Exchange Rates, Natural Rates, and the Price of Risk*

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Abstract

We study the source of exchange rate fluctuations using a general equilibrium model accommodating shocks in goods and financial markets. These shocks differ in their induced comovements between exchange rates, interest rates, and quantities. A calibration matching data from the U.S. and G10 currency countries implies that persistent shocks to relative demand, reflected in persistent interest rate differentials, account for 75% of the variance in the dollar/G10 exchange rate. Shocks to currency intermediation are important, however, in generating deviations from uncovered interest parity at high frequencies and explaining the dollar appreciation in crises.

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1 Introduction

What drives exchange rates? This question is particularly challenging because exchange rates are very difficult to predict (Meese and Rogoff (1983)); appear inconsistent with uncovered interest parity, at least at short horizons (Hansen and Hodrick (1980), Fama (1984)); and are inconsistent with standard macroeconomic models with complete markets (Backus and Smith (1993)). In this context, recent advances in international macroeconomics and finance have identified shocks to intermediary constraints, "noise trader" demand, and convenience yields, all ultimately entering as wedges in the uncovered interest parity (UIP) condition, as possibly important drivers of exchange rates, and have made tractable their study in general equilibrium.

In this paper, we investigate the source of exchange rate fluctuations using a general equilibrium model accommodating shocks to currency intermediation as well as shocks to demand and supply in each country. The key distinction between currency intermediation shocks and other shocks is the induced comovement between interest rate differentials and the exchange rate. The distinction between supply shocks and other shocks is the induced comovement between relative consumption and the exchange rate. Between the U.S. and G10 currency countries over 1991-2020, U.S. interest rates and relative U.S. consumption have been high when the dollar is strong, suggesting an important role for demand shocks. In a calibration disciplining demand shocks using bond yields, we find that these shocks indeed account for roughly 75% of the variance in the dollar/G10 exchange rate and generate the observed comovements. Demand shocks generate much larger volatility of exchange rates than macroeconomic aggregates because they are persistent, reflected in persistent yield differentials, and exchange rates are forward-looking. While demand shocks have a predominant role in accounting for the dollar/G10 exchange rate, we also find currency intermediation shocks are important to account for UIP deviations at high frequencies, and to explain the dollar appreciation in crises when the yield on dollar bonds is relatively low.

We study these questions using a two-country exchange economy. We build on the recent literature studying exchange rates in segmented markets in which U.S. households trade a bond denominated in their own consumption bundle (a "dollar bond"), Foreign households trade a bond denominated in their consumption bundle, and arbitrageurs with limited risk-bearing capacity trade with both groups of households. Demand shocks are shocks to the discount factors of households, supply shocks are shocks to endowments, and currency intermediation shocks are shocks to the risk tolerance of arbitrageurs. In terms of their macroeconomic implications, currency intermediation shocks can equivalently be modeled as shocks to the demand for foreign currency assets (on the part of "noise traders", or from households as reflected in convenience yields);[1] they are distinguished by shocks to discount factors because the latter affect the demand of households in a given country for *all* assets.

We first analytically clarify how the shocks in our model generate distinct comovements between exchange rates, interest rates, and macroeconomic aggregates. The key distinction between currency intermediation shocks and other shocks is the induced comovement between the real exchange rate and real interest rates. An intermediation shock that causes the dollar to appreciate also causes a relative increase in the Foreign interest rate, while the other shocks do the opposite. The key distinction between supply shocks and other shocks is the induced comovement between the real exchange rate and relative consumption. A supply shock that causes the dollar to appreciate also causes a relative increase in Foreign consumption, while demand and intermediation shocks do the opposite. It follows that demand shocks are the only ones which simultaneously imply that a strong dollar is associated with a relatively high U.S. interest rate and relatively high U.S. consumption.

We also characterize the role of persistence in shaping the exchange rate effects of demand shocks, anticipating their important role in the quantitative analysis which follows. Consider the comparative statics with respect to more persistent relative demand. In this case, an innovation in relative demand will imply more persistent interest rate differentials going forward. Since exchange rates are forward-looking, this increases the impact effect on the exchange rate provided that deviations from UIP are small, which will be the case when steady-state currency intermediation frictions are small. This generates larger volatility in exchange rates than interest rates and macroeconomic quantities. At the same time, since innovations in the expected change in the exchange rate are simply equal to the innovations in interest rates, its variance falls relative to the overall variance of the change in the exchange rate. This generates a lower R^2 in regressions of future changes in exchange rates on interest rates or other macroeconomic aggregates. The role of persistence in delivering

[1]The only distinction between shocks to risk tolerance and shocks to the demand for foreign currency assets is in the volume of intermediation done by arbitrageurs. The effects on exchange rates, interest rates, and aggregate consumption are qualitatively the same.

these results relies on incomplete markets: if instead risk sharing was efficient across countries, the persistence of demand shocks would be irrelevant in determining their impact effect on the exchange rate.

We next measure comovements of the dollar/G10 exchange rate in the data over the 1991-2020 period. We measure nominal yields in each currency and interpret these yield differences as counterparts to real yield differences in our model, given relatively low and stable expected inflation for the countries in our sample over this period. Higher U.S. yields than G10 yields are associated with a stronger dollar, both in changes and in levels. These comovements hold across tenors, which range from three months to 10 years in our data, and are statistically significantly different from zero using longer tenors. The explanatory power of yield differentials is especially strong when we condition on proxies for risk or convenience yields, such as the excess bond premium of Gilchrist and Zakrajsek (2012). We also measure real consumption per capita in each country. Higher real consumption per capita in the U.S. than G10 countries is statistically significantly associated with a stronger dollar, both in changes and in levels. Despite the strong contemporaneous comovements with yield differentials and relative consumption, these variables have much lower explanatory power for changes in exchange rates in the future. In light of our analytical results, we conclude from these empirical findings that persistent shocks to relative demand may be an important driver of exchange rates.

We turn to formally quantifying the model to see how far it can go in accounting for the dollar/G10 exchange rate. We discipline demand and supply shocks to match the stochastic properties of 10-year yields and output per capita in the data. Consistent with the dollar funding among financial institutions in practice, steady-state discount factors are such that currency arbitrageurs are net short the dollar. We discipline currency intermediation shocks by assuming that the excess bond premium is a direct measure of arbitrageur risk aversion, up to scale. We discipline the volatility of these shocks so that the model generates the same volatility of real exchange rate changes as in the data. We discipline the steady-state degree of currency intermediation frictions to match the volatility of trade flows (as, by the balance of payments, these flows in goods correspond to flows in financial assets). An important implication is that demand shocks require high persistence of 0.98 at a quarterly frequency to match the autocorrelation of the 10-year yield differential of 0.93 in the data.

The calibrated model implies that shocks to relative demand and thus interest rate differentials account for most of the variation in the dollar/G10 exchange rate. Demand shocks generate roughly 75% of the variance in the exchange rate. The comovements between the real exchange rate, yield differentials, and relative consumption are all within the 90% confidence intervals in the data; in particular, the model-implied dollar is strong when U.S. yields and U.S. consumption are relatively high. The R^2 in regressions predicting changes in the exchange rate are much lower, consistent with the weak predictability of the exchange rate in the data. Indeed, the exchange rate exhibits near random walk behavior, with a high autocorrelation in levels (0.91) and close to zero autocorrelation in levels (-0.04) comparable to their values in the data (0.94 and 0.04, respectively). We also invert the sequence of shocks required by our model to rationalize the exact path of 10-year yields, output per capita, and excess bond premium in the data over 1991-2020. Simulating the resulting real exchange rate reveals that the model accounts for both the trend and cycles in the observed exchange rate, and most of this is due to relative demand shocks.

At the same time, because currency intermediation shocks induce the opposite comovement of exchange rates and yield differentials as demand shocks, they play a complementary role in matching the data. First, intermediation shocks reduce the explanatory power of yield differentials for exchange rates to be comparable with the data; in their absence, yield differentials would account for a counterfactually high share of contemporaneous exchange rate fluctuations. Second, they allow the model to match deviations from UIP at short horizons, especially because they are more transitory than persistent demand shocks and thus account for half of the variance in the exchange rate in quarterly changes. Third, in the model simulation over 1991-2020, these shocks explain why the dollar appreciated in crises (the early 2000s, 2008, and 2020) even though U.S. interest rates were relatively low.

We conclude with an interpretation of our findings and directions for future work. First, to the extent that persistent shocks to relative demand, reflected in persistent differences in real interest rates and thus natural rates of interest across countries, are a primary driver of exchange rates, it begs the question of where these shocks come from. In richer economies, changes in credit conditions, demographics, or uncertainty would propagate like discount factor shocks in our reduced form exchange economy. Persistent shocks to growth rates would also generate movements in relative demand and natural rates. Using additional moments in the data to identify the relative importance of these shocks seems a fruitful avenue for future work. Second, there is heterogeneity along the cross-section of currencies to explore. When we extend our empirical investigation to emerging markets, we in fact find that their yields tend to be relatively *low* when their currencies are strong versus the dollar. This suggests that intermediation shocks may play a more important role in driving the dollar/EM exchange rate. More broadly, an interesting question is whether heterogeneous exposures to demand and intermediation shocks can account for the factor structure in bilateral exchange rates documented in the asset pricing literature.

Related literature Our paper contributes to an active literature studying exchange rates in segmented financial markets. We build most directly on the work of Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021), who demonstrate that shocks to currency intermediation can generate volatile exchange rates and predictable deviations from UIP.[2] We argue that, while these shocks play a meaningful role at high frequencies, the comovements between interest rates, consumption, and exchange rates call for a predominant role for demand shocks in driving the dollar/G10 exchange rate.[3] In this sense, our analysis harkens back to an earlier literature emphasizing the propagation of conventional "macroeconomic" shocks for exchange rates in segmented markets, as in Alvarez, Atkeson, and Kehoe (2002, 2009) and Heathcote and Perri (2002), though the demand shocks which we emphasize are not studied in these papers. Like us, Devereux, Engel, and Wu (2023) study the dollar exchange rate in an environment featuring demand, supply, and intermediation shocks, but they focus on the propagation of these shocks through an endogenous convenience yield. Gourinchas, Ray, and Vayanos (2024) and Greenwood, Hanson, Stein, and Sunderam (2023) share our focus on long term yields and exchange rates. However, they focus on the relationship between term premiums and exchange rates in partial equilibrium settings, whereas we focus on the relationship between persistent interest rate differentials and exchange rates in general equilibrium.

In this respect, our analysis relates this literature to an older one emphasizing the relationship between exchange rates and interest rate differentials. The "asset market view" of exchange rates, reflected for instance in Dornbusch (1976), emphasized the

[2]Related recent models of exchange rates in frictional financial markets include Adrian, Erceg, Linde, Zabczyk, and Zhou (2020), Basu, Boz, Gopinath, Roth, and Unsal (2020), Fanelli and Straub (2021), Akinci, Kalemli-Ozcan, and Queralto (2022), and Fukui, Nakamura, and Steinsson (2023).

[3]The segmentation emphasized by these papers remains crucial because it links the persistence of demand shocks to the exchange rate response, echoing Corsetti, Dedola, and Leduc (2008).

role of expectations regarding future macroeconomic fundamentals as determinants of the exchange rate, and an enormous empirical literature studying exchange rates and interest rates followed.[4] Our empirical work extends this investigation to data over the past 30 years, and our interpretation through the model focuses on shocks to real interest rates arising from non-monetary factors. Our focus on demand shocks in particular allows us to overcome the consumption-real exchange rate anomaly highlighted by Chari, Kehoe, and McGrattan (2002), and builds on the work of Stockman and Tesar (1995) and Pavlova and Rigobon (2007) who find that demand shocks can help account for other features of international business cycles and asset prices. Also in this context, two especially close precursors to our paper are Engel and West (2005) and Engel (2016). The former demonstrates that persistent innovations to fundamentals can generate realistic exchange rate properties; we make use of this insight in the context of shocks to relative demand and thus interest rates in general equilibrium. The latter argues that models of UIP deviations have difficulty in accounting for longer run properties of exchange rates; in our calibrated model, UIP indeed holds better at longer horizons than shorter horizons because risk premia exhibit more transitory fluctuations than interest rate differentials.

Since the shocks in our reduced form exchange economy likely stand in for deeper sources of fluctuations in richer production economies, our analysis also relates to wedge accounting exercises studying properties of exchange rates.[5] Itskhoki and Mukhin (2023) demonstrate that, in the limit of autarky, shocks to the demand for foreign currency assets are the only ones which can simultaneously generate a disconnect between exchange rates and other macroeconomic or financial variables. We argue that the dollar/G10 exchange rate is not disconnected from bond yields or relative consumption, especially at lower frequencies, and that demand shocks reflected in bond yields deliver realistic comovements and volatility in the exchange rate.[6] Chernov, Haddad, and Itskhoki (2024), Jiang, Krishnamurthy, and Lustig (2024b),

[4]See, for instance, Frankel (1979), Campbell and Clarida (1987), Meese and Rogoff (1988), and Clarida and Gali (1994).

[5]These complement wedge accounting in closed economies, as in Chari, Kehoe, and McGrattan (2007) and Berger, Bocola, and Dovis (2023), as well as in open economies not focused on exchange rates, as in Eaton, Kortum, and Neiman (2016a) and Eaton, Kortum, Neiman, and Romalis (2016b).

[6]To be clear, we do not challenge the idea that the exchange rate may have limited effects on the economy due to features such as high home bias, a low trade elasticity, or pricing to market. We do, however, challenge the idea that the exchange rate is driven by shocks disconnected from those driving other asset prices and macroeconomic quantities.

and Jiang, Krishnamurthy, Lustig, and Sun (2024c) study the ability of unspanned risks, cross-bond Euler equation wedges, and convenience yields to make progress on exchange rate puzzles.[7] These are similar to the currency intermediation shocks in our framework. Demand shocks are instead wedges between consumption growth and stochastic discount factors, and are not the focus of these papers.

Finally, our analysis provides a structural counterpart to a growing empirical literature arguing that the dollar exchange rate is in fact not disconnected from other macroeconomic and financial variables. Obstfeld and Zhou (2023) document that yield differentials have substantial explanatory power for the exchange rate between the dollar and advanced economies, and Engel and Wu (2024) demonstrate that macroeconomic fundamentals and proxies for risk and liquidity account for a substantial share of variation in dollar/G10 exchange rates. Our model can make sense of the comovements with the dollar exchange rate uncovered in these papers. Hau and Rey (2006) and Camanho, Hau, and Rey (2022) document that equity returns are related to exchange rates.[8] While we do not price equity claims in the present paper, the link between bond yields and exchange rates which we emphasize echoes their message that the exchange rate is not disconnected from these other asset prices. Chahrour, Cormun, De Leo, Guerron-Quintana, and Valchev (2024) argue using a vector autoregression that the dominant driver of exchange rates are news shocks about future TFP.[9] Since growth shocks would propagate like discount factor shocks in being effective demand shocks, our results are consistent with their findings.

Outline In section 2 we outline the model, and in section 3 we analytically characterize the comovements induced by each shock and the role of model ingredients in shaping the propagation of demand shocks. In section 4 we empirically characterize these comovements for the U.S. and G10 currency countries. Motivated by this evidence, in section 5 we calibrate the model to the data and assess its ability to account for exchange rate volatility and comovements. Finally, in section 6 we discuss the

[7]These papers in turn build on a "preference-free" tradition in the determination of exchange rates, as in the work of Brandt, Cochrane, and Santa-Clara (2006) and Lustig and Verdelhan (2019).

[8]Relatedly, using a demand system approach, Koijen and Yogo (2022) argue that most of the variation in the dollar exchange rate is due to macroeconomic and policy variables, and Richmond, Jiang, and Zhang (2024) argue that exchange rate movements are linked to capital flows. Lilley, Maggiori, Neiman, and Schreger (2020) document an increase in the R^2 of the dollar/G10 exchange rate with S&P 500 returns and other measures of risk appetite since the global financial crisis.

[9]Stavrakeva and Tang (2024) argue that, more generally, the response to macroeconomic news announcements accounts for most of the variation in the exchange rate.

interpretation and implications of our findings, and in section 7 we conclude.

2 Model

We first outline a two-country economy featuring shocks to demand, supply, and currency intermediation.

2.1 Environment

We study an exchange economy, though we accommodate a distinction between intermediate goods (endowments) and final goods to allow for incomplete pass-through of the exchange rate into domestic prices. There are two countries, Home (the U.S.) and Foreign, comprised of a measure 1 and ζ^* households.

U.S. households The U.S. representative household has time separable preferences with flow utility

$$u(c_t) = \frac{(c_t)^{1-1/\psi} - 1}{1 - 1/\psi},$$

discount factor β_t, and a CES aggregator over bundles of final goods from each country

$$c_t = \left(\left(\frac{1}{1+\zeta^*} + \frac{\zeta^*}{1+\zeta^*}\varsigma \right)^{\frac{1}{\sigma}} (c_{Ht})^{\frac{\sigma-1}{\sigma}} + \left(\frac{\zeta^*}{1+\zeta^*}(1-\varsigma) \right)^{\frac{1}{\sigma}} (c_{Ft})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Here, ψ denotes the household's elasticity of intertemporal substitution, $\sigma > \frac{1}{2}$ denotes its elasticity of substitution across domestic goods and imports,[10] and $\varsigma \in [0, 1)$ indexes the degree of consumption home bias. U.S. households can trade a bond denominated in the U.S. consumption bundle paying a risk-free interest rate r_t at $t + 1$, henceforth referred to as a "dollar bond". They receive an endowment of z_t intermediate goods, transfers π_t from domestic firms which they own, and net transfers π_t^a from arbitrageurs, described further below. Hence, letting b_t denote the representative household's saving in the dollar bond at t, the budget constraint of the

[10] The assumption that $2\sigma > 1$ is referred to as the Marshall-Lerner condition. This assumption is satisfied given most estimates of the trade elasticity σ in the literature, so we make it here.

U.S. representative household is

$$p_{Ht}c_{Ht} + p_{Ft}c_{Ft} + \frac{1}{1+r_t}b_t = p_t z_t + \pi_t + \pi_t^a + b_{t-1},$$

where p_{Ht} (p_{Ft}) is the price of the domestic (imported) bundle, and p_t the price of its endowment, all relative to its overall consumption bundle.

Foreign households Symmetrically, the Foreign representative household has time separable preferences with flow utility

$$u(c_t^*) = \frac{(c_t^*)^{1-1/\psi} - 1}{1-1/\psi},$$

discount factor β_t^*, and a CES aggregator over consumption

$$c_t^* = \left(\left(\frac{1}{1+\zeta^*}(1-\varsigma) \right)^{\frac{1}{\sigma}} (c_{Ht}^*)^{\frac{\sigma-1}{\sigma}} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{1}{1+\zeta^*}\varsigma \right)^{\frac{1}{\sigma}} (c_{Ft}^*)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Denoting b_t^* the household's saving in its domestic bond which pays r_t^* at $t+1$, z_t^* its endowment, and π_t^* its transfers from domestic firms, its budget constraint is

$$p_{Ht}^* c_{Ht}^* + p_{Ft}^* c_{Ft}^* + \frac{1}{1+r_t^*} b_t^* = p_t^* z_t^* + \pi_t^* + b_{t-1}^*,$$

where p_{Ft}^* (p_{Ht}^*) is the price of the domestic (imported) bundle, and p_t^* the price of its endowment, relative to its overall consumption bundle.

Arbitrageurs A third set of agents, arbitrageurs, trade in both dollar and Foreign bonds and transfer their profits or losses to the representative U.S. household. Each period t, the representative arbitrageur chooses a portfolio of dollar- and Foreign-denominated bonds $\{b_t^a, b_t^{a*}\}$ to maximize mean-variance preferences

$$\mathbb{E}_t \pi_{t+1}^a - \frac{\gamma_t}{2} Var_t \pi_{t+1}^a,$$

subject to the resource constraint at t

$$0 = \frac{1}{1+r_t} b_t^a + q_t^{-1} \frac{1}{1+r_t^*} b_t^{a*}$$

and realized profits/losses at $t+1$

$$\pi^a_{t+1} = b^a_t + q^{-1}_{t+1} b^{a*}_t.$$

Here, q_t is the price of the U.S. consumption bundle relative to the Foreign bundle (the real exchange rate), and γ_t denotes arbitrageurs' risk aversion and is time-varying. The fact that arbitrageurs transfer their profits/losses to U.S. rather than Foreign households does not meaningfully change any of the effects of shocks which we characterize later in the paper.

Final good firms In each country, a continuum of monopolistically competitive final good firms transform the domestic intermediate good into varieties sold globally. We include these final good firms in our model only to accommodate incomplete pass-through of the exchange rate into prices faced by consumers, as in the data. We follow Alessandria (2009) in capturing pricing to market in reduced form,[11] by assuming that U.S. households' aggregators over domestic and imported varieties are

$$c_{Ht} = \left(\int_0^1 c_{Ht}(i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$
$$c_{Ft} = \left(\int_0^1 c_{Ft}(i^*)^{\frac{\theta_t-1}{\theta_t}} \right)^{\frac{\theta_t}{\theta_t-1}},$$

respectively, where the elasticity of substitution across imported varieties is $\theta_t \equiv \theta(q_t/q)^{-\xi}$ and throughout we denote steady-state values without time subscripts. Analogously, Foreign aggregators over imported and domestic varieties are

$$c^*_{Ht} = \left(\int_0^1 c^*_{Ht}(i)^{\frac{\theta^*_t-1}{\theta^*_t}} \right)^{\frac{\theta^*_t}{\theta^*_t-1}},$$
$$c^*_{Ft} = \left(\int_0^1 c^*_{Ft}(i^*)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where the elasticity of substitution across imported varieties is $\theta^*_t \equiv \theta(q_t/q)^{\xi}$. Facing the usual residual demand curves in each country, each U.S. firm i then chooses prices

[11] This can stand in for nominal rigidities together with local currency pricing, as in Devereux and Engel (2002); strategic complementarities with CES demand, as in Atkeson and Burstein (2008); or departures from a CES demand system, as in Gopinath and Itskhoki (2010).

$\{p_{Ht}(i), p^*_{Ht}(i)\}$ to maximize profits

$$\pi_t(i) = (p_{Ht}(i) - p_t)\left(\frac{p_{Ht}(i)}{p_{Ht}}\right)^{-\theta} c_{Ht} + \left(q_t^{-1}p^*_{Ht}(i) - p_t\right)\left(\frac{p^*_{Ht}(i)}{p^*_{Ht}}\right)^{-\theta^*_t} \zeta^* c^*_{Ht}.$$

Each Foreign firm i^* analogously chooses prices $\{p_{Ft}(i^*), p^*_{Ft}(i^*)\}$ to maximize profits

$$\pi^*_t(i^*) = (q_t p_{Ft}(i^*) - p^*_t)\left(\frac{p_{Ft}(i^*)}{p_{Ft}}\right)^{-\theta_t} c_{Ft} + (p^*_{Ft}(i^*) - p^*_t)\left(\frac{p^*_{Ft}(i^*)}{p^*_{Ft}}\right)^{-\theta} \zeta^* c^*_{Ft}.$$

The aggregate profits earned by households per capita in each country are then

$$\pi_t = \int_0^1 \pi_t(i)di,$$
$$\pi^*_t = \frac{1}{\zeta^*}\int_0^1 \pi^*_t(i^*)di^*.$$

Market clearing and driving forces The environment is closed with standard market clearing conditions in goods and bonds. The driving variables are discount factors $\{\beta_t, \beta^*_t\}$, endowments $\{z_t, z^*_t\}$, and the risk aversion of arbitrageurs γ_t. For each driving variable $x_t \in \{\beta_t, \beta^*_t, z_t, z^*_t, \gamma_t\}$, we assume $\log x_t$ follows an AR(1) process around $\log x$ with mean reversion ρ^x, standard deviation of shocks σ^x, and correlation of shocks with another driving variable x'_t denoted $\rho^{xx'}$.

Taken together, this environment builds most closely on that in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). The main innovation is our focus on demand shocks $\{\beta_t, \beta^*_t\}$, which we will argue play an important role in driving the exchange rate in the data.[12]

2.2 Equilibrium and solution

The equilibrium has a recursive representation in the endogenous state variable b^*_t and exogenous state variables $\{\beta_t, \beta^*_t, z_t, z^*_t, \gamma_t\}$. The full set of equilibrium conditions is outlined in appendix A.

Following Itskhoki and Mukhin (2021) and Borovicka, Hansen, and Sargent (2023), we study a first-order approximation of the system around a steady-state in which

[12] Demand shocks are not modeled in Itskhoki and Mukhin (2021). Gabaix and Maggiori (2015) accommodate overall demand shocks via innovations in their $\{\theta_t, \theta^*_t\}$, but their analysis focuses on shocks to import demand, currency intermediation, endowments, and monetary policy.

the currency risk premium does not disappear. In particular, consider arbitrageurs' optimal portfolio choice condition

$$\mathbb{E}_t \frac{q_t}{q_{t+1}}(1+r_t^*) - (1+r_t) = \gamma_t \left(q_t^{-1} \frac{1}{1+r_t^*} b_t^{a*} \right) Var_t \frac{q_t}{q_{t+1}}(1+r_t^*).$$

Scaling the volatility of all driving forces by the perturbation parameter σ, we assume that

$$\gamma_t = \gamma(\sigma) \exp\left(\hat{\gamma}_t\right),$$

where $\gamma(\sigma)\sigma^2$ remains positive and finite as $\sigma \to 0$, and where $\hat{\gamma}_t$, an AR(1) process around zero, also features shocks scaled by σ. Under this assumption, appendix A shows that the first-order approximation of the above condition in σ is

$$\hat{r}_t^* - \mathbb{E}_t \Delta \hat{q}_{t+1} - \hat{r}_t = \frac{q^{-1} b^{a*} \Gamma}{1 - 2q^{-1} b^{a*} \Gamma} \left(\hat{\gamma}_t + \hat{r}_t - \hat{q}_t - \hat{r}_t^* + \frac{1}{b^{a*}} \hat{b}_t^{a*} \right), \tag{1}$$

where $\Gamma \equiv \lim_{\sigma \to 0} \gamma(\sigma) Var \log q_{+1}$, and $\hat{}$ denotes log deviations from steady-state for all variables except bond positions (which can be negative), in which case it denotes level deviations. The associated steady-state condition is

$$(1+r^*) - (1+r) = q^{-1}(1+r^*) b^{a*} \Gamma,$$

which clarifies that arbitrageurs' limited risk-bearing capacity can sustain a non-zero interest rate differential in steady-state.[13] The first-order approximation to all other equilibrium conditions, and their steady-state implications, remain standard.

Since in particular $1+r^* = (\beta^*)^{-1}$ and $1+r = \beta^{-1}$, it is clear that in steady-state arbitrageurs will intermediate capital flows from the U.S. to Foreign ($b^{a*} > 0$) provided $\beta^* < \beta$, that is, Foreign households are more impatient than U.S. households. In this case, arbitrageurs are short the dollar and long Foreign bonds.

We view the idea that arbitrageurs are short the dollar as realistic, given the dominant role of dollar funding among global intermediaries.[14] However, one unrealistic implication in the present model is that this would mean the U.S. has a positive net foreign asset position. This counterfactual implication is easily resolved

[13] As a result, the present environment may be a promising one to account for the patterns in excess currency returns described in Hassan, Mertens, and Wang (2024).

[14] See, for instance, Adrian, Etula, and Shin (2010), Bruno and Shin (2015a,b), and Avdjiev, Du, Koch, and Shin (2019).

by allowing Foreign households to also hold dollar bonds and assuming that they receive a non-pecuniary value of doing so, consistent with the Foreign demand for safe dollar assets.[15] We view this as a promising environment to study the implications of the shocks in this paper for *gross* flows and valuation effects in current account adjustment, since (together with $\beta^* < \beta$) it could be consistent with the U.S. being short dollar bonds, long Foreign bonds, and having a negative net foreign asset position. However, since this would not qualitatively change the comovements of exchange rates, interest rates, and consumption — our primary focus in this paper — we refrain from complicating the model along these dimensions here.

We finally note that the results which follow also do not depend on the specific form of segmented markets we have assumed here. We could alternatively consider an environment without intermediaries and in which households in each country trade a domestic and foreign bond, subject to adjustment costs in the foreign bond.[16] Shocks to these adjustment costs are essentially equivalent to the intermediation shocks in the present environment. Shocks to discount factors and endowments still propagate as they do here. Both our analytical results and quantitative findings are thus unaffected.

