which sharpens their incentive to report by increasing their perceived pivotality.

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A Appendix A

Proof of Lemma 2. Let $B := \max_{a,\omega} |u^p(a,\omega)|$. As the equilibrium is symmetric, we analyze the incentives of individual 1. Conditional on the state ω, the reports of the other $n-1$ individuals are i.i.d. Define $x_{n,\omega} := \sum_{i=2}^{n} r_i \sim \text{Bin}(n-1, p_\omega)$, where $p_\omega = \Pr(r_i = 1 \mid \omega)$.

Let $U^k = \mathbb{E}(u^p(a,\omega) \mid \sum_{i=1}^{n} r_i = k, \omega)$ be the individual's expected utility when k reports are made, where the expectation is included to allow for a (possibly) random action by the decision-maker. The individual's expected marginal benefit of reporting, $\Delta(\omega)$, is:

$$
\begin{aligned}
|\Delta(\omega)| := {} & \sum_{k=1}^{n} U^k \Pr(x_{n,\omega} = k-1) - \sum_{k=0}^{n-1} U^k \Pr(x_{n,\omega} = k) \\
= {} & U^n \Pr(x_{n,\omega} = n-1) - U^0 \Pr(x_{n,\omega} = 0) + \sum_{k=1}^{n-1} U^k \big(\Pr(x_{n,\omega} = k-1) - \Pr(x_{n,\omega} = k)\big) \\
\leq {} & B \left(\Pr(x_{n,\omega} = n-1) + \Pr(x_{n,\omega} = 0) + \sum_{k=1}^{n-1} \left(\Pr(x_{n,\omega} = k-1) - \Pr(x_{n,\omega} = k) \right) \right),
\end{aligned}
$$

which follows from the definition of B and the triangle inequality. Let k^* be the mode of $x_{n,\omega}$, then $\Pr(x_{n,\omega} = k)$ is increasing in k for $k \leq k^*$ and decreasing if $k \geq k^*$. Therefore:

$$
\begin{aligned}
\sum_{k=1}^{n-1} \left(\Pr(x_{n,\omega} = k-1) - \Pr(x_{n,\omega} = k) \right) &= \sum_{k=1}^{k^*} \big(\Pr(x_{n,\omega} = k) - \Pr(x_{n,\omega} = k-1)\big) \\
&\quad + \sum_{k=k^*+1}^{n-1} \big(\Pr(x_{n,\omega} = k-1) - \Pr(x_{n,\omega} = k)\big) \\
&= 2\Pr(x_{n,\omega} = k^*) - \Pr(x_{n,\omega} = 0) - \Pr(x_{n,\omega} = n-1),
\end{aligned}
$$

as both of these are telescoping series. Substituting this into the previous bound implies:

$$|\Delta(\omega)| \leq 2B \max_k \Pr(x_{n,\omega} = k) \leq \frac{2BC}{\sqrt{(n-1)}} \leq \frac{2\sqrt{2}BC}{\sqrt{n}} := \frac{\tilde{M}}{\sqrt{n}},$$

where C is the constant derived in Proposition C1. $\square$

The following lemma is used to prove the subsequent results.

Lemma A1. *Consider two sequences of binomial random variables:* $\mathrm{Bin}(n, q_n)$ *and* $\mathrm{Bin}(n, p_n)$. *Suppose* $q_n, p_n \in [\lambda, 1-\lambda]$ *for some* $\lambda > 0$ *for all* n. *Then, there exists two constants* $0 < c_1 < c_2$ *for which*

$$c_1 n(p_n - q_n)^2 \leq \mathrm{KL}(\mathrm{Bin}(n, p_n), \mathrm{Bin}(n, q_n)) \leq c_2 n(p_n - q_n)^2, \tag{8}$$

where $\mathrm{KL}(\cdot)$ *denotes the Kullback-Leibler divergence between two distributions.*

Proof of Lemma A1.

$$\mathrm{KL}(\mathrm{Bin}(n, p_n), \mathrm{Bin}(n, q_n)) = n\left(p_n \ln \frac{p_n}{q_n} + (1-p_n) \ln \frac{1-p_n}{1-q_n}\right) := f(p_n, q_n) \tag{9}$$

Therefore, by Taylor's theorem

$$f(p_n, q_n) = f(q_n, q_n) + \frac{\partial f}{\partial p_n}\Big|_{p_n = q_n} (p_n - q_n) + \frac{\partial^2 f}{\partial p_n^2}\Big|_{p_n = \alpha_n q_n + (1-\alpha_n) p_n} \frac{(p_n - q_n)^2}{2}, \tag{10}$$

for some $\alpha_n \in [0, 1]$. Further, observe that $f(q_n, q_n) = 0$ and

$$\frac{\partial f}{\partial p_n}\Big|_{p_n = q_n} = n \ln \frac{p_n(1-q_n)}{q_n(1-p_n)}\Big|_{p_n = q_n} = 0 \tag{11}$$

$$\frac{\partial^2 f}{\partial p_n^2}\Big|_{p_n = \alpha_n q_n + (1-\alpha_n) p_n} = \frac{n}{p_n(1-p_n)}\Big|_{p_n = \alpha_n q_n + (1-\alpha_n) p_n}. \tag{12}$$

Using these calculations,

$$\mathrm{KL}(\mathrm{Bin}(n, p_n), \mathrm{Bin}(n, q_n)) = \frac{n(p_n - q_n)^2}{2(\alpha_n q_n + (1-\alpha_n)p_n)(1 - \alpha_n q_n - (1-\alpha_n)p_n)}. \tag{13}$$

As q_n, p_n are bounded away from 0 and 1, the denominator has a finite upper bound and non-zero lower bound, implying the existence of c_1, c_2, completing the proof. $\square$

Proof of Proposition 3. Statements 1 and 2 follow from Proposition 4, which proves a strict generalization of these statements.

