A Currency Premium Puzzle*

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Abstract

We show that quantitative asset pricing models, built to address the equity premium and risk-free rate puzzles, systematically fail when applied to open economies: they cannot generate the large and persistent interest rate differentials we observe between risky and safe currencies. This failure stems from the key mechanism that produces their success in matching both closed-economy puzzles: In these models, the mean of the stochastic discount factor offsets its variance nearly one-for-one, which immobilizes interest rates — so that differences in expected currency returns across countries must arise almost exclusively from predictable changes in exchange rates, at odds with the data. We prove this result in a broad class of models that allows for market incompleteness and exchange rate disconnect, requiring only that each country's risk-free asset is priced by investors who demand compensation for risk in line with observed risk premia. We argue this tension between canonical asset pricing and international macroeconomic models is a key reason researchers have struggled to reconcile the observed behavior of exchange rates, interest rates, and capital flows across countries.

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Interest in the role of risk premia in shaping international capital markets has surged in recent years. Numerous studies have examined the origins of risk premia and investigated their impact on exchange rates, interest rates, capital flows, and financial stability. This line of research addresses critical phenomena such as the violation of uncovered interest parity (Salomao and Varela, 2022; di Giovanni et al., 2022; Kalemli-Özcan and Varela, 2021), contagion (Forbes, 2013), the global financial cycle (Miranda-Agrippino and Rey, 2020; Morelli, Ottonello, and Perez, 2022; Boehm and Kroner, 2026; Bai et al., 2025; Oskolkov, 2024), the sustainability of trade deficits (Gourinchas and Rey, 2007; Atkeson, Heathcote, and Perri, 2025), and events such as flight to safety, capital retrenchments (Forbes and Warnock, 2021; Kekre and Lenel, 2024b), and sudden stops (Mendoza, 2010), which are of paramount concern for policymakers.

Despite the intense interest in these topics, constructing quantitative models that reconcile the observed behavior of exchange rates with large and persistent differences in interest rates across countries has proven difficult. For example, to our knowledge, no dynamic quantitative model has been able to reproduce the fact that New Zealand has, for multiple decades, had a risk-free interest rate and currency returns five percentage points above that of Japan – or that the cost of capital in Thailand is on average several percentage points lower than that in Mexico. The lack of a unifying model that reconciles unpredictable exchange rates with large and persistent interest rate differentials between risky and safe currencies is a major impediment to understanding the effect of risk, uncertainty, and market turmoil on the allocation of capital across countries.

In this paper, we show these challenges are rooted in a fundamental tension between canonical asset pricing models, which have been highly successful in addressing closed-economy asset pricing puzzles (Bansal and Yaron, 2004; Campbell and Cochrane, 1999), and the empirically observed behavior of interest rates: strong forces in these models require that any large differences in risk premia across countries manifest themselves not as differences in interest rates, but as highly predictable exchange rates. In the data, however, we see exchange rates that are notoriously hard to predict (Meese and Rogoff, 1983) as well as large and persistent differences in interest rates across countries (Hassan and Mano, 2019). We term this tension the "currency premium puzzle" and argue it is a fundamental reason the literature has struggled to construct a dynamic quantitative model that can match the behavior of exchange rates, interest rates, and macroeconomic quantities across countries. The core question is: can heterogeneity in countries' risk characteristics generate quantitatively large and persistent differences in their interest rates?[1]

[1]In standard macroeconomic models, interest rates also differ because of heterogeneity in time preferences or consumption growth. Such differences are reversed by predictable depreciations of the high-interest-rate currency — uncovered interest parity holds with respect to them — so they cannot generate the currency returns we see in the data. We examine them, and their possible interaction with risk, in Section 6.

The classical quantitative challenge in closed-economy asset pricing is to reconcile a high equity premium (Mehra and Prescott, 1985) with a low and stable risk-free rate (Weil, 1989). The former requires the stochastic discount factor (SDF) to be highly volatile (Hansen and Jagannathan, 1991), whereas the latter requires the sum of the mean and variance of the logarithm of the SDF (log SDF) to be low and stable. The preference structures used in canonical long-run risk and habit models jointly resolve these puzzles by introducing an offsetting functional relationship between the mean and the variance of the log SDF, such that any increase in variance is nearly or completely offset by a corresponding decrease in the mean.

We argue that this key feature of recursive preferences used in long-run risk and habit models is also the fundamental reason why they struggle in an open economy setting: If anything that raises the variance of the log SDF lowers its mean by nearly the same amount, interest rates are effectively immobilized, and almost fully determined by preference parameters. As a result, these models imply that variation in risk exposure across countries cannot result in quantitatively meaningful differences in interest rates across countries.

Instead, quantitative models that use these preferences tend to generate large amounts of exchange rate predictability to match the wide variation in currency returns, and thus violations of uncovered interest parity, we see in the data. We show this tension is inherent in the preference formulation itself, and therefore carries over to a broad class of models with complete and incomplete asset markets.

To illustrate this tension, consider an economy with two representative agents (one in the home country and one in the foreign country), identical preferences, and log-normal shocks. If there is no arbitrage and households in each country can buy their own country's risk-free asset – but not necessarily any other assets – we get the textbook result that each country's risk-free rate is pinned down by the mean and variance of its log SDF.

Canonical Epstein and Zin (1989) and habit preferences imply the mean of the log SDF is a linear function of half its variance, with a slope FXS that is entirely determined by preference parameters. Calibrations that resolve the equity premium and risk-free rate puzzles require FXS close to one. Combining the no-arbitrage condition with this preference-implied relation yields

$$r^{\star} - r = (1 - FXS)\,\frac{1}{2}(\mathrm{var}(m) - \mathrm{var}(m^{\star})), \tag{1}$$

where r and m denote the risk-free rate and log SDF, respectively. The key issue is that, when FXS is close to one, even large cross-country differences in risk translate into interest rate differentials that are far too small relative to the data. This tension arises directly from the preference structures in canonical long-run risk and habit models, and thus exists regardless of the degree of market incompleteness or the details of the economic environment, as long as risk-free rates are priced by a marginal investor's Euler equation who demands meaningful compensation for risk — conditions we make precise below.

We first outline this tension in the context of complete-markets models with symmetric preferences, which represent the vast majority of the existing quantitative work. If markets are complete, the difference between two countries' expected log SDFs equals the predicted depreciation of one currency relative to the other, while the expected currency return (borrowing in the home country and lending in the foreign country) is proportional to the difference in their variances – the second term in the expression above (Backus, Foresi, and Telmer, 2001). It is then easy to show that FXS is the share of the expected currency return coming from predicted appreciation of the exchange rate, whereas $(1 - FXS)$ is the share of the predicted currency return coming from the interest rate differential.

The data suggest FXS is zero or negative (exchange rates are unpredictable). By contrast, canonical long-run risk and habit models typically feature $FXS \approx 1$, implying that the lion's share of currency returns should arise from predicted appreciations.

For example, while not built to address the currency premium puzzle, the influential two-country long-run risk model by Bansal and Shaliastovich (2013) implies $FXS = 0.96$. If we augment their model to allow the two countries to differ in their risk characteristics, so that one country's currency is riskier than the other, then 96% of any difference in expected currency returns between the two countries manifests as a predictable appreciation of the exchange rate, with only the remaining 4% coming from interest rate differentials. Notably, this 96:4 split is fully determined by the parameters of the Epstein-Zin (EZ) utility function.

This finding is all the more problematic since leading theories of why risk premia and interest rates might differ persistently across countries typically point to differences in the economic environment, such as differences in country size (Martin, 2011; Hassan, 2013), their role in global trade (Richmond, 2019), resource endowments (Ready, Roussanov, and Ward, 2017b; Oskolkov and Perez, 2026), their level of indebtedness (Wiriadinata, 2021), fiscal conditions (Jiang, 2021), or the volatility of shocks (Menkhoff et al., 2012; Colacito et al., 2018a). Canonical long-run risk and habit models prevent all of these drivers of currency risk from generating quantitatively meaningful interest rate differentials. Instead, if currencies differ in their risk characteristics and the allocation is efficient, differences in currency returns must manifest as large predictable changes in exchange rates.

In other words, we demonstrate formally that currency risk premia arising in canonical long-run risk and habit models cannot fit the international data, even if they were modified to allow for heterogeneity in risk exposures across countries. To corroborate our theoretical results, we simulate widely used versions of long-run risk and habit models and show that the currency premium puzzle arises in each of them.

Moreover, our analytical results show stochastic volatility as in Bansal and Yaron (2004), and departures from log-normality do not substantially alleviate the problem. We show the same issue also arises in models that combine disaster risk with Epstein-Zin preferences (Gourio, 2012; Gourio, Siemer, and Verdelhan, 2013) and in other canonical applications of the rare disaster paradigm.

Next, we go beyond the set of existing quantitative models and ask whether incomplete markets might more easily allow large interest rate differentials to emerge between safe and risky currencies. We show the currency premium puzzle persists in a broad class of models satisfying three conditions: (i) a risk-free asset exists in each country and is priced by the Euler equation of some marginal investor; (ii) this investor demands compensation for risk in line with observed risk premia ($\text{var}(m) \geq 0.25$); and (iii) preferences — or the equilibrium interactions among investors — imply an offsetting linear relationship between the mean and the variance of the log SDF, giving rise to (1). Notably, these conditions place no restrictions on market structure: they allow for extreme forms of segmentation, in which exchange rates may be partially or fully disconnected from stochastic discount factors (Gabaix and Maggiori, 2015; Lustig and Verdelhan, 2019; Itskhoki and Mukhin, 2021).

For any model in this class, the dilemma is then governed by the slope of this relationship, FXS: calibrations that match a low and stable risk-free rate in the time series require FXS close to one, which immobilizes interest rate differentials across countries; moving FXS away from one frees the differentials, but makes the risk-free rate counterfactually high or volatile. The puzzle thus hinges neither on the completeness of asset markets nor on any particular preference specification, but on the mechanism by which quantitatively successful models reconcile large risk premia with stable interest rates.

We argue that major quantitative models in macro-finance either belong to this class, and thus face the dilemma, or violate condition (ii) — as do prominent models of exchange rate disconnect, in which the investors pricing bonds are risk-neutral, have constant marginal utility by construction, or demand compensation for risk an order of magnitude below observed premia. In such models, a country's riskiness cannot affect its interest rate — in effect, by construction.

In the paper's final section, we explore how other departures from standard modeling choices could potentially help to address the currency premium puzzle.

One possibility is to introduce systematic differences in time preference rates across countries, where, for example, households in New Zealand might be persistently more impatient than those in Japan. However, we find that such heterogeneity in preferences again has counterfactual implications: Although a country with more impatient households naturally has higher interest rates, any differences in interest rates induced by differences in time preference rates must be reversed by predictable depreciations of the high-interest-rate currency and therefore again require counterfactually high exchange rate predictability.[2] We find the same is true for any other theory of interest rate differentials that relies purely on heterogeneity in the unconditional mean of the log SDFs (rather than their variances).[3]

One possible avenue might be to construct models that combine heterogeneity in countries' risk characteristics with a second source of heterogeneity that results in persistent differences in the means of their log SDFs. We highlight some promising early work in this area.

We conclude that large interest rate differentials and currency risk premia pose a fundamentally different quantitative challenge than the classical closed-economy asset pricing puzzles: heterogeneity in risk cannot move interest rates meaningfully, while heterogeneity in tastes and growth can move interest rates but cannot generate currency returns. We do not claim that there is no solution to the currency premium puzzle. But meeting this challenge will likely require renewed efforts to integrate the study of risk premia with the models of interest rates and exchange rates widely used in international macroeconomics.

Our paper contributes to a large and growing literature at the intersection of international macroeconomics and asset pricing. One important strand of this literature studies the effects of time variation in global and country-specific risk premia, where shocks to global and local risk-bearing capacity motivate variation in global and local borrowing costs, retrenchments, capital flight, and the global financial cycle (Mendoza, 2010; Forbes, 2013; Rey, 2015; Miranda-Agrippino and Rey, 2020; Chari, Dilts Stedman, and Lundblad, 2020; di Giovanni et al., 2022; Forbes and Warnock, 2021; Morelli, Ottonello, and Perez, 2022; Bai et al., 2025).

Several important papers have attempted to microfound such time variation in global risk premia by integrating international macro models with the state-of-the-art asset pricing (e.g., Colacito and Croce, 2011, 2013; Colacito et al., 2018a,b; Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2017; Gourio, Siemer,

[2] Economists are traditionally skeptical of purely preference-based explanations (Stigler and Becker, 1977; Becker, 1976), because, ultimately, any observed economic behavior can be explained by appealing to sufficiently heterogeneous preferences. Therefore, in the absence of direct evidence of such differences in time preference across countries, such an explanation would likely be viewed as unappealing by the wider profession, even if it were able to fit the data.

[3] This problem of uncovered interest parity holding is of course familiar from a large literature in international macroeconomics where, absent differences in countries' risk characteristics, any variation in interest rates across countries generally requires off-setting, predictable, depreciations of the high-interest-rate currency – so that expected returns are equalized across currencies.

and Verdelhan, 2013). While these models are designed to address a variety of quantitative puzzles, ranging from the forward premium puzzle to exchange rate disconnect, none of them are able to match the empirical fact that currency premia are large, whereas exchange rates are largely unpredictable. We contribute to this literature by highlighting this tension as an obstacle to its further development.

Our paper also speaks to the growing international macroeconomics literature that has documented large and persistent differences in currency risk, currency returns, and interest rates across countries.[4] This risk-based view of currency returns has become the dominant paradigm in the literature based on two key empirical observations: First, interest rate differentials between countries are large and long-lasting, and not offset by predictable depreciations, so that uncovered interest parity fails and investors can make money by borrowing in the low interest rate country and lending in the high interest rate country (Fama, 1984; Hassan and Mano, 2019). Second, there is clear evidence of risk factors in currency markets, where currencies with low interest rates tend to appreciate at times of stress in the global economy (Lustig and Verdelhan, 2007; Campbell, Serfaty-De Medeiros, and Viceira, 2010; Lustig, Roussanov, and Verdelhan, 2011; Lettau, Maggiori, and Weber, 2014). Several papers mentioned above have related these differences in currency risk to features of the economic environment, such as differences in country size, trade centrality, or commodity endowments. We contribute to this literature by showing all of these forces, when evaluated through the lens of a standard quantitative model will result in currency premia that arise from large predictable appreciations of the high-interest-rate currencies, but not the large interest rate differentials we observe in the data.

Lastly, our paper relates to the growing literature emphasizing the role of incomplete international asset markets. One strand of this work characterizes in reduced-form to what extent market incompleteness can help resolve puzzles in international asset pricing (Lustig and Verdelhan, 2019; Sandulescu, Trojani, and Vedolin, 2021; Chernov, Haddad, and Itskhoki, 2023; Jiang, 2023). We add to this literature by identifying general conditions under which a currency premium puzzle arises under complete or incomplete markets. A second strand debates what moves exchange rates, attributing their fluctuations to shocks to intermediation capacity, noise, or fundamentals (Alvarez, Atkeson, and Kehoe, 2009; Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021; Chahrour et al., 2024; Kekre and Lenel, 2024a). We contribute to this debate by showing that it does not, by itself, touch the currency premium puzzle: even when exchange rates are entirely driven by noise, the problem remains that large differences in countries' riskiness produce no economically significant variation in their interest rates — the margin on which currency premia manifest in the data.

[4]See Hassan and Zhang (2021) for a survey of this literature.

The remainder of this paper is organized as follows. Section 1 lays out the basic concepts and framework for analysis. Sections 2 and 3 discuss the long-run risk and habit models. Sections 4, 5, and 6 expand the argument by allowing for departures from log-normal distributions, incomplete markets, and heterogeneity in the non-risk determinants of interest rates (time preferences and long-run growth). Section 7 concludes.

1 Basic Framework and Data

This section introduces a simple graphical representation of the currency premium puzzle. For ease of exposition, we focus on log-normally distributed stochastic discount factors (SDFs). Section 4 shows the puzzle also arises in more complex settings with non-normal shocks.

1.1 Interest Rates and the SDF

We begin by demonstrating the currency premium puzzle in its most general form, which arises in a broad class of models with complete or incomplete asset markets.

Assume the SDF M_{t+1} is log-normally distributed. If households can trade their own country's risk-free bond (but not necessarily any other assets), absence of arbitrage requires

$$\mathbb{E}_t(M_{t+1}R_t) = 1$$

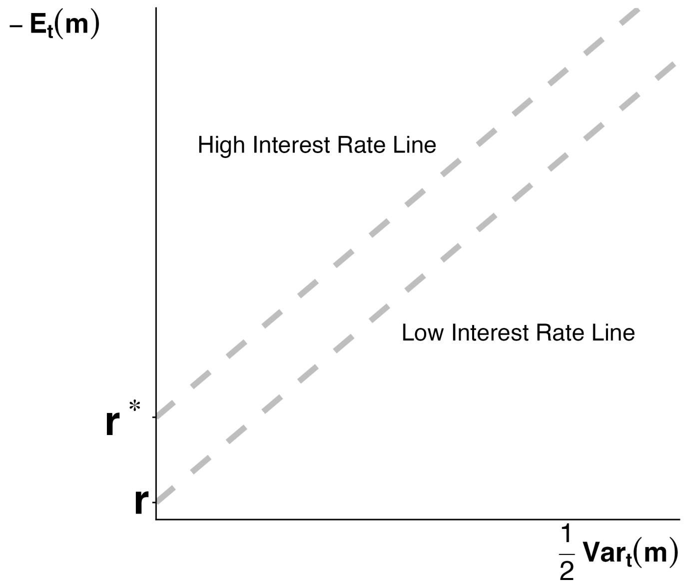
where $\mathbb{E}_t$ denotes the mathematical expectation conditional on the information set at time t and R_t denotes the risk-free rate. Taking logs and rearranging yields

$$-\mathbb{E}_t(m_{t+1}) = \frac{1}{2}\operatorname{var}_t(m_{t+1}) + r_t. \tag{2}$$

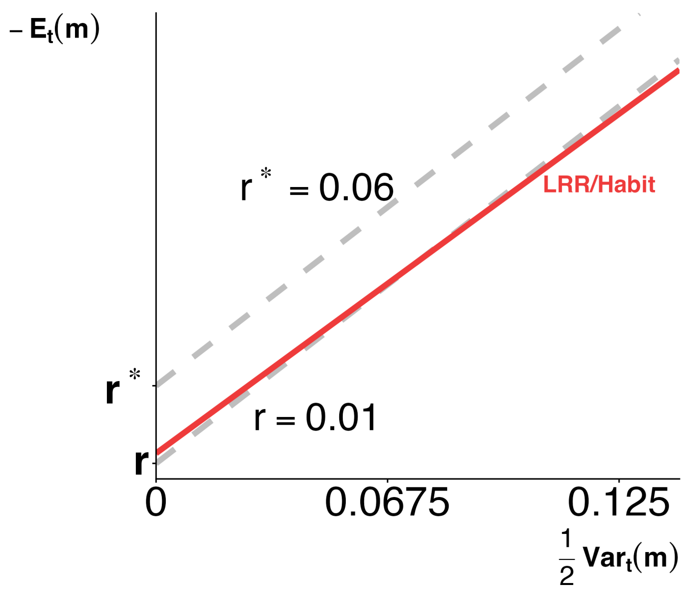
where, throughout the paper, lowercase letters denote the logarithm of a variable such that $x = \log(X)$.