3 Analytical insights

We now analytically characterize the comovements between exchange rates, interest rates, and macroeconomic aggregates induced by the shocks in our model. We also characterize the role of model ingredients in shaping the effects of relative demand shocks, anticipating their important role in the quantitative analysis which follows.

3.1 Parameter space of interest

We characterize these results for high consumption home bias ς, a small steady-state price of risk Γ, and small interest rate r^*.[17] These parametric conditions simplify the proofs and are also the relevant cases in our quantitative analysis later in the paper. For some of our analytical results, we follow Itskhoki and Mukhin (2021) in

[15] See, for instance, Eichenbaum, Johannsen, and Rebelo (2021) and Jiang, Krishnamurthy, and Lustig (2021, 2024a).

[16] These costs are consistent with the home currency bias in international portfolios (e.g., Maggiori, Neiman, and Schreger (2020)). In the language of Jiang et al. (2024b), they give rise to cross-bond Euler equation wedges.

[17] We refer to Γ as the steady-state price of risk because it depends on the limiting $\gamma(\sigma)$.

considering the autarkic limit ($\varsigma \to 1$). Since the response of the exchange rate to shocks is well defined for any $\varsigma < 1$, it is also well defined in the limit.

To further simplify the proofs without affecting the qualitative results, we also make the following parametric assumptions in this section alone:

Assumption 1 (A1)**.** *The U.S. and Foreign have the same population ($\zeta^* = 1$), the law of one price holds for each final good ($\xi = 0$), and endowments are such that the steady-state exchange rate is one ($q = 1$).*

3.2 Contrasting the comovements induced by shocks

Our first result contrasts the comovements between the exchange rate, interest rate differential, and macroeconomic quantities induced by the shocks in our model:

Proposition 1. *Assume A1, ς close to one, and Γ and r^* small. Given fluctuations induced by each shock alone, we obtain the following comovements:*

	ϵ_t^γ *(currency intermediation)*	$\epsilon_t^\beta, \epsilon_t^{\beta^*}$ *(demand)*	$\epsilon_t^z, \epsilon_t^{z^*}$ *(supply)*
$Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t)$	$(+)$	$(-)$	$(-)$
$Cov(\Delta\hat{q}_t, \hat{r}_{t-1}^* - \hat{r}_{t-1})$	$(-)$	$(+)$	$(+)$
$Cov(\hat{q}_t, \hat{c}_t^* - \hat{c}_t)$	$(-)$	$(-)$	$(+)$

The comovement between the exchange rate and interest rate differential distinguishes currency intermediation shocks from other shocks. In particular, consider an increase in arbitrageur risk aversion in the context of the UIP condition (1). For concreteness, assume $\beta^* < \beta$ in steady-state so that, as discussed in the prior section, arbitrageurs are long Foreign bonds financed by dollar bonds ($b^{a*} > 0$) in steady-state. When they get more risk averse ($\hat{\gamma}_t$ rises), they require higher excess returns on Foreign bonds relative to dollar bonds going forward. This is achieved both by an immediate dollar appreciation and thus expected dollar depreciation ($\mathbb{E}_t \Delta\hat{q}_{t+1}$ falls) and by a rise in the Foreign real interest rate relative to the dollar real interest rate ($\hat{r}_t^* - \hat{r}_t$ rises). This intuition underlies why, given a model with only currency intermediation shocks, a strong dollar is accompanied by a high Foreign interest rate relative to dollar interest rate. It also explains why such shocks are able to rationalize a negative Fama (1984) coefficient: a high Foreign interest rate relative to dollar interest rate forecasts an expected dollar depreciation. We note that these comovements are also obtained in the case when $\beta^* > \beta$ and thus arbitrageurs are long dollar bonds: in this case, an increase in risk aversion would induce a dollar depreciation and increase in the dollar interest rate relative to the Foreign interest rate.

By contrast, consider a relative increase in U.S. demand (decline in β_t or rise in β_t^*) or relative decrease in U.S. supply (decline in z_t or rise in z_t^*). Each of these shocks causes a relative increase in the U.S. interest rate on impact ($\hat{r}_t^* - \hat{r}_t$ falls), given the domestic Euler equations in each country. By UIP, this implies an immediate dollar appreciation and thus expected dollar depreciation ($\mathbb{E}_t \Delta \hat{q}_{t+1}$ falls).[18] Hence, given a model with only demand or supply shocks, a strong dollar is accompanied by a *low* Foreign interest rate relative to dollar interest rate, and the Fama (1984) coefficient is consistent with the sign of UIP.

The comovement between the exchange rate and relative quantities distinguishes currency intermediation and demand shocks from supply shocks. In particular, an increase in arbitrageur risk aversion or increase in relative U.S. demand induce capital flows toward the U.S., underpinning an increase in relative U.S. consumption. In the case of currency intermediation, the intuition is because arbitrageurs are less willing to intermediate capital flows to Foreign; in the case of demand shocks, the intuition is because U.S. households wish to borrow from Foreign households. By contrast, a decline in relative U.S. supply which appreciates the dollar is mechanically accompanied by relatively low U.S. consumption, in light of consumption home bias. Thus, a strong dollar is accompanied by relatively low Foreign consumption in the first two cases, but relatively high Foreign consumption in the last case.

Taken together, to the extent a strong dollar is accompanied by a relatively high U.S. interest rate and high U.S. consumption in the data, it suggests an important role for relative demand shocks. Anticipating that this is indeed the case in our empirical and quantitative analysis later in the paper, the next subsection characterizes the role of model features in shaping the effects of demand shocks in particular.

3.3 Further understanding relative demand shocks

We focus on the role of two key model features: the persistence of relative demand shocks, and the role of the price of risk Γ. For these results it is expositionally

[18] There is a complementary intuition focused on the goods market: the relative increase in U.S. demand or decrease in U.S. supply requires that the dollar appreciates to clear the goods market.

convenient to focus on the autarkic limit ($\varsigma \to 1$), though by continuity the insights also apply away from this limit too.

We first clarify the role of persistence, assuming the price of risk $\Gamma \to 0$. We study the magnitude of the impact effects of a demand shock, as well as the R^2 of regressions predicting the change in the exchange rate:

Proposition 2. *Assume A1, the autarkic limit ($\varsigma \to 1$), and the limit in which the price of risk $\Gamma \to 0$. Then on impact of an increase in U.S. demand ($\hat{\epsilon}_t^{\beta} < 0$),*

- *the U.S. real interest rate rises by $\hat{r}_t = (-\hat{\epsilon}_t^{\beta})$;*
- *the dollar appreciates by $\hat{q}_t = \frac{1}{1+r^*-\rho^{\beta}}(-\hat{\epsilon}_t^{\beta})$;*
- *all other variables are unchanged.*

Given demand shocks, the R^2 of a regression of the change in the exchange rate $\Delta\hat{q}_{t+1}$ on the interest rate differential $\hat{r}_t^* - \hat{r}_t$ is $Var(\mathbb{E}_t \Delta\hat{q}_{t+1})/Var(\Delta\hat{q}_{t+1}) = \frac{1}{1+\frac{1-(\rho^{\beta})^2}{(1+r^*-\rho^{\beta})^2}}$. If $r^* = 0$, the dollar appreciation becomes infinite and the R^2 falls to zero as $\rho^{\beta} \to 1$.

As described in the prior subsection, an increase in U.S. demand raises the domestic interest rate (by the Euler equation) and appreciates the dollar (by UIP). In the autarkic limit, these are the only variables which are affected, since consumptions are simply equal to the endowments, there are no capital flows, and Foreign is thus unaffected by the shock. The fact that the exchange rate response is rising in the persistence of the shock reflects that the U.S. interest rate remains persistently high relative to the Foreign interest rate, and the exchange rate is forward-looking.

As relative demand gets more persistent, the effect of future demand innovations on the future exchange rate similarly grow relative to the predictable component of the exchange rate. Provided $r^* = 0$, the R^2 of the predictability regression also falls. As relative demand approaches a unit root, the impact effect on the exchange rate grows without bound and the R^2 of the predictability regression approaches zero.

These results build on those in Itskhoki and Mukhin (2021), Engel and West (2005), and Corsetti et al. (2008). Itskhoki and Mukhin (2021) provide similar results for the effects of UIP shocks on the exchange rate;[19] we extend them to demand shocks. UIP shocks affect the exchange rate in the autarkic limit purely via the

[19] See their Propositions 1 and 5.

currency risk premium, while demand shocks affect the exchange rate when $\Gamma \to 0$ purely via the interest rate differential. The common ingredient, however, is that persistent shocks can induce a large exchange rate response (and thus low R^2 in predictability regressions) because exchange rates are forward-looking. In these respects, we also build directly on Engel and West (2005), who demonstrate that asset prices will exhibit near-random walk behavior if fundamentals are persistent and the discount factor is close to one (mirroring the condition $r^* = 0$ referenced at the end of Proposition 2). The fundamentals which drive exchange rates in the models studied by Engel and West (2005) include money supplies and inflation rates; we instead focus on natural rates in general equilibrium. A critical reason why persistent demand shocks induce large effects on exchange rates is that asset markets are incomplete: given our assumed preferences, the exchange rate response to a demand shock would not depend on its persistence if markets were complete. The important interaction between shock persistence and incomplete markets echoes Corsetti et al. (2008).

We finally clarify the role of the price of risk Γ for the propagation of demand shocks. We again study the magnitude of the impact effects of a demand shock as well as the Fama (1984) regression predicting the change in the exchange rate:

Proposition 3. *Assume A1, the autarkic limit ($\varsigma \to 1$), r^* small, and $\beta^* < \beta$. Then, at least around $\Gamma = 0$, an increase in Γ:*

- *dampens the dollar appreciation on impact of an increase in U.S. demand;*
- *reduces the expected dollar depreciation on impact of the same shock;*
- *and, given only demand shocks, lowers the Fama (1984) coefficient $Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)/Var(\hat{r}_t^* - \hat{r}_t)$ below one.*

Given a positive price of risk, the currency risk premium on the right-hand side of (1) also responds to demand shocks. In particular, consider an increase in U.S. demand (β_t falls) which, as described in the discussion of Proposition 1, induces capital flows to the U.S.[20] Since this reduces arbitrageurs' position in Foreign bonds ($\hat{b}_t^{a*}$ falls), it reduces the risk premium on Foreign bonds. This mitigates the expected dollar depreciation and thus initial appreciation, and generates a deviation from UIP.

[20] The endogenous responses of interest rates and the exchange rate also feed back to the risk premium, evident from the right-hand side of (1). The parametric conditions in the proposition ensure that these effects reinforce or do not overturn the effect via capital flows described here.

These results imply a tension: to the extent the endogenous risk premium response to demand shocks can account for deviations from UIP, it will also dampen the magnitude of exchange rate volatility induced by these shocks. Note that no such tradeoff is present for currency intermediation shocks, for which exchange rate volatility is precisely a consequence of deviations from UIP.

These results generalize to an infinite horizon setting those in Gabaix and Maggiori (2015), who characterize the effects of a non-zero currency risk premium (also controlled by Γ in their framework) on deviations from UIP in a three-period model.[21]

Taking stock We conclude this section by summarizing the main insights from our analytical results. Shocks to currency intermediation are distinguished from the other shocks by the comovement between the real exchange rate and real interest rates. Shocks to supply are distinguished from the other shocks by the comovement between the real exchange rate and relative consumption. More persistent demand increases the magnitude of the exchange rate response to a demand innovation, and dampens the R^2 in predictability regressions. A higher price of risk generates larger endogenous UIP deviations from demand shocks, though this dampens the magnitude of exchange rate fluctuations induced by these shocks.

4 Empirical analysis

We now measure comovements of the dollar/G10 exchange rate in the data over the 1991-2020 period. U.S. interest rates and U.S. consumption have been relatively high when the dollar is strong. At the same time, changes in exchange rates are difficult to predict. In light of our analytical results, these patterns are consistent with an important role for persistent demand shocks in driving the exchange rate.

4.1 Data

Our data covers the period 1991 Q2 through 2020 Q1.[22] We obtain data for the U.S. and G10 currency countries, which are Australia, Canada, Denmark, the Euro Area

[21] See their Proposition 8.

[22] The start date is dictated by the availability of the Libor-based interest rate data we use. The end date is chosen so that second moments are not driven excessively by the sharp recession during the pandemic, while still capturing the asset price comovements at the start of the crisis.

(aggregated across members), Japan, New Zealand, Norway, Sweden, Switzerland, and the U.K. At the end of the paper we provide a brief discussion of data for emerging market countries as well.

We obtain end-of-quarter nominal yields by currency and tenor from Bloomberg.[23] We map nominal yields in the data to real yields in our model given generally low and stable expected inflation among the countries in our sample over this period. Our primary measure of nominal yields are Libor rates (measured at the three-month tenor) combined with interest rate swaps (to obtain longer tenors) in each currency. We also investigate robustness using the government bond yield curve in each currency.

We obtain end-of-quarter nominal exchange rates, the consumer price index, national accounts, and population from the IMF International Financial Statistics. We construct per capita measures by dividing the relevant series by population.[24] Where relevant, we use seasonally adjusted measures as reported by the IMF.

We compute moments using simple averages across the G10. Since we do not have a balanced panel for some series, we compute these averages by computing quarterly changes at the country level, averaging these changes across countries with available data in each quarter, and cumulating.

4.2 Comovements with the real exchange rate

Table 1 reports contemporaneous comovements with the real exchange rate between the U.S. and G10 countries. As in the model, an increase in the real exchange rate corresponds to a real dollar appreciation. For any variable x_t, the first three columns report the slope coefficients in regressions of the form

$$\log q_{t+h} - \log q_t = \alpha + \beta(x_{t+h} - x_t) + \epsilon_{t+h},$$

for horizons $h \in \{1, 4, 12\}$ quarters and q denoting the real exchange rate, and the last column reports the slope coefficient in a regression of the form

$$\log q_t = \alpha + \beta x_t + \epsilon_t.$$

[23]In particular, we use data compiled by Wenxin Du and we thank her for sharing it with us.

[24]Since population is reported annually, we linearly interpolate it to obtain a quarterly series. We construct population for the Euro area as the sum of population in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain.

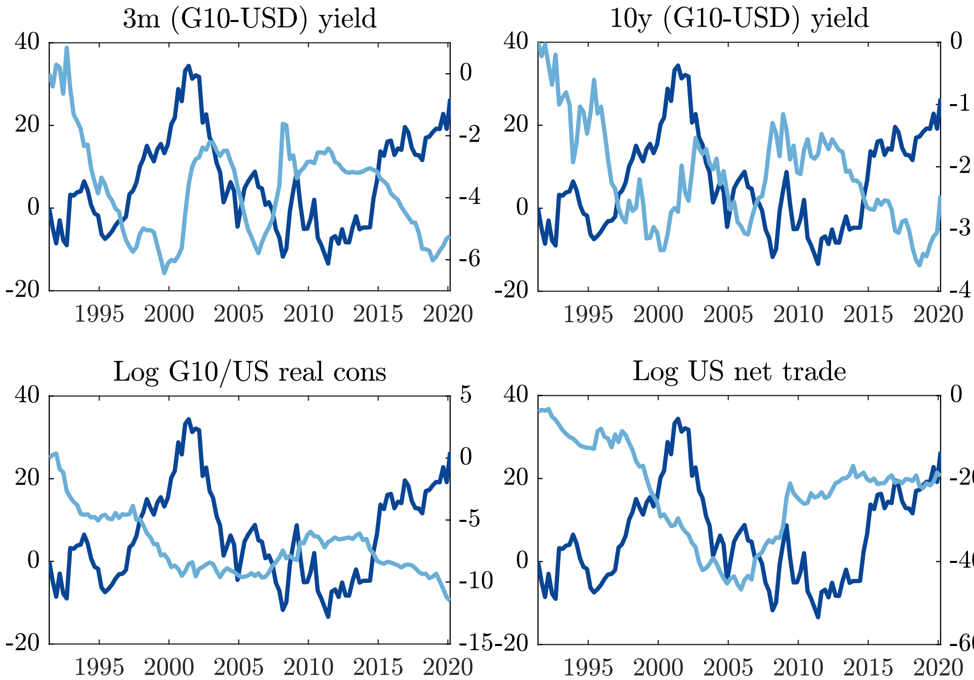

Figure 1: comovements with log real exchange rate between U.S. and G10

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series except log U.S. net trade are normalized to zero in 1991 Q2. All yields are annualized.

Figure 1 depicts the real exchange rate and other key variables over our sample period, with the former appearing in dark blue in every panel and the latter appearing in light blue and in the panel title.

Since both exchange rates and many of the right-hand side variables are highly persistent, we conduct inference using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b). Appendix C demonstrates that this approach yields much more conservative confidence intervals than using Newey and West (1987) standard errors given typical truncation parameters and standard normal critical values, and is comparable to results obtained from bootstrap methods as in Engel (2016).

The dollar is strong when G10 interest rates are relatively low versus U.S. interest rates. These relationships are evident in both changes and levels in the first two panels of Table 1, and especially strong in levels, as is clear from the first two panels of Figure

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m (G10 - USD) yield	-2.17	-1.40	-0.64	-3.28
	[-4.54,0.19]	[-4.03,1.22]	[-2.75,1.46]	[-6.78,0.23]
N	115	112	104	116
Adj R^2	0.05	0.03	-0.00	0.20
10y (G10 - USD) yield	-2.77	-4.21	-5.87	-8.48
	[-6.42,0.87]	[-6.52,-1.89]	[-9.28,-2.46]	[-13.35,-3.61]
N	115	112	104	116
Adj R^2	0.04	0.07	0.13	0.36
Log G10/U.S. real cons.	-1.75	-3.02	-3.91	-2.59
	[-2.70,-0.80]	[-4.23,-1.80]	[-6.25,-1.58]	[-4.61,-0.56]
N	115	112	104	116
Adj R^2	0.03	0.13	0.27	0.29
Log U.S. net trade	-0.08	-0.10	-0.26	-0.21
	[-0.33,0.17]	[-0.51,0.30]	[-0.75,0.24]	[-0.74,0.31]
N	115	112	104	116
Adj R^2	-0.01	-0.01	0.02	0.04

Table 1: comovements with log real exchange rate between U.S. and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

1. The point estimates in levels imply that a 1*pp* annualized decline in the G10 yield relative to the dollar yield is associated with a 3–8*pp* stronger dollar, increasing in tenor; the confidence intervals for the 10-year yield differential in particular exclude zero in all cases besides one-quarter changes. Variation in yield differentials account for 20–36% of the variation in the exchange rate, though the bootstrap results in appendix C make clear that the high persistence of these variables implies considerable uncertainty about these R^2 values. Figure 1 also shows that the comovement reverses around the onset of recessions, namely the early 2000s, 2008, and 2020, when the dollar appreciates even though G10 interest rates rise versus U.S. interest rates.

For that reason, the explanatory power of yield differentials is especially strong when we also condition on proxies for risk or convenience yields which spike in these periods. The first panel of Table 2 reports the univariate comovement with the excess bond premium of Gilchrist and Zakrajsek (2012). A higher excess bond premium is associated with a stronger dollar. In changes, it accounts for a slightly higher share of the variation in the exchange rate as yield differentials; in levels, it accounts for less of the variation. Notably, however, in a bivariate specification including both yield differentials and the excess bond premium in Table 2, the R^2 is greater than the sum of the univariate specifications, consistent with the comovement between the exchange rate and yield differential flipping sign if risk is high. The R^2 values in fact exceed 50%; in the bootstrapped confidence intervals for these R^2 values reported in appendix C, even the lower bounds reach nearly 30%. Appendix C also demonstrates similar results using yield differentials and other proxies for risk or convenience yields such as the VIX, the global factor in risky asset prices of Miranda-Agrippino and Rey (2020), and the three-month Treasury basis from Du, Im, and Schreger (2018).[25]

Turning from asset prices to quantities, the dollar is also strong when real G10 consumption per capita is relatively low versus its U.S. counterpart. The relationship is significant in both changes and levels in the third panel of Table 1. A 1*pp* decline in G10 consumption per capita relative to the U.S. is associated with a 3*pp* stronger dollar. Variation in relative consumption accounts for 29% of the variation in the exchange rate, though again there is uncertainty about this univariate R^2. Unlike with yield differentials, the comovement with relative consumption is not different around the start of recessions, as is evident from the third panel of Figure 1.

Finally, the dollar is only weakly negatively related to U.S. net trade, defined as the ratio of exports to imports.[26] This result is true in both changes and levels in the last panel of Table 1, and evident from the last panel of Figure 1. It is consistent with the weak unconditional comovement of the exchange rate and trade balance documented in the prior literature.

Turning from contemporaneous comovements to predictability, Table 3 reports

[25] We find similar results using convenience yields from Jiang et al. (2021) or Engel and Wu (2023).

[26] We focus on net trade rather than net exports to GDP because the latter also reflects secular trends in the magnitude of exports and imports relative to GDP which are not our focus in this paper (see Alessandria and Choi (2021)). With that said, we obtain similarly weak contemporaneous comovements between the exchange rate and net exports to GDP.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
EBP	2.40	3.66	5.89	5.27
	[0.23,4.58]	[2.16,5.15]	[-0.43,12.20]	[-2.83,13.37]
N	115	112	104	116
Adj R^2	0.06	0.13	0.20	0.08
3m (G10 - USD) yield	-2.28	-1.90	-1.62	-3.70
	[-4.77,0.21]	[-4.87,1.06]	[-3.65,0.41]	[-7.26,-0.15]
EBP	2.51	4.07	6.76	6.66
	[0.38,4.64]	[3.03,5.10]	[1.15,12.37]	[-0.47,13.79]
N	115	112	104	116
Adj R^2	0.11	0.18	0.25	0.33
10y (G10 - USD) yield	-5.41	-7.28	-9.14	-9.36
	[-6.80,-4.02]	[-10.67,-3.89]	[-13.80,-4.49]	[-16.11,-2.62]
EBP	4.20	5.42	8.28	7.09
	[2.42,5.98]	[4.35,6.50]	[4.22,12.35]	[1.95,12.23]
N	115	112	104	116
Adj R^2	0.19	0.32	0.50	0.51

Table 2: additional comovements using proxy for risk

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016). Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

the slope coefficient from the regression

$$(\log E_{t+h} - \log E_t) = \alpha + \beta \left(i_t^{(h)*} - i_t^{(h)} \right) (h/4) + \epsilon_{t+h},$$

where E_t denotes the nominal exchange rate (foreign currency per dollar) and $i_t^{(h)*}, i_t^{(h)}$ denote annualized h-quarter nominal yields abroad and in the U.S., respectively. We use the nominal exchange rate on the left-hand side to be consistent with the large literature following Fama (1984), but very similar results are obtained using the real exchange rate given its tight comovement with the nominal exchange rate.

	$\log E_{+1} - \log E$	$\log E_{+4} - \log E$
3m (G10 - USD) yield / 4	-0.74	
	[-2.73,1.25]	
1y (G10 - USD) yield		-1.49
		[-6.71,3.72]
N	115	92
Adj R^2	-0.00	0.03

Table 3: predicting changes in log nominal exchange rate

Notes: yields are annualized (so first row uses quarterly rates). Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

These results indicate that while the evidence against UIP has weakened, what remains the case is that interest rate differentials account for little variation in subsequent exchange rate movements. The negative point estimates imply that higher G10 yields than dollar yields forecast an expected dollar depreciation, inconsistent with UIP. However, the confidence intervals are so wide that we cannot reject UIP (a point estimate of 1) at conventional levels. This is in contrast to earlier sample periods over which there was more precise evidence against UIP (e.g., Lustig, Roussanov, and Verdelhan (2014)). However, what remains the case even in the more recent data is that the R^2 in these predictability regressions is low — an order of magnitude lower than in the regressions of exchange rates on contemporaneous yields.

4.3 Additional findings

Appendix C reports additional findings in the data which we summarize here.

First, we replace the private sector interest rate differentials in our analysis — constructed from Libor rates and interest rate swaps — with government bond yield differentials. We similarly find that the dollar is strong when G10 government bond yields are low versus U.S. government bond yields, and the share of variation explained by these yield differentials is especially high when also conditioning on proxies for risk.

Second, we study the comovement between bilateral real exchange rates and interest rate differentials for each of the G10 countries versus the U.S. For all currencies except the Japanese yen, a lower foreign yield than the dollar yield is associated with a stronger real dollar. This indicates that the comovement which we document between the broad dollar/G10 exchange rate and yield differential is robust across currencies.

Third, we replace the U.S. with three alternative base currencies and countries: the pound and U.K., the euro and Euro Area, and the yen and Japan. The comovements which we study are not U.S.-specific: when we use the pound as an alternative base currency, we obtain very similar results. Low G10 interest rates (which now includes the U.S. and excludes the U.K.) relative to U.K. interest rates and low G10 consumption per capita relative to its U.K. counterpart are associated with a stronger pound in real terms versus the G10, both in changes and in levels. These results are not as sharp for the Euro area, though this reflects the shorter sample period of available data. They are also again different for Japan in the context of yield differentials.

Fourth, we extract the low frequency components of each time series and focus on the comovements between these, following Müller and Watson (2018). An advantage of their approach is that it is robust to nonstationarity.[27] We still find that the low frequency component of the exchange rate is significantly negatively related to the low frequency component of G10 less U.S. yield differentials, and to the low frequency component of log G10 consumption relative to U.S. consumption. This analysis remains relevant to understand most of the variation in the exchange rate, since 85% (70%) of it is in periodicities above 6 (12) years.

Fifth, we assess the out-of-sample explanatory power of these variables for the exchange rate, following Meese and Rogoff (1983). In contrast to their findings, we find that over the last 30 years, yield differentials, measures of risk, and relative consumption can beat a random walk in accounting for the exchange rate out of sample. This builds on the recent findings of Engel and Wu (2024), who show that the out-of-sample performance of such variables has improved over time.

Taking stock We conclude this section by summarizing the takeaways from our empirical analysis. Both in changes and levels, U.S. interest rates and U.S. consumption have been relatively high when the dollar is strong. The explanatory power of yield differentials is especially strong when we also condition on proxies for risk. At the same time, changes in exchange rates remain difficult to predict, at least at short horizons. The comovements with the exchange rate are broadly robust to the defini-

[27] With that said, the recent literature appears to have settled on the real exchange rate in fact being stationary (see for instance Engel (2016) and Engel, Kazakova, Wang, and Xiang (2022)), motivating our baseline empirical specifications.

tion of yields used and the possibility of non-stationarity; they hold across currencies and do not appear U.S.-specific; and they are strong enough to beat a random walk out of sample. In light of our analytical results, we conclude from these empirical findings that persistent innovations in relative demand, reflected in persistent yield and consumption differentials, may be an important driver of exchange rates.

5 Quantitative analysis

We now quantify the model's ability to account for the volatility of and comovements with the dollar/G10 exchange rate in the data. Demand shocks account for roughly 75% of the variance in the dollar/G10 exchange rate, and generate contemporaneous comovements between the exchange rate, interest rate differential, and relative consumption as in the data. Currency intermediation shocks are also important, however, to account for deviations from uncovered interest parity at high frequencies, and to explain the dollar appreciation in crises.

5.1 Parameterization

We first discuss the parameterization of our model.

We set the IES ψ of households to 1 (log preferences) for simplicity. We also set three parameters which are all normalizations: U.S. steady-state output per capita is $z = 1$, Foreign steady-state output per capita is $z^* = 1$, and the elasticity of substitution across varieties from a given country is $\theta = 4$. The first defines the scale of the economy and the others are without loss of generality because only the ratios $\zeta^* z^*/z$ and $\xi/(\theta - 1)$ are relevant.

We calibrate the remaining parameters to match moments in the data, as summarized in Table 4. We first describe the calibration of stochastic processes, and then describe the few remaining parameters of the model. In all cases we compute moments in the data over our maintained 1991 Q2 - 2020 Q1 sample period. We discuss the moment in the data which most directly informs each parameter, informed by our analytical results from section 3.