Statement 3: That $\limsup_{n\to\infty} \Pi(n) < \Pi^{FB}$ is shown more generally in Proposition 4. I will now show $\Pi^{SQ} < \liminf_{n\to\infty} \Pi(n)$.

Note that conditional on the state $\omega \in \{0,1\}$, there exist threshold values $\nu^1(n) > 0 > \nu^0(n)$ for which the individuals will report $r_i = 1$ where the superscript indexes the realization of ω. Given these thresholds denote $p_n := \Pr(r_i = 1|\omega = 1)$ and $q_n := \Pr(r_i = 1|\omega = 0)$, where $1 > p_n > q_n > 0$. Given p_n, q_n, and to provide a lower bound on the decision-maker's payoff, I assume the decision-maker utilizes the ex-post efficient decision rule:

$$a = 1 \iff \sum r_i \geq T_n^* \text{ where } T_n^* = \frac{\log \tau + n \log\left(\frac{1-q_n}{1-p_n}\right)}{\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)}, \tag{14}$$

where τ is a constant depending on both the prior and the utility function. I provide the proof for this expression in Appendix C.

Claim 1: It is sufficient to show that there exists some sequence of equilibria where $\liminf_{n\to\infty} \sqrt{n}(p_n - q_n) > 0$. Claim 1 is sufficient as (i) given Claim 1, Lemma A1 implies that the KL-divergence is bounded away from 0 in the limit and (ii) the decision-maker uses the ex-post efficient rule, and thus improves her utility with this information.

Claim 2: There exists some constant c_3 for which $\limsup_{n\to\infty} \sqrt{n}(p_n - q_n) < c_3$. Lemma 2 implies $\frac{-c_4}{\sqrt{n}} < \nu^0(n) < 0 < \nu^1(n) \leq \frac{c_5}{\sqrt{n}}$ for some strictly positive constants c_4, c_5. Using these bounds, and a change of limit with the condition on $F(\cdot)$ in Statement 3 implies this claim.

Claim 3: If $T_n^*/n = q_n + O(\frac{1}{\sqrt{n}})$, then $\liminf_{n\to\infty} \sqrt{n}(p_n - q_n) > 0$. To show Claim 3, first note that if $T_n^*/n = q_n + O(\frac{1}{\sqrt{n}})$, then combining this observation with Claim 2 implies $T_n^*/n = p_n + O(\frac{1}{\sqrt{n}})$. Further, as p_n is bounded away from 0 and 1, this implies that there exists some bounded sequence $\bar{c}_n$ such that $T_n^* = (n-1)p_n + \bar{c}_n\sqrt{(n-1)p_n(1-p_n)}$. We can now write down the probability that an individual is pivotal when $\omega = 1$ as

$$\Pr\left(\text{Bin}(n-1, p_n) = (n-1)p_n + \bar{c}_n\sqrt{(n-1)p_n(1-p_n)}\right). \tag{15}$$

Define $\bar{c} = \sup_n |\bar{c}_n|$, then the probability of being pivotal is greater than

$$\Pr\left(\text{Bin}(n-1, p_n) = (n-1)p_n + \bar{c}\sqrt{(n-1)p_n(1-p_n)}\right) \geq \frac{c_6}{\sqrt{n-1}}, \tag{16}$$

because replacing $\bar{c}_n$ with $\bar{c}$ lowers the probability by the definition of $\bar{c}$ and that the density is decreasing in the distance from the mode for the Binomial distribution. The second inequality holds for some strictly positive constant c_6 that depends only on $\lim_{n\to\infty} p_n = F(0)$ and $\bar{c}$, by the De Moivre-Laplace applied to Binomial distributions.

Therefore, $\nu^1(n) \geq \frac{c_7}{\sqrt{n}}$, for some positive constant $c_7 > 0$, given the lower bound for the pivotal probability above. As $\nu^0(n) < 0$,

$$\liminf_{n\to\infty} \sqrt{n}(p_n - q_n) \geq \liminf_{n\to\infty} \sqrt{n}(F(\frac{c_7}{\sqrt{n}}) - F(0)) = \liminf_{\delta\to 0^+} c_7 \frac{F(\delta) - F(0)}{\delta} > 0,$$

completing the proof of Claim 3.

These claims imply it suffices to show that if $\liminf_{n\to\infty} \sqrt{n}(p_n - q_n) > 0$, then $t_n := T_n^*/n = q_n + O(\frac{1}{\sqrt{n}})$. This is sufficient, as for any T_n^* there exists an equilibrium determined by the p_n, q_n thresholds. Therefore, choosing the T_n^* from Claim 3 results in an equilibrium, and the following claim implies that the resulting p_n, q_n, imply that T_n^* is ex-post optimal.