Figure 1 illustrates the currency premium puzzle graphically. It plots the scaled (conditional) variance of the log SDF on the horizontal axis and the negative conditional mean on the vertical axis, which we refer to as "SDF space". Each point in this SDF space determines a log SDF represented by its first two moments. Equation (2) implies any SDF with a combination of mean and variance that lies on the same 45-degree line produces the same risk-free rate. For ease of reference, we refer to these lines as "iso-risk-free rate" or "iso-rf" lines.

Figure 1: The SDF Space and Iso-risk-free Rate Lines



(a)



(b)

Panel (a) plots two iso-rf lines (grey dashed line) in the SDF space. The higher line represents a higher risk-free rate. The intercept of each line with the y-axis represents the corresponding risk-free rate. Panel (b) adds a red line with a slope of 0.96 (consistent with a long-run risk model with a risk aversion of 12 and an intertemporal substitution of 1.81), representing a typical long run risk or habit model.

The dashed lines in Figure 1 show two examples of iso-rf lines. The intercept of the line with the vertical axis represents the risk-free rate, and higher lines represent higher risk-free rates. To fix ideas, consider the higher line to represent a country like New Zealand with a risk-free rate that persistently exceeds that of Japan (the lower line) by around five percentage points.

Now consider the classical challenge in closed-economy asset pricing: Resolving the equity premium puzzle of Mehra and Prescott (1985) requires that, at least in some circumstances, $\text{var}_t(m_{t+1})$ reaches 0.5^2 to accommodate a Sharpe ratio in equity markets of about 0.5 (Hansen and Jagannathan, 1991). To illustrate, the horizontal axis shows this range, ending with $\frac{1}{2}0.5^2 = 0.125$, which represents the rough range of variances of m_{t+1} found in quantitative asset pricing models.

At the same time, risk-free interest rates are low and stable (Weil, 1989), so that a given country must stay on or near a given iso risk-free rate line, even in response to large shocks.

As we show in later sections, long-run risk (LRR) and habit preferences resolve this tension by imposing an offsetting functional relationship between the two moments of the log SDF, so that anything that raises the scaled variance of m_{t+1} reduces its conditional mean by approximately the same amount.

In our figure, this offsetting functional relationship is represented by an upward-sloping line with a slope near one, depicted by the solid red line. Specifically, we depict the parameterization from Bansal and Shaliastovich (2013) where both countries' representative agents have relative risk aversion of 12 and an intertemporal elasticity of substitution of 1.81. With symmetric preferences and in the absence of arbitrage, both countries' SDFs must lie on this red line at all times.

For reasonable differences in $\text{var}_t(m_{t+1})$ between the two countries, the LRR or habit model depicted cannot rationalize large differences in interest rates between the two countries: With a slope close to one, the red line cannot intersect both iso-rf lines within the range depicted. For example, if we made the extreme assumption that Japan's SDF has a high variance of 0.5^2 and New Zealand's SDF a variance of 0, we would at most get an interest differential of 0.5% (equation (1)).

This failure to translate even large differences in risk exposures across countries into moderately sized interest rate differentials is the essence of the currency premium puzzle: Because canonical asset pricing models require a near one-for-one offsetting relationship between the mean and the variance of the log SDF to stabilize interest rates in the time series, the cross-section of interest rates is unresponsive to differences in risk across countries. As long as preferences are symmetric across the two countries, the same forces that resolve the equity premium and risk-free rate puzzles in these models thus also prevent them from generating the type of large risk-free interest rate differentials we see in the data.

Notably, this challenge arises even if markets are highly incomplete between the two countries. All that is required is that there is no arbitrage, that each country's risk-free asset is priced by the Euler equation of some marginal investor — who need not trade any other assets — and that these investors demand compensation for risk in line with observed risk premia, with preferences that give rise to the offsetting relationship depicted by the red line — conditions we make precise in Section 5.

1.2 Exchange Rates and the SDF

Equilibrium models in which the allocation of resources across countries is efficient also predict a link between the mean of countries' log SDFs and expected depreciations.

In particular, if markets are complete, Backus, Foresi, and Telmer (2001) show the expected change in exchange rates, quoted as units of home currency per foreign currency, is given by[5]

$$\mathbb{E}(\Delta s_{t+1}) = [-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})], \tag{3}$$

where Δs_{t+1} is the change in log exchange rates (appreciation rate of the foreign currency), and we use $\star$ to denote the foreign country.

From equation (2), the difference in risk-free rates between the two countries is given by

$$\begin{aligned}\mathbb{E}(r^{\star}_t - r_t) = {} & \mathbb{E}(m_{t+1}) - \mathbb{E}(m^{\star}_{t+1}) \\ & + \frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^{\star}_{t+1})).\end{aligned} \tag{4}$$

Analogous to the equity premium, which is defined as the unconditional difference between returns on equity and the risk-free rate, we define the currency premium as the expected return a home investor would earn if she invests in the foreign risk-free bond versus the home bond.[6] She earns the difference in interest rates net of the depreciation rate of the foreign currency. Using (3) and (4), we obtain the currency premium as

$$\mathbb{E}(rx_{t+1}) = \mathbb{E}(r^{\star}_t - r_t) + \mathbb{E}(\Delta s_{t+1}) = \frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^{\star}_{t+1})), \tag{5}$$

[5] All equations in this subsection also hold conditionally. We show unconditional equations here because we can only estimate the unconditional moments given data on exchange rates, and we emphasize the cross-country variations of unconditional moments.

[6] In the literature, this return is sometimes referred to as "currency risk premia" or simply "currency returns." We call it currency premia, analogously to equity premium.

where rx_{t+1} denotes the currency return earned between t and $t+1$. For ease of exposition, we assume the home country has a lower risk-free rate, so that the currency return coincides with the return on the "carry trade," where the investor borrows in the low-interest-rate currency and lends in the high-interest-rate currency.

Note that we can decompose currency premia into an interest rate differential and an expected change in the exchange rate. The relative importance of these two terms is key to our analysis. To facilitate later discussion, we label the share of the expected change in the exchange rate in the currency premium, $\frac{\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})}$, the "FX-share." The FX-share measures the fraction of the currency premium that comes from predicted changes in the exchange rate, rather than the difference in interest rates. In particular, a positive FX-share means investors expect the high-interest-rate currency to appreciate on average.

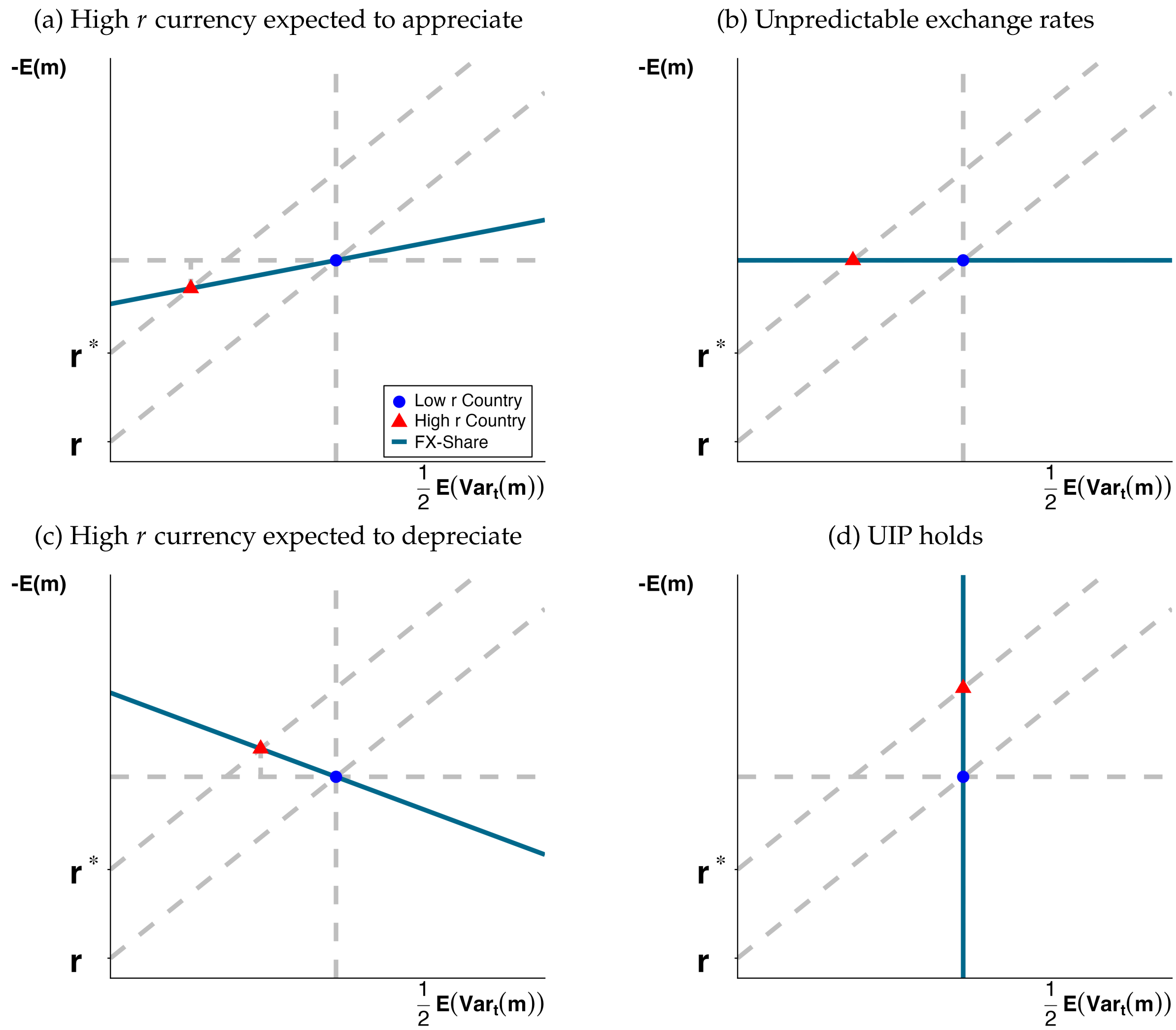
Equations (3) and (5) show exchange rates and currency premia are tightly linked to means and variances of log SDFs. Each country's SDF is characterized by one mean-variance pair $(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})), -\mathbb{E}(m))$, which we represent as a point in the SDF space. Data on exchange rates and currency premia between any two countries provide information on the relative position of their corresponding points. In particular, expected changes in the exchange rate determine vertical differences (equation (3)) and currency premia govern horizontal differences (equation (5)).

Figure 2 visualizes properties of exchange rates and currency premia in SDF space under various configurations. Panel (a) shows the high-interest-rate currency appreciating on average relative to the low-interest-rate currency. The high-interest-rate foreign currency (red triangle) lies on a higher iso-rf line and is expected to appreciate ($\mathbb{E}(\Delta s_{t+1}) = [-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})] > 0$) relative to the low-interest-rate home currency (blue dot). The slope of the blue line connecting the two dots measures the FX-share – the vertical distance between two countries (the expected rate of appreciation) divided by their horizontal distance (the currency premium).

In panel (b), exchange rates are unpredictable, namely, $\mathbb{E}(\Delta s) = 0$, and the means of the log SDFs are consequently equalized across countries, so that the blue line is horizontal, with an FX-share of zero. Panel (c) shows the situation where the high-interest-rate currency is expected to depreciate — the blue line has a negative slope. If uncovered interest rate parity holds, interest-rate differences are exactly offset by expected change in exchange rates, and currency premia (differences in variances) vanish, so that the blue line is vertical (panel (d)).

The main takeaway from Figure 2 is that the slope of the solid blue line that connects the two countries

Figure 2: Currency Premium in SDF Space



This figure plots four scenarios of currency premia in SDF space. In each panel, grey dashed lines represent iso-rf lines and vertical/horizontal lines. The blue dot represents the low-interest-rate country, and the red triangle represents the high-interest-rate country. Panel (a) represents the case when the high-interest-rate currency is expected to appreciate; panel (b) represents the case when exchange rates are unpredictable; panel (c) represents the case when the high-interest-rate currency is expected to depreciate; and panel (d) represents the case when uncovered interest rate parity holds.

measures the FX-share:

$$\begin{aligned} \text{slope} &= \frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^\star_{t+1})]}{\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m^\star_{t+1}))} \\ &= \frac{\mathbb{E}(\Delta s)}{\mathbb{E}(rx)} = \text{FX-share}. \end{aligned} \tag{6}$$

It pins down the split between expected depreciations and interest-rate differentials. As we discuss below, the currency-premium puzzle states that long-run risk and habit models generate an FX-share that is at odds with the data, i.e., they imply a currency premium that is of the wrong composition.

What do the data tell us about the FX-share? A voluminous literature going back to Meese and Rogoff (1983) has documented that exchange rates are largely unpredictable. That is, we already know from this literature that the FX-share must be close to zero in the presence of currency premia.

To estimate the FX-share more directly, we build on a key result in Hassan and Mano (2019), who estimate the share of (conditional and unconditional) currency returns that are attributable to predictable depreciations. Table 1 summarizes their results for a baseline sample of countries.[7]

The top row shows the returns on a simple (unconditional) version of the carry trade that goes long currencies that generally have high interest rates (such as New Zealand) and goes short currencies that have persistently low interest rates (such as Japan).

$$\mathbb{E}[rx^{st}] \equiv \frac{1}{T}\sum_t \sum_i rx_{i,t+1}(\bar{r}_i - \bar{r})$$

The portfolio weights each currency i in proportion to its average interest differential with the U.S. in the pre-sample (the 15 years prior to 1995, $\bar{r}_i - \bar{r}$), without allowing any adjustments of weights thereafter. Hassan and Mano (2019) refer to this strategy as the "Static Trade," because it mimics the conditional carry trade, but without allowing for rebalancing of the portfolio in response to short-term fluctuations in the interest rate.

The table shows two main insights. First, unconditional differences in currency returns are large, with a mean return to this strategy of 3.46% annually and a Sharpe ratio of 0.39. Second, the vast majority of these returns are due to long-lasting differences in interest rates between countries (4.76 percentage points), whereas, if anything, high-interest-rate currencies depreciate on average, deducting 1.30 percentage points from the overall return. Hassan and Mano (2019) show how to estimate this share more formally using regression analysis and how to construct appropriate standard errors, which are given in square brackets below.[8] Moreover, they show both features of the data (large interest-rate differentials and a slightly negative FX-share) are robust to a wide range of samples and estimation techniques. Appendix C gives details and also updates their estimates using more recent data.[9]

[7]The data range from 1983–2010. Our results are robust to including newer data. We keep our sample identical to Hassan and Mano (2019) to facilitate comparison. The table shows results from their "1 Rebalance" sample, which consists of 16 countries/regions: Australia, Canada, Switzerland, Denmark, Hong Kong SAR, Japan, Kuwait, Malaysia, Norway, New Zealand, Saudi Arabia, Sweden, Singapore, United Kingdom, South Africa, and the US. The empirical results below are robust to alternative means of portfolio construction, and the general pattern is confirmed in many related works (e.g., Lustig, Roussanov, and Verdelhan (2011)).

[8]Formally, and using the notation in their paper, the FX-share in the unconditional SDF space is $1-1/\hat{\beta}^{stat}$, whereas the equivalent FX-share in conditional SDF space is $1-1/\hat{\beta}^{ct}$.

[9]The larger deviations in covered interest parity that have arisen since 2008 suggest that forward premia are a worse measure of risk-free interest rate differentials in the later part of our sample (Du, Tepper, and Verdelhan, 2018). However, even with these large

Table 1: FX-Share in the Data

	Return (%)	Change in FX (%)	Interest Rate Diff (%)	FX-share
	$\mathbb{E}(rx^{st})$	$\mathbb{E}(\Delta s^{st})$	$\mathbb{E}(r^{\star,st} - r^{st})$	$\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$
Static Trade	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]
	$\mathbb{E}(rx^{ct})$	$\mathbb{E}(\Delta s^{ct})$	$\mathbb{E}(r^{\star,ct} - r^{ct})$	$\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$
(Conditional) Carry Trade	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]

We use the 1-rebalance sample of Hassan and Mano (2019). All moments are annual. Static trade returns are calculated by first sorting the unconditional forward premia (interest rates), and then always shorting a weighted portfolio of the currencies with below-average unconditional forward premia, and being long a weighted portfolio of the rest. Carry trade returns are calculated by sorting the forward premia (interest rates) period by period, and then shorting a weighted portfolio of the currencies with below-average forward premia, and being long a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C.1. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. Alternative regression-based estimations can be found in Appendix C and in Hassan and Mano (2019).

On average, investing in high-interest-rate currencies earns money on the interest-rate differential and loses money on the exchange rate. The FX-share is negative (-37%), as in panel (c) in Figure 2, with a confidence interval of (-1.16,0.23), which includes a horizontal relationship (unpredictable exchange rates as in panel (b)). However, the data clearly exclude the possibility that investors on average expect high-interest-rate currencies to appreciate drastically (panel (a)) or that UIP might hold (panel (d)).

The bottom row in Table 1 shows the same metrics for an alternative trading strategy, the conditional carry trade, which aims at exploiting time-varying differences in interest rates across countries. The conditional carry trade goes long high-interest-rate currencies and short low-interest-rate currencies, just like the static trade. It differs, however, in that the conditional carry trade compares risk-free rates period by period and rebalances the portfolio accordingly. Because of this period-by-period rebalancing, conditional carry trade returns also contain conditional information, which we use to discipline the FX-share in models that allow

deviations, the cross-currency basis is an order of magnitude smaller than the interest rate differentials we study — tens of basis points against several percentage points, so that they are unlikely to affect our results.

for time variation in the variance of the log SDF below.[10]

$$\mathbb{E}[rx^{ct}] \equiv \frac{1}{T} \sum_t \sum_i rx_{i,t+1}(r_{it} - \bar{r}_t), \tag{7}$$

where $\bar{r}_t$ denotes the average interest rate across countries at time t.