In the first panel, we discipline the discount factors in each country to match moments on interest rates. The steady-state U.S. discount factor β is set to target an annualized U.S. real interest rate of 0.97%, equal to the average three-month dollar

	Description	Value	Moment	Target	Model
β	U.S. disc. fact.	0.99758	$r^{(1)}$	0.97%	0.97%
β^*	Foreign disc. fact.	0.99708	$r^{(40)*} - r^{(40)}$	0.20%	0.20%
$\sigma^\beta, \sigma^{\beta^*}$	s.d. β, β^* shocks	0.002	$\sigma(r^{(40)*} - r^{(40)})$	0.81%	0.80%
$\rho^\beta, \rho^{\beta^*}$	persistence β, β^*	0.98	$\rho_{-1}(r^{(40)*} - r^{(40)})$	0.93	0.93
ρ^{β,β^*}	corr. β, β^* shocks	0.84	$\sigma(\Delta r^{(40)})$	0.53%	0.53%
σ^z, σ^{z^*}	s.d. z, z^* shocks	0.005	$\sigma(\log y^* - \log y)$	1.73%	1.76%
ρ^z, ρ^{z^*}	persistence z, z^*	0.95	$\rho_{-1}(\log y^* - \log y)$	0.91	0.91
ρ^{z,z^*}	corr. z, z^* shocks	0.10	$\sigma(\Delta \log y)$	0.49%	0.51%
σ^γ	s.d. γ shocks	11	$\sigma(\Delta \log q)$	3.91%	4.00%
ρ^γ	persistence γ	0.85	$\rho_{-1}(\log \gamma)$	0.80	0.82
ξ	pricing to market	1.3	$\sigma(\log s)/\sigma(\log q)$	0.27	0.27
Γ	arb risk pricing	$6E-4$	$\sigma(\log ex/im)$	11.41%	11.88%
σ	trade elasticity	0.9	$\sigma(\log q)$	11.48%	10.39%
ς	home bias	0.8	$(ex+im)/y$	0.25	0.23
ζ^*	rel. population	1.35	$\zeta^* q^{-1} y^*/y$	1.35	1.34

Table 4: targeted moments and calibrated parameters

Notes: data moments are estimated over 1991 Q2 – 2020 Q1. Model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Externally set parameters in model are $\psi = 1$, $z = 1$, $z^* = 1$, and $\theta = 4$, the latter three all normalizations.

yield (2.97%) less expected inflation which we assume to be 2%.[28] The steady-state Foreign discount factor β^* is set to target the average 10-year G10/U.S. yield differential of 0.20% over this period.[29] It follows from this that an increase in arbitrageur risk aversion will appreciate the dollar, as discussed in section 3. We assume for simplicity that the volatility of discount factor shocks $\sigma^\beta, \sigma^{\beta^*}$ and the persistence of discount factors $\rho^\beta, \rho^{\beta^*}$ in each country are the same. We jointly discipline these values and the correlation ρ^{β,β^*} to target the volatility and autocorrelation of the 10-year G10/U.S. yield differential and the volatility of changes in the 10-year U.S.

[28]In keeping with our notation for longer maturity bonds, we denote $r^{(1)}$ as the annualized yield on a one-quarter bond, i.e., $4r$. Analogously, we define $r^{(1)*} \equiv 4r^*$.

[29]We price 10-year bonds in the model by assuming households in each country can trade these bonds with other households in the same country. Because households in each country are identical, there is no trade in these bonds and their presence does not affect the allocation. Under our first-order approximation, the expectations hypothesis holds along each country's yield curve.

yield. We focus on 10-year yields in our baseline calibration as longer tenor bonds are less affected by the short-run interest rate smoothing of central banks which are not likely important drivers of the exchange rate (in light of our analytical results). We note that the persistence of the yield differential, evident from Figure 1, implies a high persistence of $\rho^{\beta} = \rho^{\beta^*} = 0.98$.

In the second panel, we discipline the endowments in each country to match moments on output per capita. In the model, this is the per capita value of final goods expressed in units of consumption in each country:

$$
\begin{aligned}
y_t &\equiv p_{Ht} c_{Ht} + \zeta^* q_t^{-1} p_{Ht}^* c_{Ht}^*, \\
y_t^* &\equiv \frac{1}{\zeta^*} q_t p_{Ft} c_{Ft} + p_{Ft}^* c_{Ft}^*.
\end{aligned}
$$

In the data, we analogously define output per capita as nominal GDP less investment and government spending, scaled by the consumption deflator and population.[30] Again for simplicity we assume that the volatility of endowment shocks σ^z, σ^{z^*} and the persistence of endowments ρ^z, ρ^{z^*} are the same. We jointly discipline these values and the correlation ρ^{z,z^*} to target the volatility and autocorrelation of log relative output per capita and the volatility of changes in log U.S. output per capita.

We note that ρ^{β,β^*} and ρ^{z,z^*} are the only non-zero correlation coefficients in our model; we assume all others are zero. There are clearly common components of global yields and output, and these correlation parameters allow our model to capture them. We could have equivalently modeled discount factors and endowments in our model as the sum of global and differential components which are independent.

In the third panel, we discipline the risk aversion of arbitrageurs. We treat the excess bond premium as an empirical counterpart of arbitrageurs' log risk aversion, up to scale. We thus discipline the persistence ρ^{γ} to match the persistence of the excess bond premium. We note that the persistence of 0.85 implies a half life of the $\log \gamma$ process around one year, much less than the half-life of the $\log \beta - \log \beta^*$ process of more than eight years. Using alternative proxies for $\log \gamma$ such as the VIX, global factor in risky asset prices, or three-month Treasury basis would imply the same result. We then discipline the volatility of arbitrageur risk aversion shocks σ^{γ}

[30]We take out investment and government spending to be consistent between model and data, because our model does not feature these variables. Since some of net exports in the data are also used for investment or government spending, we also take out a share of net exports proportional to the ratio between investment and government spending and these variables plus consumption.

to match the volatility of quarterly changes in the dollar/G10 real exchange rate.

In the final panel, we discipline the remaining five parameters of the model. We set the pricing-to-market coefficient ξ to match the ratio of the volatility of the U.S.' terms of trade to the volatility of the dollar/G10 real exchange rate, where the terms of trade

$$s \equiv q^{-1} p_H^* / p_F$$

is the price of U.S. exports relative to imports. The calibrated value $\xi = 0.27$ is consistent with estimates of long-run exchange rate pass-through.[31] We jointly set the steady-state price of risk Γ and trade elasticity σ to target the volatility of log U.S. net trade and the dollar/G10 real exchange rate in levels. Recall that net trade is the ratio of exports to imports, where U.S. exports and imports in the model are (in units of the U.S. consumption bundle)

$$\begin{aligned} ex_t &\equiv \zeta^* q_t^{-1} p_{Ht}^* c_{Ht}^*, \\ im_t &\equiv p_{Ft} c_{Ft}. \end{aligned}$$

The price of risk matters for the volatility of net trade because the risk-bearing capacity of arbitrageurs governs the elasticity of capital flows, and thus trade flows, to macroeconomic shocks. The trade elasticity determines how much exchange rate volatility is consistent with the observed trade volatility. The calibrated value $\sigma = 0.9$ is in line with other studies jointly matching aggregate prices and trade volumes.[32] We set the degree of home bias ς to match the average ratio of U.S. exports plus imports relative to output. Finally, we set the relative population of Foreign ζ^* to match the average ratio of cumulative G10 output to U.S. output.

5.2 Exchange rate volatility and comovements

With the calibrated model in hand, we now use it to study the volatility of and comovements with the real exchange rate. A systematic discussion of other untargeted second moments of interest rates and quantities is provided in appendix D.

[31] Firms' optimal price setting implies $\hat{p}_{Ht}^* - \hat{p}_{Ht} = (1 - \xi/(\theta - 1))\, \hat{q}_t$ and $\hat{p}_{Ft} - \hat{p}_{Ft}^* = -(1 - \xi/(\theta - 1))\, \hat{q}_t$. Given $\theta = 4$ and $\xi = 1.3$, exchange rate pass-through is 0.57. By comparison, Burstein and Gopinath (2014) report long-run pass-through into import price indices of 0.51 for the U.S. and 0.71 on average across G10 economies (their Table 7.4).

[32] Backus, Kehoe, and Kydland (1994) propose $\sigma = 1.5$, while Heathcote and Perri (2014) estimate $\sigma = 0.6$ (their Table 9.4) and Alessandria and Choi (2021) estimate $\sigma = 1$ (their Table 1).

		σ		ρ_{-1}	
	Variable	Data	Model	Data	Model
$\log q$	log exchange rate	11.48%	10.39%	0.94	0.91
$\Delta \log q$	Δ log exchange rate	3.91%	4.00%	0.04	-0.04

Table 5: exchange rate volatility and autocorrelation in data and model

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

Table 5 demonstrates that the model accounts well for the volatility and autocorrelation of the log exchange rate both in levels and in one-quarter changes. While the volatility moments were calibration targets, the autocorrelation moments were not, so it is notable that the model implies near random-walk behavior in the exchange rate (a high autocorrelation in levels and close to zero in changes), as in the data.

Figure 2 demonstrates that demand shocks are the predominant driver of exchange rates in the model. It decomposes the variance of changes in the exchange rate at increasing horizons into the contribution from each set of shocks. Demand shocks and intermediation shocks account for comparable shares of the variance in quarterly changes. As the horizon increases, demand shocks account for more and intermediation shocks for less of the variance. In levels, demand shocks account for 77% and intermediation shocks account for 21% of the variance in the exchange rate. At all horizons and in levels, supply shocks account for little of the variance.

Persistence and low steady-state currency intermediation frictions are the key mechanisms generating large exchange rate volatility from demand shocks. Figure 3 depicts the impulse responses to a one standard deviation shock. In response to a 20*bp* decline in the U.S. discount factor, the dollar appreciates by roughly 500*bp*, the Foreign interest rate falls by roughly 15*bp* relative to the U.S. interest rate, Foreign consumption falls by roughly 150*bp* relative to U.S. consumption, U.S. net trade falls by nearly 600*bp*, and the expected return to Foreign bonds less dollar bonds falls very slightly. These results are consistent with the impact effects of demand shocks characterized in section 3. The exchange rate response is an order of magnitude larger than the response of the interest rate differential because the latter is persistently low and the exchange rate is forward-looking. Indeed, if we lower persistence to $\rho^\beta = 0.9$, the same shock induces a dollar appreciation which is less than a third of that in the baseline calibration, consistent with Proposition 2. The exchange rate response is

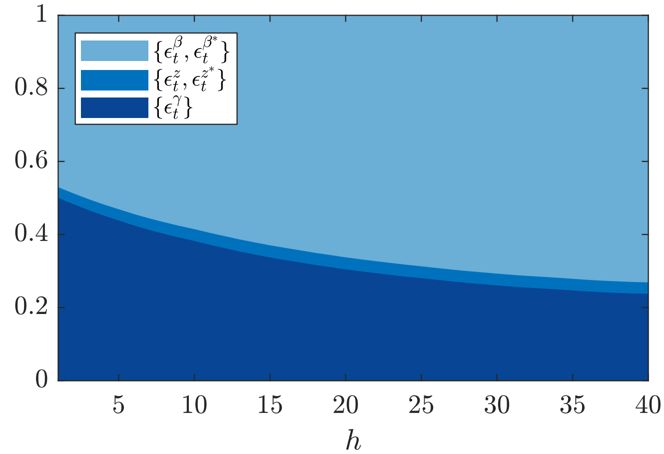

Figure 2: variance decomposition of exchange rate changes over h quarters

Notes: figure depicts, for each h, the share of $\sigma^2(\log q_{t+h} - \log q_t)$ in the model due to each set of driving forces alone. Variances are averaged over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

also large because the steady-state intermediation frictions controlled by the price of risk Γ are small, so the decline in the currency risk premium does not dampen much the exchange rate response in (1). If we raise the steady-state price of risk by a factor of 10 to $\Gamma = 6 \times 10^{-3}$, the same shock induces a dollar appreciation which is only two thirds of that in the baseline calibration, consistent with Proposition 3.

Table 6 compares comovements with the exchange rate between data and model, all of which were untargeted in the calibration.[33] The first column reports 90% confidence intervals from the levels specification in Table 1. The second column reports the model-generated comovements, and the remaining columns report comovements eliminating two of the three sets of shocks to focus on each set alone.

The first two panels indicate that the model is successful in accounting for the comovement between the exchange rate and yield differentials. At both tenors, the model-generated regression coefficients are near the center of the empirical confidence intervals, the R^2 coefficients are of a similar order of magnitude as the data, and the R^2 coefficients are rising in tenor as in the data. Consistent with Proposition 1, demand or supply shocks are necessary to match the sign of the empirical comovements: with only intermediation shocks, the dollar would be strong when U.S. yields are

[33] Appendix D compares the specifications in changes between data and model, as well as the out-of-sample analysis in data and model.

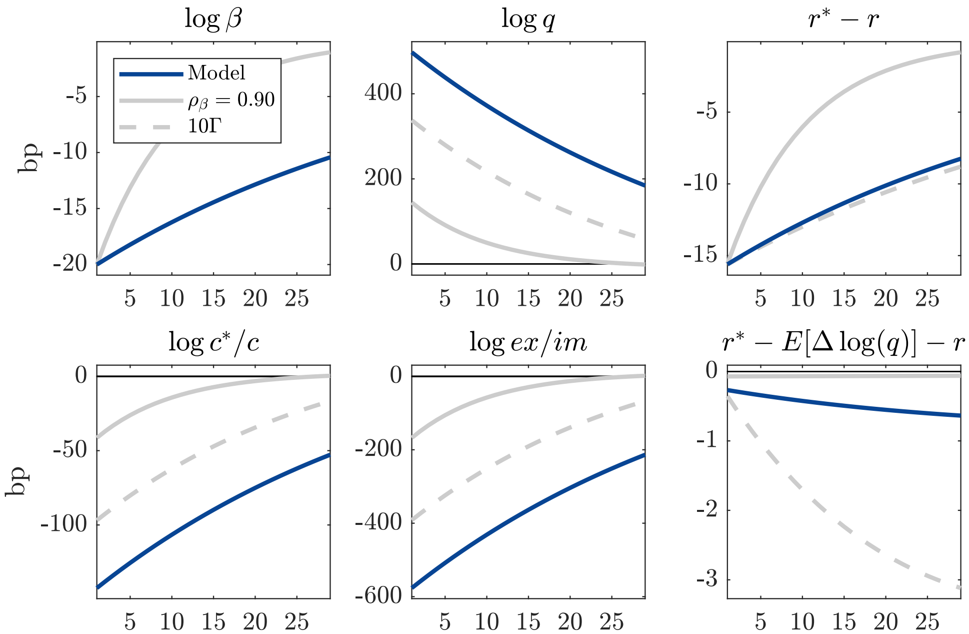

Figure 3: impulse responses to demand shock

relatively low. At the same time, the presence of intermediation shocks is important to explain why the R^2 of these regressions is below one: with only demand or supply shocks, the yield differential would account for too much of the exchange rate. Since intermediation shocks are more transitory than demand shocks, they dampen the comovements and R^2 for short tenors more so than longer tenors.[34]

The third panel indicates that the model is also successful in accounting for deviations from UIP and weak predictability of the exchange rate at high frequencies. Using the interest rate differential to predict the appreciation of the dollar over the next quarter (similar results are obtained using the one-year yield differential and exchange rate appreciation over the next year), the model implies predictable deviations from UIP, but also that the interest rate differential explains essentially none of the variation in the subsequent change in the exchange rate. Both of these results are consistent with the data. Consistent with Proposition 1, here intermediation shocks are essential because they imply that the dollar would be expected to depreciate when U.S. rates are relatively low. In their absence, a relatively low dollar yield would fore-

[34]The presence of both shocks also implies that exchange rates are not "spanned" by bond returns. In appendix D, we replicate the regressions of Chernov et al. (2024) on model-generated data.

	Data	Model	Only $\{\epsilon_t^{\beta}, \epsilon_t^{\beta^*}\}$	Only $\{\epsilon_t^{z}, \epsilon_t^{z^*}\}$	Only $\{\epsilon_t^{\gamma}\}$
$\log q$ on $r^{(1)*} - r^{(1)}$					
Coefficient	[-6.78,0.23]	-2.59	-7.36	-5.20	5.56
Adj R^2	0.20	0.20	0.88	0.99	0.99
$\log q$ on $r^{(40)*} - r^{(40)}$					
Coefficient	[-13.35,-3.61]	-9.40	-11.06	-11.95	33.66
Adj R^2	0.36	0.52	0.87	0.99	0.99
$\log q_{+1} - \log q$ on $(r^{(1)*} - r^{(1)})/4$					
Coefficient	[-2.73,1.25]	-0.29	2.14	1.81	-4.13
Adj R^2	-0.00	-0.00	0.03	0.04	0.09
$\log q$ on $\log c^* - \log c$					
Coefficient	[-4.61,-0.56]	-2.34	-3.49	1.05	-3.49
Adj R^2	0.29	0.58	1.00	0.99	1.00
$\log q$ on $\log ex/im$					
Coefficient	[-0.74,0.31]	-0.86	-0.86	5.86	-0.86
Adj R^2	0.04	0.96	1.00	0.44	1.00

Table 6: exchange rate comovements and predictability in data and model

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

cast an expected dollar appreciation by an amount which is inconsistent with the data.[35] At the same time, the low R^2 in such predictability regressions is not unique to intermediation shocks: even persistent demand shocks imply a lower R^2 than in contemporaneous regressions, consistent with Proposition 2.

The fourth panel indicates that the model is again successful in accounting for the comovement between the exchange rate and relative consumption. As in the data, the dollar is strong when Foreign consumption is relatively low versus U.S. consumption. Consistent with Proposition 1, both demand and intermediation shocks deliver this comovement. One shortcoming of the model is that relative consumption explains

[35]Relatedly, because of intermediation shocks, the model generates realistic returns on the dollar carry trade, a strategy paying $\text{sign}(r_t^* - r_t)\,(r_t^* - \Delta \log q_{t+1} - r_t)$ each period $t+1$ (following Lustig et al. (2014)). The annualized average return and Sharpe ratio of this strategy in the model are 1.6% and 0.10, comparable to our estimates of 3.1% and 0.21 in the data and the estimates for the dollar trade in Hassan and Mano (2019).

double the variation in the exchange rate in the model relative to the data.

The fifth panel indicates that the model counterfactually implies a tighter link between the dollar and net trade than exists in the data (though the sign is consistent between data and model). This is because both demand and intermediation shocks imply that a weak dollar induces a contemporaneous expenditure switch toward U.S. goods.[36] Extending the model to feature investment in capital and/or dynamic trade would be one way of weakening the contemporaneous link between the exchange rate and trade balance, as in Backus et al. (1994), Alessandria and Choi (2007), and Drozd and Nosal (2012). Appendix D demonstrates that adding trade shocks along the lines of Alessandria and Choi (2021) and Mac Mullen and Woo (2024) also resolves this model shortcoming. Since these features dampen the comovement between the exchange rate and trade flows, they also dampen the share of exchange rate variation accounted for by relative consumption above. Disciplining the stochastic properties of trade shocks to match these moments, it remains that relative demand shocks account for most of the variation in the exchange rate.

Our conclusion that relative demand shocks account for most of the variation in the exchange rate contrasts with that of Itskhoki and Mukhin (2021). That paper does not include demand shocks and concludes that intermediation shocks drive nearly all the volatility in the exchange rate. Appendix D compares moments generated by our model and that in Itskhoki and Mukhin (2021). While both models generate comparable moments in regards to exchange rate volatility, autocorrelation, predictability (or lack thereof), and comovement with relative consumption, the dominant role of intermediation shocks in Itskhoki and Mukhin (2021) counterfactually implies that the dollar is strong when U.S. yields are relatively low. Figure 4 depicts histograms of these comovements across many simulations from both models, making evident that this difference between models is clear even accounting for sampling uncertainty.[37] By

[36]Supply shocks also induce such an expenditure switch, but there is a countervailing effect operating through incomes: a relative increase in U.S. supply (which depreciates the dollar) raises relative U.S. income and thus import demand. At our calibrated parameter values, this effect dominates. However, because supply shocks are a minor driver of the exchange rate, they do not much affect the unconditional comovement between the dollar and net trade.

[37]These histograms are also useful in addressing the concern that the estimated relationship between the exchange rate and yield differential in the data may reflect a spurious correlation among highly persistent variables. If the true data generating process was one without relative demand shocks, as in Itskhoki and Mukhin (2021), we would expect to estimate a comovement between the exchange rate and three-month yield differential (10-year yield differential) as or more negative than what we estimate in the data only 3% (2%) of the time.

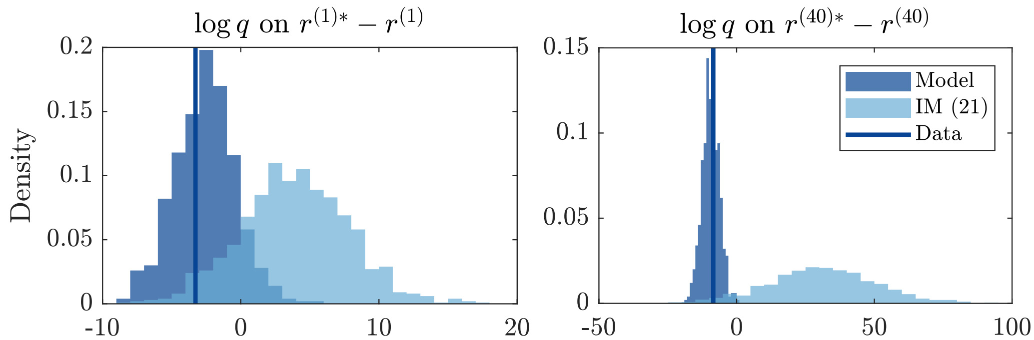

Figure 4: comovement between exchange rate and yield differentials

Notes: for our model and that in Itskhoki and Mukhin (2021), histograms depict estimated coefficients from 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Itskhoki and Mukhin (2021) model simulated using online replication package and calibration "IRBC+" (column 7 of Table 1 in that paper). We price long bonds using expectations hypothesis and we use nominal yield differential for $r_t^{(\tau)} - r_t^{(\tau)*}$, as we do in data. Dark blue lines depict our estimates in data.

adding demand shocks and disciplining them using observed yield differentials, we are able to build on the successes of Itskhoki and Mukhin (2021) while also matching the comovement between the exchange rate and yield differentials in the data.

5.3 The exchange rate over 1991-2020

The prior subsection studied model properties using averages over many simulations of 116 quarters each. Here we instead use the model to invert the sequence of shocks required to rationalize the observed sequence of 10-year bond yields, output per capita, and the excess bond premium over 1991-2020, and we ask how the model-implied paths of other variables compare to the data.

We proceed as follows. Since we treat the excess bond premium as our data counterpart to arbitrageur risk aversion up to scale, this series alone delivers the sequence of $\{\epsilon_t^\gamma\}$. The observed sequence of output per capita largely disciplines the sequence of $\{\epsilon_t^z, \epsilon_t^{z^*}\}$.[38] We finally recover $\{\epsilon_t^\beta, \epsilon_t^{\beta^*}\}$ from the observed sequence of 10-year bond yields. We then feed these shocks into our model. In the simulation, we initialize the model's exogenous state variables to be consistent with the 10-year

[38]We detrend output per capita by taking out a linear trend obtained as the weighted average of linear trends in U.S. output per capita and G10 output per capita, with weights equal to the average output shares of the U.S. and G10 (1/2.35 and 1.35/2.35, respectively).

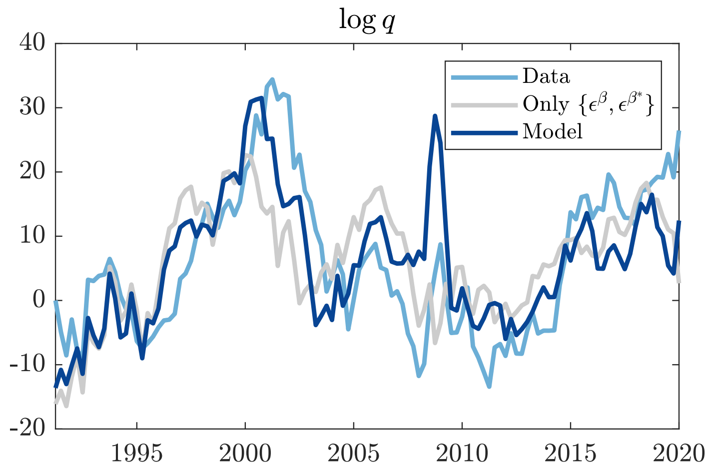

Figure 5: dollar/G10 exchange rate in data and model

Notes: model-implied exchange rate simulated by inverting $\{\beta_t, \beta_t^*, z_t, z_t^*, \gamma_t\}$ to match 10-year yields and output per capita in U.S. and G10, as well as excess bond premium of Gilchrist and Zakrajsek (2012). All series plotted in *pp*. Constant is added to model-generated series to match same average value as data series.

yields, output per capita, and excess bond premium in 1991 Q2, relative to their time-series averages through 2020 Q1. We initialize the model's endogenous state variables to their steady-state values for simplicity and in the absence of a clear alternative.

Figure 5 demonstrates that the simulated model is remarkably successful in accounting for the trend and cycles in the dollar/G10 exchange rate over this period. We emphasize that the exchange rate was not at all used in recovering the model's shocks, except that the intermediation shocks are scaled so that the model's unconditional volatility of exchange rate changes equals that in the data. With only demand shocks, we find that the simulated path continues to account for much of the data, consistent with the variance decomposition in Figure 2. This underscores our finding that demand shocks as reflected in persistent interest rate differentials are the predominant driver of the dollar/G10 exchange rate.

This result may be surprising in light of present value decompositions which typically conclude that expected interest rate differentials account for little of the variation in exchange rates (see, for instance, Froot and Ramadorai (2005)). Appendix D uses our model to clarify that small-sample bias renders the conclusions from such decompositions misleading. In small samples, there is downward bias in the estimated autocorrelation of the interest rate differential, given its high persistence. Vector autoregressions thus interpret too much of the volatility of the exchange rate as "excess volatility" which cannot be explained by expected interest rate differentials.[39]

At the same time, Figure 5 also makes clear that demand shocks alone are unable to explain the dollar appreciation at the onset of recessions: in the early 2000s, 2008, and 2020. As discussed in the context of our empirical results, in these periods, the dollar strengthened even as dollar interest rates fell relative to G10 interest rates. For this reason the full model, which includes positive γ_t shocks in these periods disciplined by the excess bond premium, better matches the data in Figure 5.[40]

Appendix D reports model simulations of other variables of interest. The model generates the low-frequency movement in relative consumption observed in the data, though it is more volatile at higher frequencies than the data. The model-implied net trade series is similarly too volatile in changes and appears to lead the data. As discussed in the prior subsection, accounting for endogenous production, dynamic trade, and/or trade shocks would improve the model along these dimensions.[41]

Taking stock In summary, our parsimonious model accounts for a substantial share of the volatility in the dollar/G10 exchange rate and its comovements. Demand shocks, reflected in persistent interest rate differentials between the U.S. and G10 economies, are the predominant driver of the exchange rate. These shocks also generate contemporaneous comovements between the exchange rate, interest rate differential, and relative consumption comparable to the data. At the same time, currency intermediation shocks, disciplined by the excess bond premium in our implementation, are also important to account for deviations from uncovered interest parity at high frequencies, and to explain the dollar appreciation in crises.

Appendix D demonstrates that our quantitative conclusions are robust to two alternative calibrations. The first allows relative demand, relative supply, and intermediation shocks to be correlated with one another. This calibration underscores that

[39] This conclusion echoes a point made in Engel et al. (2022). It is also similar to that of Atkeson, Heathcote, and Perri (2024) in the context of news about dividend growth and stock price volatility.

[40] Appendix D indeed depicts a tight comovement between the unexplained component of the exchange rate using only demand shocks and the excess bond premium, VIX, or global factor in risky asset prices of Miranda-Agrippino and Rey (2020), echoing the discussion around Table 2.

[41] The appendix also illustrates that, even when trade shocks are set to exactly rationalize net trade in the data over 1991-2020, it does not much change the model-implied exchange rate series, or the conclusion that most of its variation is driven by demand shocks.

we do not view the shocks in our framework as structural; they are better thought of as wedges which may be correlated because of the propagation of deeper structural shocks.[42] Nonetheless, even in this case demand wedges account for most of the variation in the exchange rate: the implied correlations are not large enough to change our results. The second alternative calibration targets the stochastic properties of the three-month rather than 10-year yield differential. The role of demand shocks again is quite comparable to our baseline, because the calibration implies even more persistent relative demand, but less volatile shocks.

6 Interpretation and directions for future work

We conclude with an interpretation of our findings and directions for future work.

What drives movements in relative demand? Our analysis implies that persistent shocks to relative demand, reflected in persistent differences in real interest rates and thus natural rates of interest, are a primary driver of exchange rates. This is important because it qualifies the idea of "exchange rate disconnect": while the exchange rate may have limited effects on the economy due to features such as high home bias, a low trade elasticity, or pricing to market, the shocks driving most exchange rate variation are *not* disconnected from those driving other asset prices and macroeconomic quantities. With that said, we have modeled demand shocks in reduced form, and an obvious next question is what exactly such shocks are.