Let us simplify t_n as follows

$$t_n = \frac{\log\tau}{n\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)} + \frac{\log\left(\frac{1-q_n}{1-p_n}\right)}{\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)}. \tag{17}$$

Further, recall that both p_n and q_n are bounded away from 0 and 1 by Assumption 1. Hence, we can use a first-order Taylor expansion on the following terms in Equation (17),

$$\begin{aligned} \log(1-q_n) - \log(1-p_n) &= \frac{p_n - q_n}{1-q_n} + O((p_n-q_n)^2) \\ \log(\frac{p_n(1-q_n)}{q_n(1-p_n)}) &= \frac{p_n - q_n}{q_n(1-q_n)} + O((p_n-q_n)^2). \end{aligned} \tag{18}$$

Let us first simplify the second term in (17):

$$\frac{\log\left(\frac{1-q_n}{1-p_n}\right)}{\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)} = \frac{\frac{1}{1-q_n} + O(p_n - q_n)}{\frac{1}{q_n(1-q_n)} + O(p_n - q_n)} = q_n + O(p_n - q_n).$$

Let us now simplify the first term in (17):

$$\begin{aligned} \frac{\log\tau}{n\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)} &= \frac{\log\tau}{n(p_n-q_n)} \frac{1}{\frac{1}{q_n(1-q_n)} + O(p_n - q_n)} \\ &= \frac{\log\tau}{n(p_n-q_n)} \big(q_n(1-q_n) + O(p_n-q_n)\big) = \frac{O(1)}{\sqrt{n}} \big(q_n(1-q_n) + O(p_n-q_n)\big), \end{aligned}$$

where the first equality follows from Equation (18), the second comes from q_n being bounded away from 0 and 1, and the final equality comes from (i) $0 < \liminf_{n\to\infty} \sqrt{n}(p_n - q_n) <$

$\limsup_{n\to\infty} \sqrt{n}(p_n - q_n)$ and (ii) τ being strictly positive. Finally, we can further simplify

$$\frac{O(1)}{\sqrt{n}}\big(q_n(1-q_n) + O(p_n - q_n)\big) = \frac{O(1)}{\sqrt{n}} + \frac{O(p_n - q_n)}{\sqrt{n}} = O(\frac{1}{\sqrt{n}}), \tag{19}$$

where these inequalities come from q_n being bounded away from 0 and 1 and that $p_n - q_n$ converges to zero. Therefore,

$$t_n = q_n + O(p_n - q_n) + O(\frac{1}{\sqrt{n}}) = q_n + O(\frac{1}{\sqrt{n}}), \tag{20}$$

as $0 < \liminf_{n\to\infty} \sqrt{n}(p_n - q_n) < \limsup_{n\to\infty} \sqrt{n}(p_n - q_n)$.

Statement 4. As before, let us have the decision-maker employ the ex-post efficient decision rule. That $\nu^0(n) < 0 < \nu^1(n)$ implies that $\limsup_{n\to\infty} \sqrt{n}(p_n - q_n) = \infty$ as there exists a mass point at 0. Therefore, Lemma A1 implies that the decision-maker can perfectly infer the state as $n \to \infty$. Finally, that she utilizes the ex-post efficient decision rule implies she obtains her first-best payoff in the limit. □

Proof of Proposition 4. Statement 1: Pick n^* such that $\frac{\tilde{M}}{\sqrt{n^*}} < \delta$.

Statement 2: By Lemma 2, the probability that r_i contains information about ω is bounded above by $F(\frac{\tilde{M}}{\sqrt{n}}) - F(\frac{-\tilde{M}}{\sqrt{n}})$. As a further bound, let us assume that with this probability, the individual's information always was perfectly revealing about ω. For instance, let Ω_1, Ω_2 be some partition of Ω and that if $\nu_i \in (-\frac{\tilde{M}}{\sqrt{n}}, \frac{\tilde{M}}{\sqrt{n}})$ then $r_i = 1$ if and only if $\omega \in \Omega_1$. Therefore,

$$\Pr(r_i = 1|\omega) = F(-\frac{\tilde{M}}{\sqrt{n}}) + (F(\frac{\tilde{M}}{\sqrt{n}}) - F(-\frac{\tilde{M}}{\sqrt{n}}))\mathbf{1}_{\omega\in\Omega_1}. \tag{21}$$

Assumption 1 implies the existence of a $\lambda > 0$ needed to apply Lemma A1. Lemma A1 implies that the KL divergence in the distribution of $\sum r_i$ when $\omega \in \Omega_1$ as opposed to $\omega \in \Omega_2$ is bounded above by

$$c_2 n\Big(F(\frac{\tilde{M}}{\sqrt{n}}) - F(-\frac{\tilde{M}}{\sqrt{n}})\Big)^2. \tag{22}$$

Further, this expression converges to zero as $n \to \infty$ whenever $F'(0) = 0$. Therefore, an upper-bound on $\Pi(n)$ converges to Π^{SQ}, implying $\lim_{n\to\infty} \Pi(n) = \Pi^{SQ}$, proving Statement 2.