In sum, an efficient (complete-markets) model that rationalizes the large and persistent differences in interest rates that we see in the data must satisfy two key properties:

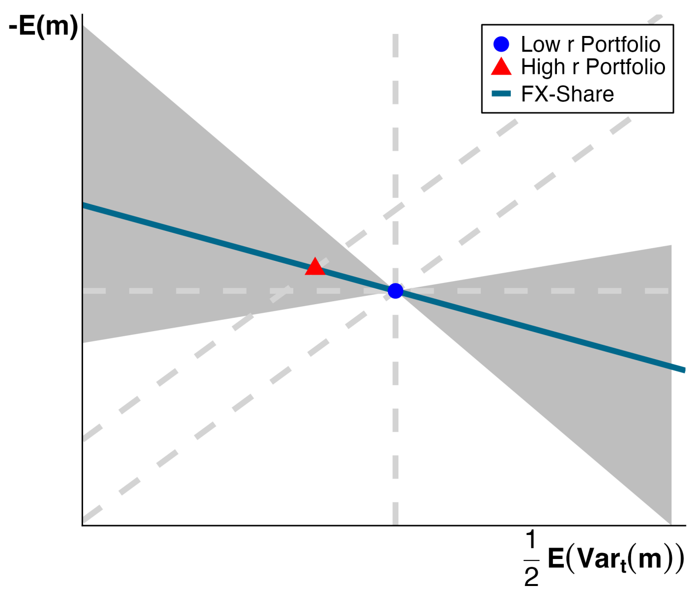
Property 1. *A large difference in the variances of log SDFs $\mathbb{E}(var_t(m) - var_t(m^\star))$ of at least 0.07.*[11]

Property 2. *The FX-share is non-positive*

$$\frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^\star_{t+1})]}{\frac{1}{2}\mathbb{E}(var_t(m) - var_t(m^\star))} \leq 0$$

which means the high-interest-rate currency depreciates on average.

Figure 3: Static Trade in SDF Space



This figure plots the relative position of the low-interest-rate portfolio and the high-interest-rate portfolio implied by the static trade returns in the data, as well as the confidence intervals of the FX-share. The position of the low-interest-rate portfolio is arbitrarily chosen because only the relative positions of these dots matter for our discussion. The position of the high-interest-rate portfolio is inferred from the data using equations (3) and (5). The shaded area represents confidence intervals for the FX-share.

[10]In these models, the FX-share measures the relative position of the portfolio that we long each period and the portfolio that we short each period, as we explain below.

[11]To put things into perspective, the Hansen and Jagannathan (1991) bound implies $\text{var}_t(m) \geq 0.25$. The threshold follows from applying equation (5) to the static trade return in Table 1: $2 \times 3.46\% \approx 0.07$.

We incorporate these empirical results in the graphical representation of SDFs in Figure 3. The FX-share (slope of the blue line) is weakly negative, with the shaded region representing a 95% confidence interval.

In the following sections, we show risk premia generated by canonical long-run risk and habit models cannot satisfy both properties simultaneously. They either generate unpredictable exchange rates with small currency returns (Property 2 but not Property 1) or generate large currency returns through predictable changes in exchange rates (Property 1 but not Property 2); that is, risk premia from both models are incompatible with large interest rate differentials that are the primary drivers of currency returns. We show theoretically that the fundamental reason these models fail to account for the two properties is that they introduce an off-setting functional relationship between means and variances of log SDFs.

2 Long-Run Risk Models

In this section, we set up and analyze a canonical long-run risk model under complete markets, with a structure similar to that of many cutting-edge quantitative frameworks in the literature. We derive in closed form the relationship between the first two moments of the log SDFs and argue that such a relationship is at odds with data from currency markets. We then numerically solve and simulate a number of prominent versions of the long-run risk model from the literature and show the currency premium puzzle quantitatively for each of them.

A representative agent derives utility according to

$$U_t = \left((1-\delta)C_t^{1-\frac{1}{\psi}} + \delta\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{8}$$

U_t denotes utility at time t and C_t denotes consumption. δ is the subjective time discount factor, ψ governs the elasticity of intertemporal substitution, and γ scales risk aversion. Following the long-run risk approach, we further assume consumption follows

$$\begin{aligned}\Delta c_{t+1} &= \mu + z_t + \sigma\varepsilon_{t+1}\\ z_t &= \rho z_{t-1} + \sigma_{LR}\varepsilon_{LR,t},\end{aligned}$$

where μ is the mean growth rate of consumption, z_t is a long-run process that moves the mean of consumption growth, and ρ denotes its persistence. σ governs the volatility of short-run shocks and σ_{LR} governs the volatility of long-run shocks. ε_{t+1} and $\varepsilon_{LR,t}$ are short-run and long-run shocks, respectively. For ease of exposition, we set $\sigma = 0$ for our derivation in the main text. Appendix A.1.2 shows the full solution including short-run shocks, and Appendix A.2 generalizes our results to settings with stochastic volatility.

Under these preferences and with complete markets, the log SDF is given by

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log\left(\mathbb{E}_t[\exp((1-\gamma)u_{t+1})]\right)\right). \tag{9}$$

Assuming u_{t+1} is normally distributed, we solve for the first and second moment of the log SDF as

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\text{var}_t(u_{t+1})) \tag{10}$$

$$\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\mathbb{E}(\text{var}_t(u_{t+1})). \tag{11}$$

These two equations show the first and second moments of the log SDF are tightly linked whenever utility is not time separable ($\gamma \neq \frac{1}{\psi}$). In fact, when we substitute out $\mathbb{E}(\text{var}_t(u_{t+1}))$ from these conditions, we get a functional relationship between the mean and variance:[12]

$$-\mathbb{E}(m_{t+1}) = \frac{1}{2}\frac{1-\gamma}{\frac{1}{\psi} - \gamma}\mathbb{E}(\text{var}_t(m_{t+1})) + \left(\frac{1}{\psi}\mu - \log(\delta)\right) \tag{12}$$

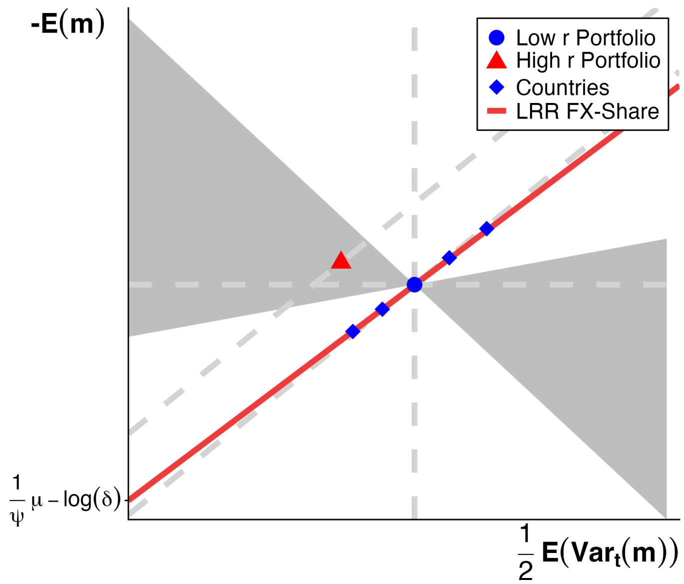
Standard calibrations assume a preference for early resolution of uncertainty, where $\gamma > 1$ and $\gamma > \frac{1}{\psi}$. In this case, the EZ utility function (10) implies a positive coefficient on the second moment of the log SDF. As a result, any shock that increases the variance lowers the mean log SDF, as shown in (12) above.

Importantly, all SDFs that satisfy the relationship in equation (12) lie on a line in SDF space. We add this line to Figure 4 as the red line with a common calibration of $\gamma = 6.5$ and $\psi = 1.6$. The red line has a slope of 0.94 and thus has a positive slope close to that of the iso risk-free lines (a slope of 1). For a given country, this feature of the long-run risk model helps resolve both the equity premium puzzle and the risk-free rate puzzle because it ensures a stable risk-free rate even when the variance of the SDF is large or changes over time.

However, if all countries share the same preference parameters and steady-state growth rate (μ), all SDFs have to lie on the same red line. The slope of this line represents the long-run risk model-implied FX-share (LRR FX-share) across countries. Given that the LRR FX-share is close to 1, no specification of the model

[12] We put $-\mathbb{E}(m_{t+1})$ on the left-hand side to facilitate graphical representation in SDF space.

Figure 4: The LRR FX-share in SDF Space



This figure plots the long-run risk FX-share with $\gamma = 6.5$ and $\psi = 1.6$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area represents confidence intervals inferred from the data. Blue squares are examples of countries that differ in the variance of their log SDFs.

exists that would lead to a large difference in interest rates without generating substantial exchange rate predictability. Specifically, the model cannot generate a high-interest-rate portfolio with little or no exchange rate predictability, such as the red triangle in the figure. Put differently, the LRR FX-share lies outside of the admissible grey cone and is thus clearly rejected by the data.

To illustrate this point, in Figure 4, we plot five representative countries (blue rhomboids) that differ in the variances of their log SDFs. These differences in variances could be driven by various forms of cross-country heterogeneities in risk characteristics proposed in the literature (different loadings on a global shock, country size, trade centrality, etc.). As long as all countries share the same preference parameters, they lie on the same line and no linear combination of them would match the empirical pattern established in Section 1.2 (the red triangle).[13]

The long-run risk model thus pins down the slope of the relationship between means and variances of log SDFs via equation (12), and thus the FX-share, whenever $\psi \neq 1/\gamma$.

Proposition 1. *Let γ, ψ, μ, and δ be identical across countries and assume absence of short-run risk ($\sigma = 0$). Further*

[13]For example, we confirm that in the calibrated long-run risk model of Colacito et al. (2018a) (a heterogeneous country general equilibrium model where countries differ in their loading on a global endowment shock), all countries indeed lie on the same LRR line. We confirm this by solving their model using risk-adjusted-affine approximation (log-linearization with risk-adjustments. See, e.g., Chen and Palomino (2026), Malkhozov (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015)). We use their calibration in Table II (in particular, $\gamma = 6.5$, $\psi = 1.6$), and the heterogeneous loadings on the global shock are evenly spaced between [-0.65,0.65], as in the paper's calibration on page 18. We then numerically solve for the unconditional variances and means of each country's SDF.

assume u_{t+1} is normally distributed. Then, for any two countries, the long-run risk model implies[14]

$$\textit{FX-share} = \frac{\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{1-\gamma}{\frac{1}{\psi}-\gamma}. \tag{13}$$

If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, and we assume $\gamma > 1$, the FX-share is positive and the model cannot satisfy Property 2. In particular, if $\gamma > 2 - \frac{1}{\psi}$, FX-share $\geq \frac{1}{2}$, so that the expected change in the exchange rate $\mathbb{E}(\Delta s_{t+1})$ accounts for more than 50% of the currency premium.

Proof. Implied by equations (3), (5), and (12). □

Equation (13) shows the LRR FX-share is determined purely by preference parameters. It is thus deeply connected with the assumption of recursive preferences: when agents prefer an early (or late) resolution of uncertainty ($\gamma \neq \frac{1}{\psi}$), the expected continuation utility enters non-linearly in (8), so that its higher-order moments (in the case of log-normal shocks, its second moment) move not just the variance but also the conditional mean of the log SDF. In this sense, the FX-share, and thus the composition of currency premia, is independent of the economic environment and, instead, directly follows from the choice of preferences. As we show in Appendix A.2 and Section 4, this logic is very general, and this relationship is robust to adding volatility shocks and relaxations of the log-normality assumption.

To show the implications of this theoretical result, we solve and simulate five state-of-the-art long-run risk models from the literature and summarize the results in Table 2. While none of them were built to solve the currency premium puzzle, we check how they fare in regards to international asset return data. These models are replications of the seminal works in Colacito et al. (2018a), Colacito et al. (2018b), Bansal and Shaliastovich (2013), and Colacito and Croce (2013), as well as the original long-run risk model in Bansal and Yaron (2004).[15] The most recent of the five models (Colacito et al., 2018a) allows countries to differ in their loadings on global productivity shocks, so that some countries (currencies) are permanently riskier than others in equilibrium. Through this channel, the model is able to generate sizable currency premia, matching empirical Property 1. However, consistent with the currency premium puzzle, more than 90% of these currency premia arise from predictable appreciation of the high-interest-rate currencies, implying an FX-share close to 1 and contradicting empirical Property 2.[16] The remaining models do not explicitly model

[14] The expression should be read as the slope of the mean–variance locus; when countries are symmetric, both numerator and denominator are zero and the FX-share is defined as the limit (see the note to Table 2).

[15] We thank Ric Colacito, Max Croce, Federico Gavazzoni, and Robert Ready for providing their code.

[16] In the linear relationship between interest rate differentials and currency premia, using l'Hôpital's rule, the FX-share can be computed even for the limit when interest rate differentials converge to zero.

Table 2: Static Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	1.12	0.93	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.00	0.98*	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.00	0.96*	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.00	0.95*	No	No
Bansal and Yaron (2004) JF	-	-	-	0.96*	No	No

This table shows the simulation results of four long-run risk models. For Colacito et al. (2018a), we simulate the model in their section II, with calibrations in their Table II (γ = 6.5, ψ = 1.6, exactly the same model as the heterogeneous EZ model in their Table III). For Colacito et al. (2018b), we simulate the model in their section II, with calibrations in their Table 2 (γ = 10, ψ = 1.2, the same model as EZ-BKK in their Table 3). For Bansal and Shaliastovich (2013), we simulate the model in their Section 2, with calibrations in their Table 4 and section 4.4 (γ = 12, ψ = 1.81, the same model as Table 7), we report the simulation results for the real model to be consistent with other models. For Colacito and Croce (2013), we simulate the model in their sections II and III, with calibrations in their Table II, Panel A (γ = 8, ψ = 1.5,the same model as model (1) in their Table II, Panel B). For Colacito et al. (2018a), we follow their approach and simulate 100 economies, each with 320 periods and 100 burn-in periods (discarded). FX-share are calculated for each sample and averaged across different simulations. For all the other models, we simulate 100 economies of 10,000 periods. Static trade returns are computed using the same method in our empirical analysis. All moments are averaged across periods and simulations. Because Colacito et al. (2018b), Bansal and Shaliastovich (2013) and Colacito and Croce (2013) feature symmetric countries and currency premia are 0, we show the theoretical FX-shares for these models using Proposition 1 instead. We mark these theoretical FX-shares with asterisks. We also show the theoretical FX-share for the calibration of Bansal and Yaron (2004) for comparison (with γ = 10, ψ = 1.5). The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively. All moments are annual.

heterogeneity in risk across countries. (In this sense, they fail to match Property 1.) However, the table shows they would suffer from the currency premium puzzle even if they did. In each of the four models, going back to Bansal and Yaron (2004), the FX-share is close to 1 (the slope of the iso-rf line), suggesting predictable appreciation would again make up well over 90% of any currency returns the models could produce.

These conclusions are essentially unchanged when we consider the conditional carry trade (7) within these models. Relative to the static trade, the conditional carry trade re-sorts portfolios each month based on contemporaneous interest differentials; in the data, the two strategies deliver similar returns because interest differentials are highly persistent, so the incremental gain from monthly re-sorting is limited (Hassan and Mano, 2019). For example, in Colacito et al. (2018a) the conditional carry trade earns a mean annual return of 4.47%, but the average long–short interest rate differential remains far too small (1.71%, against 7.11% in the data), and the implied FX-share, at 0.62, remains of the wrong sign, though smaller than the 0.93 of the static trade.[17] This modest reduction in the FX-share is instructive for our discussion below: temporary differences in expected consumption growth $(z_t - z_t^\star)$ generate temporary interest rate differentials that are fully reversed by predictable changes in exchange rates — uncovered interest parity holds for any variation in interest rates induced by differences in the mean, rather than the variance, of the log SDF. The measured conditional FX-share is therefore attenuated towards zero relative to the structural slope, because its numerator contains additional predictable variation in exchange rates that is unrelated to returns.[18]

To summarize, long-run risk models with recursive (EZ) preferences imply a tight functional relationship between the mean and the variance of the log SDF. Under standard calibrations in which agents prefer early resolution of uncertainty (Bansal and Yaron, 2004), this restriction implies a positive FX-share and hence currency premia that arise predominantly through expected appreciation of the high-interest-rate currency, contrary to the data.

3 External Habit Models

Next, we show external habit models have similar difficulties in matching the data as long-run risk models. We illustrate these challenges using the pioneering work of Verdelhan (2010), which extends the classical habit model in Campbell and Cochrane (1999) to an open economy framework and allows for a closed-form solution. We study further variations of the habit model numerically below.

Agents feature habit utility of the form

$$\mathbb{E}\sum_{t=0}^{\infty}\left(\delta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}\right.,$$

where H_t is an externally given habit level, δ denotes the time discount factor, and γ governs risk aversion as before. The surplus consumption ratio is given by $X_t \equiv \frac{C_t - H_t}{C_t}$. Log consumption follows a random walk with drift given by

$$\Delta c_{t+1} = \mu + \sigma \varepsilon_{t+1},$$

where μ is again the mean growth rate and σ is the conditional volatility. ε_{t+1} is an i.i.d. normal shock.

[17] See Appendix Table A1 for details.

[18] Appendix A.2.2 derives the conditional expressions formally, including with stochastic volatility, and shows innovations to $(z_t - z_t^\star)$ are, up to second order, unrelated to differences in $\mathbb{E}_t(rx_{t+1})$ (equations (A10) and (A11). Also see equations (A12) and (A5), and the discussion therein.).

Moreover, Campbell and Cochrane (1999) and Verdelhan (2010) assume the process for the log surplus consumption ratio follows

$$x_{t+1} = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu),$$

where ϕ governs persistence, the logarithm of the upper bound x_{max} is given by $x_{max} = \bar{x} + \frac{1-(\bar{X})^2}{2}$ with steady state of the surplus consumption ratio as $\bar{X} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$, and the sensitivity function $\lambda(x_t)$ is specified as

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1-2(x_t-\bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere.} \end{cases}$$

Note $\gamma(1-\phi) - B > 0$ by construction.[19] Using the process for the log surplus consumption ratio x_t, one can derive the log SDF as

$$m_{t+1} = \log(\delta) - \gamma(\Delta c_{t+1} + \Delta x_{t+1}).$$

This specification implies the first two moments of the log SDF are

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \gamma\mu + \gamma(1-\phi)(x_t - \bar{x}) \tag{14}$$
$$\frac{1}{2}\text{var}_t(m_{t+1}) = \frac{1}{2}(\gamma(1-\phi) - B) - (\gamma(1-\phi) - B)(x_t - \bar{x}). \tag{15}$$

Both the conditional mean and the variance of the log SDF depend on the surplus consumption ratio $x_t - \bar{x}$. This state dependence is critical to the habit model's ability to match asset pricing moments. Since unconditional moments are constant, we derive the model in terms of conditional moments and study the returns to the conditional carry trade.