Potential candidates include shocks to credit conditions, demographics, or uncertainty, which generate comovements like discount factor shocks.[43] It may be that nominal rigidities are an important transmission mechanism shaping these comovements. However, we do not interpret monetary shocks themselves as plausible candidates to drive most exchange rate volatility. This is in part because standard estimates of nominal rigidities would imply monetary shocks should not have such persistent effects on real interest rates, but also because monetary shocks induce comovements

[42]For instance, these correlations imply that a country experiencing a fall in its endowment also sees a decline in demand (a higher discount factor), which may arise from nominal rigidities and monetary policy which does not perfectly track the natural rate in the short run.

[43]See, for instance, Gourio (2012), Eggertsson and Krugman (2012), Carvalho, Ferrero, and Nechio (2016), Guerrieri and Lorenzoni (2017), and Mian, Straub, and Sufi (2021) in closed economies, and Gourio, Siemer, and Verdelhan (2013), Kollmann (2016), Auclert, Malmberg, Martenet, and Rognlie (2021), and Carvalho, Ferrero, Mazin, and Nechio (2023) in the open economy context.

like supply shocks: a U.S. monetary easing induces a weak dollar because it generates (temporarily) high U.S. output. Consistent with the conclusions of Chari et al. (2002) as well as our Proposition 1, such shocks cannot account for the comovement between the exchange rate and relative consumption we observe in the data. This distinguishes the perspective of the present paper from the older literature emphasizing monetary shocks as the main drivers of exchange rate volatility, as in Dornbusch (1976).

Persistent shocks to growth rates could also generate persistent movements in relative demand and natural rates which drive the exchange rate. Like a decline in β, an increase in expected U.S. growth would raise the U.S. interest rate and U.S. consumption, driving a dollar appreciation. Persistent growth rate shocks would again have a large effect on exchange rates in our model because asset markets are incomplete. In fact, it is because markets are incomplete that the transmission of growth rate shocks would differ from their transmission in existing exchange rate models emphasizing "long run risk" with recursive preferences in complete markets.[44] Discount factor and growth shocks would be distinguished by the implied predictive content in exchange rates for future consumption growth, and relatedly by the contemporaneous comovement between the exchange rate and claims on future output (such as equities). Using these features of the data to discriminate between these shocks as drivers of relative demand and natural rates seems a fruitful avenue for future work.

Heterogeneity across currencies and countries While we have focused our analysis on the broad dollar/G10 exchange rate, it may be that different economies are subject to a different mix of shocks, another interesting direction for future study.

This idea is particularly relevant when we contrast the comovements obtained between the U.S. and G10 economies with those obtained between the U.S. and emerging markets (EMs).[45] Figure 6 depicts the comovements between the average real exchange rate, average 10-year yield differential, and average measure of relative consumption per capita between the U.S. and EM economies. Unlike the case with the G10 economies, EM yields tend to be relatively *high* when the dollar is strong.

[44]See, for instance, Colacito and Croce (2011, 2013). In these models, an increase in expected U.S. growth which lowers U.S. marginal utility *lowers* U.S. consumption as part of the efficient risk-sharing arrangement and *depreciates* the dollar, while still raising the U.S. interest rate. One implication of this is that the conditional comovement of the interest rate differential and exchange rate is distinct from that induced by growth shocks with incomplete markets.

[45]In our sample these are Brazil, Chile, China, Colombia, Hungary, Indonesia, Israel, India, South Korea, Mexico, Malaysia, Peru, Philippines, Poland, Russia, Thailand, Turkey, and South Africa.

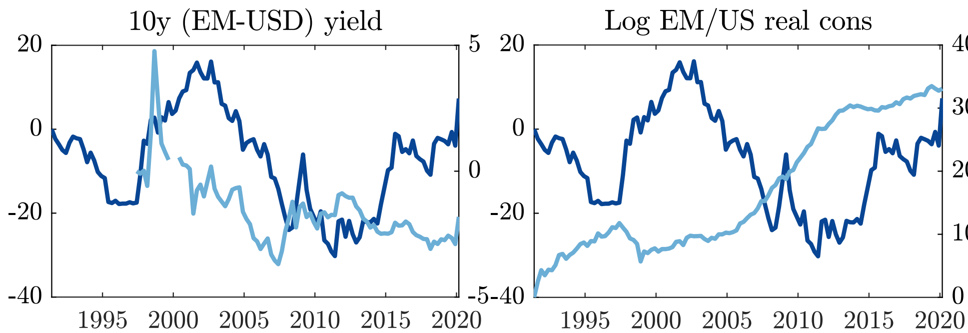

Figure 6: comovements with log real exchange rate between U.S. and EMs

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series are normalized to zero in first quarter. All yields are annualized.

At the same time, it remains the case that (around a clear trend) EM consumption tends to be relatively low when the dollar is strong. These comovements suggest that shocks to the intermediation of capital flows, as captured by γ_t in our framework, may play a more important role in driving EM exchange rates.[46]

More broadly, an interesting question is whether heterogeneous exposures to demand and intermediation shocks can account for the factor structure in bilateral exchange rates documented in the empirical literature. Lustig et al. (2014) demonstrate that there is a carry factor, and Verdelhan (2018) demonstrates there is a dollar factor, priced in the cross-section of excess currency returns. Miranda-Agrippino and Rey (2022) argue that there is also a two-factor structure in global capital flows. Shocks to demand and to the intermediation of capital flows may be promising candidates to account for these risk factors in asset markets.

7 Conclusion

We began this paper by outlining a set of exchange rate puzzles: exchange rates are inconsistent with standard macroeconomic models (Backus and Smith (1993)); are very difficult to predict (Meese and Rogoff (1983)); and appear inconsistent with UIP at short horizons (Hansen and Hodrick (1980), Fama (1984)). Our argument is that at

[46]This is consistent with emerging markets' cyclicality of interest rates emphasized by Neumeyer and Perri (2005), and features of their UIP premia documented in Kalemli-Ozcan and Varela (2024).

least between the U.S. and G10 currencies, exchange rates are inconsistent with standard models because they are in fact driven predominantly by relative demand shocks, which imply that a strong currency is associated with relatively high consumption. Exchange rates are difficult to predict because these demand shocks are persistent, as reflected in the persistence of bond yields. And exchange rates are inconsistent with UIP at short horizons because of transitory shocks to currency intermediation, as reflected in financial conditions such as the excess bond premium. Our analysis raises important questions about the sources of persistent demand shocks affecting exchange rates and heterogeneous exposures to demand and intermediation shocks across currencies. We intend to explore these questions in future work.

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Appendix for Online Publication

A Equilibrium

We first outline the equilibrium conditions pertaining to the model described in section 2 and the first-order approximation studied throughout the paper.

A.1 Equilibrium conditions

The optimality conditions for the U.S. representative household are:

$$c_{Ht} = \left(\frac{1}{1+\zeta^*} + \frac{\zeta^*}{1+\zeta^*}\varsigma \right) (p_{Ht})^{-\sigma} c_t, \tag{2}$$

$$c_{Ft} = \left(\frac{\zeta^*}{1+\zeta^*}(1-\varsigma) \right) (p_{Ft})^{-\sigma} c_t, \tag{3}$$

$$1 = \mathbb{E}_t \beta_t \left(\frac{c_{t+1}}{c_t} \right)^{-1/\psi} (1+r_t). \tag{4}$$

The household's resource constraint is implied by Walras' Law so we omit it here.

The optimality conditions for the Foreign representative household are:

$$c^*_{Ht} = \left(\frac{1}{1+\zeta^*}(1-\varsigma) \right) (p^*_{Ht})^{-\sigma} c^*_t, \tag{5}$$

$$c^*_{Ft} = \left(\frac{\zeta^*}{1+\zeta^*} + \frac{1}{1+\zeta^*}\varsigma \right) (p^*_{Ft})^{-\sigma} c^*_t, \tag{6}$$

$$1 = \mathbb{E}_t \beta^*_t \left(\frac{c^*_{t+1}}{c^*_t} \right)^{-1/\psi} (1+r^*_t). \tag{7}$$

Its resource constraint, using as well Foreign firms' aggregate profits, the symmetry across varieties, and goods market clearing, is

$$c^*_t + \frac{1}{1+r^*_t} b^*_t = \frac{1}{\zeta^*} q_t p_{Ft} c_{Ft} + p^*_{Ft} c^*_{Ft} + b^*_{t-1}. \tag{8}$$

Arbitrageurs' optimality conditions are:

$$\mathbb{E}_t \frac{q_t}{q_{t+1}}(1+r^*_t) - (1+r_t) = \gamma_t \left(q_t^{-1} \frac{1}{1+r^*_t} b^{a*}_t \right) Var_t \frac{q_t}{q_{t+1}}(1+r^*_t), \tag{9}$$

$$0 = \frac{1}{1+r_t} b_t^a + q_t^{-1} \frac{1}{1+r_t^*} b_t^{a*}. \tag{10}$$

Firms' optimal pricing conditions imply:

$$p_{Ht} = \frac{\theta}{\theta - 1} p_t, \tag{11}$$

$$q_t^{-1} p_{Ht}^* = \frac{\theta_t^*}{\theta_t^* - 1} p_t, \tag{12}$$

$$q_t p_{Ft} = \frac{\theta_t}{\theta_t - 1} p_t^*, \tag{13}$$

$$p_{Ft}^* = \frac{\theta}{\theta - 1} p_t^*. \tag{14}$$

Since prices faced by households in each country have been expressed relative to the overall consumption bundles, the aggregate price indices imply:

$$1 = \left(\left(\frac{1}{1+\zeta^*} + \frac{\zeta^*}{1+\zeta^*} \varsigma \right) (p_{Ht})^{1-\sigma} + \frac{\zeta^*}{1+\zeta^*} (1-\varsigma)(p_{Ft})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{15}$$

$$1 = \left(\frac{1}{1+\zeta^*} (1-\varsigma)(p_{Ht}^*)^{1-\sigma} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{1}{1+\zeta^*} \varsigma \right) (p_{Ft}^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{16}$$

Combining the intermediate and final goods market clearing conditions yields

$$c_{Ht} + \zeta^* c_{Ht}^* = z_t, \tag{17}$$

$$c_{Ft} + \zeta^* c_{Ft}^* = \zeta^* z_t^*. \tag{18}$$

Finally, bond market clearing requires

$$b_t + b_t^a = 0, \tag{19}$$

$$b_t^{a*} + \zeta^* b_t^* = 0. \tag{20}$$

By Walras' Law, we have that the U.S. household's resource constraint

$$c_t + \frac{1}{1+r_t} b_t = p_t z_t + \left[(p_{Ht} - p_t) c_{Ht} + (q_t^{-1} p_{Ht}^* - p_t) \zeta^* c_{Ht}^* \right] + \left[b_{t-1}^a + q_t^{-1} b_{t-1}^{a*} \right] + b_{t-1}$$

is satisfied as well.

Equations (2)-(20) define a dynamical system of 19 equations in 19 unknowns

$$\{c_t, c_{Ht}, c_{Ft}, b_t, c_t^*, c_{Ht}^*, c_{Ft}^*, b_t^*, b_t^a, b_t^{a*}, p_t, p_{Ht}, p_{Ft}, p_t^*, p_{Ht}^*, p_{Ft}^*, q_t, r_t, r_t^*\},$$

given the endogenous state variable b_{t-1}^*, exogenous state variables $\{\beta_t, \beta_t^*, z_t, z_t^*, \gamma_t\}$, and the evolution of exogenous state variables.

A.2 First-order approximation with currency risk premium

We now characterize the first-order approximation employed in the paper. Since the approximation for all equilibrium conditions except arbitrageurs' portfolio choice condition is standard, we focus on the latter here.

The first-order approximations for $\log q_t$, $\log(1 + r_t^*)$, $\log(1 + r_t)$, and b_t^{a*} are

$$\begin{aligned} \log q_t &= \log q + \boldsymbol{\delta^q} \left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right), \\ \log(1 + r_t^*) &= \log(1 + r^*) + \boldsymbol{\delta^{r^*}} \left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right), \\ \log(1 + r_t) &= \log(1 + r) + \boldsymbol{\delta^{r}} \left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right), \\ b_t^{a*} &= b^{a*} + \boldsymbol{\delta^{b^{a*}}} \left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right), \end{aligned}$$

where $\boldsymbol{\delta^x}$ for $x \in \{q, r^*, r, b^{a*}\}$ is a row vector of coefficients on the state variables and $\boldsymbol{\theta}_t$ is the column vector of state variables. The evolution of state variables is given by

$$\boldsymbol{\theta}_t = \boldsymbol{\theta} + \boldsymbol{\Delta^\theta} \left(\boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}\right) + \boldsymbol{\delta^\theta_\epsilon} \sigma \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{\Delta^\theta}$ is a matrix of coefficients on lagged state variables, $\boldsymbol{\delta^\theta_\epsilon}$ is a matrix of coefficients on shocks, and $\boldsymbol{\epsilon}_t$ is the column vector of shocks. We scale all shocks by the perturbation parameter σ, and ignore coefficients on this parameter itself anticipating that these will equal zero around the point of approximation.

Now by the property of lognormally distributed variables, it follows that

$$\begin{aligned} &Var_t \exp\left(-\left[\log q + \boldsymbol{\delta^q}\left(\boldsymbol{\theta}_{t+1} - \boldsymbol{\theta}\right)\right]\right), \\ &\quad = Var_t \exp\left(-\left[\log q + \boldsymbol{\delta^q}\boldsymbol{\Delta^\theta}\left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right) + \boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\sigma\boldsymbol{\epsilon}_{t+1}\right]\right), \\ &\quad = \exp\left(-2\left[\log q + \boldsymbol{\delta^q}\boldsymbol{\Delta^\theta}\left(\boldsymbol{\theta}_t - \boldsymbol{\theta}\right)\right]\right)\left(\exp\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\Sigma\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\right)'\sigma^2\right) - 1\right)\exp\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\Sigma\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\right)'\sigma^2\right), \end{aligned}$$

where Σ is the covariance matrix of the shocks (scaled appropriately by σ). At the point of approximation featuring $\sigma \to 0$, let us assume that

$$\gamma(\sigma)\left(\exp\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\Sigma\left(\boldsymbol{\delta^q}\boldsymbol{\delta^\theta_\epsilon}\right)'\sigma^2\right)-1\right)\to\Gamma$$

for some Γ finite. Then around that point of approximation, substituting the first order approximations for all variables into (9) yields

$$\begin{aligned}&\exp\left(\left(\boldsymbol{\delta^q}-\boldsymbol{\delta^q}\boldsymbol{\Delta^\theta}\right)(\boldsymbol{\theta}_t-\boldsymbol{\theta})+\log(1+r^*)+\boldsymbol{\delta^{r*}}(\boldsymbol{\theta}_t-\boldsymbol{\theta})\right)-\exp\left(\log(1+r)+\boldsymbol{\delta^r}(\boldsymbol{\theta}_t-\boldsymbol{\theta})\right)\\ &=\exp\left(\log q+\boldsymbol{\delta^q}(\boldsymbol{\theta}_t-\boldsymbol{\theta})+\log(1+r^*)+\boldsymbol{\delta^{r^*}}(\boldsymbol{\theta}_t-\boldsymbol{\theta})-2\left[\log q+\boldsymbol{\delta^q}\boldsymbol{\Delta^\theta}(\boldsymbol{\theta}_t-\boldsymbol{\theta})\right]+\hat{\gamma}_t\right)\\ &\qquad\times\left(b^{a*}+\boldsymbol{\delta^{b^{a*}}}(\boldsymbol{\theta}_t-\boldsymbol{\theta})\right)\Gamma.\end{aligned}$$

Hence, when $\boldsymbol{\theta}_t=\boldsymbol{\theta}$ and $\hat{\gamma}_t=0$ (i.e., at the point of approximation), it must be that

$$(1+r^*)-(1+r)=q^{-1}(1+r^*)b^{a*}\Gamma.$$

Differentiating the above condition with respect to each state variable and evaluating at the point of approximation, the coefficients will solve

$$(1+r^*)\left(\hat{r}^*_t-\mathbb{E}_t\Delta\hat{q}_{t+1}\right)-(1+r)\hat{r}_t=q^{-1}(1+r^*)b^{a*}\Gamma\left(\hat{q}_t-2\mathbb{E}_t\hat{q}_{t+1}+\hat{r}^*_t+\frac{1}{b^{a*}}\hat{b}^{a*}_t+\hat{\gamma}_t\right),$$

for $\hat{\cdot}$ denoting log/level deviations from the point of approximation. Combining the last two results, we can equivalently write the previous condition as

$$\hat{r}^*_t-\mathbb{E}_t\Delta\hat{q}_{t+1}-\hat{r}_t=\frac{q^{-1}b^{a*}\Gamma}{1-2q^{-1}b^{a*}\Gamma}\left(\hat{r}_t-\hat{q}_t-\hat{r}^*_t+\frac{1}{b^{a*}}\hat{b}^{a*}_t+\hat{\gamma}_t\right),$$

which is the form stated in the main text and used in the proofs in the next section.

B Proofs

We now provide the proofs of the analytical results described in section 3.

B.1 Log-linearized system and solution around autarkic limit

We first outline the log-linearized equilibrium system and we characterize the solution around the autarkic limit. In the following subsections we use these results to prove

Propositions 1-3.

B.1.1 Log-linearized system

Under assumption A1, log-linearizing (2)-(20) yields:

$$
\begin{aligned}
&\hat{c}_{Ht} = -\sigma\hat{p}_{Ht} + \hat{c}_t,\\
&\hat{c}_{Ft} = -\sigma\hat{p}_{Ft} + \hat{c}_t,\\
&\frac{1}{\psi}\mathbb{E}_t\Delta\hat{c}_{t+1} = \hat{\beta}_t + \hat{r}_t,\\
&\hat{c}^*_{Ht} = -\sigma\hat{p}^*_{Ht} + \hat{c}^*_t,\\
&\hat{c}^*_{Ft} = -\sigma\hat{p}^*_{Ft} + \hat{c}^*_t,\\
&\frac{1}{\psi}\mathbb{E}_t\Delta\hat{c}^*_{t+1} = \hat{\beta}^*_t + \hat{r}^*_t,\\
&c^*\hat{c}^*_t - \frac{1}{1+r^*}b^*\hat{r}^*_t + \frac{1}{1+r^*}\hat{b}^*_t = c_F\left(\hat{q}_t + \hat{p}_{Ft} + \hat{c}_{Ft}\right) + c^*_F\left(\hat{p}^*_{Ft} + \hat{c}^*_{Ft}\right) + \hat{b}^*_{t-1},\\
&\hat{r}^*_t - \mathbb{E}_t\Delta\hat{q}_{t+1} - \hat{r}_t = \frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\left(\hat{r}_t - \hat{q}_t - \hat{r}^*_t + \frac{1}{b^{a*}}\hat{b}^{a*}_t + \hat{\gamma}_t\right),\\
&0 = -\frac{1}{1+r}b^a\hat{r}_t + \frac{1}{1+r}\hat{b}^a_t + \frac{1}{1+r^*}b^{a*}\left(-\hat{q}_t - \hat{r}^*_t\right) + \frac{1}{1+r^*}\hat{b}^{a*}_t,\\
&\hat{p}_{Ht} = \hat{p}_t,\\
&-\hat{q}_t + \hat{p}^*_{Ht} = \hat{p}_t,\\
&\hat{q}_t + \hat{p}_{Ft} = \hat{p}^*_t,\\
&\hat{p}^*_{Ft} = \hat{p}^*_t,\\
&0 = \frac{1}{2}(1+\varsigma)\hat{p}_{Ht} + \frac{1}{2}(1-\varsigma)\hat{p}_{Ft},\\
&0 = \frac{1}{2}(1-\varsigma)\hat{p}^*_{Ht} + \frac{1}{2}(1+\varsigma)\hat{p}^*_{Ft},\\
&c_H\hat{c}_{Ht} + c^*_H\hat{c}^*_{Ht} = z\hat{z}_t,\\
&c_F\hat{c}_{Ft} + c^*_F\hat{c}^*_{Ft} = z^*\hat{z}^*_t,\\
&\hat{b}_t + \hat{b}^a_t = 0,\\
&\hat{b}^{a*}_t + \hat{b}^*_t = 0.
\end{aligned}
$$

The solution is given by

$$\hat{x}_t = \delta^x_\beta\hat{\beta}_t + \delta^x_{\beta^*}\hat{\beta}^*_t + \delta^x_z\hat{z}_t + \delta^x_{z^*}\hat{z}^*_t + \delta^x_\gamma\hat{\gamma}_t + \delta^x_{b^{a*}}\hat{b}^{a*}_{t-1}$$

for each endogenous variable x and coefficients $\delta_{\cdot}^{\cdot}$. Note that we write this with $\hat{b}_{t-1}^{a*}$ rather than $\hat{b}_{t-1}^{*}$ as an endogenous state variable, but the latter is just the negative of the former by market clearing.

We characterize these coefficients around the autarkic limit. Note that at the limit, there would be no trade in goods, thus no trade in financial assets, and thus a zero currency risk premium. To offset this, we assume that as ς approaches one, the price of risk Γ approaches infinity, so that the ratio $b^{a*}\Gamma$ remains finite. The conditions and comparative statics with respect to Γ described in the main text should thus be understood to refer to this composite term.

For those coefficients which are non-zero in the autarkic limit, we characterize the limit. For those coefficients which are zero in the limit, we characterize their limiting behavior proportional to $1-\varsigma$. Before solving for the coefficients, we combine the above conditions to obtain a simpler system, and we use that in the autarkic limit $c = c^* = z = z^*$ (under assumption A1) to simplify certain coefficients. We do, however, retain some terms multiplied by $1-\varsigma$ so that we can properly characterize the limiting behavior. Finally, without loss of generality we assume $z = 1$.

The domestic Euler equations and arbitrageurs' optimality condition are as above:

$$\frac{1}{\psi}\mathbb{E}_t\Delta\hat{c}_{t+1} = \hat{\beta}_t + \hat{r}_t, \tag{21}$$

$$\frac{1}{\psi}\mathbb{E}_t\Delta\hat{c}^*_{t+1} = \hat{\beta}^*_t + \hat{r}^*_t, \tag{22}$$

$$\hat{r}^*_t - \mathbb{E}_t\Delta\hat{q}_{t+1} - \hat{r}_t = \frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\left(\hat{r}_t - \hat{q}_t - \hat{r}^*_t + \frac{1}{b^{a*}}\hat{b}^{a*}_t + \hat{\gamma}_t\right). \tag{23}$$

Combining Foreign goods market clearing, bond market clearing, and definitions of relative prices with the Foreign resource constraint yields

$$\hat{c}^*_t + \frac{1}{1+r^*}b^{a*}\hat{r}^*_t - \frac{1}{1+r^*}\hat{b}^{a*}_t = -\frac{1}{2}(1-\varsigma)\hat{q}_t + \hat{z}^*_t - \hat{b}^{a*}_{t-1}. \tag{24}$$

Finally, combining households' intratemporal optimality conditions and the definitions of relative prices with the goods market clearing conditions yields

$$-\frac{1}{2}(1+\varsigma)(1-\varsigma)\sigma\hat{q}_t + \frac{1}{2}(1+\varsigma)\hat{c}_t + \frac{1}{2}(1-\varsigma)\hat{c}^*_t = \hat{z}_t, \tag{25}$$

$$\frac{1}{2}(1+\varsigma)(1-\varsigma)\sigma\hat{q}_t + \frac{1}{2}(1-\varsigma)\hat{c}_t + \frac{1}{2}(1+\varsigma)\hat{c}^*_t = \hat{z}^*_t. \tag{26}$$

The system (21)-(26) is 6 equations in 6 unknowns

$$\{\hat{c}_t, \hat{c}_t^*, \hat{b}_t^{a*}, \hat{q}_t, \hat{r}_t, \hat{r}_t^*\}$$

given state variables

$$\{\hat{z}_t, \hat{z}_t^*, \hat{\beta}_t, \hat{\beta}_t^*, \hat{\gamma}_t, \hat{b}_{t-1}^{a*}\}$$

and the evolution of exogenous state variables.

B.1.2 Solution

We now characterize the coefficients $\delta_\cdot^\cdot$ solving the system (21)-(26) using the method of undetermined coefficients.

Coefficients on $\hat{b}_{t-1}^{a*}$ By the method of undetermined coefficients, these solve

$$\begin{aligned}
&\frac{1}{\psi}\delta_{b^{a*}}^{c}\left(\delta_{b^{a*}}^{b^{a*}}-1\right)=\delta_{b^{a*}}^{r},\\
&\frac{1}{\psi}\delta_{b^{a*}}^{c^*}\left(\delta_{b^{a*}}^{b^{a*}}-1\right)=\delta_{b^{a*}}^{r^*},\\
&\delta_{b^{a*}}^{c^*}+\frac{1}{1+r^*}b^{a*}\delta_{b^{a*}}^{r^*}-\frac{1}{1+r^*}\delta_{b^{a*}}^{b^{a*}}=-\frac{1}{2}\left(1-\varsigma\right)\delta_{b^{a*}}^{q}-1,\\
&\delta_{b^{a*}}^{r^*}-\delta_{b^{a*}}^{q}\left(\delta_{b^{a*}}^{b^{a*}}-1\right)-\delta_{b^{a*}}^{r}=\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\delta_{b^{a*}}^{r}-\delta_{b^{a*}}^{q}-\delta_{b^{a*}}^{r^*}+\frac{1}{b^{a*}}\delta_{b^{a*}}^{b^{a*}}\right],\\
&-\frac{1}{2}\left(1+\varsigma\right)\left(1-\varsigma\right)\sigma\delta_{b^{a*}}^{q}+\frac{1}{2}\left(1+\varsigma\right)\delta_{b^{a*}}^{c}+\frac{1}{2}\left(1-\varsigma\right)\delta_{b^{a*}}^{c^*}=0,\\
&\frac{1}{2}\left(1+\varsigma\right)\left(1-\varsigma\right)\sigma\delta_{b^{a*}}^{q}+\frac{1}{2}\left(1-\varsigma\right)\delta_{b^{a*}}^{c}+\frac{1}{2}\left(1+\varsigma\right)\delta_{b^{a*}}^{c^*}=0.
\end{aligned}$$

We conjecture a solution of the form

$$\delta_{b^{a*}}^{q}=\frac{1}{1-\varsigma}\tilde{\delta}_{b^{a*}}^{q}.$$

Substituting these into the previous system and solving yields

$$\begin{aligned}
\delta_{b^{a*}}^{b^{a*}}:=&\frac{2}{2\sigma-1}\left(1-\frac{1}{1+r^*}\delta_{b^{a*}}^{b^{a*}}\right)\left(1-\delta_{b^{a*}}^{b^{a*}}\right)\\
&-\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left(-\frac{2}{2\sigma-1}\left(1-\frac{1}{1+r^*}\delta_{b^{a*}}^{b^{a*}}\right)+\frac{1-\varsigma}{b^{a*}}\delta_{b^{a*}}^{b^{a*}}\right)=0,
\end{aligned}$$

$$\tilde{\delta}^q_{b^{a*}} = \frac{2}{2\sigma-1}\left(1 - \frac{1}{1+r^*}\delta^{b^{a*}}_{b^{a*}}\right),$$
$$\delta^c_{b^{a*}} = \frac{2\sigma}{2\sigma-1}\left(1 - \frac{1}{1+r^*}\delta^{b^{a*}}_{b^{a*}}\right),$$
$$\delta^{c^*}_{b^{a*}} = -\frac{2\sigma}{2\sigma-1}\left(1 - \frac{1}{1+r^*}\delta^{b^{a*}}_{b^{a*}}\right),$$
$$\delta^r_{b^{a*}} = \frac{1}{\psi}\delta^c_{b^{a*}}\left(\delta^{b^{a*}}_{b^{a*}} - 1\right),$$
$$\delta^{r^*}_{b^{a*}} = \frac{1}{\psi}\delta^{c^*}_{b^{a*}}\left(\delta^{b^{a*}}_{b^{a*}} - 1\right).$$

Coefficients on $\hat{\beta}^*_t$, $\hat{\beta}^*_t$, $\hat{z}_t$, $\hat{z}^*_t$, and $\hat{\gamma}_t$ We similarly use the method of undetermined coefficients to characterize the coefficients on the exogenous state variables. For brevity we summarize the solutions here.