Statement 3: When the condition in Statement 3 holds, the upper-bound on the KL divergence is bounded, implying the decision-maker cannot hold arbitrarily precise beliefs about ω, and thus she cannot obtain her first-best payoff. That the decision-maker can obtain a payoff higher than her status quo follows from Statement 3 in Proposition 3 which shows this result for the binary Volunteer's Dilemma. This is because one can simply partition the state space into two sets Ω_1, Ω_2 and have two actions a_1, a_2 corresponding to the optimal action when ω is restricted to Ω_1, Ω_2. Further, the proof from Statement 3 in Proposition 3 implies that the decision-maker who uses an ex-post efficient decision rule for deciding between a_1 and a_2 will receive a higher payoff than her status quo payoff.

Statement 4: Write $\alpha := F(0) - F(0^-) > 0$. As $\Pi(n) \leq \Pi^{FB}$ trivially, it suffices to show $\liminf_{n\to\infty} \Pi(n) \geq \Pi^{FB}$. Fix $\eta > 0$ and pick $a_1 < \cdots < a_K$ in A with $\mathbb{E}[\max_j u^p(a_j, \omega)] \geq \Pi^{FB} - \eta$ (possible by compactness and continuity of u^p). Define thresholds $T_j := \lfloor n(F(0^-) + \alpha j/K) \rfloor$ for $j = 1, \ldots, K-1$, $T_0 := 0$, $T_K := n+1$, and the nondecreasing decision rule $d_n(k) = a_j$ for $k \in I_j := [T_{j-1}, T_j)$.

Consider symmetric strategies where $\nu_i < 0$ types always report, $\nu_i > 0$ types never report, and zero-cost types report with probability $\gamma(\omega) \in [0,1]$, giving $p_n(\omega) = F(0^-) + \alpha \cdot \gamma(\omega)$. The marginal benefit of reporting for a zero-cost type is

$$\Delta(\omega; p) := \sum_{j=1}^{K-1} \Big[u^p(a_{j+1}, \omega) - u^p(a_j, \omega)\Big] \cdot \Pr\Big(\mathrm{Bin}(n-1, p) = T_j - 1\Big), \tag{23}$$

which is continuous in p and, by strictly increasing differences, strictly increasing in ω for each p. Let $j^*(\omega) := \arg\max_j u^p(a_j, \omega)$.

For $j^* \in \{2, \ldots, K-1\}$, define $\underline{p}_n := T_{j^*-1}/(n-1)$ and $\bar{p}_n := T_{j^*}/(n-1)$; for large n both lie in $(F(0^-), F(0))$. I show $\Delta(\omega; \underline{p}_n) > 0 > \Delta(\omega; \bar{p}_n)$, after which continuity and the Intermediate Value Theorem yield $p_n^*(\omega) \in (\underline{p}_n, \bar{p}_n)$ with $\Delta(\omega; p_n^*(\omega)) = 0$.

At $p = \underline{p}_n$, the mean of $X := \mathrm{Bin}(n-1, \underline{p}_n)$ is equal to T_{j^*-1}. Therefore as $n \to \infty$, the only actions that are chosen with positive probability are a_{j^*-1} and a_{j^*}, as the intervals between thresholds grows linearly with n, but the Central Limit Theorem implies the standard deviation of reporting grows with $\sqrt{n}$. However by increasing differences at ω, the individuals prefer $a_{j^*} > a_{j^*-1}$ implying $\Delta(\omega; \underline{p}_n) > 0$. Further, a symmetric argument implies $\Delta(\omega; \bar{p}_n) < 0$. Finally, for $j^* = 1$ (resp. $j^* = K$), $u^p(a_{j+1}, \omega) - u^p(a_j, \omega) \leq 0$ for all j (resp. ≥ 0), so $\Delta(\omega; p) \leq 0$ (resp. ≥ 0) for all p and $\gamma^*(\omega) = 0$ (resp. 1) is directly optimal.

Therefore, setting $\gamma^*(\omega) := (p_n^*(\omega) - F(0^-))/\alpha \in [0,1]$ constitutes an equilibrium: zero-cost types are indifferent by construction, while $\nu_i \neq 0$ types play a strict best response, implying this is an equilibrium. Finally, note that for each ω as $n \to \infty$, $j^*(\omega)$ is chosen with probability one as (i) the intervals between thresholds grows linearly with n (ii) the Central Limit Theorem implies the standard deviation of reporting grows with $\sqrt{n}$ and (iii) $np_n(\omega) \in I_{j^*(\omega)}$. $\square$

B Appendix B: Microfoundations

All proofs of information-transmission results utilize Lemma 1, which is shown by invoking Assumption 1. Assumption 1 holds under two interpretations: (i) there are some individuals who simply enjoy making reports or (ii) Quantal Response Equilibria, which is a behavioral microfoundation for mistakes or noise in decision-making. Below, I discuss additional micro-foundations which will imply Assumption 1: behavioral types and noisy communication. After this, I discuss an additional microfoundation which does not imply Assumption 1, but nonetheless may result in information aggregation failures for large groups: biased individuals. Below, I formally define these microfoundations.