Substituting for the surplus consumption ratio unveils a linear functional relationship between the first two moments, which are now conditioned on time t information:

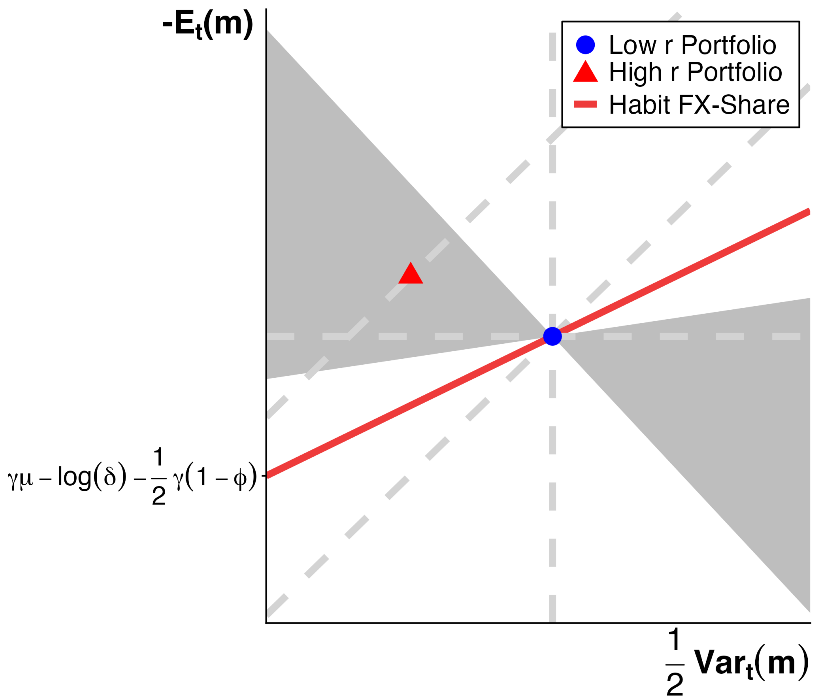
$$-\mathbb{E}_t(m_{t+1}) = \frac{1}{2}\frac{\gamma(1-\phi)}{\gamma(1-\phi) - B}\text{var}_t(m_{t+1}) - (\log(\delta) - \gamma\mu) - \frac{1}{2}\gamma(1-\phi). \tag{16}$$

Equation (16) again implies a line in the—now conditional—SDF space. That is, the first two moments of all

[19] The specification of the $\lambda()$ function strictly follows Campbell and Cochrane (1999). As they point out, the specific functional form is designed to keep the risk-free rates stable. Parameter B nests different SDFs from the literature. $B < 0$ in Verdelhan (2010), $B = 0$ in Campbell and Cochrane (1999), and $B > 0$ in Wachter (2006).

stochastic discount factors produced by the model, for a given parameterization, lie on a line in SDF space. We plot this line in Figure 5 with a calibration of $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$, taken directly from Verdelhan (2010). The red line again has a positive slope. The slope is 0.5, allowing for some degree of risk-free rate volatility while maintaining a high risk premium (high variance of log SDF).[20]

Figure 5: The Habit FX-share in SDF Space



This figure plots the FX-share of the habit model with $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area shows the confidence intervals inferred from the data.

As in the case of long-run risk models, all countries that share the same preferences and steady-state growth rates must have SDFs on this red line, so that the preference parameters chosen again fully determine the FX-share of any currency premium the model can produce, independently of any features of the economic environment that might make one country riskier than the other.

Proposition 2. *If γ, δ, ϕ, B and μ are symmetric across countries, the FX-share is given by*

$$FX\text{-}share = \frac{\mathbb{E}_t(\Delta s_{t+1})}{\mathbb{E}_t(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi) - B}.$$

[20] By setting $B = -0.01$, Verdelhan (2010) lowers the FX-share to 0.5, but makes the risk-free rate volatile in the time series — his simulated real rate has an annualized standard deviation of 2.54%, compared to 1.99% in his data (his Table III) and zero in Campbell and Cochrane (1999), where the sensitivity function is chosen precisely to keep the risk-free rate constant. Verdelhan notes his model reproduces the UIP puzzle "only at the price of potentially negative real interest rates" (p. 131). Lowering FXS thus directly trades off against the stability of the risk-free rate. We return to this general trade-off in Section 5.

Because $\gamma(1-\phi) - B > 0$ is required for stationarity, the FX-share is always strictly positive and the model cannot satisfy Property 2. Furthermore, if $\gamma(1-\phi) > -B$, then FX-share $> \frac{1}{2}$. Thus, an appreciation of the high-interest-rate currency accounts for more than 50% of the currency premium.

Proof. Implied by equations (3), (5), and (16). □

Table 3: Conditional Carry Trade Returns under Habit Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Verdelhan (2010) JF	4.54	2.19	2.35	0.48	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.17	1.95	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.43	0.88	Yes	No
Campbell and Cochrane (1999) JPE	-	-	-	1.00*	No	No

This table shows the simulation results of three habit models. For Verdelhan (2010), we simulate the model in section I, with calibrations in Table II. For Stathopoulos (2017), we simulate the model in section 1, with calibrations in Table 1. For Verdelhan (2010) and Stathopoulos (2017), we simulate 100 economies of 10,000 periods. All moments are averaged across periods and simulations. Results for Heyerdahl-Larsen (2014) are obtained by using the same simulated samples that generated his Table 10. Details on conditional carry trade return construction can be found in Appendix B, and the same method is used for both the empirical and the simulation results. We also show the theoretical FX-share for the calibration of Campbell and Cochrane (1999) for comparison. Numbers with an asterisk are obtained from Proposition 2 rather than simulated. The last two columns summarize whether the simulated results can match our empirical properties 1 and 2, respectively. All moments are annual.

To show that the currency premium puzzle applies to a wider set of habit models, we numerically solve and simulate several prominent and state-of-the-art models in the literature. We summarize our results in Table 3. The habit model of Verdelhan (2010) closely matches the conditional carry trade returns in the data (Property 1). Almost half of these returns, however, arise from expected appreciations of the high-interest-rate currency, contradicting Property 2. The FX-share is positive at +0.48 in this model,[21] whereas it is negative in the data (-0.43).

In the continuous-time habit model of Stathopoulos (2017), the high-interest-rate currency does depreciate on average.[22] But it depreciates so much that the expected depreciation exceeds the interest rate differential,

[21]This is consistent with the prediction of Proposition 2, which predicts an FX-share of 0.5 under the calibration of Verdelhan (2010).

[22]We thank Andreas Stathopoulos for providing us with his code.

and the resulting currency premium is negative, putting it at odds with the data and contradicting both our empirical properties. The deep habit model of Heyerdahl-Larsen (2014) features an FX-share of 0.88 and thus also fails to match Property 2.[23] We further report the FX-share for Campbell and Cochrane (1999), which is 1 exactly. In fact, the authors explicitly parameterize the model to fully immobilize the risk-free rate, so that it also does not allow differences in risk exposure to produce variation in interest rates across countries.

Analogous to the literature on long-run risks, none of these models aimed to fit the composition of currency premia, and instead focused on other salient features of the data. Our analysis shows, however, that augmenting them with heterogeneity in countries' risk characteristics would again produce predictions at odds with the data: because the FX-share is pinned down by preference parameters, any differences in currency premia would manifest as predictable appreciations of the high-interest-rate currency rather than as large and persistent interest rate differentials.

Unlike in long-run risk models, preference parameters and the economic environment are not cleanly separable in habit models: by construction, the endowment process directly shapes agents' risk aversion. This blurring offers an additional lever. Allowing the long-run habit parameter B to differ across countries generates unconditional differences in the variances of log SDFs (see equations (14) and (15)). Absent a theory of why B differs, however, this amounts to fitting the data with heterogeneous preferences. Wang (2021) suggests one source of discipline: heterogeneous loadings on a global shock endogenously induce permanently different habit levels, and thus different risk aversion, under symmetric preferences — quantitatively linking currency premia to capital-output ratios.[24] We return to such combinations of risk and non-risk heterogeneity in Section 6.

To summarize, in this section, we derived a closed-form functional relationship between conditional means and variances of the log SDFs under external habit models. We showed that under classic calibrations, standard models in this literature generate a positive FX-share and cannot satisfy Property 2, thereby running into the currency premium puzzle.

[23]We thank Christian Heyerdahl-Larsen for providing us with his simulated samples.

[24]Studies have also tied B to the slope of the term structure (Wachter, 2006; Verdelhan, 2010), but to our knowledge, no study has tied B to a specific economic source. The volatility parameter σ, by contrast, offers no traction: it has no effect on interest rates in the standard formulation of the model.

4 Relaxing Log-Normality

In addition to long-run risk and habit models, many authors have considered rare disasters and other departures from log-normal shocks as possible explanations for the equity premium puzzle and other closed-economy asset-pricing phenomena. In this section, we show that, perhaps surprisingly, the currency premium puzzle carries over to this broader class of models.

Table 4 decomposes the returns to the conditional carry trade that arise in Gourio, Siemer, and Verdelhan (2013), a quantitatively highly successful rare-disaster model in which countries' representative agents have EZ utility and differ in their exposures to global disaster risk. The table shows that the currency premium puzzle is also evident in this model, with an FX-share of 0.77. As in the long-run risk and habit models discussed above, this FX-share is of the wrong sign, and again close to one.

Table 4: Conditional Carry Trade Returns under Disaster Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1 $-\frac{1}{\text{FX-share}}$	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Gourio, Siemer and Verdelhan (2013) JIE	2.36	1.81	0.55	0.77	Yes	No
Barro (2009) AER	-	-	-	0.73*	No	No
Gourio (2012) AER	-	-	-	0.72*	No	No

The return, FX-share, and interest rate differences for Gourio, Siemer, and Verdelhan (2013) are calculated from their Tables 2 and 4. We also show the theoretical FX-share for a special case of the closed-economy models of Barro (2009) and Gourio (2012) for comparison, assuming only skewness differs across countries. We mark these theoretical predictions with an asterisk. These FX-shares are calculated by plugging the preferred calibrations ($\psi = 2, \gamma = 4$ for Barro (2009) and $\psi = 2, \gamma = 3.8$ for Gourio (2012)) into the approximated FX-share in equation (19). The last two columns summarize if the simulated results can match our empirical properties 1 and 2, respectively. All moments are annual.

To understand the driving force behind this result, we generalize our basic framework from Section 1 to a non-normal distribution and show the tension between the equity premium puzzle, the risk-free rate, and exchange rate predictability exists regardless of log-normality.

Following Backus, Foresi, and Telmer (2001), the risk-free rate can be written for general distributions as

$$
\begin{aligned}
r_t &= -\log(\mathbb{E}_t\, M_{t+1}) = -\mathbb{E}_t(m_{t+1}) - \left[\log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})\right] \\
&= -\mathbb{E}_t(m_{t+1}) - \Xi_t(m_{t+1}),
\end{aligned}
$$

where $\Xi_t(m_{t+1}) \equiv \log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})$ denotes the entropy of the SDF.

If the allocation is efficient, the expected change in exchange rates and currency premia are now given by

$$
\begin{aligned}
\mathbb{E}_t(\Delta s_{t+1}) &= [-\mathbb{E}_t(m_{t+1})] - [-\mathbb{E}_t(m^{\star}_{t+1})] \\
\mathbb{E}_t(rx_{t+1}) &= \Xi_t(m_{t+1}) - \Xi_t(m^{\star}_{t+1}).
\end{aligned}
$$

These expressions are identical to the ones in Section 1 except that the entropy $\Xi_t(m_{t+1})$ takes the place of the variance of the log SDF ($\frac{1}{2}\operatorname{var}_t(m_{t+1})$) in Figures 1 and 2. The relationship between risk-free rates, exchange rates, currency premia, and the mean, and now the entropy of the SDFs is, however, preserved. As a result, the discussion in Section 1 applies, even when we allow for disasters and other deviations from log-normality. The log-normal model emerges as a special case, and the tension between a high equity premium, a low and stable risk-free rate, and predictable exchange rates extends to the generalized setup.

To see this formally, again consider representative agents with EZ preferences as in (8); and, to simplify the exposition, assume that consumption is known one period in advance so that $\Xi_t(\Delta c_{t+1}) = 0$. For arbitrary distributions of shocks, equation (9) then yields (Appendix B shows the detailed derivation):

$$
\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}\left[\left((1-\gamma)u_{t+1}) - \log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)\right]\right(\tag{17}
$$

$$
\mathbb{E}(\Xi_t(m_{t+1})) = -\,\mathbb{E}\left[\left(\left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \log\left(\mathbb{E}_t\left(\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)\right]\right(. \tag{18}
$$

Similar to our analysis in Section 2, the mean and entropy of the log SDF are tightly linked since both depend on the continuation utility (the terms in square brackets), implied by the recursive preference structure.[25]

[25] Although the defining feature of disaster models is the existence of disaster risk, EZ preferences are commonly used. See Barro (2009), Gourio (2012), for example. The reason is that standard CRRA preferences, in which the intertemporal elasticity of substitution equals the reciprocal of risk aversion, give rise to counterfactual predictions for asset prices in addition to the well-known equity premium puzzle. For example, when risk aversion is larger than 1 under CRRA, an increase in uncertainty raises the price-dividend ratio, and an increase in growth rate lowers it, both of which contradicts the data. See Barro (2009) and Bansal and Yaron (2004) for the benefits of separating risk aversion and IES.

Using cumulant generating functions (Martin, 2013), we can re-write these two expressions as

$$\mathbb{E}(m_{t+1}) = \left(\left(\log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\left(\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots,\right.\right.$$

where $\kappa_{i,t}(u_{t+1})$ is the ith cumulant of u_{t+1}; [26] and

$$\mathbb{E}(\Xi_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots$$

We can make further analytical progress in two special cases. First, suppose countries differ in only one higher-order moment of their shock distributions. For concreteness, assume countries differ in the skewness of their shocks, as emphasized by Barro (2006). With skewness ($\kappa_{3,t}(u_{t+1})$) differing across countries and setting all other cumulants to be identical, we recover the FX-share as

$$\text{FX-share} = \frac{[-\mathbb{E}_t(m_{t+1})] - [-\mathbb{E}_t(m^\star_{t+1})]}{\Xi_t(m_{t+1}) - \Xi_t(m^\star_{t+1})} = \left(\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)^2, \tag{19}$$

which is always positive and close to 1 under standard calibrations ($\gamma > \frac{1}{\psi}$, $\gamma > 1$ and $\psi > 1$). For example, with $\gamma = 8.5$ and $\psi = 2$, as in Gourio, Siemer, and Verdelhan (2013), the FX-share is 0.88, similar to the number obtained in the simulations shown in Table 4.

To see the generality of this result, consider a second special case, where the elasticity of intertemporal substitution ψ is equal to 1, and higher-order moments can differ arbitrarily across countries. With $\psi = 1$, one can show

$$-\mathbb{E}(m_{t+1}) = \mathbb{E}(\Xi_t(m_{t+1})) - \log(\delta) + \mathbb{E}(\Delta c_{t+1}),$$

which implies an FX-share of 1. That is, countries with identical preference parameters and $\psi = 1$ must always have the same interest rate. Any differences in risk premia thus manifest entirely as predictable appreciations of one currency relative to the other.

Both special cases thus strongly suggest that departures from log-normality do not fundamentally change the challenge of addressing the currency premium puzzle, as long as (symmetric) EZ preferences are involved.

Here again, we can trace this choice of parameters directly to the original works in closed-economy asset

[26] Cumulants are functions of the usual central moments.

pricing – Barro (2009) and Gourio (2012). Table 4 shows the FX-shares implied by the preference parameters used in these two seminal papers, if one were to extend these models to allow for countries with heterogeneous skewness in the SDF. [27]

To summarize, the inability of standard models to simultaneously generate a high equity premium, a low and stable risk-free rate, and largely unpredictable exchange rates extends beyond the case of log-normality. In particular, existing disaster models are also subject to the currency premium puzzle.[28] That said, allowing multiple higher cumulants or moments to differ across countries at the same time may offer more degrees of freedom. How such a model should be constructed and what would drive these differences in cumulants is not obvious and requires further research.

5 Incomplete Markets

To what extent can market incompleteness help resolve the currency premium puzzle? In this section, we relax the assumption of complete markets and study the currency premium puzzle in its more general form. Specifically, consider a general environment in which a risk-free asset exists in each country, no-arbitrage and the law of one price hold, and, for convenience, shocks are log-normal.

In this general environment financial markets may be potentially severely incomplete, including segmentation within countries (only intermediaries can access financial markets), segmentation across countries (intermediaries cannot trade foreign assets), and limited spanning (available assets may not fully span underlying shocks). Moreover, the SDF may, but need not, be tied to consumption growth. In the presence of such financial frictions, the change in the exchange rate may be partially or fully disconnected from the ratio of SDFs across countries (Gabaix and Maggiori, 2015; Lustig and Verdelhan, 2019; Itskhoki and Mukhin, 2021), so that equation (3) no longer holds.

Nevertheless, even when exchange rates are driven by noise, cross-border asset trade is severely restricted,

[27]In this calculation, we abstract from the fact that the utility function in Gourio, Siemer, and Verdelhan (2013) includes leisure.

[28]Perhaps more surprisingly, virtually identical results hold for Farhi and Gabaix (2016), who build a disaster model with constant relative risk aversion and tradeable and nontradeable goods. Their conditional carry trade simulation results show a FX-share of 0.69 (calculated from Table III in their paper). Although the time-separable preferences in this model do not force a negative functional relationship between the mean and higher-order moments of the log SDF, this relationship appears to assert itself nevertheless when calibrating the model to reflect low and stable risk-free rates given that countries are symmetric in terms of disaster resilience. Ready, Roussanov, and Ward (2017a) also constructs a two-country disaster model with commodity and producer countries without using EZ preferences. They generate a conditional carry trade return of 4.15% with an interest rate difference of 3.41% (FX-share = 0.2). However, their framework inherits the usual implications of CRRA preferences: risk-free rates are too high, and other problems mentioned in footnote 25. Relatedly, Jurek (2014) and Farhi et al. (2009) find that even when disaster risk is hedged using options, conditional carry trade returns remain large, suggesting disasters are not the sole explanation for systematic variation in currency returns.

and pricing kernels are not necessarily tied to consumption growth, equation (2) continues to hold: As long as a risk-free asset exists in each country, no-arbitrage and the law of one price require that interest rates are pinned down by a stochastic discount factor (e.g., Campbell, 2018). Under log-normal shocks, this dependence reduces to the mean and variance of the log SDF, as shown in equation (2). The only change in this general environment is that m and $m^{\star}$ now denote the log SDFs of the marginal, rather than representative, agents in the home and foreign economies (e.g., intermediaries).