On $\hat{\beta}_t$ we obtain:

$$\delta^q_\beta = -\frac{2}{2\sigma-1}\frac{\left(1 + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\right)\frac{1}{1+r^*}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}^q_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$
$$\delta^{b^{a*}}_\beta = (1-\varsigma)\tilde{\delta}^{b^{a*}}_\beta,\quad \tilde{\delta}^{b^{a*}}_\beta = \frac{1 + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}^q_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$
$$\delta^c_\beta = (1-\varsigma)\tilde{\delta}^c_\beta,\quad \tilde{\delta}^c_\beta = -\frac{2\sigma}{2\sigma-1}\frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_\beta,$$
$$\delta^{c^*}_\beta = (1-\varsigma)\tilde{\delta}^{c^*}_\beta,\quad \tilde{\delta}^{c^*}_\beta = \frac{2\sigma}{2\sigma-1}\frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_\beta,$$
$$\delta^r_\beta := -1,$$
$$\delta^{r^*}_\beta = 0,$$

On $\hat{\beta}^*_t$ we obtain:

$$\delta^q_{\beta^*} = -\frac{2}{2\sigma-1}\frac{-\frac{1}{1+r^*} + \frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}\tilde{\delta}^q_{b^{a*}}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^{\beta^*}) + \tilde{\delta}^q_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$
$$\delta^{b^{a*}}_{\beta^*} = (1-\varsigma)\tilde{\delta}^{b^{a*}}_\beta,\quad \tilde{\delta}^{b^{a*}}_\beta = -\frac{1 + \frac{2}{2\sigma-1}\frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}(1-\rho^{\beta^*}) + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma} + 1\right]}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^{\beta^*}) + \tilde{\delta}^q_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$
$$\delta^c_{\beta^*} = (1-\varsigma)\tilde{\delta}^c_{\beta^*},\quad \tilde{\delta}^c_{\beta^*} = -\frac{2\sigma}{2\sigma-1}\left(\frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma} + \frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_{\beta^*}\right),$$

$$\delta^{c^*}_{\beta^*} = (1-\varsigma)\tilde{\delta}^{c^*}_{\beta^*}, \;\; \tilde{\delta}^{c^*}_{\beta^*} = \frac{2\sigma}{2\sigma-1}\left(\frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma} + \frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_{\beta^*}\right),$$

$$\delta^{r}_{\beta^*} = 0,$$

$$\delta^{r^*}_{\beta^*} = -1.$$

On $\hat{z}_t$ we obtain:

$$\delta^{q}_{z} = -\frac{2}{2\sigma-1}\frac{\left(1+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\right)\frac{1}{\psi}\frac{1}{1+r^*}(1-\rho^z) + \frac{1}{2}\tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{\frac{1}{2}(1-\varsigma)}{b^*}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^z) + \tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$

$$\delta^{b^{a*}}_{z} = (1-\varsigma)\tilde{\delta}^{b^{a*}}_{z}, \;\; \tilde{\delta}^{b^{a*}}_{z} = \frac{\left(-\frac{1}{2\sigma-1} + \left(1+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\right)\frac{1}{\psi}\right)(1-\rho^z) - \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{1}{2\sigma-1}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^z) + \tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$

$$\delta^{c}_{z} = 1,$$

$$\delta^{c^*}_{z} = (1-\varsigma)\tilde{\delta}^{c^*}_{z}, \;\; \tilde{\delta}^{c^*}_{z} = \frac{1}{2}\frac{1}{2\sigma-1} + \frac{2\sigma}{2\sigma-1}\frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_{z},$$

$$\delta^{r}_{z} = -\frac{1}{\psi}(1-\rho^z),$$

$$\delta^{r^*}_{z} = 0.$$

On $\hat{z}^*_t$ we obtain:

$$\delta^{q}_{z^*} = \frac{2}{2\sigma-1}\frac{\frac{1}{\psi}\frac{1}{1+r^*}(1-\rho^{z^*}) + \left(\frac{1}{2} - \frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}\frac{1}{\psi}\left(1-\rho^{z^*}\right)\right)\tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{\frac{1}{2}(1-\varsigma)}{b^{a*}}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^{z^*}) + \tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]},$$

$$\delta^{b^{a*}}_{z^*} = 2\left[\frac{\left(\frac{2}{2\sigma-1}\left(\frac{1}{2} - \frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}\frac{1}{\psi}\left(1-\rho^{z^*}\right)\right) - \frac{1}{\psi}\right)(1-\rho^{z^*})}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^{z^*}) + \tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]}\right.$$
$$\left. + \frac{\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\left(\frac{1}{2} - \frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}\frac{1}{\psi}\left(1-\rho^{z^*}\right)\right) - \frac{1}{\psi}(1-\rho^{z^*})\right]}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^{z^*}) + \tilde{\delta}^{q}_{b^{a*}} + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]}\right],$$

$$\delta^{c}_{z^*} = (1-\varsigma)\tilde{\delta}^{c}_{z^*}, \;\; \tilde{\delta}^{c}_{z^*} = \frac{1}{2}\frac{1}{2\sigma-1} - \frac{2\sigma}{2\sigma-1}\left(\frac{1}{1+r^*}\frac{b^{a*}}{1-\varsigma}\frac{1}{\psi}\left(1-\rho^{z^*}\right) + \frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_{z^*}\right),$$

$$\delta^{c^*}_{z^*} = 1,$$

$$\delta^{r}_{z^*} = 0,$$

$$\delta^{r^*}_{z^*} = -\frac{1}{\psi}(1-\rho^{z^*}).$$

Finally, on $\hat{\gamma}_t$ we obtain:

$$\delta^q_\gamma = \frac{\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{2}{2\sigma-1}\frac{1}{1+r^*}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\gamma)+\tilde{\delta}^q_{b^{a*}}+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right]},$$

$$\delta^{b^{a*}}_\gamma = (1-\varsigma)\tilde{\delta}^{b^{a*}}_\gamma,\ \ \tilde{\delta}^{b^{a*}}_\gamma = -\frac{\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\gamma)+\tilde{\delta}^q_{b^{a*}}+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right]},$$

$$\delta^c_\gamma = (1-\varsigma)\tilde{\delta}^c_\gamma,\ \ \tilde{\delta}^c_\gamma = -\frac{2\sigma}{2\sigma-1}\frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_\gamma,$$

$$\delta^{c^*}_\gamma = (1-\varsigma)\tilde{\delta}^{c^*}_\gamma,\ \ \tilde{\delta}^{c^*}_\gamma = \frac{2\sigma}{2\sigma-1}\frac{1}{1+r^*}\tilde{\delta}^{b^{a*}}_\gamma,$$

$$\delta^r_\gamma = (1-\varsigma)\tilde{\delta}^r_\gamma,\ \ \tilde{\delta}^r_\gamma = \frac{1}{\psi}\left(\tilde{\delta}^c_\gamma(\rho^\gamma-1)+\delta^c_{b^{a*}}\tilde{\delta}^{b^{a*}}_\gamma\right),$$

$$\delta^{r^*}_\gamma = (1-\varsigma)\tilde{\delta}^{r^*}_\gamma,\ \ \tilde{\delta}^{r^*}_\gamma = \frac{1}{\psi}\left(\tilde{\delta}^{c^*}_\gamma(\rho^\gamma-1)+\delta^{c^*}_{b^{a*}}\tilde{\delta}^{b^{a*}}_\gamma\right).$$

We repeatedly reference these coefficients in the proofs of Propositions 1-3 which follow.

B.2 Proposition 1

Proof. Let us first characterize $\hat{q}_t$ given a single driving force $x \in \{\beta, \beta^*, z, z^*, \gamma\}$. We have

$$\begin{aligned}
\hat{q}_t &= \delta^q_x \hat{x}_t + \delta^q_{b^{a*}}\hat{b}^{a*}_{t-1}, \\
&= \delta^q_x \hat{\epsilon}^x_t + \sum_{\tau=0}^{\infty}\left[\delta^q_x\left(\rho^x\right)^{\tau+1} + \delta^q_{b^{a*}}\delta^{b^{a*}}_x \frac{\left(\rho^x\right)^{\tau+1} - \left(\delta^{b^{a*}}_{b^{a*}}\right)^{\tau+1}}{\rho^x - \delta^{b^{a*}}_{b^{a*}}}\right]\hat{\epsilon}^x_{t-1-\tau}, \\
&= \delta^q_x \hat{\epsilon}^x_t + \sum_{\tau=0}^{\infty}\left[\delta^q_x\left(\rho^x\right)^{\tau+1} + \tilde{\delta}^q_{b^{a*}}\tilde{\delta}^{b^{a*}}_x \frac{\left(\rho^x\right)^{\tau+1} - \left(\delta^{b^{a*}}_{b^{a*}}\right)^{\tau+1}}{\rho^x - \delta^{b^{a*}}_{b^{a*}}}\right]\hat{\epsilon}^x_{t-1-\tau}.
\end{aligned}$$

Interest rate differential We now characterize the contemporaneous comovements with the interest rate differential $\hat{r}^*_t - \hat{r}_t$.

We begin with β_t shocks (an analogous argument holds for β^*_t shocks). Based on the results in section B.1.2, in the autarkic limit

$$\begin{aligned}\hat{r}_t^* - \hat{r}_t &= \hat{\beta}_t, \\ &= \sum_{\tau=0}^{\infty} \left(\rho^{\beta}\right)^{\tau} \hat{\epsilon}_{t-\tau}^{\beta}.\end{aligned}$$

Thus, for ς close to one, we have that

$$\begin{aligned}Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t) / \left(\sigma^{\beta}\right)^2 &\approx \sum_{\tau=0}^{\infty} \left[\delta_{\beta}^{q} \left(\rho^{\beta}\right)^{\tau} + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{\left(\rho^{\beta}\right)^{\tau} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau}}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}}\right] \left(\rho^{\beta}\right)^{\tau}, \\ &= \frac{1}{1 - \left(\rho^{\beta}\right)^2} \frac{1}{1 - \rho^{\beta} \delta_{b^{a*}}^{b^{a*}}} \left[\delta_{\beta}^{q} \left(1 - \rho^{\beta} \delta_{b^{a*}}^{b^{a*}}\right) + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \rho^{\beta}\right],\end{aligned}$$

where the second line follows from straightforward algebra. As $\Gamma \to 0$ and thus $\delta_{b^{a*}}^{b^{a*}} \to 1$, at the autarkic limit the expression in brackets is

$$\begin{aligned}\delta_{\beta}^{q} \left(1 - \rho^{\beta}\right) + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \rho^{\beta} &= -\frac{1 - \rho^{\beta}}{1 + r^* - \rho^{\beta}} + \frac{r^* \rho^{\beta}}{1 + r^* - \rho^{\beta}}, \\ &= -\frac{1 - (1 + r^*)\rho^{\beta}}{1 + r^* - \rho^{\beta}},\end{aligned}$$

using the coefficients characterized in section B.1.2. The numerator reflects two terms. On impact, an increase in U.S. demand appreciates the dollar and lowers the Foreign interest rate relative to the U.S. interest rate, driving a negative correlation. Going forward, the dollar is expected to depreciate, and in the medium run, the increased wealth in Foreign (due to its higher saving) would imply a weak dollar while the U.S. interest rate remains relatively high. Provided r^* is sufficiently small, for any $\rho^{\beta} < 1$ the former effect will dominate the latter and this comovement will be negative. By continuity, this result also holds away for the autarkic limit and for Γ small. We employ similar arguments for all other comovements.

In particular, next consider z_t shocks (an analogous argument holds for z_t^* shocks). In response to z_t shocks, we have at the autarkic limit

$$\begin{aligned}\hat{r}_t^* - \hat{r}_t &= \frac{1}{\psi}(1 - \rho^z)\hat{z}_t, \\ &= \frac{1}{\psi}(1 - \rho^z) \sum_{\tau=0}^{\infty} \left(\rho^z\right)^{\tau} \hat{\epsilon}_{t-\tau}^{z}.\end{aligned}$$

Using similar steps as above, it follows that as $\Gamma \to 0$ and thus $\delta_{b^{a*}}^{b^{a*}} \to 1$, in the autarkic limit

$$Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t)/\left(\sigma^z\right)^2 \propto -(1 - (1 + r^*)\rho^z).$$

Again this will be negative for r^* sufficiently small, given any $\rho^z < 1$.

We finally consider γ_t shocks. We have that

$$\begin{aligned}
\hat{r}_t^* - \hat{r}_t &= \left(\delta_\gamma^{r^*} - \delta_\gamma^r\right)\hat{\gamma}_t + \left(\delta_{b^{a*}}^{r^*} - \delta_{b^{a*}}^r\right)\hat{b}_{t-1}^{a*}, \\
&= \left(\delta_\gamma^{r^*} - \delta_\gamma^r\right)\hat{\epsilon}_t^\gamma + \sum_{\tau=0}^\infty \left[\left(\delta_\gamma^{r^*} - \delta_\gamma^r\right)\left(\rho^\gamma\right)^{\tau+1} + \left(\delta_{b^{a*}}^{r^*} - \delta_{b^{a*}}^r\right)\delta_\gamma^{b^{a*}} \frac{\left(\rho^\gamma\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}}\right]\hat{\epsilon}_{t-1-\tau}^\gamma.
\end{aligned}$$

Further note that

$$\begin{aligned}
\delta_\gamma^{r^*} &= -\delta_\gamma^r, \\
\delta_{b^{a*}}^{r^*} &= -\delta_{b^{a*}}^r,
\end{aligned}$$

so we can write the above as

$$\hat{r}_t^* - \hat{r}_t = 2\delta_\gamma^{r^*}\hat{\epsilon}_t^\gamma + \sum_{\tau=0}^\infty \left[2\delta_\gamma^{r^*}\left(\rho^\gamma\right)^{\tau+1} + 2\delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}} \frac{\left(\rho^\gamma\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}}\right]\hat{\epsilon}_{t-1-\tau}^\gamma.$$

Thus, we have that

$$\begin{aligned}
&Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t)/\left(2\left(\sigma^\gamma\right)^2\right) \\
&= \sum_{\tau=0}^\infty \left[\delta_\gamma^q\left(\rho^\gamma\right)^\tau + \tilde{\delta}_{b^{a*}}^q\tilde{\delta}_\gamma^{b^{a*}} \frac{\left(\rho^\gamma\right)^\tau - \left(\delta_{b^{a*}}^{b^{a*}}\right)^\tau}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}}\right]\left[\delta_\gamma^{r^*}\left(\rho^\gamma\right)^\tau + \delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}} \frac{\left(\rho^\gamma\right)^\tau - \left(\delta_{b^{a*}}^{b^{a*}}\right)^\tau}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}}\right], \\
&= \frac{1}{1 - \left(\rho^\gamma\right)^2}\left[\delta_\gamma^q + \tilde{\delta}_{b^{a*}}^q\tilde{\delta}_\gamma^{b^{a*}} \frac{\rho^\gamma}{1 - \left(\rho^\gamma\delta_{b^{a*}}^{b^{a*}}\right)}\right]\delta_\gamma^{r^*} \\
&\quad + \frac{1}{1 - \left(\rho^\gamma\right)^2}\left[\delta_\gamma^q + \tilde{\delta}_{b^{a*}}^q\tilde{\delta}_\gamma^{b^{a*}} \frac{\rho^\gamma}{1 - \left(\rho^\gamma\delta_{b^{a*}}^{b^{a*}}\right)}\right]\delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}} \frac{1}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}} \\
&\quad - \frac{1}{1 - \rho^\gamma\delta_{b^{a*}}^{b^{a*}}}\left[\delta_\gamma^q + \tilde{\delta}_{b^{a*}}^q\tilde{\delta}_\gamma^{b^{a*}} \frac{\delta_{b^{a*}}^{b^{a*}}}{1 - \left(\delta_{b^{a*}}^{b^{a*}}\right)^2}\right]\delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}} \frac{1}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}},
\end{aligned}$$

where the last line follows from straightforward algebra.

Now since

$$\delta_{b^{a*}}^{r^*} = \frac{1}{\psi}\delta_{b^{a*}}^{c^*}\left(\delta_{b^{a*}}^{b^{a*}} - 1\right),$$

we can write this as

$$\begin{aligned}
&Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t)/\left(2\left(\sigma^\gamma\right)^2\right) \\
&\quad = \frac{1}{1-(\rho^\gamma)^2}\left[\delta_\gamma^q + \tilde{\delta}_{b^{a*}}^q \tilde{\delta}_\gamma^{b^{a*}} \frac{\rho^\gamma}{1-\left(\rho^\gamma \delta_{b^{a*}}^{b^{a*}}\right)}\right]\delta_\gamma^{r^*} \\
&\qquad + \frac{1}{1-(\rho^\gamma)^2}\left[\delta_\gamma^q + \tilde{\delta}_{b^{a*}}^q \tilde{\delta}_\gamma^{b^{a*}} \frac{\rho^\gamma}{1-\left(\rho^\gamma \delta_{b^{a*}}^{b^{a*}}\right)}\right]\delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}}\frac{1}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}} \\
&\qquad - \frac{1}{1-\rho^\gamma \delta_{b^{a*}}^{b^{a*}}}\left[\delta_\gamma^q\right]\delta_{b^{a*}}^{r^*}\delta_\gamma^{b^{a*}}\frac{1}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}} \\
&\qquad + \frac{1}{1-\rho^\gamma \delta_{b^{a*}}^{b^{a*}}}\tilde{\delta}_{b^{a*}}^q \tilde{\delta}_\gamma^{b^{a*}}\frac{\delta_{b^{a*}}^{b^{a*}}}{1+\delta_{b^{a*}}^{b^{a*}}}\frac{1}{\psi}\delta_{b^{a*}}^{c^*}\delta_\gamma^{b^{a*}}\frac{1}{\rho^\gamma - \delta_{b^{a*}}^{b^{a*}}}.
\end{aligned}$$

As $\Gamma \to 0$, we have that $\delta_{b^{a*}}^{r^*} \to 0$, so we focus on the first and last terms alone. Substituting in the values of the other coefficients as $\Gamma \to 0$ and again focusing on the case with r^* small, we have that the sign of $Cov(\hat{q}_t, \hat{r}_t^* - \hat{r}_t)/(2(\sigma^\gamma)^2)$ is governed by the impact effect of the shock ($\delta_\gamma^q \delta_\gamma^{r^*}$), which implies a positive comovement.

Lagged interest rate differential We now characterize the comovements with the lagged interest rate differential $\hat{r}_{t-1}^* - \hat{r}_{t-1}$, reflected in the Fama (1984) regression coefficient.

In response to β_t shocks, we already prove in Proposition 3 that $Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)/\left(\sigma^\beta\right)^2 > 0$.

In response to z_t shocks, the same steps as in the proof of Proposition 3 imply

$$Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)/\left(\sigma^z\right)^2 \propto -\delta_z^q + \tilde{\delta}_{b^{a*}}^q \tilde{\delta}_z^{b^{a*}} \frac{1}{1-\delta_{b^{a*}}^{b^{a*}}\rho^z}.$$

For $\Gamma \to 0$, this is positive.

Finally, in response to γ_t shocks, for r^* small we similarly have

$$Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)/\left(\sigma^\gamma\right)^2 \propto -\delta_\gamma^{r^*}\delta_\gamma^q,$$

which is negative.

Consumption differential Let us start with z_t shocks, since it is clear that in the autarkic limit $\hat{c}_t^* - \hat{c}_t = \hat{z}_t^* - \hat{z}_t$. We have that

$$\hat{z}_t^* - \hat{z}_t = \sum_{\tau=0}^{\infty} \left(\rho^{z^*}\right)^{\tau} \hat{\epsilon}_{t-\tau}^{z^*} - \sum_{\tau=0}^{\infty} \left(\rho^{z}\right)^{\tau} \hat{\epsilon}_{t-\tau}^{z}.$$

Thus, using similar steps as above, in response to z_t shocks we have that as $\Gamma \to 0$

$$Cov(\hat{q}_t, \hat{z}_t^* - \hat{z}_t)/\sigma^2 \propto 1 - (1 + r^*)\rho^z,$$

which is positive for r^* sufficiently small, given any $\rho^z < 1$.

Now let us focus on β_t and γ_t shocks. For $x \in \{\beta, \gamma\}$, we have that

$$\begin{aligned}
&\hat{c}_t^* - \hat{c}_t \\
&= \left(\delta_x^{c^*} - \delta_x^{c}\right) \hat{\epsilon}_t^x + \sum_{\tau=0}^{\infty} \left[\left(\delta_x^{c^*} - \delta_x^{c}\right) (\rho^x)^{\tau+1} + \left(\delta_{b^{a*}}^{c^*} - \delta_{b^{a*}}^{c}\right) \delta_x^{b^{a*}} \frac{(\rho^x)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^x - \delta_{b^{a*}}^{b^{a*}}} \right] \hat{\epsilon}_{t-1-\tau}^x.
\end{aligned}$$

Note that

$$\begin{aligned}
\delta_{b^{a*}}^{c} &= -\delta_{b^{a*}}^{c^*}, \\
\delta_{\beta}^{c} &= -\delta_{\beta}^{c^*}, \\
\delta_{\gamma}^{c} &= -\delta_{\gamma}^{c^*}.
\end{aligned}$$

Hence, we have that

$$\hat{c}_t^* - \hat{c}_t = 2\delta_x^{c^*} \hat{\epsilon}_t^x + \sum_{\tau=0}^{\infty} \left[2\delta_x^{c^*} (\rho^x)^{\tau+1} + 2\delta_{b^{a*}}^{c^*} \delta_x^{b^{a*}} \frac{(\rho^x)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^x - \delta_{b^{a*}}^{b^{a*}}} \right] \hat{\epsilon}_{t-1-\tau}^x.$$

Now consider β_t shocks in particular. Using similar steps as above, it follows that

$$\begin{aligned}
&Cov(\hat{q}_t, \hat{c}_t^* - \hat{\tilde{c}}_t)/\left(2\left(\sigma^{\beta}\right)^2\right) \\
&\quad = \sum_{\tau=0}^{\infty} \left[\delta_{\beta}^{q} \left(\rho^{\beta}\right)^{\tau} + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{\left(\rho^{\beta}\right)^{\tau} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau}}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \right] \left[\delta_{\beta}^{c^*} \left(\rho^{\beta}\right)^{\tau} + \delta_{b^{a*}}^{c^*} \delta_{\beta}^{b^{a*}} \frac{\left(\rho^{\beta}\right)^{\tau} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau}}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \right], \\
&\quad = \frac{1}{1 - \left(\rho^{\beta}\right)^2} \left[\delta_{\beta}^{q} + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{\rho^{\beta}}{1 - \left(\rho^{\beta} \delta_{b^{a*}}^{b^{a*}}\right)} \right] \delta_{\beta}^{c^*}
\end{aligned}$$

$$+\frac{1}{1-(\rho^\beta)^2}\left[\delta^q_\beta+\tilde{\delta}^q_{b^{a*}}\tilde{\delta}^{b^{a*}}_\beta\frac{\rho^\beta}{1-\left(\rho^\beta\delta^{b^{a*}}_{b^{a*}}\right)}\right]\delta^{c^*}_{b^{a*}}\delta^{b^{a*}}_\beta\frac{1}{\rho^\beta-\delta^{b^{a*}}_{b^{a*}}}$$
$$-\frac{1}{1-\rho^\beta\delta^{b^{a*}}_{b^{a*}}}\left[\delta^q_\beta+\tilde{\delta}^q_{b^{a*}}\tilde{\delta}^{b^{a*}}_\beta\frac{\delta^{b^{a*}}_{b^{a*}}}{1-\left(\delta^{b^{a*}}_{b^{a*}}\right)^2}\right]\delta^{c^*}_{b^{a*}}\delta^{b^{a*}}_\beta\frac{1}{\rho^\beta-\delta^{b^{a*}}_{b^{a*}}}.$$

As $\Gamma\to 0$, terms multiplying $\delta^{b^{a*}}_\beta$ are of order r^* and can be ignored for r^* small. Note that this also includes

$$\frac{\delta^{b^{a*}}_\beta}{1-\left(\delta^{b^{a*}}_{b^{a*}}\right)^2}$$

in the last term. Recall that

$$\delta^{b^{a*}}_{b^{a*}}:=\frac{2}{2\sigma-1}\left(1-\frac{1}{1+r^*}\delta^{b^{a*}}_{b^{a*}}\right)\left(1-\delta^{b^{a*}}_{b^{a*}}\right)$$
$$-\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left(-\frac{2}{2\sigma-1}\left(1-\frac{1}{1+r^*}\delta^{b^{a*}}_{b^{a*}}\right)+\frac{1-\varsigma}{b^{a*}}\delta^{b^{a*}}_{b^{a*}}\right)=0.$$

It follows that around $\Gamma=0$ (which implies $\delta^{b^{a*}}_{b^{a*}}=1$) and $r^*=0$,

$$\frac{1-\delta^{b^{a*}}_{b^{a*}}}{r^*}\propto\frac{\Gamma}{(r^*)^2}.$$

In other words

$$\frac{r^*}{1-\delta^{b^{a*}}_{b^{a*}}}\propto\frac{(r^*)^2}{\Gamma}.$$

Hence, if r^* and Γ go to zero at the same rate,

$$\frac{r^*}{1-\left(\delta^{b^{a*}}_{b^{a*}}\right)^2}\to 0.$$

It follows that the sign of $Cov(\hat{q}_t,\hat{c}^*_t-\hat{c}_t)/(2(\sigma^\beta)^2)$ is dominated by $\delta^q_\beta\delta^{c^*}_\beta$, which again reflects the impact effect of a β_t innovation. For small Γ, this is negative.

The same argument applies for γ_t shocks. □

B.3 Proposition 2

Proof. We again evaluate the coefficients characterized in section B.1.2 in the autarkic limit.

The impact effect on the U.S. real interest rate is

$$\delta_\beta^r = -1.$$

The impact effect on the real exchange rate is

$$\delta_\beta^q = -\frac{2}{2\sigma-1}\frac{\left(1+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\right)\frac{1}{1+r^*}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta)+\tilde{\delta}^q_{b^{a*}}+\frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right]}.$$

Then, as $\Gamma \to 0$, we have that

$$\delta_\beta^q \to -\frac{1}{1+r^*-\rho^\beta}.$$

It is immediate that this is increasing in absolute magnitude as ρ^β rises.

It is clear from all other coefficients characterized in section B.1.2 that the impact effect on other variables is zero in the autarkic limit.

We now characterize $Var(\mathbb{E}_t\Delta\hat{q}_{t+1})/Var(\Delta\hat{q}_{t+1})$, the share of variance in exchange rate innovations that is accounted for the predictable part. In the autarkic limit and with $\Gamma \to 0$, we have that

$$\mathbb{E}_t\Delta\hat{q}_{t+1} = \hat{r}_t^* - \hat{r}_t = \hat{\beta}_t - \hat{\beta}_t^*,$$

and

$$\Delta\hat{q}_{t+1} = (\Delta\hat{q}_{t+1} - \mathbb{E}_t\Delta\hat{q}_{t+1}) + \mathbb{E}_t\Delta\hat{q}_{t+1} = \delta_\beta^q\hat{\epsilon}_{t+1}^\beta + \delta_{\beta^*}^q\hat{\epsilon}_{t+1}^{\beta^*} + \hat{\beta}_t - \hat{\beta}_t^*.$$

Focusing on β_t shocks alone for simplicity (equivalently, β_t/β_t^* shocks), it follows that

$$\begin{aligned}
Var(\hat{\beta}_t) &= \frac{1}{1-(\rho^\beta)^2}\left(\sigma^\beta\right)^2, \\
Var(\delta_\beta^q\hat{\epsilon}_t^\beta) &= \left(\frac{1}{1+r^*-\rho^\beta}\right)^2\left(\sigma^\beta\right)^2, \\
Var(\mathbb{E}_t\Delta\hat{q}_{t+1})/Var(\Delta\hat{q}_{t+1}) &= \frac{\frac{1}{1-\left(\rho^\beta\right)^2}\left(\sigma^\beta\right)^2}{\left(\frac{1}{1+r^*-\rho^\beta}\right)^2\left(\sigma^\beta\right)^2+\frac{1}{1-\left(\rho^\beta\right)^2}\left(\sigma^\beta\right)^2}, \\
&= \frac{1}{1+\frac{1-(\rho^\beta)^2}{(1+r^*-\rho^\beta)^2}}.
\end{aligned}$$

If $r^* = 0$, note that this becomes

$$\frac{1}{1+\frac{1+\rho^\beta}{1-\rho^\beta}}.$$

The comparative statics with respect to ρ^β are then immediate. $\square$

B.4 Proposition 3

Proof. We again reference the coefficients characterized in section B.1.2 in the autarkic limit.