B.1 Behavioral Types:

In addition to the rational types described in the model, there exist two behavioral types who put no weight on $u^p(a, \omega)$, and each type either always reports one or zero. Define by ϵ_1 the probability an individual is a behavioral type who always report 1 and ϵ_0 the probability of a behavioral type who always report 0. Therefore, with probability $1-\epsilon_1-\epsilon_0$, an individual is a rational type as described in the model. Lemma 1 holds under this microfoundation upon taking $\epsilon = \min\{\epsilon_1, \epsilon_0\} > 0$.

B.2 Noisy Communication:

As motivated by Boudreau et al. (2023), consider the following adapted model. The agent's utility and decision-maker's utility remain as in the main model. However, now the agent chooses $\tilde{r}_i \in \{0, 1\}$ and receives utility $u^p(a, \omega) - \nu_i \tilde{r}_i$. The decision-maker observes $\sum r_i$, where $r_i = \tilde{r}_i$ with probability $p \in (1/2, 1)$ and the decision-maker can commit to any decision rule as a function of $\sum r_i$. If p were equal to 1 then this would coincide with the main model where an individual's report is transmitted without noise. In contrast, if p were equal to 1/2 then the decision-maker observes reports that are independent of what an agent reports. One can now calculate

$$1 - p = \Pr(r_i = 1|\tilde{r}_i = 0) \leq \Pr(r_i = 1) \leq \Pr(r_i = 1|\tilde{r}_i = 1) = p \tag{24}$$

$$1 - p = \Pr(r_i = 0|\tilde{r}_i = 1) \leq \Pr(r_i = 0) \leq \Pr(r_i = 0|\tilde{r}_i = 0) = p, \tag{25}$$

where these inequalities hold independent of ω. Therefore, defining $\epsilon = 1 - p > 0$, Lemma 1 continues to hold with noisy communication as defined above.

Finally, the simplest interpretation of p is an intentional garbling of an agent's report.

In more applied contexts, one could also view p as possible misinterpretations deriving from language barriers, communication barriers, or miscommunication. While formal modeling of such miscommunication is sparse (for the reasons described in Lipman, 2025), this could serve as an informal microfoundation.

B.3 Biased Individuals

In this subsection, I specialize to the case of binary decisions as in the case of the Volunteer's Dilemma with a binary state. I assume that in addition to the rational types described in the model there exist upward-biased individuals who aim to maximize a and downward-biased individuals who aim to maximize $-a$. I assume an individual is upwards biased with probability $p_u \in (0, 1)$, downward biased with probability $p_d \in (0, 1-p_u)$ and shares the same preferences as the decision-maker (henceforth, unbiased) with probability $1 - p_u - p_d > 0$. I begin with a result in the case of monotone decision rules: defined as a decision rule where the action is monotone in the number of reports.

Proposition B1. *If $p_u, p_d > 0$, then for any monotone decision rule there exists an $\epsilon > 0$ such that in any equilibrium where the individuals do not play weakly dominated actions $\Pr(r_i = 1|\omega) \in [\epsilon, 1 - \epsilon]$.*

Proof of Proposition B1. Without loss, consider a weakly increasing decision rule, where a symmetric argument holds for decreasing decision rules. With a weakly increasing decision rule, the downward biased agent has a weakly dominant strategy to select $r_i = 0$ and the upward biased agent to choose $r_i = 1$. Therefore, setting $\epsilon = \min\{p_u, p_d\}$ completes the proof. □

Given this result, Proposition 3 in the main text continues to hold when $\Pi(n)$ is redefined as the maximum of all monotone decision rules and among all equilibria where the individuals do not play weakly dominated actions. I now show that a very natural class of decision rules happens to be monotone. I define an ex-post efficient decision rule as one where the decision-maker cannot commit to a decision rule, but rather the action must be optimal given her beliefs from the reports.

Remark 1. *Any ex-post efficient decision rule is monotone.*

Proof of Remark 1. Let q^1, q^0 denote the reporting probabilities when the state is high and low, respectively. If $q^1 = q^0$, then the decision-maker's beliefs will be independent of the number of messages received. Therefore, the decision-maker's optimal action is her prior action following any number of reports, which is monotone. If $q^1 \neq q^0$, then consider when the decision-maker observes the total number of reports $k = \sum r_i$ and updates her beliefs. Because $q^1 \neq q^0$, the binomial distributions $\text{Bin}(n, q^1)$ and $\text{Bin}(n, q^0)$ satisfy the Monotone Likelihood Ratio Property. The likelihood ratio is:

$$\Lambda(k) = \frac{\Pr(k \mid \omega = 1)}{\Pr(k \mid \omega = 0)} = \left(\frac{q^1}{q^0}\right)^k \left(\frac{1 - q^1}{1 - q^0}\right)^{n-k}$$

Because $\frac{q^1(1-q^0)}{q^0(1-q^1)} \neq 1$, $\Lambda(k)$ is strictly monotone in k. This implies her decision rule is monotone in the reports. □

Finally, I show that even if one allows the decision-maker to use non-monotone decision rules, if one assumes that the biased agents break indifference in a specific manner that the probability of a report remains intermediate.