The key issue we highlighted in the context of long-run risk and habit models, is that these models reconcile high equity premia with low and stable risk-free rates by imposing a linear relationship

$$\mathbb{E}_t(m_{t+1}) = const - FXS\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) \tag{20}$$

with $FXS \approx 1$. We denote the coefficient that links the mean of the log SDF to its variance as FXS, as it serves the same role that the FX-share played in complete markets models in terms of linking these two objects. Under complete markets, FXS can be empirically measured as the FX-share. When markets are incomplete, FXS is not directly observable from exchange rate data. We can, however, compute it within the confines of a given model.

Combining (2) with (20) yields

$$r_t = -\mathrm{const} - (1 - FXS)\,\tfrac{1}{2}\,\mathrm{var}_t(m_{t+1}), \tag{21}$$

where $(1 - FXS)$ governs the sensitivity of the risk-free rate to the variance of the log SDF: the closer FXS is to one, the less r_t moves with $\mathrm{var}_t(m_{t+1})$, and in the limit ($FXS = 1$) the risk-free rate is constant.

That is, even with incomplete markets and stochastic discount factors that are divorced from aggregate consumption, any model that relies on an off-setting functional relationship (20) to address the equity premium and risk-free rate puzzles must therefore feature FXS close to one. A large equity premium requires a log SDF with a large, potentially time-varying conditional variance; keeping the risk-free rate low and stable in the face of such variation requires the mean of the log SDF to offset its variance nearly one-for-one (Hansen and Jagannathan, 1991). $FXS \approx 1$ provides such an offset.

Proposition 3 (The currency premium puzzle under incomplete markets)**.** *Suppose that in each country $i \in \{\ ,\star\}$: (i) a one-period risk-free asset denominated in local currency exists, the law of one price holds, and there is no arbitrage, so that a SDF M^i_{t+1} exists such that $\mathbb{E}_t\left(M^i_{t+1}\right)R^i_t = 1$; (ii) the log SDF m^i_{t+1} is conditionally normally* *distributed; and (iii) its conditional moments satisfy the offsetting relation (20), where $const$ and FXS are identical across countries.*[29] *Then the interest rate differential satisfies*

$$r_t^\star - r_t = [1 - FXS]\,\frac{1}{2}(var_t(m_{t+1}) - var_t(m_{t+1}^\star)) \tag{22}$$

In particular, the magnitude of the interest rate differential between the two countries can never exceed $|1 - FXS|$ times half the largest conditional variance of the log SDF the model can produce, and as $FXS \to 1$ interest rate differentials vanish for any configuration of risk across countries.[30]

Proof. Conditions (i) and (ii) imply equation (2) holds in each country. Substituting condition (iii) into (2) for each country and differencing yields (22). □

Under these general conditions, the proposition thus derives the relationship we previewed as equation (1) in the introduction. It shows a general tension between generating a low and stable interest rate in the time series and generating dispersion in interest rates across countries: As long as a stochastic discount factor prices domestic risk-free assets, large and persistent differences in interest rates require correspondingly large differences in the variance of m across countries, and these required differences increase in size as FXS approaches one. At the same time, matching the equity premium and risk-free rate in the time series requires that the sum of the mean and variance of m remains stable over time, and this stability is easiest to achieve with $FXS \approx 1$.[31]

In this sense, the tension between matching the equity premium and risk-free rate puzzles in the time series and generating large and persistent interest rate differentials across countries arises even with highly incomplete markets and does not necessarily rely on the SDF being tied to consumption growth.

Proposition 3 also gives us a simple way to evaluate any candidate model. All we need to know is the FXS implied by the model's structure and the differences in the variances of log SDFs it can plausibly generate.

For example, if the marginal investors pricing the risk-free asset in each country have symmetric EZ utility as in Section 2, equation (20) holds with $FXS = (1-\gamma)/(\frac{1}{\psi}-\gamma)$. Using equation (12), and taking unconditional

[29] Models in which $const$ differs across countries are theories of interest rate differentials based on non-risk forces, which we take up in Section 6.

[30] For models in which the offsetting relation in condition (iii) holds only approximately or locally, equation (22) holds approximately or locally as well.

[31] We should note that, implicitly, for the tension between equity premium and risk-free rate to exist, equity and the risk-free asset must be priced by some common SDF. However, when a risk-free asset exists, it is only natural that whoever have access to the stock market must also be able to trade the risk-free asset. Under incomplete markets the SDF pricing a given bond is not necessarily unique. Condition (ii) should therefore be read as applying to the marginal utility growth of the marginal investor, rather than to an arbitrary admissible discount factor.

expectations of (22) yields

$$\mathbb{E}(r_t^\star - r_t) = \left[1 - \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right]\left(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1}^\star))\right). \tag{23}$$

Similarly, if the marginal investors have habit preferences, equation (20) holds with $FXS = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B}$, so that

$$r_t^\star - r_t = \left[1 - \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B}\right]\left(\frac{1}{2}\,\text{var}_t(m_{t+1}) - \frac{1}{2}\,\text{var}_t(m_{t+1}^\star)\right).$$

Using these relationships, and ignoring the need to match exchange rate data, what would the difference in the variances of log SDFs have to be to generate the observed interest rate differential of 4.8% between our portfolios of high and low interest currencies (Table 1)? Typical calibrations studied in Sections 2 and 3, FXS range from 0.94 (Bansal and Yaron, 2004) to 1.00 (Campbell and Cochrane, 1999), so that the required difference in variances ranges from 1.6 to infinity.

To provide a benchmark, it has generally proven difficult to rationalize stochastic discount factors with a variance exceeding 0.25, the level required to clear the Hansen-Jagannathan bound. For example, the variance of the log SDF is 0.53 in Bansal and Yaron (2004), and 0.25 in Campbell and Cochrane (1999),[32] representing typical values in the literature. One should thus expect *differences* in the variances of SDFs across countries to be substantially smaller than 0.25 — an order of magnitude below what these preferences require to match the observed differences in interest rates across countries.

Taken together, canonical long-run risk and habit preferences thus imply severe limitations in generating large cross-country interest rate differentials, even when augmented with heterogeneity in risk exposure and substantial market incompleteness.

Importantly, these limitations are also not confined to the representative-agent setting. As it turns out, leading heterogeneous-agent models with incomplete markets that quantitatively match both the equity premium and a stable risk-free rate also feature an effective FXS close to one, imposing (20) either through preferences or through equilibrium interactions between different agents.

For example, in Kekre and Lenel (2022), households feature Epstein–Zin preference and heterogeneous risk aversion. With a common intertemporal elasticity of substitution of 0.8 and risk aversions between 10 and

[32]For Bansal and Yaron (2004), see Table V, which reports a volatility of the log SDF being 0.73, implying a variance of 0.53. For Campbell and Cochrane (1999), see Table 2, which reports a Sharpe ratio of 0.5. Given that the maximum Sharpe ratio is approximately the standard deviation of the log SDF (equation (7) in their paper) and that the simulated Sharpe ratios are very close to efficient (Figure 6 in their paper), we use the simulated Sharpe ratio to calculate the approximate variance of log SDF and report 0.25.

25.5 — parameters at which each type's preferences individually imply an offsetting relationship with slope near one — the savings-weighted pricing condition inherits $FXS \approx 1.01$ directly from equation (23).[33] In Gârleanu and Panageas (2015), agents differ in both their IES and risk aversion parameters, and no individual agent delivers the offset — the two types' preferences individually imply FXS of 7.0 and −0.9 — but the joint distribution of preference parameters is chosen so that the consumption-share-weighted blend of their Euler equations does: shocks that raise the conditional variance of the pricing SDF shift the blend across the two types in a way that leaves the interest rate locally unchanged (their Proposition 4), implementing $FXS \approx 1$ at the stationary mean. Chien, Cole, and Lustig (2011) obtain an equilibrium $FXS \approx 0.97$ from CRRA preferences, where the mean reversion of the active traders' wealth share ties the two moments together: the states in which concentrated risk makes the conditional variance of their SDF rise are also those in which these traders expect rapid consumption growth as their wealth share recovers.[34]

Hypothetical multi-country extensions of these heterogeneous-agent models would thus, to the extent that the offset survives aggregation in a multi-country equilibrium, face the same limitations as their representative-agent counterparts: with FXS between 0.97 and 1.01, plausible differences in risk across countries would move interest rates by at most a few basis points — and where FXS exceeds one, in the wrong direction.

By contrast, segmented-markets and intermediary models that price bonds without an offset of the form (20) behave exactly as Proposition 3 predicts in its absence. For example, in Alvarez, Atkeson, and Kehoe (2009), bonds are priced by the Euler equations of the households that actively participate in asset markets, and whose consumption is therefore far more variable than aggregate consumption. Because these active households have time-separable preferences, no offsetting relation links the mean and the variance of their log SDF, and interest rate differentials are free to respond to time-varying risk. The flip side of this freedom is that the variance of the log SDF must remain small for interest rates to remain well-behaved: at their calibration, the implied maximal Sharpe ratio is far below the Hansen–Jagannathan bound.[35]

[33] With a common ψ, the individual slopes $(1-\gamma^i)/(1/\psi-\gamma^i)$ lie between 1.01 and 1.03 for all three types: for risk aversion large relative to $1/\psi$, the slope is close to one regardless of γ^i, so heterogeneity in risk aversion scales the size of the premium without affecting the offset.

[34] Equation (21) implies $\text{std}(r_t) = |1-FXS| \cdot \text{std}\left(\frac{1}{2}\,\text{var}_t(m_{t+1})\right)$. Chien, Cole, and Lustig (2011) report a conditional market price of risk, $\varpi_t \equiv \text{std}_t(M_{t+1})/\mathbb{E}_t(M_{t+1})$, that varies between 0.35 and 0.60 across states. Under conditional log-normality, $\text{var}_t(m_{t+1}) = \log(1+\varpi_t^2)$, so $\frac{1}{2}\,\text{var}_t(m_{t+1})$ ranges from 0.06 to 0.15, implying a standard deviation on the order of 0.03. Combined with their reported $\text{std}(r_t) = 0.10\%$, this yields $|1-FXS| \approx 0.001/0.03 \approx 0.03$. Technically, $FXS \approx 0.97$ or $FXS \approx 1.03$, we report 0.97 in the main text to avoid confusion as this detail does not affect our argument.

[35] At their baseline calibration, the conditional variance of the log SDF is roughly $0.14^2 \approx 0.02$ in annualized terms (their equation (38)). In our own work, we use the same mechanism — concentrating aggregate risk on a subset of active market participants with time-separable preferences (Hassan, 2013; Hassan, Mertens, and Zhang, 2023) — and scale it up to match observed interest rate

Models that do allow the variance of the log SDF to reach empirically relevant levels face the opposite problem. In He and Krishnamurthy (2013), where the specialists pricing bonds also have time-separable preferences, the riskless rate is oversensitive to the aggregate state — falling to −8.81 percent in the state in which the risk premium reaches 12 percent. The authors write that "the quantitative implications regarding the interest rate are the least credible results of our analysis" (p. 755). In Di Tella (2017), agents have Epstein–Zin preferences, but the offsetting relation (20) nevertheless fails to hold in equilibrium, because shocks to idiosyncratic uncertainty depress the interest rate without raising the variance of the aggregate pricing SDF, decoupling the two moments. As a result, a large uncertainty shock sends the riskless rate to "an unrealistic −105.91 percent" (p. 2065).

Matching both closed-economy moments thus requires the offset — whether imposed by preferences or engineered in equilibrium — and with it, the limits on interest rate differentials it entails.

In essence, the same force that allows these models to reconcile high equity premia with smooth risk-free rates in the time series makes it challenging to fit the large, apparently risk-induced, differences in interest rates in the cross-section.

Of course, one can instead assume that risk plays no role in the determination of interest rates. This is, in effect, the route taken by some of the leading models of exchange rate disconnect (Gabaix and Maggiori, 2015; Fanelli and Straub, 2021; Itskhoki and Mukhin, 2021). Most of these are deliberately stylized theories that abstract from risk premia in bond markets altogether; even in the quantitative model of Itskhoki and Mukhin (2021), where the risk-free rate is priced by the household's Euler equation, the model is solved to first order, leaving the household SDF nearly deterministic. These are deliberate modeling choices that sidestep the equity premium and risk-free rate puzzles, allowing these models to focus on other features of the data. But they also imply that, by construction, a country's riskiness cannot affect its interest rate — so that within these models, the large and persistent interest rate differentials we observe must reflect something other than compensation for risk.[36]

The natural candidates are the traditional macroeconomic determinants of interest rates: differences in time preference and long-run growth. We turn to these next, and show they face a symmetric problem —

differentials in Hassan et al. (2025). Doing so is feasible there because the model is static: over multiple periods, the same calibration would produce counterfactually volatile risk-free rates.

[36]A related route relaxes condition (i) of Proposition 3 directly: Jiang et al. (2022) and Jiang, Krishnamurthy, and Lustig (2023) argue that matching the joint behavior of exchange rates and interest rates requires wedges in the bond Euler equations themselves — convenience yields, home currency bias, or financial repression. In such environments, observed safe rates no longer reveal any investor's marginal willingness to save, and the mapping from interest rate differentials to SDF moments need not hold. The challenge then shifts to explaining why these wedges are large, persistent, and aligned with the risk factors in currency markets.

they can move interest rates, but cannot generate currency returns.

6 Heterogeneity in Time Preferences and Long-Term Growth

So far, we have asked whether heterogeneity in countries' risk characteristics can generate large and persistent differences in their interest rates. The answer is a dilemma: models in which the investors pricing bonds demand compensation for risk in line with observed risk premia either impose an offsetting relationship between the mean and the variance of their log SDFs, which immobilizes interest rates, or forgo the offset and fail to match the level and stability of the risk-free rate.

In this section, we ask whether heterogeneity in the non-risk determinants of interest rates can help. We extend the basic model to allow countries to differ in their time preference rates or long-run growth rates — the traditional macroeconomic determinants of interest rates. As we show below, neither force resolves the currency premium puzzle by itself: each moves interest rates, but the differentials it produces are reversed by predictable movements in exchange rates, generating no currency returns. We then consider a more promising avenue, in which heterogeneity in growth rates is correlated with differences in countries' risk exposures.

First, consider simply assigning each country a different time preference rate, δ_i, while keeping the exposure to risk $\mathbb{E}(\text{var}_t(m_{i,t+1}))$ symmetric. For example, in the long-run risk model of Section 2, allowing for a country-specific δ_i in equation (10) shifts the intercept, but not the slope, of the relationship between the mean and the variance of the log SDF:

$$-\mathbb{E}(m_{i,t+1}) = \frac{1}{2}\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\,\mathbb{E}(\text{var}_t(m_{i,t+1})) + \frac{1}{\psi}\mu - \log(\delta_i). \tag{24}$$

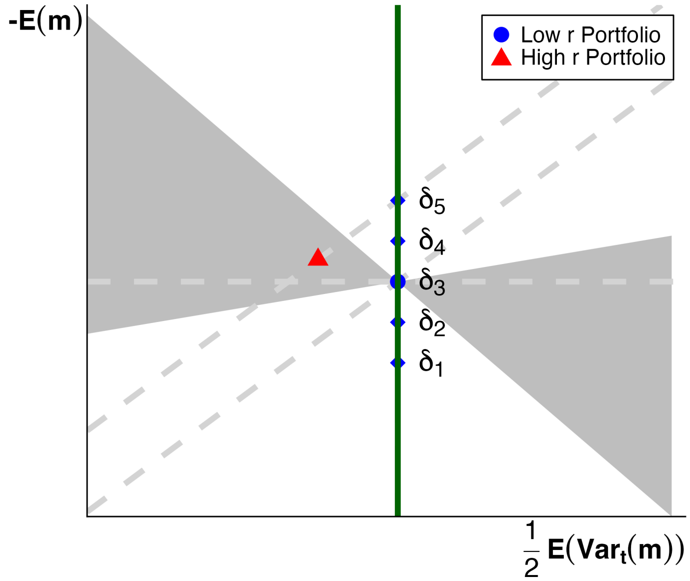
That is, variation in δ_i moves the mean of the log SDF, but not its variance.[37]

Figure 6 shows what this implies in SDF space: because their variances are identical, countries that differ only in their time preferences stack vertically on top of each other.

These vertical differences produce interest rate differentials — more impatient countries have higher interest rates — but no differences in currency returns: by equations (3) and (5), differences in the means of log SDFs move interest rates one-for-one with expected depreciations, while leaving currency premia unchanged. Any interest rate differential induced by heterogeneous time preferences is thus fully reversed

[37] The same is true in the model with external habits; see equations (14) and (15).

Figure 6: Countries with Heterogeneous Time Preferences



This figure plots five countries with different time preferences δ, all of which lie on the vertical heterogeneous time preference (HTP) line. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area represents confidence intervals inferred from the data. Blue squares are examples of countries that only differ in their time preferences. Countries with only growth rate differences also lie on the same vertical line.

by a predictable depreciation of the high-interest-rate currency: uncovered interest parity holds with respect to these differences, just as it does in traditional macroeconomic models that abstract from differences in currency risk (Backus, Kehoe, and Kydland, 1992; Galí and Monacelli, 2005).

The same logic applies to permanent differences in growth rates. A country with a persistently higher growth rate μ_i has a lower mean log SDF (equation (10)) and thus a higher interest rate, but its variance is again unchanged — the country simply moves down the same vertical line in Figure 6. Growth differentials therefore also produce interest rate differentials that are reversed by predictable depreciations, and no currency premia.

Neither of the two macro-based sources of heterogeneity, time preference rates and long-term growth rates, can therefore produce long-lasting differences in interest rates while keeping exchange rates unpredictable. On the other hand, we have seen that differences in risk can produce sizable currency premia, but struggle to generate sizable differences in interest rates across countries.