Let us first characterize the comparative static of δ^q_β with respect to Γ. First note that

$$\begin{aligned}
\frac{1}{b^{a*}}\frac{d\delta^q_\beta}{d\Gamma}|_{\Gamma=0} \propto & -\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta)+\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}\right] \\
& + \frac{1}{b^{a*}}\frac{d\tilde{\delta}^q_{b^{a*}}}{d\Gamma}|_{\Gamma=0} + \left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right], \\
= & \frac{2}{2\sigma-1}\frac{1}{1+r^*}\rho^\beta - \frac{2}{2\sigma-1}\frac{r^*}{1+r^*} + \frac{1-\varsigma}{b^{a*}} + \frac{1}{b^{a*}}\frac{d\tilde{\delta}^q_{b^{a*}}}{d\Gamma}|_{\Gamma=0}.
\end{aligned}$$

The condition defining $\tilde{\delta}^q_{b^{a*}}$ implies

$$\frac{d\tilde{\delta}^q_{b^{a*}}}{d\Gamma}|_{\Gamma=0} = -\frac{2}{2\sigma-1}\frac{1}{1+r^*}\frac{d\delta^{b^{a*}}_{b^{a*}}}{d\Gamma}|_{\Gamma=0}.$$

Finally, implicitly differentiating the condition defining $\delta^{b^{a*}}_{b^{a*}}$ yields

$$-\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}\frac{d\delta^{b^{a*}}_{b^{a*}}}{d\Gamma}|_{\Gamma=0} - b^{a*}\left(-\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right) = 0.$$

It follows that

$$\frac{d\delta^{b^{a*}}_{b^{a*}}}{d\Gamma}|_{\Gamma=0} = -\frac{1}{\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}}b^{a*}\left(-\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right),$$

so

$$\frac{d\tilde{\delta}^q_{b^{a*}}}{d\Gamma}|_{\Gamma=0} = \frac{1}{r^*}b^{a*}\left(-\frac{2}{2\sigma-1}\frac{r^*}{1+r^*}+\frac{1-\varsigma}{b^{a*}}\right).$$

Thus, for r^* small we have that

$$\frac{d\delta_\beta^q}{d\Gamma}|_{\Gamma=0} \propto b^{a*}\frac{1-\varsigma}{b^{a*}} > 0.$$

Recalling that $\delta_\beta^q < 0$ around the autarkic limit, it follows that an increase in Γ dampens the magnitude of the exchange rate response upon a $\hat{\epsilon}_t^\beta$ innovation. For instance, it reduces the magnitude of the dollar appreciation upon a negative $\hat{\epsilon}_t^\beta$ innovation.

The impact effect on the expected carry trade return in the autarkic limit is

$$\begin{aligned}
&\frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\left[\delta_\beta^r - \delta_\beta^q - \delta_\beta^{r*} + \frac{1-\varsigma}{b^{a*}}\tilde{\delta}_\beta^{b^{a*}}\right] \\
&\qquad = \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{\frac{2}{2\sigma-1}\frac{1}{1+r^*}\rho^\beta - \tilde{\delta}_{b^{a*}}^q + \frac{1-\varsigma}{b^{a*}}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}_{b^{a*}}^q + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]}.
\end{aligned}$$

The impact effect on the expected dollar appreciation in the autarkic limit is

$$\begin{aligned}
&-\delta_\beta^r - \frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\left[\delta_\beta^r - \delta_\beta^q - \delta_\beta^{r*} + \frac{1-\varsigma}{b^{a*}}\tilde{\delta}_\beta^{b^{a*}}\right] \\
&\quad = 1 - \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\frac{\frac{2}{2\sigma-1}\frac{1}{1+r^*}\rho^\beta - \tilde{\delta}_{b^{a*}}^q + \frac{1-\varsigma}{b^{a*}}}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}_{b^{a*}}^q + \frac{b^{a^*}\Gamma}{1-2b^{a^*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]}.
\end{aligned}$$

Around $\Gamma = 0$, the expression multiplying $\frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}$ in each expression is positive for r^* small and $b^{a*} > 0$ (which is implied by $\beta^* < \beta$). It follows that an increase in Γ amplifies the response of the expected carry trade return and dampens the response of the expected dollar appreciation. In the latter case, this means it reduces the expected dollar depreciation upon a negative $\hat{\epsilon}_t^\beta$ innovation.

Finally, consider the Fama (1984) coefficient. We have that

$$\begin{aligned}
\hat{b}_t^{a*} &= \delta_\beta^{b^{a*}}\hat{\beta}_t + \delta_{b^{a*}}^{b^{a*}}\hat{b}_{t-1}^{a*}, \\
&= \delta_\beta^{b^{a*}}\hat{\beta}_t + \delta_{b^{a*}}^{b^{a*}}\delta_\beta^{b^{a*}}\hat{\beta}_{t-1} + \left(\delta_{b^{a*}}^{b^{a*}}\right)^2\hat{b}_{t-2}^{a*}, \\
&\vdots \\
&= \delta_\beta^{b^{a*}}\sum_{\tau=0}^{\infty}\sum_{s=0}^{\tau}\left(\delta_{b^{a*}}^{b^{a*}}\right)^s\left(\rho^\beta\right)^{\tau-s}\hat{\epsilon}_{t-\tau}^\beta,
\end{aligned}$$

$$= \delta_{\beta}^{b^{a*}} \sum_{\tau=0}^{\infty} \frac{\left(\rho^{\beta}\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \hat{\epsilon}_{t-\tau}^{\beta}.$$

Thus, for $x \in \{q, r, r^*\}$,

$$\begin{aligned}
\hat{x}_t &= \delta_{\beta}^{x} \hat{\beta}_t + \delta_{b^{a*}}^{x} \hat{b}_{t-1}^{a*}, \\
&= \delta_{\beta}^{x} \hat{\epsilon}_t^{\beta} + \sum_{\tau=0}^{\infty} \left[\delta_{\beta}^{x} \left(\rho^{\beta}\right)^{\tau+1} + \delta_{b^{a*}}^{x} \delta_{\beta}^{b^{a*}} \frac{\left(\rho^{\beta}\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1}}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \right] \hat{\epsilon}_{t-1-\tau}^{\beta}.
\end{aligned}$$

In the autarkic limit, it follows that

$$\begin{aligned}
&\Delta \hat{q}_{t+1} = \delta_{\beta}^{q} \hat{\epsilon}_{t+1}^{\beta} \\
&+ \sum_{\tau=0}^{\infty} \left[\delta_{\beta}^{q} \left(\left(\rho^{\beta}\right)^{\tau+1} - \left(\rho^{\beta}\right)^{\tau} \right) + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{\left(\left(\rho^{\beta}\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1} \right) - \left(\left(\rho^{\beta}\right)^{\tau} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau} \right)}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \right] \hat{\epsilon}_{t-\tau}^{\beta}
\end{aligned}$$

and

$$\hat{r}_t^* - \hat{r}_t = \sum_{\tau=0}^{\infty} \left(\delta_{\beta}^{r^*} - \delta_{\beta}^{r} \right) \left(\rho^{\beta}\right)^{\tau} \hat{\epsilon}_{t-\tau}^{\beta}.$$

Then

$$\begin{aligned}
&Cov(\Delta \hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t) / \left(\sigma^{\beta}\right)^2 \\
&= \sum_{\tau=0}^{\infty} \left[\delta_{\beta}^{q} \left(\left(\rho^{\beta}\right)^{\tau+1} - \left(\rho^{\beta}\right)^{\tau} \right) + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{\left(\left(\rho^{\beta}\right)^{\tau+1} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau+1} \right) - \left(\left(\rho^{\beta}\right)^{\tau} - \left(\delta_{b^{a*}}^{b^{a*}}\right)^{\tau} \right)}{\rho^{\beta} - \delta_{b^{a*}}^{b^{a*}}} \right] \times \\
&\qquad \left[\left(\delta_{\beta}^{r^*} - \delta_{\beta}^{r} \right) \left(\rho^{\beta}\right)^{\tau} \right], \\
&= \left(\delta_{\beta}^{r^*} - \delta_{\beta}^{r} \right) \left[-\delta_{\beta}^{q} \frac{1}{1+\rho^{\beta}} + \tilde{\delta}_{b^{a*}}^{q} \tilde{\delta}_{\beta}^{b^{a*}} \frac{1}{1+\rho^{\beta}} \frac{1}{1 - \delta_{b^{a*}}^{b^{a*}} \rho^{\beta}} \right].
\end{aligned}$$

And

$$\begin{aligned}
Var(\hat{r}_t^* - \hat{r}_t) / \left(\sigma^{\beta}\right)^2 &= \left(\delta_{\beta}^{r^*} - \delta_{\beta}^{r} \right)^2 \sum_{\tau=0}^{\infty} \left(\rho^{\beta}\right)^{2\tau}, \\
&= \left(\delta_{\beta}^{r^*} - \delta_{\beta}^{r} \right)^2 \frac{1}{1 - \left(\rho^{\beta}\right)^2}.
\end{aligned}$$

Hence,

$$\frac{Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)}{Var(\hat{r}_t^* - \hat{r}_t)} = -\frac{\delta_\beta^q}{\delta_\beta^{r^*} - \delta_\beta^r}(1-\rho^\beta) + \frac{\tilde{\delta}_{b^{a*}}^q \tilde{\delta}_\beta^{b^{a*}}}{\delta_\beta^{r^*} - \delta_\beta^r}\frac{1-\rho^\beta}{1-\delta_{b^{a*}}^{b^{a*}}\rho^\beta}.$$

Now substituting in for δ_β^q, $\tilde{\delta}_\beta^{b^{a*}}$, and $\delta_\beta^{r^*} - \delta_\beta^r = 1$ yields the Fama (1984) coefficient

$$\frac{Cov(\Delta\hat{q}_{t+1}, \hat{r}_t^* - \hat{r}_t)}{Var(\hat{r}_t^* - \hat{r}_t)} = \frac{\left(1 + \frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\right)\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}_{b^{a*}}^q \frac{1-\rho^\beta}{1-\delta_{b^{a*}}^{b^{a*}}\rho^\beta}\right]}{\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \tilde{\delta}_{b^{a*}}^q + \frac{b^{a*}\Gamma}{1-2b^{a*}\Gamma}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]}.$$

The derivative of this expression with respect to Γ evaluated at $\Gamma = 0$ is proportional to

$$\begin{aligned}
&\frac{d\tilde{\delta}_{b^{a*}}^q}{d\Gamma}|_{\Gamma=0} + \frac{2}{2\sigma-1}\frac{r^*}{1+r^*}\frac{\rho^\beta}{1-\rho^\beta}\frac{d\delta_{b^{a*}}^{b^{a*}}}{d\Gamma}|_{\Gamma=0} + b^{a*}\left(\frac{2}{2\sigma-1}\frac{1}{1+r^*}(1-\rho^\beta) + \frac{2}{2\sigma-1}\frac{r^*}{1+r^*}\right) \\
&\qquad - \left(\frac{d\tilde{\delta}_{b^{a*}}^q}{d\Gamma}|_{\Gamma=0} + b^{a*}\left[\frac{2}{2\sigma-1}\frac{1}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right]\right) \\
&\qquad = b^{a*}\left[-\frac{1}{1-\rho^\beta}\left(-\frac{2}{2\sigma-1}\frac{r^*}{1+r^*} + \frac{1-\varsigma}{b^{a*}}\right) - \frac{2}{2\sigma-1}\frac{1}{1+r^*}\rho^\beta\right],
\end{aligned}$$

which is negative for $b^{a*} > 0$ and r^* small. Thus, the Fama (1984) coefficient falls as Γ rises. □

C Additional empirical analysis

We now provide additional empirical results accompanying those in section 4.

C.1 Alternative approaches to inference

Since both exchange rates and many of the other variables in our analysis are highly persistent, conventional inference can substantially understate the degree of uncertainty in our estimates, especially in small samples. Our preferred method of inference in this context is to use Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b). Here we compare this approach to alternatives. For brevity we report results for three of our main specifications: the univariate comovement of the exchange rate with the 10-year yield differential (Table 7) and with relative consumption (Table 8), and the bivariate comovement with the 10-year yield differential and excess bond premium (Table 9). Similar conclusions are obtained for our other specifications.

We consider four versions of inference using Newey and West (1987) standard errors. The first sets the truncation parameter S equal to $0.75N^{1/3}$ where N is the sample size (which evaluates to 4 in our context), a widely followed rule of thumb since Andrews (1991), and standard normal critical values. We next increase the truncation parameter to $1.3N^{1/2}$ (which evaluates to 14 in our context), an updated rule of thumb proposed by Lazarus, Lewis, Stock, and Watson (2018), and then to the sample size itself, following Kiefer and Vogelsang (2002a,b). Rather than using standard normal critical values, we also use fixed-b critical values (where $b = S/N$) using the formula provided in Kiefer and Vogelsang (2005). As is evident from Tables 7-9, the confidence intervals for our estimated coefficients widen as we increase the truncation parameter and use fixed-b critical values.

We next consider three versions of inference using the moving block bootstrap. For each specification, we create 5,000 pseudo-samples by using the estimated coefficients and drawing blocks with replacement from the estimated residuals. For each pseudo-sample, we re-estimate the specification, and we use the distribution of estimates as our confidence intervals. We consider block sizes of 1; 4-5; or 23-29, in the latter two cases choosing a value which evenly divides the sample size. We find that increasing the block size increases the width of the confidence intervals, consistent with the simulation results of Kiefer and Vogelsang (2005) when variables are highly persistent.

We finally consider a VAR-based bootstrap, following Engel (2016). We estimate a two-lag VAR in the log real exchange rate, three-month yield differential, 10-year yield differential, log relative consumption, log U.S. net trade, and excess bond premium over our maintained sample period, 1991 Q2 – 2020 Q1.[47] Given the estimated coefficients, we then create 5,000 pseudo-samples by drawing from the VAR residuals with replacement.[48] For each pseudo-sample, we estimate the comovements of interest, and we again use the distribution of estimates as our confidence intervals.

We settle on Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values as our baseline form of inference in

[47]The results which follow are not meaningfully affected using three lags.

[48]We initialize the state variables of the VAR to their average values in our sample, and we draw pseudo-samples which are 1,000 quarters longer than our observed sample so we can discard the first 1,000 quarters as a burn-in period.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
10y (G10 - USD) yield	-2.77	-4.21	-5.87	-8.48
NW: $S = 0.75N^{1/3}$, N	[-4.85,-0.69]	[-7.37,-1.04]	[-9.18,-2.56]	[-10.51,-6.44]
NW: $S = 1.3N^{1/2}$, N	[-4.78,-0.76]	[-6.75,-1.66]	[-8.64,-3.11]	[-11.06,-5.90]
NW: $S = 1.3N^{1/2}$, b	[-5.11,-0.43]	[-7.18,-1.23]	[-9.14,-2.60]	[-11.51,-5.45]
NW: $S = N$, b	[-6.42,0.87]	[-6.52,-1.89]	[-9.28,-2.46]	[-13.35,-3.61]
Bootstrap, $\ell = 1$	[-4.71,-0.82]	[-6.50,-1.91]	[-8.25,-3.52]	[-10.21,-6.74]
Bootstrap, $\ell = 4, 5$	[-4.61,-0.87]	[-7.64,-0.87]	[-9.92,-1.72]	[-11.48,-5.35]
Bootstrap, $\ell = 23 - 29$	[-4.65,-0.79]	[-8.40,-0.16]	[-12.53,0.36]	[-13.33,-3.18]
Bootstrap, VAR	[-4.98,-0.88]	[-7.90,-0.16]	[-14.89,-0.78]	[-18.28,-1.88]
N	115	112	104	116
Adj R^2	0.04	0.07	0.13	0.36
Bootstrap, $\ell = 1$	[-0.00,0.12]	[0.01,0.16]	[0.05,0.25]	[0.25,0.47]
Bootstrap, $\ell = 4, 5$	[-0.00,0.12]	[-0.00,0.22]	[0.00,0.35]	[0.17,0.57]
Bootstrap, $\ell = 23 - 29$	[-0.00,0.12]	[-0.01,0.25]	[-0.01,0.48]	[0.06,0.64]
Bootstrap, VAR	[-0.00,0.13]	[-0.01,0.22]	[-0.00,0.47]	[0.01,0.60]

Table 7: comovements with log real exchange rate between U.S. and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Rows in brackets report 90% confidence intervals using different methods. NW refers to Newey and West (1987) standard errors with truncation parameter S; N refers to critical values from standard normal distribution, while b refers to critical values from asymptotics with fixed $b = S/N$ as reported by Kiefer and Vogelsang (2002a,b, 2005). Bootstrap uses moving blocks with length ℓ or VAR described in text.

the paper. It is more conservative than Newey and West (1987) standard errors with lower truncation parameters or standard normal critical values, and is comparable to the bootstrap methods with large block size or a VAR to address persistence.

We also note that using any of these approaches, a higher 10-year yield differential or relative G10/U.S. consumption is significantly associated with a weaker dollar. The bootstrap specifications imply there is substantial uncertainty about the univariate R^2 of these variables, but in the bivariate specification with the 10-year yield differential and excess bond premium, the lower bound of the R^2 reaches nearly 30%.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
Log G10/U.S. real cons.	-1.75	-3.02	-3.91	-2.59
NW: $S = 0.75N^{1/3}$, N	[-2.80,-0.70]	[-4.29,-1.75]	[-5.38,-2.45]	[-3.49,-1.68]
NW: $S = 1.3N^{1/2}$, N	[-2.71,-0.79]	[-4.02,-2.02]	[-5.80,-2.03]	[-3.85,-1.32]
NW: $S = 1.3N^{1/2}$, b	[-2.86,-0.63]	[-4.19,-1.85]	[-6.14,-1.69]	[-4.07,-1.10]
NW: $S = N$, b	[-2.70,-0.80]	[-4.23,-1.80]	[-6.25,-1.58]	[-4.61,-0.56]
Bootstrap, $\ell = 1$	[-3.12,-0.35]	[-4.19,-1.80]	[-4.94,-2.88]	[-3.20,-1.97]
Bootstrap, $\ell = 4, 5$	[-3.10,-0.39]	[-4.81,-1.20]	[-5.75,-2.06]	[-3.76,-1.40]
Bootstrap, $\ell = 23 - 29$	[-3.06,-0.35]	[-4.80,-1.08]	[-6.45,-1.57]	[-4.65,-0.59]
Bootstrap, VAR	[-3.16,-0.41]	[-5.55,-0.76]	[-8.29,-0.93]	[-8.75,-1.64]
N	115	112	104	116
Adj R^2	0.03	0.13	0.27	0.29
Bootstrap, $\ell = 1$	[-0.01,0.11]	[0.04,0.23]	[0.16,0.40]	[0.18,0.40]
Bootstrap, $\ell = 4, 5$	[-0.01,0.11]	[0.02,0.30]	[0.09,0.52]	[0.11,0.50]
Bootstrap, $\ell = 23 - 29$	[-0.01,0.10]	[0.01,0.26]	[0.05,0.56]	[0.01,0.64]
Bootstrap, VAR	[-0.01,0.10]	[-0.00,0.29]	[0.00,0.57]	[0.03,0.70]

Table 8: comovements with log real exchange rate between U.S. and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Rows in brackets report 90% confidence intervals using different methods. NW refers to Newey and West (1987) standard errors with truncation parameter S; N refers to critical values from standard normal distribution, while b refers to critical values from asymptotics with fixed $b = S/N$ as reported by Kiefer and Vogelsang (2002a,b, 2005). Bootstrap uses moving blocks with length ℓ or VAR described in text.

C.2 Exchange rates and financial conditions

We next provide additional results on the comovement between the real exchange rate and proxies for risk or convenience yields. The main text reported results using the excess bond premium of Gilchrist and Zakrajsek (2012), while here we report results using the VIX, the global factor in risky asset prices of Miranda-Agrippino and Rey (2020), and the three-month Treasury basis from Du et al. (2018).[49] Tables 10-12 provide univariate and bivariate comovements with the dollar/G10 exchange rate.

The results echo those for the excess bond premium in the main text. A higher

[49] We find similar results using convenience yields from Jiang et al. (2021) or Engel and Wu (2023).

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
10y (G10 - USD) yield	-5.41	-7.28	-9.14	-9.36
NW: $S = 0.75N^{1/3}$, N	[-7.24,-3.58]	[-10.74,-3.82]	[-12.34,-5.95]	[-11.46,-7.27]
NW: $S = 1.3N^{1/2}$, N	[-6.94,-3.88]	[-10.46,-4.10]	[-12.14,-6.14]	[-12.39,-6.34]
NW: $S = 1.3N^{1/2}$, b	[-7.19,-3.63]	[-11.00,-3.56]	[-12.69,-5.60]	[-12.92,-5.81]
NW: $S = N$, b	[-6.80,-4.02]	[-10.67,-3.89]	[-13.80,-4.49]	[-16.11,-2.62]
Bootstrap, $\ell = 1$	[-7.40,-3.42]	[-9.45,-5.20]	[-11.04,-7.27]	[-10.88,-7.80]
Bootstrap, $\ell = 4, 5$	[-7.32,-3.45]	[-10.30,-4.31]	[-12.31,-6.00]	[-12.15,-6.59]
Bootstrap, $\ell = 23 - 29$	[-7.35,-3.31]	[-10.97,-3.63]	[-14.22,-4.29]	[-14.05,-4.58]
Bootstrap, VAR	[-7.59,-3.75]	[-10.87,-4.44]	[-16.72,-4.61]	[-19.31,-6.11]
EBP	4.20	5.42	8.28	7.09
NW: $S = 4$, N	[2.96,5.44]	[3.90,6.94]	[5.41,11.15]	[2.37,11.81]
NW: $S = 1.3N^{1/2}$, N	[2.99,5.41]	[4.21,6.63]	[4.81,11.75]	[1.66,12.52]
NW: $S = 1.3N^{1/2}$, b	[2.79,5.61]	[4.01,6.84]	[4.18,12.38]	[0.71,13.47]
NW: $S = N$, b	[2.42,5.98]	[4.35,6.50]	[4.22,12.35]	[1.95,12.23]
Bootstrap, $\ell = 1$	[2.74,5.65]	[4.01,6.84]	[6.74,9.79]	[5.24,9.00]
Bootstrap, $\ell = 4, 5$	[2.82,5.62]	[3.41,7.34]	[5.66,10.91]	[4.02,10.14]
Bootstrap, $\ell = 23 - 29$	[2.71,5.67]	[3.15,7.67]	[4.78,11.86]	[3.45,11.20]
Bootstrap, VAR	[2.85,5.53]	[3.88,7.38]	[4.21,12.05]	[5.32,13.56]
N	115	112	104	116
Adj R^2	0.19	0.32	0.50	0.51
Bootstrap, $\ell = 1$	[0.10,0.32]	[0.22,0.44]	[0.41,0.61]	[0.42,0.61]
Bootstrap, $\ell = 4, 5$	[0.10,0.32]	[0.19,0.49]	[0.36,0.67]	[0.37,0.69]
Bootstrap, $\ell = 23 - 29$	[0.09,0.31]	[0.16,0.50]	[0.29,0.73]	[0.27,0.77]
Bootstrap, VAR	[0.10,0.33]	[0.21,0.54]	[0.22,0.68]	[0.28,0.77]

Table 9: additional comovements using proxy for risk

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016). Rows in brackets report 90% confidence intervals using different methods. NW refers to Newey and West (1987) standard errors with truncation parameter S; N refers to critical values from standard normal distribution, while b refers to critical values from asymptotics with fixed $b = S/N$ as reported by Kiefer and Vogelsang (2002a,b, 2005). Bootstrap uses moving blocks with length ℓ or VAR described in text.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
VIX	0.14	0.26	0.30	0.34
	[-0.02,0.29]	[0.16,0.37]	[-0.17,0.78]	[-0.43,1.11]
N	115	112	104	116
Adj R^2	0.06	0.08	0.05	0.05
3m (G10 - USD) yield	-2.13	-1.83	-1.16	-3.28
	[-4.12,-0.14]	[-4.92,1.27]	[-3.81,1.50]	[-7.53,0.97]
VIX	0.14	0.30	0.36	0.34
	[-0.00,0.27]	[0.16,0.44]	[-0.05,0.76]	[-0.16,0.85]
N	115	112	104	116
Adj R^2	0.11	0.13	0.07	0.25
10y (G10 - USD) yield	-4.87	-6.28	-7.86	-8.69
	[-6.03,-3.71]	[-9.99,-2.57]	[-14.72,-1.00]	[-16.43,-0.95]
VIX	0.21	0.38	0.49	0.39
	[0.09,0.33]	[0.21,0.55]	[0.17,0.81]	[0.05,0.72]
N	115	112	104	116
Adj R^2	0.17	0.22	0.27	0.42

Table 10: additional comovements using proxy for risk

Notes: columns marked “Changes” regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked “Levels” regresses log real exchange rate on given variable. Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
Global factor	-3.73	-3.66	-3.67	-3.58
	[-5.35,-2.11]	[-5.42,-1.91]	[-6.98,-0.36]	[-8.96,1.81]
N	111	108	100	112
Adj R^2	0.19	0.18	0.11	0.07
3m (G10 - USD) yield	-2.35	-1.99	-2.76	-4.06
	[-3.81,-0.88]	[-3.97,-0.01]	[-5.01,-0.51]	[-5.18,-2.93]
Global factor	-3.81	-3.98	-6.18	-5.77
	[-4.81,-2.81]	[-4.99,-2.97]	[-10.01,-2.35]	[-7.87,-3.67]
N	111	108	100	112
Adj R^2	0.25	0.24	0.22	0.36
10y (G10 - USD) yield	-6.67	-8.06	-11.53	-9.77
	[-8.24,-5.10]	[-10.73,-5.38]	[-14.12,-8.93]	[-13.47,-6.06]
Global factor	-5.45	-5.15	-7.60	-5.76
	[-6.26,-4.63]	[-6.24,-4.06]	[-11.76,-3.44]	[-9.49,-2.04]
N	111	108	100	112
Adj R^2	0.40	0.41	0.50	0.54

Table 11: additional comovements using proxy for risk

Notes: columns marked “Changes” regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked “Levels” regresses log real exchange rate on given variable. Global factor in risky asset prices is from Miranda-Agrippino and Rey (2020). Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m Treasury basis	0.92	0.17	-0.20	-0.51
	[-0.24,2.08]	[-1.37,1.72]	[-5.14,4.74]	[-9.51,8.49]
N	115	112	104	116
Adj R^2	0.00	-0.01	-0.01	-0.01
3m (G10 - USD) yield	-2.09	-1.40	-0.68	-3.44
	[-4.43,0.25]	[-4.09,1.28]	[-2.55,1.20]	[-6.60,-0.28]
3m Treasury basis	0.35	0.23	-0.70	-3.46
	[-0.79,1.49]	[-2.09,2.55]	[-5.65,4.25]	[-5.19,-1.73]
N	115	112	104	116
Adj R^2	0.04	0.02	-0.01	0.20
10y (G10 - USD) yield	-2.67	-4.28	-5.87	-8.56
	[-6.62,1.29]	[-6.53,-2.03]	[-9.31,-2.44]	[-12.89,-4.23]
3m Treasury basis	0.71	0.75	-0.15	-2.08
	[-0.55,1.97]	[-0.80,2.31]	[-6.09,5.80]	[-5.07,0.90]
N	115	112	104	116
Adj R^2	0.03	0.06	0.12	0.35

Table 12: additional comovements using proxy for risk

Notes: columns marked “Changes” regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked “Levels” regresses log real exchange rate on given variable. 3m Treasury basis is from Du et al. (2018), updated through 2020 using data shared with us by Wenxin Du. Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

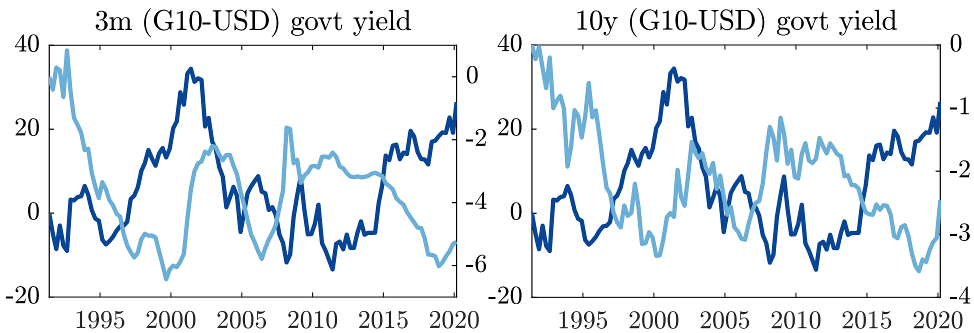

Figure 7: comovements with log real exchange rate and government yield differentials

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series are normalized to zero in 1991 Q2. All yields are annualized.