Proposition B2. *Suppose $p_u, p_d > 0$ and that if the upward biased (respectively, downward biased) is indifferent between reporting and not, they chose to report (respectively, not report). For any decision rule there exists an $\epsilon > 0$ such that in any equilibrium* $\Pr(r_i = 1|\omega) \in [\epsilon, 1 - \epsilon]$.

Proof of Proposition B2. For any decision rule, denote by

$$\Delta_n(\omega) = \sum_{k=0}^{n-1} \Pr(X = k \mid \omega)\mathbb{E}(a(k+1) - a(k)), \tag{26}$$

which captures whether another report increases or decreases the action in expectation.

If $\Delta_n(\omega) \neq 0$, then the upward biased agents choose $r_i = \mathbf{1}_{\Delta_n(\omega)>0}$ and the downward biased agents choose $r_i = \mathbf{1}_{\Delta_n(\omega)<0}$. In contrast, if $\Delta_n(\omega) = 0$, then the tie-breaking rule imposed implies that $r_i = 1$ for the upward biased agents and $r_i = 0$ for the downward biased agents. Therefore, in either case, one can set $\epsilon = \min\{p_u, p_d\}$. □

Therefore, even when restricting to non-monotone decision rules any information aggregation must rely on the biased agents breaking ties in a way that benefits the decision-maker. For instance, a mechanism whereby $a = 1 \iff \sum r_i \geq T > 1$ has the following equilibrium: when $\omega = 0$, no individual reports, and when $\omega = 1$ the upward biased individuals always report. This is an equilibrium, as when $\omega = 0$, no individual has an incentive to report. However, this equilibrium relies on (i) the upward biased individual using a weakly dominated strategy and (ii) for their equilibrium strategy to depend on the realization of ω, which is not payoff relevant for the biased types.

C Mathematical Appendix

Proposition C1. *Let $X \sim Bin(n, p)$ where $p \in [\epsilon, 1-\epsilon]$ for a fixed $\epsilon \in (0, 1/2]$. There exists a constant $C_\epsilon > 0$, dependent only on ϵ, such that* $\sup_k \Pr(X = k) \leq C_\epsilon/\sqrt{n}$ *for all $n \geq 1$.*

Proof of Proposition C1. Let F_n be the cumulative distribution function of the sum of n independent draws of a variable X, and let Φ be the standard normal cumulative distribution function. In this case X is bernoulli with variance $\sigma^2 = p(1-p)$ and third absolute central moment $\rho = p(1-p)[p^2 + (1-p)^2]$. By the Berry-Esseen theorem, there exists an absolute constant A such that

$$\sup_{x \in \mathbb{R}} \left| F_n(x) - \Phi\left(\frac{x - np}{\sigma\sqrt{n}}\right) \right| \leq \frac{A\rho}{\sigma^3\sqrt{n}}. \tag{27}$$

Next, $\Pr(X = k) = F_n(k) - F_n(k-1)$. Substituting the Berry-Esseen uniform bound gives

$$\begin{aligned} \Pr(X = k) &\leq \Phi\left(\frac{k - np}{\sigma\sqrt{n}}\right) - \Phi\left(\frac{k - 1 - np}{\sigma\sqrt{n}}\right) + 2\frac{A\rho}{\sigma^3\sqrt{n}} \\ &\leq \sup_x \Phi'(x) \cdot \frac{1}{\sqrt{n}\sigma} + 2\frac{A\rho}{\sigma^3\sqrt{n}} = \frac{1}{\sqrt{2\pi\sigma^2 n}} + \frac{2A\rho}{\sigma^3\sqrt{n}}, \end{aligned} \tag{28}$$

where the second inequality follows from the Mean Value Theorem and the final equality follows from the maximum of the normal pdf being $1/\sqrt{2\pi}$. Finally, using the values for σ, ρ we derive

$$\Pr(X = k) \leq \frac{1}{\sqrt{2\pi\sigma^2 n}} + \frac{2A\rho}{\sigma^3\sqrt{n}} = \frac{1}{\sqrt{n p(1-p)}}\left(\frac{1}{\sqrt{2\pi}} + 2A(p^2 + (1-p)^2)\right) \tag{29}$$

On the domain $p \in [\epsilon, 1-\epsilon]$, the quadratic $p(1-p)$ is concave and minimized at the boundaries, providing the uniform lower bound $p(1-p) \geq \epsilon(1-\epsilon)$. Furthermore, the convex function $p^2 + (1-p)^2$ is maximized at the boundaries, providing the uniform upper bound $p^2 + (1-p)^2 \leq \epsilon^2 + (1-\epsilon)^2$. Applying these bounds implies

$$\Pr(X = k) \leq \frac{1}{\sqrt{n\epsilon(1-\epsilon)}}\left(\frac{1}{\sqrt{2\pi}} + 2A(\epsilon^2 + (1-\epsilon)^2)\right) \tag{30}$$

which completes the proof as A is a universal constant. □

Proposition C2. *Fix $n \in \mathbf{N}$ and parameters $0 < q_n < p_n < 1$. Let $X \in \{0, 1, \ldots, n\}$ be the observed number of successes. Consider testing*

$$H_0: \ X \sim \text{Bin}(n, q_n) \qquad vs \qquad H_1: \ X \sim \text{Bin}(n, p_n).$$