A plausible solution to this conundrum might be to combine the two forces, pairing heterogeneity in risk characteristics with offsetting differences in long-term growth rates. Recall from equation (5) that the high-premium, risky currency belongs to the country with the lower variance of the log SDF. In the canonical models, with $FXS \approx 1$, this currency earns its premium through a predictable appreciation. But if countries with risky currencies (low $\text{var}(m)$) also grow at faster rates or are less patient (low $\mathbb{E}(m)$), they move up the vertical dimension of the SDF space, transforming the predictable appreciation into a persistent interest rate differential. In the extreme case in which the two forces exactly offset each other, such a model would produce persistent differences in interest rates and fully unpredictable exchange rates.

Andrews et al. (2024) propose a partial equilibrium long-run risk model with this feature — risky countries exogenously have higher consumption growth rates for long periods of time. They start from a long-run risk model very similar to the one we discuss in Section 2, where currencies vary in their expected returns because some countries' currencies are riskier than others. Following our discussion above, their model would ordinarily produce an FX-share close to one. However, they point out that, in the data, countries whose log SDFs have a high variance — those with safe currencies — also exhibit lower consumption growth rates over long periods of time. Effectively, they show that the link between differences in growth rates and differences in SDF variances is well approximated by

$$\hat{\mu}^i = -0.21 \left(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) \right),$$

where $\hat{\mu}^i$ is the difference between country i's long-run consumption growth rate and the world average, and $\mathbb{E}(\text{var}_t(m_{t+1}))$ refers to the world average variance of the log SDF.[38] Although their focus is on other features of the data, this link between long-lasting heterogeneity in the first and second moments of log SDFs significantly reduces the FX-share of currency returns: Because the growth differential shifts the mean of the log SDF by $\frac{1}{\psi}\hat{\mu}^i$, the effective FX-share equals the preference-implied FX-share plus $\frac{1}{\psi}$ times the coefficient in this relation (see Appendix A.4). In their calibration, $\psi = 1$, so the preference-implied FX-share is exactly one and the coefficient passes through one-for-one: the estimated -0.21 results in an FX-share of 0.79 for the static trade. Extended versions of their model that also include heterogeneous rates of inflation can produce FX-shares of 0.38.

Matching the data along this dimension, however, requires a substantially stronger link between the two moments: the FX-share of -0.37 we measure for the static trade corresponds to a coefficient of roughly -1.4 — about seven times the -0.21 implied by observed growth differentials. Any such theory must also explain why the two moments are linked and confront the nonstationarity implied by permanent differences in growth rates.

Kekre and Lenel (2024a) suggest a complementary route in a model with segmented bond markets along

[38] See Appendix A.4 for a detailed derivation.

the lines of Section 5: countries differ in their patience — a non-risk force that would ordinarily be reversed by predictable depreciations — but arbitrageurs with limited risk-bearing capacity must absorb the resulting imbalances, so that a persistent interest rate differential and a currency premium survive in steady state. The differential is priced by the arbitrageurs' capacity rather than by any investor's SDF; whether this mechanism can quantitatively account for the composition of currency premia we document is, as the authors themselves note, a promising open question.

Taken together with the results of the previous sections, the currency premium puzzle thus remains intact: heterogeneity in risk cannot meaningfully move interest rates, heterogeneity in the non-risk determinants of interest rates cannot generate currency returns, and attacking both margins at once is, at observed magnitudes, still too weak, though a promising avenue for further investigation.

7 Conclusion

In this paper, we highlight a fundamental tension between canonical asset pricing models, which have been highly successful in explaining closed-economy puzzles, and the observed behavior of interest rates and exchange rates across countries. To our knowledge, no quantitative model currently explains why New Zealand has paid interest rates five percentage points above Japan's for decades — and paid investors returns to match. We show this is no accident: the same mechanism that allows canonical models to reconcile large equity premia with low and stable risk-free rates — an offsetting relationship between the mean and the variance of the stochastic discount factor — prevents differences in risk from appearing in interest rates.

In canonical asset pricing models, heterogeneity in risk therefore cannot meaningfully move interest rates, while the non-risk forces that move interest rates in canonical macroeconomic models cannot generate currency returns. We term this tension the "currency premium puzzle." The puzzle is robust to stochastic volatility, disaster risk, and other departures from log-normality, and incomplete markets alone do not resolve it: our analysis covers a broad class of environments with segmentation within and across countries, limited spanning, and exchange rates partially or fully disconnected from stochastic discount factors. Across these settings, any model in which the investors pricing bonds demand compensation for risk in line with observed premia faces the same dilemma: impose an offsetting relationship between the mean and the variance of the log SDF and immobilize interest rate differentials, or forgo the offset and fail to match the stability of the risk-free rate in the time series.

To be clear, we do not claim that the currency premium puzzle cannot be solved. Our point is that the standard resolution of the closed-economy asset pricing puzzles does not translate to the standard models of international macroeconomics. Two large literatures have each resolved their own puzzles with tools the other cannot accommodate — which is why a synthesis has been missing. Our results spell out what such a synthesis must deliver: an investor who prices bonds while bearing risk at observed levels, without the offset, and yet with a well-behaved risk-free rate — or a second source of heterogeneity that converts predictable appreciations into interest rate differentials.

Understanding the effect of risk premia on international capital markets requires models in which risk can move interest rates and the price of capital. The currency premium puzzle shows that the current generation of quantitative models does not allow this. We hope that highlighting this puzzle spurs renewed efforts to integrate risk premia into the models of interest rates and exchange rates used in international macroeconomics; such integration is essential for understanding critical phenomena, such as the violation of uncovered interest parity, the global financial cycle, flights to safety, and sudden stops — which all arise from the interplay of risk premia, interest rates, and the allocation of capital across countries.

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A Details on the LRR Models

A.1 Derivation of moments of log SDF

Start from Epstein-Zin (EZ) preferences.

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A1}$$

The stochastic discount factor (SDF) is given by

$$M_{t+1} = \frac{\frac{\partial U_t}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} = \frac{\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}},$$

where the derivatives are

$$\begin{aligned}
\frac{\partial U_t}{\partial U_{t+1}} &= \delta \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
&= \delta U_t^{\frac{1}{\psi}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
\frac{\partial U_t}{\partial C_t} &= (1-\delta) \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} C_t^{-\frac{1}{\psi}} \\
&= (1-\delta) U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}.
\end{aligned}$$

It immediately follows that the SDF is

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.$$

Taking logs on both side, we get the log SDF as

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\left(u_{t+1} - \frac{1}{1-\gamma}\log(\mathbb{E}_t\left[\exp((1-\gamma)u_{t+1})\right]\right)\right).$$

A.1.1 Model without short-run shocks

The consumption process is given by

$$\Delta c_{t+1} = \mu + z_t \tag{A2}$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t. \tag{A3}$$

Assumption 1. *Assume u_{t+1} is normal.*

Plugging these expressions into the log SDF from the previous section results in

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\operatorname{var}_t(u_{t+1})$$

$$\frac{1}{2}\operatorname{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \operatorname{var}_t(u_{t+1}).$$

This implies

$$-\mathbb{E}_t(m_{t+1}) = \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\cdot\frac{1}{2}\operatorname{var}_t(m_{t+1}) - (\log(\delta) - \frac{1}{\psi}\mu) + \frac{1}{\psi}z_t \tag{A4}$$

and the risk-free rate is given by

$$\begin{aligned} r_t &= -\mathbb{E}_t(m_{t+1}) - \frac{1}{2}\operatorname{var}_t(m_{t+1}) \\ &= \frac{1}{\psi}z_t - (\log(\delta) - \frac{1}{\psi}\mu) + \frac{1}{2}\left(\left(\frac{1}{\psi} - \gamma\right)\left(\left(1 - \frac{1}{\psi}\right)\operatorname{var}_t(u_{t+1})\right.\right.\end{aligned} \tag{A5}$$

Long-run shocks z_t move the risk-free rate. Equation (12) follows from unconditional moments

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathbb{E}(\operatorname{var}_t(u_{t+1})) \tag{A6}$$

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\operatorname{var}_t(u_{t+1})). \tag{A7}$$

A.1.2 Model with short-run shocks

Assuming the presence of short-run shocks $\varepsilon_{SR,t}$

$$\Delta c_{t+1} = \mu + z_t + \sigma_{SR}\varepsilon_{SR,t+1},$$
$$z_t = \rho z_{t-1} + \sigma\varepsilon_t.$$

where all shocks are i.i.d. standard normal shocks.

To facilitate the computation of the second moment, we define $V_t = \frac{U_t}{C_t}$. Then

$$V_t = \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(\frac{U_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} = \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(V_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

Note that the conditional covariance of v_{t+1} and Δc_{t+1} is zero. To see this, suppose that after some terminal date T, all shocks are zero and $z_T = 0$. Then, we have

$$\begin{aligned} v_T &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma)\left(v_{T+1} + \Delta c_{T+1}\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma)\left(v_{T+1} + \mu\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right). \end{aligned}$$

Since $v_T = v_{T+1}$, we can solve for v_T as a constant. At $T-1$, we have

$$\begin{aligned} v_{T-1} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma)\left(v_T + \Delta c_T\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma)\left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T}\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right). \end{aligned}$$

$\mathbb{E}_{T-1}\left[\exp(1-\gamma)\left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T}\right)\right]$ is only a function of z_{T-1}, and thus is v_{T-1}. Similarly,

$$\begin{aligned} v_{T-2} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma)\left(v_{T-1} + \Delta c_{T-1}\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma)\left(v_{T-1} + \mu + z_{T-2} + \varepsilon_{SR,T-1}\right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} . \right) \end{aligned}$$

As a result, v_{T-2} is also only a function of z_{T-2}. Using backward induction, we conclude that for any t, v_{t+1} is a function of z_{t+1}, and thus,

$$\mathrm{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0.$$

The reason is that consumption growth depends on z_t, which is known at time t, and the remaining innovation to Δc_{t+1} is uncorrelated with v_{t+1} because the short-run and long-run innovations are independent.

The stochastic discount factor takes the form

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}\frac{C_{t+1}}{C_t}}{\left\{\mathbb{E}_t\left[\left(V_{t+1}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

Taking logs results in

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(v_{t+1} + \Delta c_{t+1} - \frac{1}{1-\gamma}\log \mathbb{E}_t\left[\exp((1-\gamma)(v_{t+1} + \Delta c_{t+1}))\right]\right).$$

Now, assuming v_{t+1} is normal, we have

$$\begin{aligned}\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(\mathbb{E}_t(v_{t+1}) - \mathbb{E}_t(v_{t+1}) - \frac{1}{2}(1-\gamma)\,\mathrm{var}_t(v_{t+1} + \Delta c_{t+1})\right) \\ &= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathrm{var}_t(v_{t+1} + \Delta c_{t+1}).\end{aligned}$$

And the conditional variance of the log SDF is given by

$$\begin{aligned}\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \mathrm{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\mathrm{cov}_t(\Delta c_{t+1}, v_{t+1} + \Delta c_{t+1}) \\ &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \mathrm{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\mathrm{var}_t(\Delta c_{t+1}).\end{aligned}$$

The second equality uses the fact that $\text{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0$. Substituting in the time-series processes, we get

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\mu + z_t) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)(\text{var}_t(v_{t+1} + \Delta c_{t+1}))$$
$$\frac{1}{2}\text{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \text{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\sigma_{SR}^2.$$

Unconditional moments are thus

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}\left(\text{var}_t(v_{t+1} + \Delta c_{t+1})\right)$$
$$\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\text{var}_t(v_{t+1} + \Delta c_{t+1})) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\sigma_{SR}^2.$$

The variance of short-run shocks shifts the intercept in this relation between the mean and the variance. Keeping short-run shock volatilities identical across countries produces the same results as in the main text; if we do allow them to differ across countries, resulting currency premia are given by

$$\frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi} + \gamma\right)(\sigma_{SR}^2 - (\sigma_{SR}^\star)^2).$$

Given that consumption growth volatility is low in the data, differences in the variance of short-run shocks are small and this term is small. We confirm this claim with our quantitative results in Section 2.

A.2 Long-run risk model with stochastic volatility

A.2.1 Main results

In this section, we explore if adding time variation in the second moments might help in mitigating the currency premium puzzle. Section 2 abstracts from this mechanism. Following Bansal and Yaron (2004), many long-run risk models feature stochastic volatility to generate time-varying risk premia. Here, we examine the role of stochastic volatility on the FX-share.

With stochastic volatility, the endowment process for the long-run risk model is given by

$$\Delta c_{t+1} = \mu + z_t + \sigma_{SR}\varepsilon_{SR,t+1}$$
$$z_t = \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t}$$
$$w_t^2 = (1-\phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t},$$

where w_t^2 induces stochastic volatility and ϕ governs its persistence and σ_w its volatility.[39] The remaining setup is identical to the model in Section 2. While we keep short-run shocks in the setup for now, we ultimately abstract from them by setting $\sigma_{SR} = 0$.

When including stochastic volatility, a closed-form solution is only available for approximations of the model. Using a log-linearization with risk adjustments to solve the model, we show that[40]

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)(A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2) \tag{A8}$$
$$\frac{1}{2}\mathbb{E}(\text{var}_t(\hat{m}_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 (A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2). \tag{A9}$$

The various coefficients labeled A are constants and the variables with a hat denote deviations from the deterministic steady state. Substituting out the right-hand side shows the tight link between first and second moments of the log SDFs:

$$-\mathbb{E}(\hat{m}_{t+1}) = \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\cdot\frac{1}{2}\mathbb{E}(\text{var}_t(\hat{m}_{t+1})).$$

As a consequence, adding stochastic volatility to the model does not help to resolve the currency premium puzzle. There remains a tight link between the mean and the variance of the log SDF.

A.2.2 Detailed derivation

To derive the link between the first and second moments of the log SDF under stochastic volatility, we use risk-adjusted affine approximation.Dividing both sides of the EZ preference by C_t and defining $V_t = U_t/C_t$,

[39]As in Bansal and Yaron (2004), the linear-Gaussian volatility process can violate positivity in extreme draws; we follow the standard practice of treating it as a local approximation (truncating at zero in simulations).

[40]See a detailed proof in Section A.2.2.

we have

$$
\begin{aligned}
\exp\left(\left(1-\frac{1}{\psi}\right)v_t\right) &= 1-\delta+\delta\exp\left(\left(1-\frac{1}{\psi}\right)q_t\right)\\
q_t &= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(v_{t+1}+\Delta c_{t+1})\right)^{1-\gamma}\right]\right\}\\
m_{t+1} &= \log(\delta)-\frac{1}{\psi}\Delta c_{t+1}+\left(\frac{1}{\psi}-\gamma\right)\left(v_{t+1}-q_t+\Delta c_{t+1}\right).
\end{aligned}
$$

Log-linearizing the system around the deterministic steady state, we get

$$
\begin{aligned}
\Delta\hat{c}_{t+1} &= z_t+\sigma_{SR}\varepsilon_{SR,t+1}\\
z_t &= \rho z_{t-1}+w_{t-1}\varepsilon_{LR,t}\\
w_t^2 &= (1-\phi)w_0^2+\phi w_{t-1}^2+\sigma_w\varepsilon_{w,t}\\
\hat{v}_t &= \delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)\hat{q}_t\\
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1}+\left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1}-\hat{q}_t+\Delta\hat{c}_{t+1}.\right)
\end{aligned}
$$

Using the log SDF and the link between $\hat{q}_t$ and $\hat{v}_t$, we obtain the following equation

$$
\hat{q}_t = \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(\hat{v}_{t+1}+\Delta\hat{c}_{t+1})\right)^{1-\gamma}\right]\right\}.
$$

We guess the functional form of the solution as

$$
\hat{v}_t = A_{v0}+A_{vz}z_t+A_{vw}w_t^2
$$

and plug it into the equation immediately above to obtain

$$
\begin{aligned}
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\left(A_{v0}+A_{vz}z_t+A_{vw}\hat{w}_t^2\right) &= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0}+A_{vz}z_{t+1}+A_{vw}\hat{w}_{t+1}^2+z_t+\sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0}+A_{vz}\rho z_t+A_{vz}\hat{w}_t\varepsilon_{LR,t+1})\right)^{1-\gamma}\right]\right\} \\
&\quad+\frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{vw}(1-\phi)w_0^2+A_{vw}\phi w_t^2+A_{vw}\sigma_w\varepsilon_{w,t+1})\right)^{1-\gamma}\right]\right\} \\
&\quad+\frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(z_t+\sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= A_{v0}+A_{vz}\rho z_t+\frac{1}{2}(1-\gamma)A_{vz}^2w_t^2+A_{vw}(1-\phi)w_0^2+A_{vw}\phi w_t^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+z_t+\frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
&= A_{v0}+A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2+(1+A_{vz}\rho)z_t+\left(\frac{1}{2}(1-\gamma)A_{vz}^2+A_{vw}\phi\right)w_t^2.
\end{aligned}
$$

Matching coefficients results in

$$
\begin{aligned}
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{v0} &= A_{v0}+A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vz} &= 1+A_{vz}\rho \\
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vw} &= \frac{1}{2}(1-\gamma)A_{vz}^2+A_{vw}\phi.
\end{aligned}
$$

As a result, the three coefficients are given by

$$
\begin{aligned}
A_{vz} &= \frac{1}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-\rho} \\
A_{vw} &= \frac{\frac{1}{2}(1-\gamma)A_{vz}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-\phi} \\
A_{v0} &= \frac{A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-1}.
\end{aligned}
$$

We can then solve for the pricing kernel:

$$
\begin{aligned}
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t + \Delta\hat{c}_{t+1}\right) = -\gamma\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\hat{v}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(A_{v0} + A_{vz}(\rho z_t + w_t\varepsilon_{LR,t+1}) + A_{vw}((1-\phi)w_0^2 + \phi w_t^2 + \sigma_w\varepsilon_{w,t+1})\right) \\
&\quad - \left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}(A_{v0} + A_{vz}z_t + A_{vw}w_t^2)\right)
\end{aligned}
$$

This expression simplifies to

$$
\begin{aligned}
\hat{m}_{t+1} = &-\left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2\right) \\
&- \frac{1}{\psi}z_t + \left(\frac{1}{\psi} - \gamma\right)\left(-\frac{1}{2}(1-\gamma)A_{vz}^2\right)w_t^2 + \left(\frac{1}{\psi} - \gamma\right)A_{vz}w_t\varepsilon_{LR,t+1} \\
&+ \left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1} + \left(\frac{1}{\psi} - \gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right).
\end{aligned}
$$

The conditional mean of the log SDF is thus

$$
\mathbb{E}_t(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vz}^2w_t^2 \qquad \text{(A10)}
$$

and the conditional variance

$$
\frac{1}{2}\operatorname{var}_t(\hat{m}_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vz}^2w_t^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vw}^2\sigma_w^2. \qquad \text{(A11)}
$$

Note that the conditional variance is independent of z_t.