VIX or lower level of global risky asset prices is associated with a stronger dollar (interestingly, this is not consistently the case for a higher Treasury basis). In changes, these variables account for a comparable or higher share of the variation than yield differentials; in levels, they account for much less of the variation in the exchange rate. And again, in bivariate specifications, the explanatory power together is greater than the sum of their parts, and we can account for as much as 50% of the exchange rate variation both in changes and levels.

C.3 Exchange rates and government bond yield differentials

We next replace the private sector interest rate differentials — constructed from Libor rates and interest rate swaps — with government bond yield differentials. The first two panels of Table 13 report the analogs of the first two panels of Table 1 in the main text using government bond yields. Figure 7 depicts the comovements of the dollar/G10 real exchange rate with these government bond yield differentials. The third and fourth panels of Table 13 report the analogs of the latter two panels of Table 2 using government bond yield differentials and the excess bond premium.

The results echo those with private sector interest rate differentials. The dollar is strong when G10 government bond yields are low versus U.S. government bond yields. This is not the case in the early 2000s, 2008, and 2020. So when we also condition on a proxy for risk such as the excess bond premium, the comovement of the exchange rate with government bond yield differentials strengthens, and the share of variation

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m (G10 - USD) yield	-1.40	-1.49	-0.63	-3.52
	[-3.86,1.05]	[-3.69,0.71]	[-2.77,1.51]	[-7.02,-0.02]
N	115	112	104	116
Adj R^2	0.01	0.03	-0.00	0.18
10y (G10 - USD) yield	-2.00	-2.98	-4.89	-7.55
	[-6.45,2.46]	[-5.39,-0.58]	[-9.22,-0.56]	[-10.37,-4.72]
N	115	112	104	116
Adj R^2	0.01	0.03	0.08	0.25
3m (G10 - USD) yield	-1.71	-2.26	-2.14	-4.29
	[-4.43,1.01]	[-5.01,0.48]	[-4.34,0.07]	[-7.78,-0.81]
EBP	2.60	4.37	7.26	7.39
	[0.12,5.07]	[3.53,5.22]	[2.14,12.39]	[0.87,13.91]
N	115	112	104	116
Adj R^2	0.08	0.20	0.27	0.34
10y (G10 - USD) yield	-4.73	-6.68	-9.46	-8.83
	[-6.21,-3.25]	[-10.37,-3.00]	[-14.64,-4.28]	[-12.71,-4.95]
EBP	4.02	5.50	8.76	7.43
	[2.26,5.79]	[4.32,6.68]	[4.41,13.11]	[1.47,13.40]
N	115	112	104	116
Adj R^2	0.15	0.27	0.47	0.42

Table 13: comovements with log real exchange rate and government yield differentials

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016). Rows in brackets report 90% confidence intervals using Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

in the exchange rate we can explain rises.

C.4 Bilateral exchange rates and interest rate differentials

We now study the comovement between bilateral real exchange rates and interest rate differentials for each of the G10 countries versus the U.S. Table 14 reports these comovements focusing on the levels specifications for brevity, and Figures 8 and 9 visually depict them.

For all currencies except the Japanese yen, a lower foreign yield than the dollar yield is associated with a stronger real dollar versus that currency. Hence, the comovement which we document between the broad dollar/G10 exchange rate and yield differential in the main text is robust across currencies. The distinct comovement with the yen reflects the fact that over our sample period, Japanese yields have tended to rise relative to U.S. yields (since Japan hit the zero lower bound early in the sample while U.S. yields tended toward zero), and yet Japan has experienced a trend real depreciation versus the U.S. We note that around these trends, low Japanese yields versus U.S. yields have also been associated with a relatively strong dollar, as depicted in Figure 9.

Interestingly, the share of variation in the bilateral exchange rates accounted for by yield differentials differs substantially across countries. The 10-year yield differential, for instance, accounts for only 2% of the variation in the real exchange rate between the U.S. and Canada, but 37% of the variation in the real exchange rate between the U.S. and Sweden. There is substantial uncertainty about these R^2 values in population given the persistence of the variables in question. Nonetheless, exploring these differences across currencies would be an interesting direction for future work, related to the discussion in section 6.

C.5 Alternative base currencies

In this subsection we replace the U.S. with three alternative base currencies and countries: the pound and the U.K., the euro and Euro Area, and the yen and Japan. Tables 15-17 report the analogs to Table 1 for these base currencies and countries, and Figures 10-12 depict the analogs to Figure 1.

These results indicate that the comovements which we document in the main text are not U.S.-specific: when we use the pound as an alternative base currency, we

	3m	10y		3m	10y
AUD	-5.33	-11.45	GBP	-3.99	-7.19
	[-8.70,-1.97]	[-19.60,-3.30]		[-8.75,0.77]	[-16.33,1.94]
N	115	113	N	116	113
Adj R^2	0.22	0.33	Adj R^2	0.37	0.33
CAD	-3.02	-2.99	JPY	1.96	3.56
	[-6.49,0.44]	[-8.35,2.37]		[-1.27,5.20]	[-4.49,11.60]
N	114	113	N	111	105
Adj R^2	0.07	0.02	Adj R^2	0.04	0.04
CHF	-2.83	-9.79	NOK	-1.77	-7.43
	[-7.96,2.29]	[-13.09,-6.49]		[-10.74,7.20]	[-23.38,8.53]
N	105	104	N	82	83
Adj R^2	0.10	0.23	Adj R^2	0.03	0.14
DKK	-2.97	-7.60	NZD	-2.68	-8.53
	[-8.44,2.50]	[-10.77,-4.43]		[-7.24,1.88]	[-21.46,4.39]
N	88	89	N	95	95
Adj R^2	0.10	0.28	Adj R^2	0.04	0.10
EUR	-3.71	-7.52	SEK	-4.24	-7.74
	[-8.86,1.45]	[-14.38,-0.67]		[-9.61,1.13]	[-15.25,-0.24]
N	84	79	N	88	98
Adj R^2	0.14	0.16	Adj R^2	0.23	0.37

Table 14: comovements of log real exchange rate and yield differential

Notes: columns regress log real exchange rate on 3m yield differential (first column) or 10y yield differential (second column) for each currency versus USD. Regressions use Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

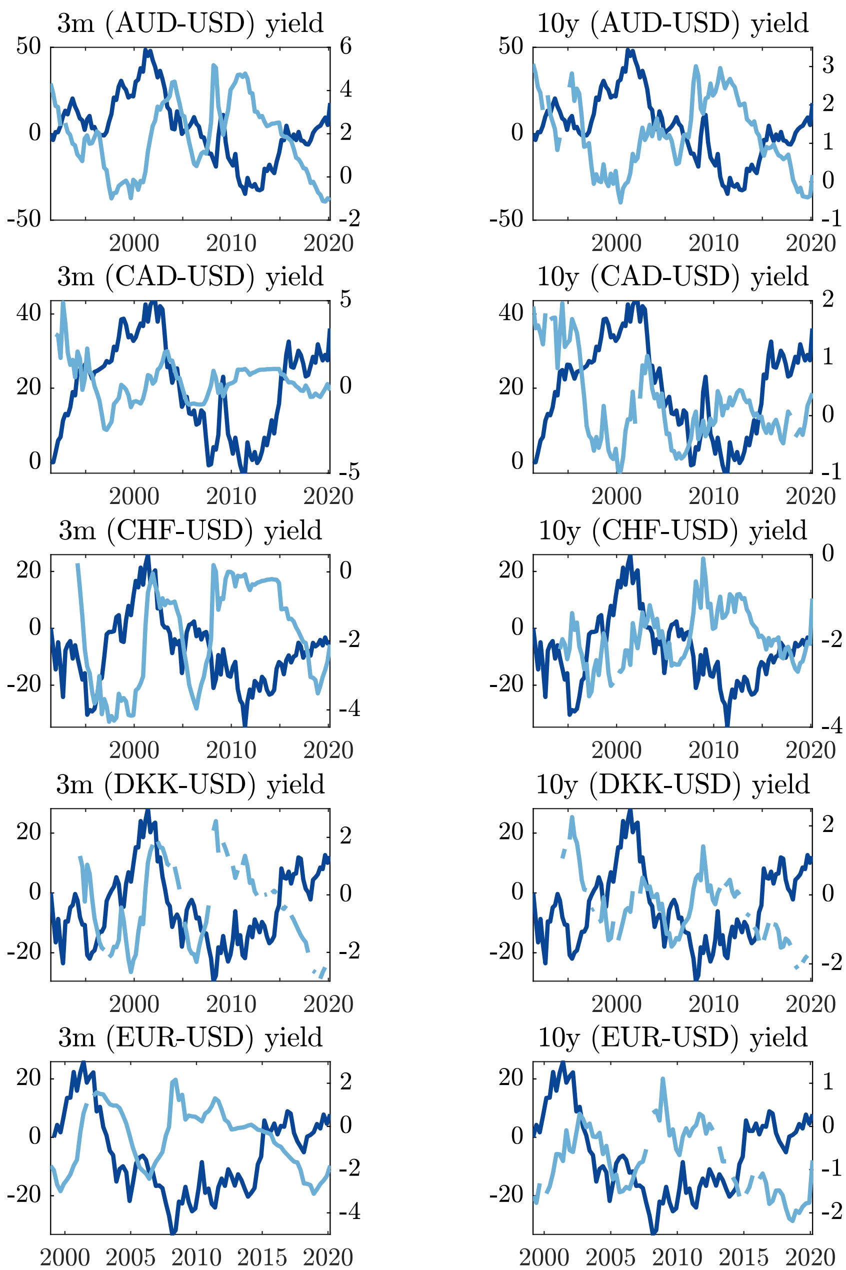

Figure 8: comovements with log real exchange rate and yield differential

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. Real exchange rate normalized to zero in 1991 Q2 except for euro, which is normalized to zero in 1999 Q1. All yields are annualized.

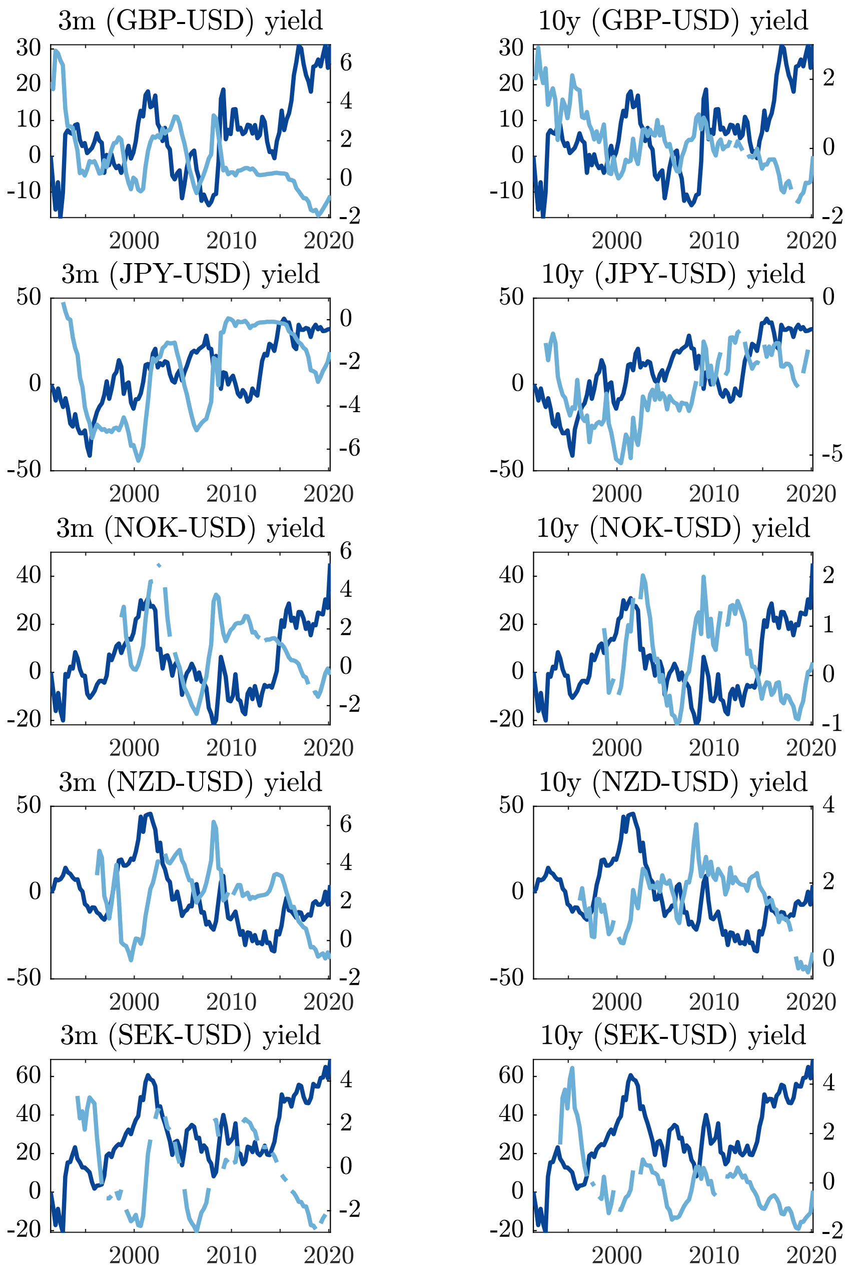

Figure 9: comovements with log real exchange rate and yield differential

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. Real exchange rate normalized to zero in 1991 Q2. All yields are annualized.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m (G10 - GBP) yield	-3.81	-3.98	-5.27	-6.60
	[-6.02,-1.60]	[-5.51,-2.46]	[-8.65,-1.89]	[-10.90,-2.31]
N	115	112	104	116
Adj R^2	0.23	0.27	0.36	0.56
10y (G10 - GBP) yield	-3.94	-7.09	-5.62	-9.14
	[-8.14,0.25]	[-18.25,4.07]	[-24.37,13.13]	[-25.65,7.38]
N	106	101	94	110
Adj R^2	0.06	0.09	0.04	0.20
Log G10/U.K. real cons.	-0.75	-1.38	-2.38	-2.46
	[-2.00,0.50]	[-3.53,0.78]	[-3.44,-1.32]	[-3.55,-1.37]
N	100	97	89	101
Adj R^2	0.02	0.08	0.40	0.52
Log U.K. net trade	-0.05	-0.09	-1.25	-0.69
	[-0.18,0.08]	[-0.61,0.42]	[-2.41,-0.09]	[-1.54,0.15]
N	115	112	104	116
Adj R^2	-0.01	-0.01	0.16	0.05

Table 15: comovements with log real exchange rate between U.K. and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Regressions use Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

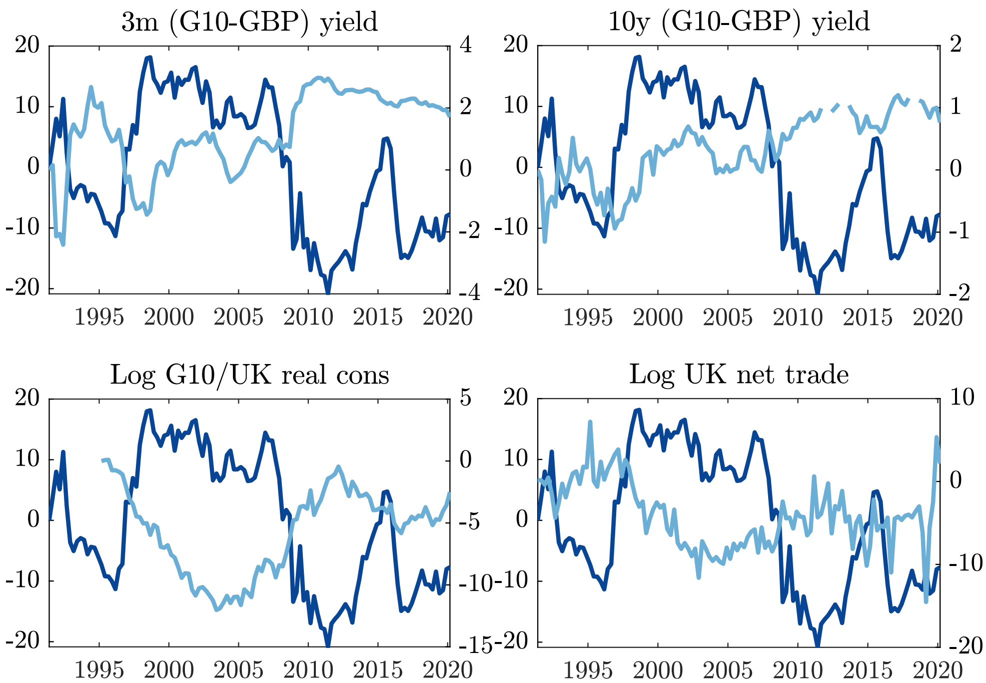

Figure 10: comovements with log real exchange rate between U.K. and G10

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series except log U.K. net trade are normalized to zero in first quarter. All yields are annualized.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m (G10 - EUR) yield	-3.19	-3.36	-3.83	-1.15
	[-6.69,0.31]	[-8.88,2.16]	[-11.03,3.37]	[-9.83,7.53]
N	81	77	70	83
Adj R^2	0.03	0.06	0.06	-0.00
10y (G10 - EUR) yield	-6.55	-4.80	-4.71	-1.10
	[-8.27,-4.83]	[-12.35,2.74]	[-19.94,10.52]	[-18.54,16.34]
N	66	64	53	73
Adj R^2	0.16	0.05	0.02	-0.01
Log G10/EA real cons.	0.20	0.09	-0.05	0.10
	[-1.63,2.03]	[-1.60,1.79]	[-2.08,1.98]	[-0.67,0.88]
N	84	81	73	85
Adj R^2	-0.01	-0.01	-0.01	-0.00
Log EA net trade	-0.12	0.07	-0.30	-0.35
	[-0.43,0.19]	[-0.49,0.64]	[-1.24,0.63]	[-1.57,0.86]
N	84	81	73	85
Adj R^2	-0.01	-0.01	-0.00	0.02

Table 16: comovements with log real exchange rate between Euro area (EA) and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Regressions use Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

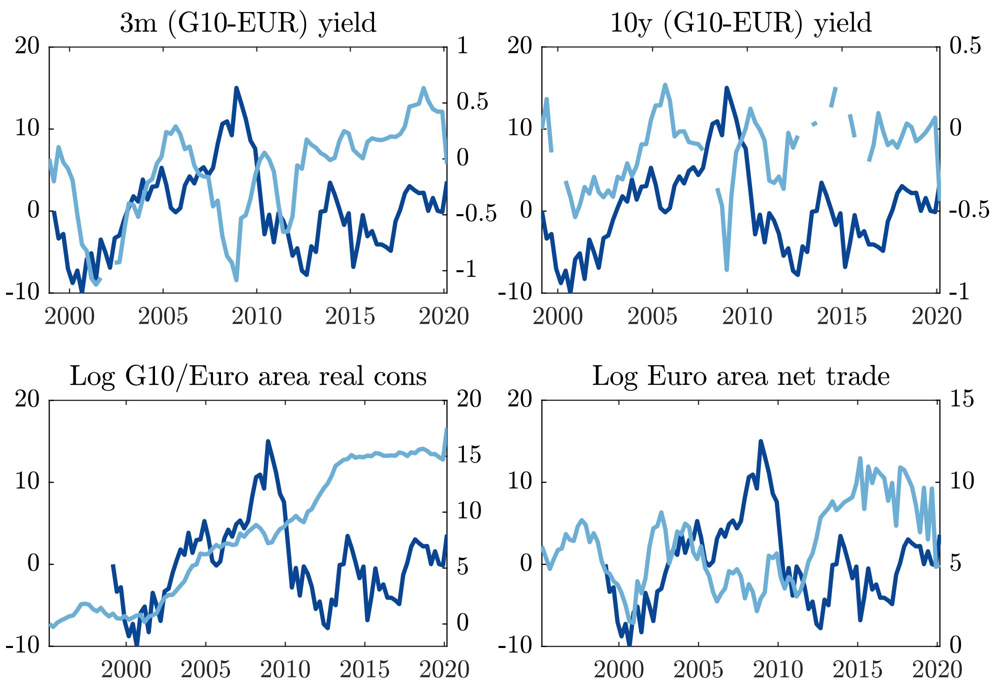

Figure 11: comovements with log real exchange rate between Euro area and G10

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series except log Euro area net trade are normalized to zero in first quarter. All yields are annualized.

	Changes			
	1 qtr	4 qtrs	12 qtrs	Levels
3m (G10 - JPY) yield	-4.02	-1.45	-2.66	6.54
	[-9.19,1.15]	[-8.63,5.74]	[-10.95,5.64]	[1.95,11.13]
N	110	107	99	111
Adj R^2	0.06	0.01	0.03	0.33
10y (G10 - JPY) yield	-4.05	-0.43	5.60	11.52
	[-11.75,3.65]	[-11.90,11.03]	[-0.08,11.28]	[8.62,14.42]
N	93	87	79	99
Adj R^2	0.06	-0.01	0.07	0.40
Log G10/Japan real cons.	0.35	1.36	1.17	-1.99
	[-2.02,2.73]	[-0.15,2.88]	[-1.21,3.54]	[-2.43,-1.55]
N	104	101	93	105
Adj R^2	-0.01	0.03	0.02	0.56
Log Japan net trade	0.00	0.05	0.35	1.08
	[-0.52,0.53]	[-0.53,0.62]	[-0.02,0.73]	[0.28,1.88]
N	104	101	93	105
Adj R^2	-0.01	-0.01	0.03	0.35

Table 17: comovements with log real exchange rate between Japan and G10

Notes: columns marked "Changes" regress 1-, 4-, or 12-quarter change in log real exchange rate on 1-, 4-, or 12-quarter change in given variable. Column marked "Levels" regresses log real exchange rate on given variable. Regressions use Newey and West (1987) standard errors with truncation parameter equal to sample size and fixed-b critical values, following Kiefer and Vogelsang (2002a,b).

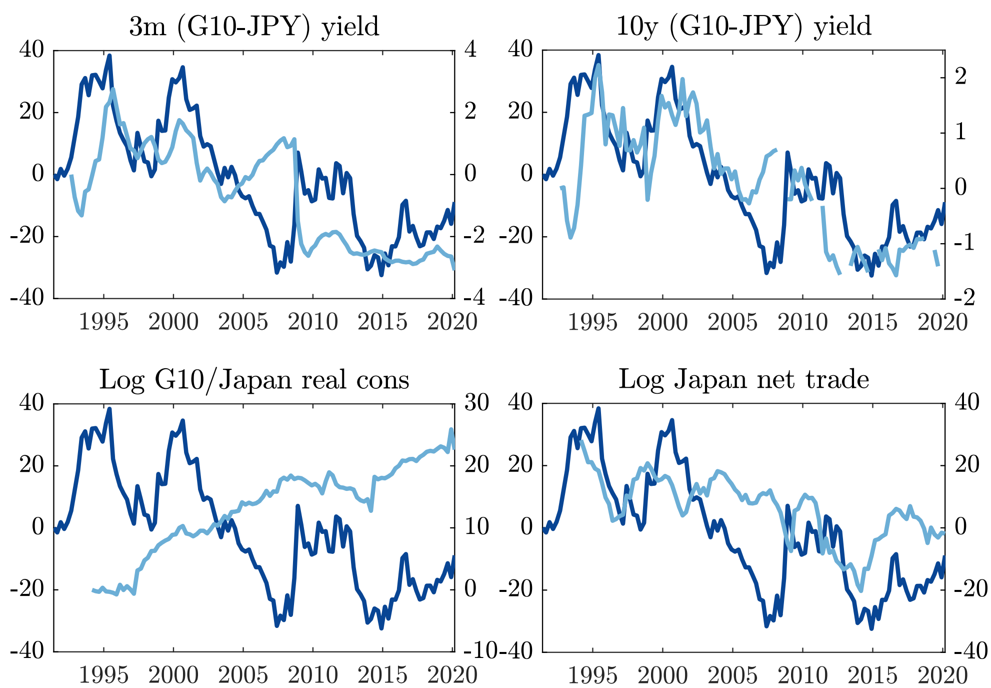

Figure 12: comovements with log real exchange rate between Japan and G10

Notes: dark blue line in each panel is log real exchange rate, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. All series plotted in *pp*. All series except log Japan net trade are normalized to zero in first quarter. All yields are annualized.

obtain very similar results. Low G10 interest rates (which now includes the U.S. and excludes the U.K.) relative to U.K. interest rates and low G10 consumption per capita relative to its U.K. counterpart are associated with a stronger pound in real terms versus the G10, both in changes and in levels. In levels, the yield differentials account for 20-56% of the variation, and relative consumption per capita accounts for 52% of the variation, in the real exchange rate.

At the same time, these results are not as sharp for the Euro area and Japan. In the case of the Euro area, this reflects the shorter sample period of available data, which puts more weight on the early 2000s period in which a strengthening euro was accompanied by falling Euro area yields and consumption relative to the rest of the G10. While the early 2000s also featured a breakdown of the typical comovement between the dollar/G10 exchange rate and yield differentials as documented in the main text, these dynamics may also reflect particularities of the early years of the euro. In the case of Japan, the comovements with yield differentials (in levels) were strikingly positive prior to the mid-2000s. Related to the discussion in section 6 and the prior subsection, understanding these comovements for Japan seems a valuable direction for future work.

C.6 Low frequency comovements

We next study the comovements between the low frequency components of the exchange rate and other variables, following Müller and Watson (2018). An advantage of their approach is that it is robust to nonstationarity.

Their methodology works as follows. We project the series depicted in Figure 1 onto cosine functions with periodicities above a particular cutoff. For instance, Figure 13 plots these projections for the log real exchange rate and 10-year yield differential given two cutoffs, 6 years and 12 years. The comovements between these projections capture the long run covariability between these series.[50] Müller and Watson (2018) provide an algorithm to conduct Bayesian inference on the regression and correlation coefficients between these series using a wide prior distribution on their degree of cointegration. We use their algorithm without modification.

Table 18 summarizes regressions of the low frequency component of the log real exchange rate on the low frequency components of other variables. It demonstrates

[50]We say "long run" because these periodicities exceed the typical U.S. business cycle, which has averaged 75 months between 1945-2020 (NBER (n.d.)).

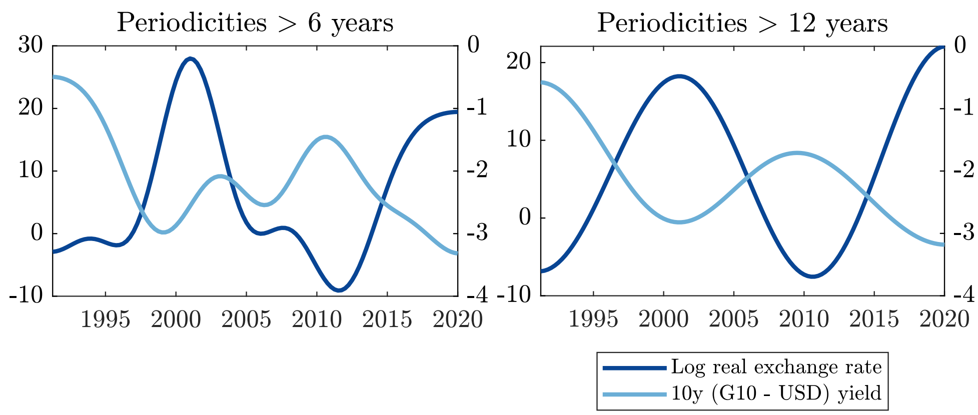

Figure 13: low frequency log real exchange rate and 10-year yield differential

Notes: panels depict projections of each series on cosine functions with periodicities above 6 years (left) or 12 years (right) using Müller and Watson (2018) algorithm. We add a scalar to each projection so that the in-sample mean equals that of the series depicted in Figure 1.

that the results from Table 1 in the main text are robust to this approach focused on low frequencies. The dollar has been strong when G10 yields have been low relative to U.S. yields, and when G10 consumption has been low relative to U.S. consumption. The 90% posterior credible sets exclude zero except for the 3-month yield differential with the 6-year periodicity cutoff, and for relative consumption with the 12-year periodicity cutoff. The share of exchange rate variation explained by these variables ranges between 18-28% for the 6-year periodicity cutoff, and higher values for the 12-year periodicity cutoff, reaching 58% for the 10-year yield differential. The 90% posterior credible sets for these values are wide, however, echoing the results we obtain using the bootstrap earlier in this appendix. There is little evidence of long run covariability between the exchange rate and log U.S. net trade.[51]

The last panel of Table 18 notes that this analysis, while focused on the low frequency components of these series, remains highly relevant to understand most exchange rate variation. It reports that 85% (70%) of the unconditional variation in the log real exchange rate is captured by the projection on periodicities above 6 (12) years. This underscores existing results in the literature that most of the variation in the exchange rate is at low frequencies (e.g., Rabanal and Rubio-Ramirez (2015)).