For any constant $\tau > 0$ (independent of n, p_n, q_n), the likelihood-ratio test that rejects H_0 when $\Lambda(X) \geq \tau$ has an upper-tail *rejection region defined by $x \geq T_n^*$ as defined below.*

$$\{\, X \geq T_n^* \,\}, \qquad T_n^* \;=\; \frac{\log \tau + n \log\left(\frac{1-q_n}{1-p_n}\right)}{\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)}.$$

Proof of Proposition C2. The binomial likelihoods at k are

$$L_1(k) = \binom{n}{k} p_n^k (1-p_n)^{n-k}, \qquad L_0(k) = \binom{n}{k} q_n^k (1-q_n)^{n-k}.$$

The likelihood ratio is

$$\Lambda(k) := \frac{L_1(k)}{L_0(k)} = \left(\frac{p_n}{q_n}\right)^k \left(\frac{1-p_n}{1-q_n}\right)^{n-k}.$$

Taking logs,

$$\log \Lambda(k) = k\left[\log\left(\frac{p_n}{q_n}\right) - \log\left(\frac{1-p_n}{1-q_n}\right)\right] + n \log\left(\frac{1-p_n}{1-q_n}\right).$$

Since $p_n > q_n$, we have

$$\frac{p_n(1-q_n)}{q_n(1-p_n)} \;=\; \frac{p_n}{q_n} \cdot \frac{1-q_n}{1-p_n} \;>\; 1,$$

hence

$$\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right) = \log\left(\frac{p_n}{q_n}\right) - \log\left(\frac{1-p_n}{1-q_n}\right) > \; 0.$$

Therefore, $\log \Lambda(k) \geq \log \tau$ if and only if

$$k \cdot \log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right) \geq \; \log \tau + n \log\left(\frac{1-q_n}{1-p_n}\right).$$

Because the left-hand coefficient is positive, this is equivalent to

$$k \;\geq\; \frac{\log \tau + n \log\left(\frac{1-q_n}{1-p_n}\right)}{\log\left(\frac{p_n(1-q_n)}{q_n(1-p_n)}\right)} \;=\; T_n^*.$$

□

D Extensions

Proposition D1 (State-Dependent Reporting Costs)**.** *Assume that the reporting costs ν_i are drawn from a distribution $F_\omega(\cdot)$ which depends on the realized state $\omega \in \{0, 1\}$, and both F_0 and F_1 satisfy Assumption 1. If $F_1(0) \neq F_0(0)$, then $\lim_{n\to\infty} \Pi(n) = \Pi^{FB}$. Therefore, suppose $F_1(0) = F_0(0)$.*

1. *If there exists a $\delta > 0$ such that $F_\omega(\delta) - F_\omega(-\delta) = 0$ for $\omega \in \{0, 1\}$, then there exists*

$n^* \in \mathbf{N}$ such that for all $n > n^*$, $\Pi(n) = \Pi^{SQ}$.

2. *If $F_1'(0) = F_0'(0) = 0$, then $\lim\limits_{n\to\infty} \Pi(n) = \Pi^{SQ}$.*

3. *If $F_1'(0) > 0$ and $F_0'(0) > 0$, then $\Pi^{SQ} < \liminf\limits_{n\to\infty} \Pi(n) \le \limsup\limits_{n\to\infty} \Pi(n) < \Pi^{FB}$.*

Proof of Proposition D1. By Lemma 2, there exists a constant $\tilde{M}$ such that the rational reporting thresholds satisfy $\nu^\omega(n) \in \left[-\frac{\tilde{M}}{\sqrt{n}}, \frac{\tilde{M}}{\sqrt{n}}\right]$ for $\omega \in \{0,1\}$. Let $p_n := \Pr(r_i = 1 \mid \omega = 1) = F_1(\nu^1(n))$ and $q_n := \Pr(r_i = 1 \mid \omega = 0) = F_0(\nu^0(n))$.

Let us begin by showing if $F_1(0) \neq F_0(0)$ then $\Pi(n) \to \Pi^{FB}$. Since $\nu^\omega(n) \to 0$ as $n \to \infty$ and F_ω is continuous, $\lim_{n\to\infty} p_n = F_1(0)$ and $\lim_{n\to\infty} q_n = F_0(0)$. If $F_1(0) \neq F_0(0)$, the difference $p_n - q_n$ converges to a non-zero constant. By Lemma A1, the KL-divergence between $\mathrm{Bin}(n, p_n)$ and $\mathrm{Bin}(n, q_n)$ scales with $n(p_n - q_n)^2$, which diverges to infinity. Consequently, the decision-maker can perfectly infer the state asymptotically and obtain her first-best payoff, $\lim_{n\to\infty} \Pi(n) = \Pi^{FB}$.

Statement 1 : Assume $F_1(0) = F_0(0)$. For Statement 1, if there exists a flat region around zero, pick n^* such that $\frac{\tilde{M}}{\sqrt{n^*}} < \delta$. For all $n > n^*$, $p_n = F_1(0)$ and $q_n = F_0(0)$. Since $p_n = q_n$, the reports convey no information about ω, implying $\Pi(n) = \Pi^{SQ}$.