Taking unconditional expectations, we get a link between the mean and variance of the log SDF

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)\left((1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vz}^2 w_0^2\right.$$

$$\frac{1}{2}\mathbb{E}(\mathrm{var}_t(\hat{m}_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vz}^2 w_0^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vw}^2\sigma_w^2$$

$$= \frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi} + \gamma\right)\left(\sigma_{SR}^2 - \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\mathbb{E}(\hat{m}_{t+1})\right.$$

Setting $\sigma_{SR} = 0$ yields (A8) and (A9).

A.3 Numerical Simulation for Carry Trade Returns

This section evaluates the quantitative performance of the LRR models in Table 2 in terms of the conditional carry trade. See Table A1 below.

Table A1: Conditional Carry Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	1.71	0.62	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	0.44	6.11	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	0.23	9.54	No	No
Colacito and Croce (2013) JF	0.05	-0.35	0.41	-7.02	No	No

This table shows the simulation results for the four long-run risk models of Table 2 with identical parameter values. Instead of the static trade, this table focuses on the conditional carry trade return. The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

Note that the general patterns are essentially unchanged compared to static trade (Table 2). Relative to the static trade, the conditional carry trade allows investors to re-sort their portfolios each month based on contemporaneous interest differentials. In the data, returns to the conditional carry trade and the static trade are typically similar, because interest differentials are large and highly persistent, so the incremental gain from monthly re-sorting is limited (Hassan and Mano, 2019). The best-performing specification in Table 2—the heterogeneous-country model of Colacito et al. (2018a)—reflects this fact, with a mean annual return on the conditional carry trade of 4.47%. However, the average long–short interest rate differential is again too small—1.71%, compared to 7.11% in the data. The model's implied FX-share for the conditional carry trade is 0.62: it remains of the wrong sign (high-interest-rate currencies are again expected to appreciate on average), though it is smaller than the corresponding FX-share of 0.93 for the static (unconditional) trade.

This modest reduction in the FX-share is mechanical. Note that (A10) and (A11) implies

$$\mathbb{E}_t(\Delta s_{t+1}) = \frac{1}{\psi}(z_t - z_t^{\star}) + \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\,\mathbb{E}_t(rx_{t+1}) \tag{A12}$$

and

$$r_t^{\star} - r_t = -\frac{1}{\psi}(z_t - z_t^{\star}) + \frac{\frac{1}{\psi}-1}{\frac{1}{\psi}-\gamma}\,\mathbb{E}_t(rx_{t+1}). \tag{A13}$$

In other words, uncovered interest parity holds for any variation in interest rates induced by $(z_t - z_t^{\star})$; and of course more generally for all variation in interest rates induced by differences in the mean, rather than the variance, of the log SDF. As a result, the conditional FX-share is equal to the unconditional FX-share, but attenuated towards zero, because the numerator now contains additional predictable variation in exchange rates that is unrelated to returns.

In the models in the bottom three rows of Table A1, currency returns are non-zero due to changes in volatility depending on the state of the economy. This stochastic volatility results in temporary differences in the variance of log SDFs across countries and thus produces small currency premia.

As a consequence of sizable expected exchange rate movements and small currency returns, the models produce very large FX-shares. In keeping with our discussion in Section 2, these small conditional currency returns are predominantly generated by predictable variation in exchange rates, generating FX shares in excess of 100%. Note that, however, if one examines the interest rate differential and change in exchange rates, one can see that they approximately offset each other, i.e., UIP holds approximately in these models. This violates our property 1.

A.4 Long-run Risk Models with Heterogeneous Growth

A.4.1 Theoretical Results

In this subsection, we study the effect of allowing countries to feature different growth rates in addition to heterogeneous risk characteristics. In particular, we study the model in Andrews et al. (2024).

The model is largely identical to the one in A.1.1,[41] except for the following two features: First, countries differ in their loadings on a global long-run process; second, different countries feature different growth rates. With these changes, consumption growth rates are given by

$$\begin{aligned} \Delta c_{i,t+1} &= \mu^i + \beta_c^i x_{c,t} \\ x_{c,t} &= \rho_c x_{c,t-1} + \sigma_{x,c} \varepsilon_{c,t}, \end{aligned}$$

where $x_{c,t}$ is the time-varying component in expected global growth (or a global long-run process) that is common to all countries. Different countries' consumption growth rates have different loadings β_c^i on this component.[42] Country-specific growth rates are related to the loadings via the following equation:

$$\mu^i = \mu + \hat{\mu}^i \tag{A14}$$

where $\hat{\mu}^i = \mu(1-\beta_c^i)$, so that countries with higher loadings on the global long-run process — those with safe currencies, such as Japan — also grow slower.

The relationship between mean and variance of log SDF is now given by

$$-\mathbb{E}(m_{i,t+1}) = \frac{1-\gamma}{\frac{1}{\psi}-\gamma} \cdot \frac{1}{2}\mathbb{E}(\text{var}_t(m_{i,t+1})) - (\log(\delta) - \frac{1}{\psi}(\mu + \hat{\mu}^i)). \tag{A15}$$

If we set $\hat{\mu}^i = 0$ for all countries, (A15) implies the familiar FX-share of $\frac{1-\gamma}{\frac{1}{\psi}-\gamma}$. In fact, if we use the calibration of Andrews et al. (2024) and set $\psi = 1$, symmetric growth rates imply identical risk-free rates across countries and an FX-share of 1 such that all currency premia are accounted for by expected changes in exchange rates.

[41] Andrews et al. (2024) do feature short-run shocks, but the volatilities are assumed to be the same across countries, thus do not affect currency premium, exchange rates, or interest rate differentials. One can thus safely set them to 0 for simplicity.

[42] Andrews et al. (2024) also feature an inflation process similar to that of consumption growth. We discuss this more complicated model in our quantitative analysis and focus on the real model for simplicity for now.

With heterogeneous $\hat{\mu}^i$, the FX-share is given by

$$\begin{aligned}\text{FX-share}^{\text{LRR}}_{\text{Heter Growth}} &= \frac{(-\mathbb{E}(m^i)) - (-\mathbb{E}(m))}{\frac{1}{2}\mathbb{E}(\text{var}_t(m^i)) - \frac{1}{2}\mathbb{E}(\text{var}_t(m))} \\ &= \frac{1-\gamma}{\frac{1}{\psi}-\gamma} + \frac{1}{\psi}\frac{\hat{\mu}^i}{\frac{1}{2}\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}))}\end{aligned}$$

where the country without a superscript denotes the average country with $\hat{\mu}^i = 0$. This implies that one can pin down the FX-share by examining the relationship between $\hat{\mu}^i$ and the variances (currency premia) in the model. To see this, note that re-arranging yields

$$\hat{\mu}^i = \psi\left(\text{FX-share} - \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)\cdot\left(\frac{1}{2}\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}))\right). \tag{A16}$$

If we set $\psi = 1$ as in Andrews et al. (2024), the above equation further simplifies to

$$\hat{\mu}^i = (\text{FX-share} - 1)\cdot\left(\frac{1}{2}\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}))\right) \tag{A17}$$

Thus, one can approximate the model implied FX-share by running a regression of $\hat{\mu}^i$ on $\frac{1}{2}\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}))$. Using the calibration of Andrews et al. (2024), the estimated coefficient is -0.206, which implies an FX-share of 0.79. As we will show in our quantitative exercises, this approximation is quite accurate.

A.4.2 Quantitative Results

We solve and simulate the full model in this subsection. The model is partial equilibrium and can be solved analytically. We thus show theoretical results along with simulation results whenever possible. Theoretical results are obtained by solving the object in closed form and then plugging in the calibration.

Agents feature the following preference

$$U_{i,t} = (1-\delta)\log C_{i,t} + \delta\frac{1}{1-\gamma}\log\mathbb{E}_t\, e^{(1-\gamma)U_{i,t+1}}$$

The elasticity of substitution is equal to one, i.e., $\psi = 1$. Absent differences in growth rates or inflation, these preferences imply that all countries share the same unconditional risk-free rate (see the theoretical results in Section 2) and the FX-share is one.

Table A2: ACCG Results: Static Trade

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	-	-
Real Simulation	5.76	4.87	0.89	0.85	Yes	No
Nominal Simulation	3.95	2.61	1.34	0.66	Yes	No
Real: Theory	4.41	3.49	0.92	0.79	Yes	No
Nominal: Theory	3.90	2.26	1.64	0.58	Yes	No
Nominal Sim: EW	3.13	1.81	1.31	0.77	Yes	No

This table shows the static trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. We also report the real and nominal theoretical returns. Portfolios are constructed in the same way as our data (see Appendix C). All moments are annual.

There is a long-run component in both consumption and inflation, and we have

$$\begin{bmatrix} x_{\pi,t} \\ x_{c,t} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ \rho_{c\pi} & \rho_c \end{bmatrix} \cdot \begin{bmatrix} x_{\pi,t-1} \\ x_{c,t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{x,\pi} & 0 \\ 0 & \sigma_{x,c} \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi_t} \\ \varepsilon_{c,t} \end{bmatrix}$$

Consumption and inflation follow

$$\Delta c_{i,t+1} = \mu^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c$$

$$\pi_{i,t+1} = \mu_\pi^i + \beta_i^\pi x_{\pi,t} + \sigma_\pi \eta_{i,t+1}^\pi$$

Table A3: ACCG Results: Conditional Carry Trade

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Real Simulation	5.63	4.74	0.89	0.84	Yes	No
Nominal Simulation	3.74	1.76	1.98	0.47	Yes	No
Nominal Sim: EW	3.17	1.19	1.98	0.38	Yes	No

This table shows the conditional carry trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. Portfolios are constructed in the same way as our data (see Appendix C). All moments are annual.

All shocks are i.i.d. normal. Country-specific mean growth rates and inflation rates

$$\mu^i = \bar{\mu}_c + \bar{\mu}_c(1 - \beta^i_c)$$
$$\mu^i_\pi = \bar{\mu}_\pi - \bar{\mu}_\pi(1 - \beta^i_\pi)$$

These equations highlight the assumed link between growth rates and loadings on shocks.

The real log SDF is

$$\begin{aligned} m_{i,t+1} = \log(\delta) - \frac{1}{2}(1-\gamma)^2\sigma_c^2 - \mu^i - \frac{1}{2}\left[(k^i_{\varepsilon c}\sigma_{x,c})^2 + (k^i_{\varepsilon\pi}\sigma_{x,\pi})^2\right]\Bigg(\\ - \beta^i_c x_{c,t} - k^i_{\varepsilon c}\sigma_{x,c}\varepsilon_{c,t+1} + k^i_{\varepsilon\pi}\sigma x, \pi\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta^c_{i,t+1}, \end{aligned}$$

from which we can compute the unconditional expectation

$$\mathbb{E}(m^{real}_{i,t+1}) = \log(\delta) - \frac{1}{2}(1-\gamma)^2\sigma_c^2 - \mu^i - \frac{1}{2}\left[(k^i_{\varepsilon c}\sigma_{x,c})^2 + (k^i_{\varepsilon\pi}\sigma_{x,\pi})^2\right]\Bigg(,$$

where

$$
\begin{aligned}
k^i_{\varepsilon c} &= (\gamma - 1)\beta^i_c \left(\frac{\delta}{1 - \delta\rho_c}\right) \\
k_\varepsilon \pi^i &= -\rho_{c\pi} k^i_{\varepsilon c} \left(\frac{\delta}{1 - \delta\rho_\pi}\right).
\end{aligned}
$$

The unconditional expectation of the nominal SDF is linked to the expectation of the real log SDF via

$$
\mathbb{E}(m^{nominal}_{i,t+1}) = \mathbb{E}(m^{real}_{i,t+1}) - \mu^i_\pi.
$$

From these expressions, we get an average risk-free rate of

$$
\mathbb{E}(r^{i,real}) = \mu^i - \log(\delta) + \left(\frac{1}{2} - \gamma\right) \sigma^2_c.
$$

The unconditional expectation of nominal risk-free rates is given by

$$
\mathbb{E}(r^{i,nominal}) = \mu^i + \mu^i_\pi - \log(\delta) + \left(\frac{1}{2} - \gamma\right) \sigma^2_c - \frac{1}{2}\sigma^2_\pi.
$$

Using these relationships, the currency premium (note that σ_π are the same across countries) is

$$
\begin{aligned}
\mathbb{E}(rx) &= r^{i,real} - r^{US,real} - \mathbb{E}\left(m^{US} - m^i\right) \\
&= r^{i,nominal} - r^{US,nominal} + m^{i,nominal} - m^{US,nominal} \\
&= \frac{1}{2}\left[(k^{US}_{\varepsilon c}\sigma_{x,c})^2 + (k^{US}_{\varepsilon\pi}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k^i_{\varepsilon c}\sigma_{x,c})^2 + (k^i_{\varepsilon\pi}\sigma_{x,\pi})^2\right].
\end{aligned}
$$

The expected change in real exchange rates is

$$
\begin{aligned}
\mathbb{E}(\Delta s) &= \mathbb{E}(m^{real}_{US}) - \mathbb{E}(m^{real}_i) \\
&= -\mu^{US} + \mu^i - \frac{1}{2}\left[(k^{US}_{\varepsilon c}\sigma_{x,c})^2 + (k^{US}_{\varepsilon\pi}\sigma_{x,\pi})^2\right] + \frac{1}{2}\left[(k^i_{\varepsilon c}\sigma_{x,c})^2 + (k^i_{\varepsilon\pi}\sigma_{x,\pi})^2\right],
\end{aligned}
$$

and the expected change in nominal exchange rates

$$\begin{aligned}\mathbb{E}(\Delta s) &= \mathbb{E}(m_{US}^{nominal}) - \mathbb{E}(m_i^{nominal}) \\ &= -\mu^{US} + \mu^i - \mu_\pi^{US} + \mu_\pi^i - \frac{1}{2}\left[(k_{\varepsilon c}^{US}\sigma_{x,c})^2 + (k_{\varepsilon\pi}^{US}\sigma_{x,\pi})^2\right] + \frac{1}{2}\left[(k_{\varepsilon c}^{i}\sigma_{x,c})^2 + (k_{\varepsilon\pi}^{i}\sigma_{x,\pi})^2\right].\end{aligned}$$

Using the model to pin down real interest rate differentials

$$r^{i,real} - r^{US,real} = \mu^i - \mu^{US}$$

and nominal interest rate differentials

$$r^{i,nominal} - r^{US,nominal} = \mu^i - \mu^{US} + \mu_\pi^i - \mu_\pi^{US},$$

we get that both nominal and real interest rates are identical when growth rates and inflation are the same across countries. This confirms our results in Section 2.

Because the model is solved in closed form, we show both theoretical and simulation results for the static trade in Table A2. The discrepancy between the two originates from the fact that when doing simulation, we follow Andrews et al. (2024) where the consumption and inflation news processes are not purely random (see the table notes for details). Allowing different countries to feature different growth rates reduces the FX-share, but it is not enough to match the empirical result. Note that the theoretical FX-share of 0.79 is well approximated by our regression (A17). Adding inflation processes further reduces FX-share, but again results in FX-shares much higher than in the data. In fact, in Andrews et al. (2024), inflation processes and growth rates are virtually equivalent in terms of their effects on the FX-share, it is thus not surprising that adding more heterogeneity would alleviate the problem.

Table A3 shows the simulation results for conditional carry trade. A similar pattern emerges: Allowing growth rates and inflation to differ reduces the FX-share, but not by enough to match the data.[43]

[43] In their table 11, Andrews et al. (2024) reports an FX-share of $\frac{0.85}{2.75} = 0.31$. This is due to a different weighting scheme (GDP-weighted vs forward-premium-weighted).

B Details on the Disaster Models

The Gourio, Siemer, and Verdelhan (2013) model features Epstein-Zin preferences, and thus, the SDF is given by

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

As a result, the mean log SDF is given by

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\mathbb{E}_t\left((1-\gamma)u_{t+1}\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right). \tag{A18}$$

To derive the entropy $\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1})$, we compute $\log \mathbb{E}_t(M_{t+1})$ via

$$\begin{aligned}
\log \mathbb{E}_t(M_{t+1}) &= \log \mathbb{E}_t\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right) \\
&= \log(\delta) + \log\left(\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right]\mathbb{E}_t\left[\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right] + \mathrm{cov}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right]\right).
\end{aligned}$$

We consider the special case where $\frac{C_{t+1}}{C_t}$ is known at time t. That is, the entropy of consumption growth is zero. This assumption is a direct extension of the "no-short-run-shocks" assumption in the long-run risk framework. We show in Table 4 that this assumption does not drive the results. With this simplification, we have

$$\begin{aligned}
\log \mathbb{E}_t(M_{t+1}) &= \log(\delta) + \log \mathbb{E}_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right) + \log \mathbb{E}_t\left(\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right) \\
&= \log(\delta) - \frac{1}{\psi}(\Delta c_{t+1}) + \left[\log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right)\right].
\end{aligned}$$

Combining with (A18), the entropy is

$$\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) = -\mathbb{E}_t\left(\left(\frac{1}{\psi} - \gamma\right) u_{t+1}\right) + \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right). \tag{A19}$$

By comparing the two terms on the right-hand side of equations (A18) and (A19), we can already see a tight link between the two. In particular, if $\psi = 1$, we have

$$-\mathbb{E}_t(m_{t+1}) = \Xi_t(m_{t+1}) - \log(\delta) + \mathbb{E}_t(\Delta c_{t+1}).$$

Taking unconditional expectations, we have

$$-\mathbb{E}(m_{t+1}) = \mathbb{E}(\Xi_t(m_{t+1})) - \log(\delta) + \mathbb{E}(\Delta c_{t+1}).$$

This is a line with a slope of 1 in the generalized SDF space with entropy on the x-axis. Consequently, as long as all countries feature the same preference and consumption growth rate, they all lie on the same line, which coincides with an iso-rf line. That is, when $\psi = 1$, all countries feature the same risk-free rates.