Statement 2: By Lemma 2, we know that $|\nu^\omega(n)| \le \frac{\tilde{M}}{\sqrt{n}}$ for $\omega \in \{0,1\}$. We want to show that $\lim\limits_{n\to\infty} n(p_n - q_n)^2 = 0$. By the triangle inequality:

$$\begin{aligned} |p_n - q_n| = |F_1(\nu^1(n)) - F_0(\nu^0(n))| &= |F_1(\nu^1(n)) - F_1(0) + F_0(0) - F_0(\nu^0(n))| \\ &\le |F_1(\nu^1(n)) - F_1(0)| + |F_0(\nu^0(n)) - F_0(0)|. \end{aligned} \tag{31}$$

Multiplying both sides by $\sqrt{n}$ gives:

$$\sqrt{n}|p_n - q_n| \le \sqrt{n}|\nu^1(n)| \quad \frac{F_1(\nu^1(n)) - F_1(0)}{\nu^1(n)} \quad + \sqrt{n}|\nu^0(n)| \quad \frac{F_0(\nu^0(n)) - F_0(0)}{\nu^0(n)} \quad , \tag{32}$$

where we adopt the convention that the ratio is zero if $\nu^\omega(n) = 0$.

Because $|\nu^\omega(n)| \le \frac{\tilde{M}}{\sqrt{n}}$, we have the strict upper bound $\sqrt{n}|\nu^\omega(n)| \le \tilde{M}$, yielding:

$$\sqrt{n}|p_n - q_n| \le \tilde{M} \quad \frac{F_1(\nu^1(n)) - F_1(0)}{\nu^1(n)} \quad + \tilde{M} \quad \frac{F_0(\nu^0(n)) - F_0(0)}{\nu^0(n)} \quad . \tag{33}$$

However, by the definition of a derivative and that $\nu^1(n), \nu^0(n) \to 0$, the limit of the above expression as $n \to \infty$ is equal to $\tilde{M}(F_1'(0) + F_0'(0)) = 0$. Finally, by Lemma A1, the Kullback-Leibler divergence between the two reporting distributions converges to zero. Consequently, the decision-maker learns nothing from the reports asymptotically, resulting in $\lim_{n\to\infty} \Pi(n) = \Pi^{SQ}$.

Statement 3: Using Equation (33) and the conditions on the derivative in Statement 3, one can see that the Kullback-Leibler divergence between the two reporting distributions is bounded above by $\tilde{M}^2(F_1'(0) + F_0'(0))^2$, which is finite. Therefore, the decision-maker can never perfectly infer the states from the reports, implying her payoff is below the first-best payoff.

Next, we show $\Pi^{SQ} < \liminf_{n\to\infty} \Pi(n)$. Suppose the decision-maker uses the ex-post efficient decision rule T_n^*. As established in Claim 3 of Proposition 3, this ensures the probability of an individual being pivotal is bounded below, which implies $\nu^1(n) \geq \frac{\tilde{c}_2}{\sqrt{n}}$ for some constant $\tilde{c}_2 > 0$. Additionally, as before, $\nu^0(n) < 0$. Now:

$$\begin{aligned} \sqrt{n}(p_n - q_n) &= \sqrt{n}\big(F_1(\nu^1(n)) - F_1(0)\big) - \sqrt{n}\big(F_0(\nu^0(n)) - F_0(0)\big) \\ &= \sqrt{n}\nu^1(n)\frac{F_1(\nu^1(n)) - F_1(0)}{\nu^1(n)} + \sqrt{n}(-\nu^0(n))\frac{F_0(\nu^0(n)) - F_0(0)}{\nu^0(n)}. \end{aligned} \tag{34}$$

Because $\nu^0(n) < 0$ and F_0 is strictly increasing around zero (implied by $F_0'(0) > 0$), the ratio for the second term is positive, making the entire second term non-negative. Therefore, we can strictly lower-bound the sum by the first term alone:

$$\sqrt{n}(p_n - q_n) \geq \sqrt{n}\nu^1(n)\frac{F_1(\nu^1(n)) - F_1(0)}{\nu^1(n)}. \tag{35}$$

Substituting the bound $\sqrt{n}\nu^1(n) \geq \tilde{c}_2$, we get:

$$\sqrt{n}(p_n - q_n) \geq \tilde{c}_2\frac{F_1(\nu^1(n)) - F_1(0)}{\nu^1(n)}. \tag{36}$$

Taking the limit infimum as $n \to \infty$:

$$\liminf_{n\to\infty} \sqrt{n}(p_n - q_n) \geq \tilde{c}_2 F_1'(0) > 0. \tag{37}$$

Thus, $\liminf_{n\to\infty} n(p_n - q_n)^2 > 0$. By Lemma A1, the KL-divergence is strictly bounded away from zero. Because the decision-maker is using the efficient rule, she successfully extracts this information to strictly exceed her status-quo payoff, establishing $\Pi^{SQ} < \liminf_{n\to\infty} \Pi(n)$. $\square$