In standard calibrations, ψ is typically close to but larger than 1. In this case, we can use cumulant generating functions and obtain

$$\begin{aligned}
\Xi_t(m_{t+1}) &= \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \mathbb{E}_t\left(\left(\frac{1}{\psi} - \gamma\right) u_{t+1}\right) \\
&= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \kappa_{2,t}(u_{t+1}) + \frac{1}{6}\left(\frac{1}{\psi} - \gamma\right)^3 \kappa_{3,t}(u_{t+1}) + \frac{1}{24}\left(\frac{1}{\psi} - \gamma\right)^4 \kappa_{4,t}(u_{t+1}) \ldots \\
\mathbb{E}_t(m_{t+1}) &= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) - \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\left(\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) - \mathbb{E}_t\left((1-\gamma)u_{t+1}\right)\right) \\
&= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) \\
&\quad - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\kappa_{2,t}(u_{t+1}) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi} - \gamma\right)\kappa_{3,t}(u_{t+1}) - \frac{1}{24}(1-\gamma)^3\left(\frac{1}{\psi} - \gamma\right)\kappa_{4,t}(u_{t+1}) + \ldots.
\end{aligned}$$

Taking unconditional expectations, we end up with

$$\begin{aligned}
\mathbb{E}(m_{t+1}) &= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots, \\
\mathbb{E}(\Xi_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi} - \gamma\right)^3 \mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots
\end{aligned}$$

If we only allow skewness ($\mathbb{E}(\kappa_{3,t}(u_{t+1}))$) to differ across countries and set all other cumulants to be identical, we get

$$\mathbb{E}(m_{t+1}) = -\frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \text{constant},$$
$$\mathbb{E}(\Xi_t(m_{t+1})) = \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \text{constant}$$

Substituting $\mathbb{E}(\kappa_{3,t}(u_{t+1}))$ out, we end up with

$$-\mathbb{E}(m_{t+1}) = \left(\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)^2 \mathbb{E}(\Xi(m_{t+1})) + \text{constant}$$

The mean-entropy pairs of all countries should satisfy the above equation. What's more, the FX-share is now given by

$$\text{FX-share} = \frac{\mathbb{E}(\Delta s)}{\mathbb{E}(\Delta rx)} = \frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})]}{\mathbb{E}(\Xi_t(m_{t+1}) - \Xi_t(m^{\star}_{t+1}))} = \left(\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)^2$$

If only unconditional skewness are allowed to differ across countries, the FX-share is again directly pinned down by preference parameters and is very close to 1 under standard calibrations.

C Portfolio Construction and Alternative Estimates

In this section, we discuss the construction of static trade and conditional carry trade portfolios in this paper, as well as estimates of the FX-share using updated data and alternative regression-based methods.

C.1 Portfolio Construction

In terms of portfolio construction, We follow the procedures in Hassan and Mano (2019).[44] We use the Hassan and Mano (2019) portfolios because of their close connection to regression-based results, on which we would base our alternative estimates of the FX-share in this section.[45] Static trade and conditional carry trade returns are constructed by forming a portfolio of currencies weighted by their forward premium relative to the US

[44]Our results are robust to alternative portfolio construction methods, for example, the equal-weighted method utilized in Lustig, Roussanov, and Verdelhan (2011). The broad pattern that carry traders lose money on the exchange rate is also evident in Table 1 of Lustig, Roussanov, and Verdelhan (2011).

[45]We only provide essential information in this section. Interested readers should refer to Hassan and Mano (2019) for details.

(equivalently, their risk-free rates). Letting $f_{i,t}$ be the log one-period forward exchange rate of currency i at time t, and let $s_{i,t}$ be the spot rate, both quoted in units of currency i per US dollar, by covered interest-rate parity, we have

$$r_{i,t} - r_{US,t} = f_{i,t} - s_{i,t} = fp_{i,t}.$$

Sorting currencies by their forward premia ($fp_{i,t}$) is thus equivalent to sorting currencies on their risk-free rates. Letting $rx_{i,t+1} = f_{i,t} - s_{i,t+1} = r_{i,t} - r_{US,t} - \Delta s_{i,t+1}$ be the currency premium of currency i relative to the US dollar, our static trade return is then given by

$$\frac{1}{T}\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)],$$

where $\overline{fp}_i^e$ denotes the estimated[46] forward premium of currency i over time and $\overline{fp}^e = \frac{1}{N}\Sigma_i \overline{fp}_i$. Intuitively, investors conducting the static trade would weight the currencies using their long-term forward premium, longing high-interest-rate currencies and shorting low-interest-rate ones. They fix their portfolio (the weights, $\overline{fp}_i^e - \overline{fp}^e$, do not change over time), thus conducting a "static" carry trade.

The conditional carry trade return is given by

$$\frac{1}{T}\Sigma_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)],$$

where $\overline{fp}_t = \frac{1}{N}\Sigma_i fp_{i,t}$. The only difference from static trade is that the weights now change over time. In each period, investors would weight currencies based on their forward premium in that period.

For both of these trades, we can view the currency portfolio that investors long as one representative country, and the currency portfolio that investors short as another representative country. Using the static trade portfolio as an example, mathematically, the currency return for the representative high-interest-rate country relative to the US is given by

$$\frac{1}{T}\Sigma_{i\in\{\text{for all} i \text{ s.t. } \overline{fp}_i^e - \overline{fp}^e > 0\},t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)].$$

The risk-free rate (or forward premium) and exchange rate of this representative country are defined in a

[46]From an investor's perspective, a currency's forward premium over the whole sample period is unknown. We assume investors simply expect $\overline{fp}_i$ to be equal to the mean of $fp_{i,t}$ across all available data prior to the investment period. We do the same thing when running all our simulations in the paper.

similar manner, simply replacing $rx_{i,t+1}$ with $fp_{i,t}$ and $-\Delta s_{i,t+1}$, respectively. Using these returns, as well as their composition, we could infer FX-shares for these representative countries as in the main text of this paper.

C.2 FX-shares using Updated Data

In the main text, we used the same data set as in Hassan and Mano (2019) for consistency, which ends at June 2010. In this section, we update our data set to June 2024. The updated empirical results are shown in Tables A4 and A5.

Table A4: FX-Share in the Data: Static Trade

	Return (%)	Change in FX (%)	Interest Rate Diff (%)	FX-share
	$\mathbb{E}(rx)$	$\mathbb{E}(\Delta s)$	$\mathbb{E}(r^\star - r)$	$\frac{\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$
1/1995 - 6/2024	2.62	-2.46	5.08	-0.94
	[-0.06,5.60]	[-5.86,0.33]	[1.07,9.41]	$[31.55, +\infty) \cup (-\infty, 0.10]$
No Fixed	3.03	-2.13	5.17	-0.70
	[0.32,5.90]	[-5.55,0.72]	[1.24,9.41]	[-7.12,0.21]
1/2000-6/2024	2.70	-2.60	5.31	-0.96
	[-0.12,6.01]	[-6.63,0.77]	[1.06,10.37]	$[15.77, +\infty) \cup (-\infty, 0.23]$
No Fixed	2.81	-2.61	5.42	-0.93
	[-0.11,6.11]	[-6.73,0.91]	[1.12,10.29]	$[23.16, +\infty) \cup (-\infty, 0.26]$

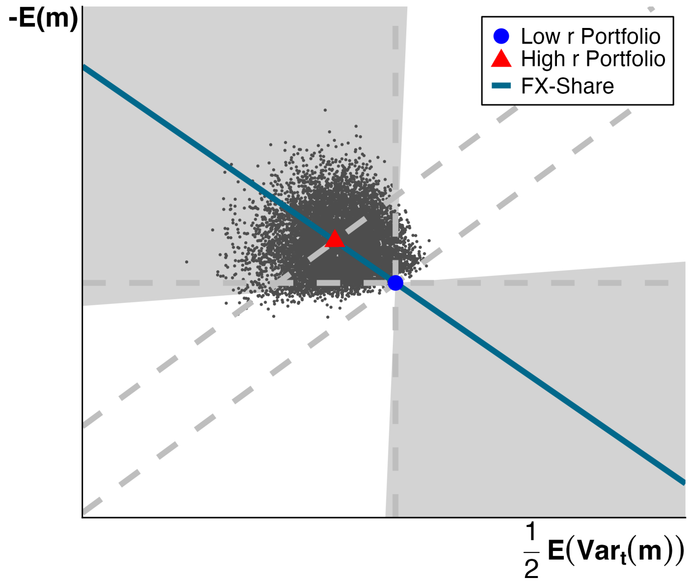
We use the same set of countries as the 1-rebalance sample in Hassan and Mano (2019), but update the data to June 2024. All moments are annual. Static trade returns are calculated by first sorting the unconditional forward premia (interest rates), and then always shorting a weighted portfolio of the currencies with below-average unconditional forward premia, and longing a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C.1. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. We show two different investment periods: 1/1995-6/2024, which starts at the same date as Hassan and Mano (2019), and 1/2000-6/2024, which give the investors more periods to form their expectations of forward premia. Results labeled "No Fixed" removes currencies with de facto fixed exchange rates (HKG, SAU, KWT) according to Ilzetzki, Reinhart, and Rogoff (2019).

Table A4 shows the static trade return, expected change in exchange rates, interest rate differences, as well as FX-share estimated using the updated sample. We show results for two alternative investment periods: 1/1995-6/2024 (same starting period as Hassan and Mano (2019)), and 1/2000- 6/2024.[47] We assume that investors use data before the investment period to form their expectations of forward premia ($\overline{fp_i}^e$): they simply used the unconditional mean of the forward premia up until the investment period. We also show results removing countries featuring de facto fixed exchange rates during some subperiods of our sample according to Ilzetzki, Reinhart, and Rogoff (2019). The broad patterns are consistent with Table 1: static returns are somewhat smaller, but still large (2.62%-3.03%, depending on the sample). Interest rate differences are large (around 5%), and investors on average lose money on the exchange rates. FX-shares are

[47]Since we have more data, we can afford to allow more periods for expectation formation.

a bit more negative compared to Table 1. All standard errors are a bit wider, but are consistent with our properties 1 and 2.

Figure A1: Static Trade in SDF Space: Updated Data



This figure plots the relative position of the low-interest-rate portfolio and the high-interest-rate portfolio implied by the static trade returns in the updated data (1/2000 - 6/2024), as well as the confidence intervals of the FX-share. The position of the low-interest-rate portfolio is arbitrarily chosen because only the relative positions of these dots matter for our discussion. The position of the high-interest-rate portfolio is inferred from the data using equations (2) and (4). The shaded area represents confidence intervals for the FX-share. Black dots are bootstrap samples.

The standard errors for FX-share is a bit non-standard because as a slope, it crosses the vertical case. Figure A1 illustrates the case when the investment period starts at 1/2000, with the red triangle showing the point estimate and each black dot representing a bootstrap sample. Graphically, the updated sample again yields similar results as in the main text, with a somewhat wider confidence interval.

Table A5 shows the updated result for carry trade. Again, the pattern is largely consistent with Table 1 and our properties 1 and 2.

C.3 Regression-based Estimates

As shown in Hassan and Mano (2019), one major advantage of forming portfolios in this way is one can easily relate these portfolios to regression results. For example, the static trade is closely related to β^{static} in the following regression:

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{static}(\overline{fp}_i^e - \overline{fp}^e) + \epsilon_{i,t+1}^{static}.$$

Table A5: FX-Share in the Data: Carry Trade

	Return (%) $\mathbb{E}(rx)$	Change in FX (%) $\mathbb{E}(\Delta s)$	Interest Rate Diff (%) $\mathbb{E}(r^\star - r)$	FX-share $\frac{\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$
1/1995 - 6/2024	4.50	-2.65	7.15	-0.59
	[1.49,7.49]	[-6.07,0.29]	[2.28,13.33]	[-1.19,0.09]
No Fixed	4.11	-2.95	7.06	-0.72
	[0.94,7.12]	[-6.21,0.03]	[1.97,13.06]	[-1.75,0.01]
1/2000-6/2024	3.90	-1.52	5.42	-0.39
	[1.32,6.57]	[-3.84,0.25]	[1.78,9.91]	[-0.97,0.10]
No Fixed	3.45	-1.81	5.27	-0.53
	[0.93,5.93]	[-4.21,0.04]	[1.56,9.49]	[-1.36,0.02]

We use the same set of countries as the 1-rebalance sample in Hassan and Mano (2019), but update the data to June 2024. All moments are annual. Carry trade returns are calculated by sorting the forward premia (interest rates) period by period, and then shorting a weighted portfolio of the currencies with below-average forward premia, and longing a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C.1. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. We show two different investment periods: 1/1995-6/2024, which starts at the same date as Hassan and Mano (2019), and 1/2000-6/2024, which give the investors more periods to form their expectations of forward premia. Results labeled "No Fixed" removes currencies with de facto fixed exchange rates (HKG, SAU, KWT) according to Ilzetzki, Reinhart, and Rogoff (2019).

More importantly, estimates of β^{static} are closely related to the FX-share:

$$\text{FX-share} = 1 - \frac{1}{\hat{\beta}^{static}}.$$

To see this, note $\hat{\beta}^{static} = \frac{\Sigma_{i,t}[(rx_{i,t+1} - \overline{rx}_{t+1})(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]}$, so we have

$$1 - \frac{1}{\hat{\beta}^{static}} = \frac{\Sigma_{i,t}[-\Delta s_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]} = \text{FX-share}.$$

A similar result can be obtained for carry-trade returns. We then use the estimates of β^{static} and β^{ct} in Hassan and Mano (2019) to construct alternative estimates of FX-share.[48]

Table A6 lists all the estimates using different sub samples in Hassan and Mano (2019). We illustrate all these estimates in our SDF space in Figure A2. Each of the lines represent an estimate of the FX-share, with the shaded area showing the widest confidence interval. One can clearly see the results are consistent with our bootstrap-based estimates: high-interest-rate currency tends to depreciate, and the FX-share tends to be negative or, at most, approximately horizontal.

[48]The identity above holds exactly only when the portfolio weights are formed from the same sample used to estimate $\hat{\beta}$. Because the weights $\overline{fp}_i^e$ are estimated from presample data (see Section C.1), the exact finite-sample equivalence between $1 - 1/\hat{\beta}^{static}$ and the portfolio FX-share no longer holds. We therefore report the values in Table A6 as regression-implied FX-shares.

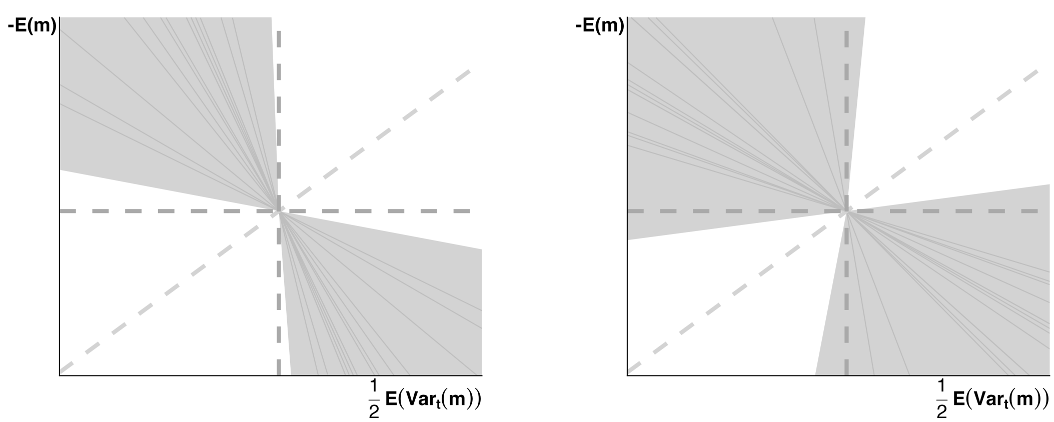
Table A6: Estimation of FX-Share: Hassan and Mano (2019)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horizons (months)	1	1	6	12	1	1	6	12
Sample	1 Reblance				3 Reblance			
Static T: β^{stat}	0.47	0.37	0.56	0.60	0.26	0.18	0.26	0.25
	[0.31, 0.63]	[0.19, 0.55]	[0.36, 0.76]	[0.40, 0.80]	[0.16, 0.36]	[0.08, 0.28]	[0.18, 0.34]	[0.13, 0.37]
Static T: FX-share	-1.13	-1.70	-0.79	-0.67	-2.85	-4.56	-2.85	-3.00
	[-2.19, -0.60]	[-4.17, -0.83]	[-1.75, -0.32]	[-1.48, -0.26]	[-5.17, -1.79]	[-11.20, -2.60]	[-4.51, -1.96]	[-6.55, -1.72]
Carry T: β^{ct}	0.68	0.55	0.62	0.71	0.57	0.45	0.42	0.43
	[0.15, 1.21]	[0.04, 1.06]	[0.05, 1.19]	[0.20, 1.22]	[0.20, 0.94]	[0.10, 0.80]	[0.01, 0.83]	[0.06, 0.80]
Carry T: FX-share	-0.47	-0.82	-0.61	-0.41	-0.75	-1.22	-1.38	-1.33
	[-5.63,0.17]	[-23.75,0.06]	[-18.38,0.16]	[-3.99,0.18]	[-4.06, -0.06]	[-9.29, -0.25]	[-118.05, -0.20]	[-16.36, -0.25]
Sample	6 Reblance				12 Reblance			
Static T: β^{stat}	0.23	0.15	0.25	0.24	0.34	0.23	0.31	0.30
	[0.13, 0.33]	[0.05, 0.25]	[0.17, 0.33]	[0.14, 0.34]	[0.18, 0.50]	[0.05, 0.41]	[0.15, 0.47]	[0.14, 0.46]
Static T: FX-share	-3.35	-5.67	-3.00	-3.17	-1.94	-3.35	-2.23	-2.33
	[-6.58, -2.05]	[-18.23, -3.03]	[-4.83, -2.05]	[-6.04, -1.96]	[-4.46, -1.01]	[-17.66, -1.46]	[-5.53, -1.14]	[-5.98, -1.19]
Carry T: β^{ct}	0.56	0.45	0.45	0.11	0.67	0.52	0.57	0.22
	[0.21, 0.91]	[0.12, 0.78]	[0.08, 0.82]	[-0.16, 0.38]	[0.36, 0.98]	[0.21, 0.83]	[0.26, 0.88]	[-0.11, 0.55]
Carry T: FX-share	-0.79	-1.22	-1.22	-8.09	-0.49	-0.92	-0.75	-3.55
	[-3.83,-0.10]	[-7.56,-0.28]	[-11.89,-0.22]	[7.08,+∞)∪(−∞,-1.60]	[-1.81,-0.02]	[-3.84,-0.20]	[-2.90,-0.13]	[9.83,+∞)∪(−∞,-0.81]

Point estimates are taken from Table III in Hassan and Mano (2019). Confidence intervals are calculated using the corresponding standard errors.

Figure A2: Alternative Estimations of the FX-share

(a) Static trade (b) Conditional Carry trade



This figure plots the point estimates and confidence intervals of the FX-shares inferred from estimates of β^{static} and β^{carry}. Solid grey lines represent point estimates across different samples. The shaded grey area represents the widest confidence interval.