[51]As discussed in the main text, this may simply reflect delayed adjustment of trade to the exchange rate. Indeed, visually, the low frequency component of the exchange rate appears to lead the low frequency component of U.S. net trade by a few years.

	Periodicities > 6 years	Periodicities > 12 years
3m (G10 - USD) yield	-3.49	-8.00
	[-8.26,1.17]	[-13.45,-2.58]
N	116	116
Adj R^2	0.18	0.58
	[-0.01,0.56]	[0.01,0.94]
10y (G10 - USD) yield	-8.86	-12.73
	[-17.13,-0.53]	[-22.63,-2.61]
N	116	116
Adj R^2	0.27	0.52
	[-0.01,0.66]	[0.00,0.93]
Log G10/U.S. real cons.	-3.16	-3.31
	[-6.12,-0.33]	[-7.83,1.18]
N	116	116
Adj R^2	0.28	0.33
	[-0.01,0.66]	[-0.01,0.84]
Log U.S. net trade	-0.31	-0.28
	[-1.15,0.49]	[-1.55,0.95]
N	116	116
Adj R^2	0.11	0.16
	[-0.01,0.40]	[-0.01,0.60]
Memo:		
% Var(log real exchange rate)	85%	70%

Table 18: low frequency comovements with log real exchange rate

Notes: each panel regresses low frequency component of log real exchange rate on low frequency component of given variable, using Müller and Watson (2018) methodology. Rows in brackets report 90% posterior credible sets. Last panel reports share of variation of log real exchange rate accounted for by its projection on cosine functions with periodicities above 6 or 12 years.

C.7 Out-of-sample explanatory power

We finally study the ability of yield differentials, measures of risk or convenience yields, and relative quantities to explain the exchange rate out of sample. We follow the methodology of Meese and Rogoff (1983) and the large subsequent literature surveyed in Rossi (2013).

In particular, we estimate the contemporaneous specification in changes

$$\log q_{t+h} - \log q_t = \alpha + \beta \left(x_{t+h} - x_t\right) + \epsilon_{t+h},$$

where x_t is a candidate explanatory variable. We estimate this specification over a rolling 60 quarter sample, beginning with 1991 Q2 through 2005 Q1. With coefficients α_{t-1} and β_{t-1} estimated using forecasts through $t-1+h$, we use these estimated coefficients and the realized change in the explanatory variable $x_{t+h} - x_t$ to form the forecast $\log q_{t+h} - \log q_t$ and then compute the error versus the data. We repeat this process for all dates t between 2005 Q2 and the last date we make a forecast in our sample, 2020 Q1 less h, and we summarize the out-of-sample fit with the root mean squared error (RMSE) of forecast errors. We compare this to the RMSE obtained using the random walk, which simply predicts that

$$\log q_{t+h} - \log q_t = \epsilon_{t+h}.$$

We conduct inference on the mean squared errors by constructing the Clark and West (2006) test statistic.

Over the past 30 years, we can do better than a random walk in explaining the exchange rate out of sample. Table 19 reports the results from the data at three forecast horizons, $h = \{1, 4, 12\}$ quarters, using yield differentials, proxies for risk or convenience yields, and these together, along with relative consumption and log U.S. net trade. The three-month yield differential, excess bond premium, VIX, and global factor in risky asset prices significantly outperform the random walk at short horizons, though their individual performances deteriorate as the horizon extends. The 10-year yield differential and relative consumption outperform the random walk at four- and 12-quarters ahead, though the differences in the first case are not statistically significant at conventional levels. Combining the 10-year yield differential with the excess bond premium, VIX, or global factor in risky asset prices significantly outperforms the random walk at all horizons. The only variables which cannot beat a random walk at any horizon are net trade and the Treasury basis, consistent with their weak in-sample comovements evident from Tables 1 and 12.

	$h = 1$		$h = 4$		$h = 12$	
	RMSE	CW	RMSE	CW	RMSE	CW
Random walk	3.9		8.0		11.7	
3m (G10 - USD yield)	3.7	2.57^{***}	8.0	0.39	12.9	-1.23
10y (G10 - USD yield)	3.9	-0.18	7.9	1.10	11.6	0.84
EBP	3.7	1.51^{*}	8.1	0.87	12.6	0.54
and 3m (G10 - USD yield)	3.5	2.66^{***}	7.6	1.45^{*}	11.4	2.67^{***}
and 10y (G10 - USD yield)	3.4	2.96^{***}	6.9	2.06^{**}	8.8	2.29^{**}
VIX	3.7	1.55^{*}	7.7	1.82^{**}	13.2	-0.52
and 3m (G10 - USD yield)	3.5	2.24^{**}	7.2	2.59^{***}	12.2	0.62
and 10y (G10 - USD yield)	3.4	2.56^{***}	6.7	3.32^{***}	9.0	1.85^{**}
Global factor	3.1	3.68^{***}	6.7	2.48^{***}	13.2	-0.82
and 3m (G10 - USD yield)	2.8	4.02^{***}	5.8	2.78^{***}	11.6	1.25
and 10y (G10 - USD yield)	2.4	4.30^{***}	4.8	3.01^{***}	8.9	1.50^{*}
3m Treasury basis	4.3	-1.44	8.4	-0.85	13.4	-1.36
and 3m (G10 - USD yield)	4.1	-0.64	8.1	0.37	13.1	-1.37
and 10y (G10 - USD yield)	4.3	-1.24	8.0	0.84	11.6	0.62
Log G10/U.S. real cons. p.c.	3.9	0.81	7.3	1.96^{**}	10.3	2.14^{**}
Log U.S. net trade	4.0	-2.09	8.4	-1.16	13.6	-0.73

Table 19: out-of-sample explanatory power for log real exchange rate

Notes: each row reports root mean squared error (RMSE) and Clark and West (2006) test statistic from specification estimated in changes over rolling 60 quarter sample beginning with 1991 Q2 – 2005 Q1, and forecasting through 2020 Q1. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016), global factor in risky asset prices is from Miranda-Agrippino and Rey (2020), and 3m Treasury basis is from Du et al. (2018), updated through 2020 using data shared with us by Wenxin Du. *, **, and *** denote statistical significance at 10%, 5%, and 1% one-sided levels, respectively.

D Additional quantitative analysis

We now provide additional quantitative results accompanying those in section 5.

D.1 Additional untargeted moments in data and model

We first compare additional untargeted moments in the data and model.

Table 20 summarizes untargeted volatilities and autocorrelations of three-month yields, 10-year yields, and consumption per capita. Recall that we used the stochastic properties of demand shocks to target the volatility and autocorrelation of the 10-year yield differential. While the second panel indicates that the model generates 10-year yields in levels and changes broadly consistent with the data, the first panel indicates that the model implies more volatile three-month yields in levels and changes than in the data. This reflects that three-month yields are smoother than 10-year yields in the data, likely a result of interest rate smoothing by central banks. Since our model does not feature such smoothing, we overstate the volatility of three-month yields; since exchange rates are forward-looking, we do not anticipate such smoothing would have much effect on the model-implied exchange rate.

The last panel indicates that one shortcoming of the model is that it generates excessive volatility in consumption growth relative to the data. This is because relative consumption across countries is too volatile at high frequencies, as reflected in a counterfactually negative correlation of consumption growth between countries in the first row of Table 21. As described in the main text, this reflects the strong contemporaneous response of trade flows to shocks in the model (even with pricing to market). We expect that generalizing the model to account for dynamic trade as in Alessandria and Choi (2007) and Drozd and Nosal (2012) would be one way of delaying this adjustment and bringing these moments closer in line with the data.

The other correlations of consumption and output reported in Table 21 are more comparable between data and model.

D.2 Additional impulse responses

In the main text we presented the model's impulse responses to a demand shock. Figures 14 and 15 depict the impulse responses to supply and intermediation shocks.

A one standard deviation decrease in the U.S. endowment causes a dollar appreciation, decline in the Foreign interest rate relative to the U.S. interest rate, increase in Foreign consumption relative to U.S. consumption, temporary increase in U.S. net trade, and temporary decline in the expected excess return on Foreign bonds less dollar bonds. These results are consistent with Proposition 1. Notably, the response of the exchange rate is an order of magnitude smaller than to discount factor shocks in Figure 3. This reflects that, given the stochastic process for output per capita in the data, endowment shocks induce changes in real interest rates which are both

		σ		ρ_{-1}	
	Variable	Data	Model	Data	Model
$r^{(1)}$	U.S. interest rate	2.19%	2.57%	0.97	0.93
$r^{(1)*}$	Foreign interest rate	2.22%	2.55%	0.98	0.93
$r^{(1)*} - r^{(1)}$	interest rate spread	1.58%	1.50%	0.96	0.89
$\Delta r^{(1)}$	Δ U.S. interest rate	0.51%	0.85%	0.27	-0.02
$\Delta r^{(1)*}$	Δ Foreign interest rate	0.40%	0.83%	0.28	-0.02
$\Delta r^{(1)*} - \Delta r^{(1)}$	Δ interest rate spread	0.43%	0.63%	0.18	-0.05
$r^{(40)}$	U.S. 10y yield	2.01%	1.66%	0.96	0.93
$r^{(40)*}$	Foreign 10y yield	2.40%	1.66%	0.99	0.93
$\Delta r^{(40)}$	Δ U.S. 10y yield	0.53%	0.53%	-0.00	-0.01
$\Delta r^{(40)*}$	Δ Foreign 10y yield	0.39%	0.53%	0.18	-0.02
$\Delta r^{(40)*} - r^{(40)}$	Δ 10y yield spread	0.30%	0.26%	-0.06	-0.02
$\Delta \log c$	Δ U.S. consumption	0.50%	0.81%	0.36	-0.04
$\Delta \log c^*$	Δ Foreign consumption	0.44%	0.70%	0.39	-0.04

Table 20: untargeted volatilities and autocorrelations in data and model

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

	Data	Model
$\rho(\Delta \log c, \Delta \log c^*)$	0.59	-0.54
$\rho(\Delta \log c, \Delta \log y)$	0.87	0.79
$\rho(\Delta \log c^*, \Delta \log y^*)$	0.74	0.76
$\rho(\Delta \log y, \Delta \log y^*)$	0.31	0.03

Table 21: untargeted comovements in data and model

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

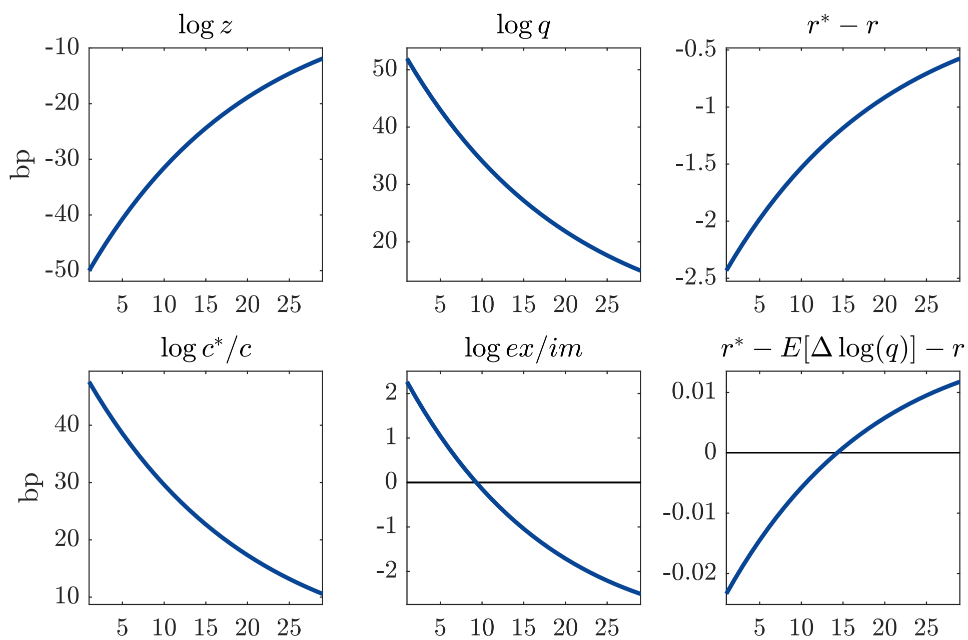

Figure 14: impulse responses to supply shock

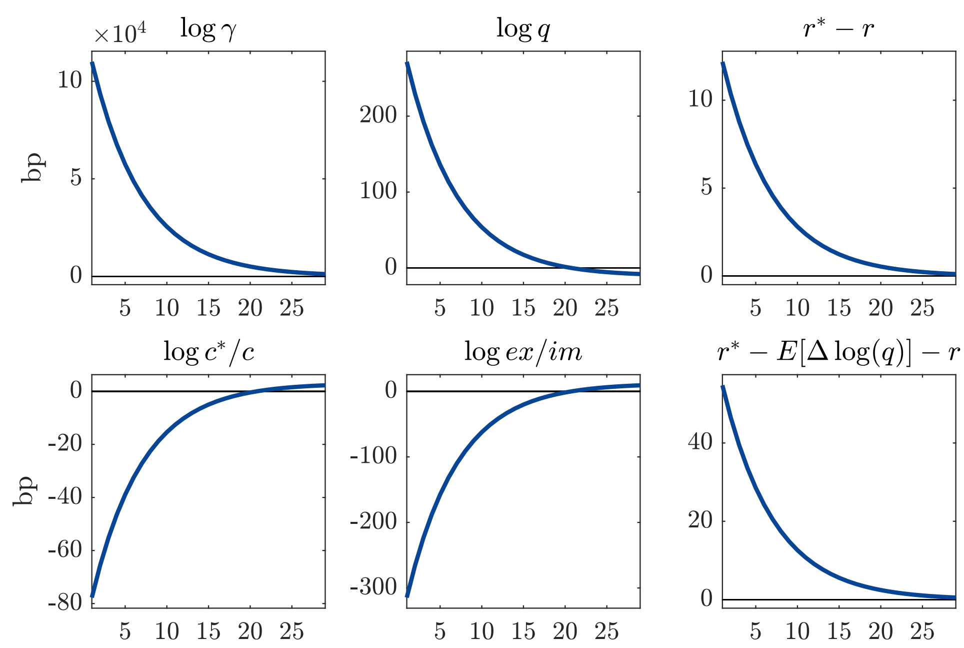

Figure 15: impulse responses to currency intermediation shock

smaller and less persistent than discount factor shocks, the latter disciplined by the stochastic properties of bond yields themselves.

A one standard deviation increase in arbitrageur risk aversion causes a dollar appreciation, increase in the Foreign interest rate relative to the U.S. interest rate, decline in Foreign consumption relative to U.S. consumption, decline in U.S. net trade, and increase in the expected excess return on Foreign bonds less dollar bonds. These results are again consistent with Proposition 1. The response of the exchange rate is less persistent than in the case of discount factor shocks. This reflects that the excess bond premium, which we use to discipline arbitrageur risk aversion shocks, is less persistent than bond yields, which discipline discount factor shocks.

D.3 Comovements of exchange rate in changes

In the main text we compared the model-implied comovements between the real exchange rate, yield differentials, and quantities to the empirical counterparts, all run in a levels specification. Table 22 compares the model and data in our changes specifications, where the latter estimates are from the first three columns of Table 1.

The model-generated comovements between changes in the exchange rate and changes in interest rate differentials get more negative, and the R^2 rises, as the horizon of changes increases, and as the tenor of the bond increases. These patterns reflect that changes in arbitrageur risk aversion are more transitory than changes in relative demand, so the specifications over longer horizons and using longer tenors increasingly reflect relative demand shocks. Most of the model-implied coefficients, though not all, are within or just outside the empirical confidence intervals, and the model-implied R^2 coefficients are not too much higher than in the data.

The model-generated comovements between changes in the exchange rate and changes in quantities reflect the results from the levels specification discussed in the main text. The model accounts well for the negative comovement between the dollar and relative consumption at all horizons, though it overstates the R^2s. The model-implied negative comovement with U.S. net trade in changes is outside the empirical confidence intervals and the model again substantially overstates the R^2s. Later in this appendix we demonstrate how an extended version of the model featuring trade shocks can make progress in resolving these inconsistencies versus the data.

	1-qtr changes		4-qtr changes		12-qtr changes	
	Data	Model	Data	Model	Data	Model
$\log q$ on $r^{(1)*} - r^{(1)}$						
Coeff	[-4.54,0.19]	0.97	[-4.03,1.22]	0.40	[-2.75,1.46]	-0.71
Adj R^2	0.05	0.02	0.03	0.02	-0.00	0.06
$\log q$ on $r^{(40)*} - r^{(40)}$						
Coeff	[-6.42,0.87]	-6.62	[-6.52,-1.89]	-7.43	[-9.28,-2.46]	-8.55
Adj R^2	0.04	0.18	0.07	0.25	0.13	0.37
$\log q$ on $\log c^* - \log c$						
Coeff	[-2.70,-0.80]	-2.35	[-4.23,-1.80]	-2.31	[-6.25,-1.58]	-2.24
Adj R^2	0.03	0.59	0.13	0.57	0.27	0.55
$\log q$ on $\log ex/im$						
Coeff	[-0.33,0.17]	-0.86	[-0.51,0.30]	-0.86	[-0.75,0.24]	-0.86
Adj R^2	-0.01	0.97	-0.01	0.96	0.02	0.96

Table 22: exchange rate comovements in changes in data and model

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

D.4 Out-of-sample fit

In the main text and prior subsection we compared the model to the data using in-sample comovements. Table 23 instead compares the model's out-of-sample performance to the data. The top panel is taken from Table 19, where we focus on the excess bond premium as a representative proxy for $\log \gamma_t$ consistent with our calibration strategy. The bottom panel replicates the empirical methodology on many model-generated samples of 116 quarters each, averaging the RMSE and Clark and West (2006) statistics across the model simulations.

Out-of-sample forecasting on model-generated data yields generally comparable results as the actual data. The forecast errors in the random walk specification have similar magnitudes as the data at all horizons, validating that the model-generated stochastic properties of the exchange rate are consistent with the data. As in the data, the forecasting ability of the one-quarter yield differential in terms of RMSE relative to the random walk deteriorates with horizon, while it improves with horizon in the case of the 10-year yield differential; arbitrageur risk aversion has significant

	$h=1$		$h=4$		$h=12$	
	RMSE	CW	RMSE	CW	RMSE	CW
Data						
Random walk	3.9		8.0		11.7	
3m (G10 - USD yield)	3.7	2.57^{***}	8.0	0.39	12.9	-1.23
10y (G10 - USD yield)	3.9	-0.18	7.9	1.10	11.6	0.84
EBP	3.7	1.51^{*}	8.1	0.87	12.6	0.54
and 3m (G10 - USD yield)	3.5	2.66^{***}	7.6	1.45^{*}	11.4	2.67^{***}
and 10y (G10 - USD yield)	3.4	2.96^{***}	6.9	2.06^{**}	8.8	2.29^{**}
Log G10/U.S. real cons. p.c.	3.9	0.81	7.3	1.96^{**}	10.3	2.14^{**}
Log U.S. net trade	4.0	-2.09	8.4	-1.16	13.6	-0.73
Model						
Random walk	4.5		8.1		13.0	
$r_t^{(1)*} - r_t^{(1)}$	4.6	-1.10	8.4	-0.27	13.2	0.30
$r_t^{(40)*} - r_t^{(40)}$	3.6	4.82^{***}	7.1	2.35^{***}	11.5	0.97
$\log \gamma_t$	3.3	6.05^{***}	6.3	2.96^{***}	7.2	1.75^{**}
and $r_t^{(1)*} - r_t^{(1)}$	0.5	7.37^{***}	1.0	4.43^{***}	2.1	2.51^{***}
and $r_t^{(40)*} - r_t^{(40)}$	0.4	7.40^{***}	1.0	4.40^{***}	2.3	2.53^{***}
$\log c_t^*/c_t$	2.9	5.62^{***}	4.7	3.76^{***}	6.5	3.09^{***}
$\log ex/im$	0.8	7.23^{***}	1.2	4.41^{***}	1.6	2.97^{***}

Table 23: out-of-sample explanatory power for log real exchange rate

Notes: on data, each row reports root mean squared error (RMSE) and Clark and West (2006) test statistic from specification estimated in differences over rolling 60 quarter sample beginning with 1991 Q2 – 2005 Q1, and forecasting through 2020 Q1. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016). On model, each row reports same statistics from same specifications estimated on 1,000 model simulations of 116 quarters each using a 1,000 quarter burn-in period. *, **, and *** denote statistical significance at 10%, 5%, and 1% one-sided levels, respectively.

forecasting power at the one-quarter horizon; the combination of risk aversion with yield differentials can significantly outperform a random walk at all horizons; and relative consumption can beat a random walk at long horizons.

There are also some differences from the data. In particular, the 10-year yield differential can significantly outperform the random walk even at short horizons; arbitrageur risk aversion can significantly outperform the random walk at long horizons;

and the magnitude of the RMSEs is quite small using the combination of risk aversion and yield differentials versus the data. These discrepancies are to be expected if the 10-year yield differential and excess bond premium are noisy measures of the expected path of interest rate differentials and arbitrageurs' risk bearing capacity in practice. Moreover, relative consumption can beat a random walk at short horizons, and net trade can also beat a random walk at all horizons, unlike the data. These are consistent with trade flows (and thus consumption) responding too quickly to relative prices in the model, as previously discussed.

D.5 Spanning regressions

Chernov and Creal (2023) and Chernov et al. (2024) find relatively low R^2 coefficients in regressions projecting exchange rate changes on local currency asset returns, implying that local currency asset returns do not span exchange rates. Here we reproduce their empirical findings and then demonstrate that our model generates comparable R^2 coefficients in such regressions, even though the dominant drivers of the exchange rate are demand shocks which also drive bond yields.

Given a set of bond maturities $\mathcal{T}$, Chernov et al. (2024) report the R^2 from regressions of the form

$$\log q_{t+1} - \log q_t = \alpha + \sum_{\tau \in \mathcal{T}} \left[\beta^{(\tau)} r^{(\tau)}_{t,t+1} + \beta^{(\tau)*} r^{(\tau)*}_{t,t+1} \right] + \epsilon_{t+1}, \tag{27}$$

where

$$\begin{aligned} r^{(\tau)}_{t,t+1} &= \log p^{(\tau-1)}_{t+1} - \log p^{(\tau)}_t, \\ r^{(\tau)*}_{t,t+1} &= \log p^{(\tau-1)*}_{t+1} - \log p^{(\tau)*}_t \end{aligned}$$

denote the dollar and foreign returns on τ-period local currency bonds from t to $t+1$, respectively. That is, they ask whether changes in the exchange rate are spanned by both U.S. and foreign bond returns over that same period. Chernov and Creal (2023) run closely related regressions, and the results which follow also hold when we reproduce their specifications.

The first panel and column of Table 24 estimates specification (27) on our data. We average the R^2 values obtained from bilateral specifications for each G10 currency; while we use Libor rates and quarterly data (as throughout the paper) whereas Cher-

$\mathcal{T}$	Data	Model	Model + τ-specific shocks
$\log q_{t+1} - \log q_t$ on $r_{t,t+1}^{(\tau)}$ and $r_{t,t+1}^{(\tau)*}$ for $\tau \in \mathcal{T}$			
{8}	0.08	0.02	0.02
{20}	0.11	0.10	0.08
{40}	0.11	0.20	0.18
{8,20,40}	0.19	0.90	0.26
{8,12,20,28,40}	0.27	0.99	0.29
$r_{t,t+1}^{(40)}$ on $r_{t,t+1}^{(\tau)}$ for $\tau \in \mathcal{T}$			
{8}	0.63	0.89	0.70
{20}	0.93	0.99	0.92
{8,20}	0.97	1.00	0.92
{8,12,20,28}	1.00	1.00	0.96
$r_{t,t+1}^{(40)*}$ on $r_{t,t+1}^{(\tau)*}$ for $\tau \in \mathcal{T}$			
{8}	0.58	0.89	0.71
{20}	0.90	0.99	0.92
{8,20}	0.95	1.00	0.93
{8,12,20,28}	0.99	1.00	0.96

Table 24: R^2 from spanning regressions in data and model

Notes: columns report R^2 from regression reported in panel title. For data, values in first and third panels are averaged across bilateral specifications for each G10 currency. For model, last column adds shocks to yield curve in each country as described in main text. All statistics averaged over 1,000 model simulations of 116 quarters each using a 1,000 quarter burn-in period.

nov et al. (2024) use government bond yields and monthly data, the R^2 values we obtain are comparable to theirs.[52] Returns on single maturities in the U.S. and abroad explain no more than 11% of the quarterly variation in the exchange rate, and even using five maturities in each country we obtain only 27% spanning.

The second column demonstrates that model-generated data features comparable

[52]Since we only observe bond yields at selected tenors, we approximate returns as follows. Given a T-quarter zero coupon bond with annualized yield $y_t(T)$ at date t, its log price is $\log P(y_t(T), T) = -(T/4)\log(1 + y_t(T))$. Assuming for simplicity that the yield on a $T-1$ quarter bond is the same as a T quarter bond at $t+1$, we can approximate the log return on a T-quarter bond between t and $t+1$ as $\log P(y_{t+1}(T-1), T-1) - \log P(y_t(T), T) \approx \log P(y_{t+1}(T), T) + (1/4)\log(1 + y_{t+1}(T)) - \log P(y_t(T), T)$.

$\mathcal{T}$	Data	Model	Model + τ-specific shocks
$\log q_{t+1} - \log q_t$ on $r_{t,t+1}^{(\tau)}$ and $r_{t,t+1}^{(\tau)*}$ for $\tau \in \mathcal{T}$			
{8}	$1E+02$	$5E+01$	$4E+01$
{20}	$8E+01$	$7E+01$	$5E+01$
{40}	$5E+01$	$7E+01$	$6E+01$
{8,20,40}	$3E+02$	$2E+03$	$1E+02$
{8,12,20,28,40}	$1E+03$	$4E+15$	$1E+02$
$r_{t,t+1}^{(40)}$ on $r_{t,t+1}^{(\tau)}$ for $\tau \in \mathcal{T}$			
{8}	$3E+02$	$3E+02$	$3E+02$
{20}	$2E+02$	$2E+02$	$2E+02$
{8,20}	$3E+02$	$2E+02$	$1E+02$
{8,12,20,28}	$2E+02$	$5E+02$	$1E+02$
$r_{t,t+1}^{(40)*}$ on $r_{t,t+1}^{(\tau)*}$ for $\tau \in \mathcal{T}$			
{8}	$3E+02$	$3E+02$	$3E+02$
{20}	$2E+02$	$2E+02$	$2E+02$
{8,20}	$2E+02$	$2E+02$	$1E+02$
{8,12,20,28}	$2E+02$	$5E+02$	$1E+02$

Table 25: 100 times maximal absolute value of spanning coefficients in data and model

Notes: columns report 100 times maximal absolute value of coefficients on dollar and foreign assets from regression reported in panel title. For data, values in first and third panels are averaged across bilateral specifications for each G10 currency. For model, last column adds shocks to yield curve in each country as described in main text. All statistics averaged over 1,000 model simulations of 116 quarters each using a 1,000 quarter burn-in period.

R^2 values for spanning regressions with a single maturity. This reflects the presence of both demand and intermediation shocks in the model, which are roughly equally important in driving exchange rate variation at short horizons. Demand shocks and intermediation shocks which have the same effect on the exchange rate have contrasting effects on local currency bond returns, implying that these bond returns explain only a limited share of the variation in the exchange rate at short horizons.

As we add more local currency bond returns to the right-hand side of these regressions, the spanning regressions on model-generated data of course feature much higher R^2 values because the model features a finite set of shocks. Importantly, however, the implied spanning portfolios involve unrealistically large positions. Table 25 reports 100 times the maximal absolute value of regression coefficients in (27), which can be interpreted as the maximal weight in the portfolio of bonds which most closely replicates the exchange rate. Using five maturities in each currency on model-generated data, this maximal portfolio weight is 10^{12} times larger than in the data.

Spanning regressions of long-maturity bond returns on shorter-maturity returns suggest that our model is missing shocks along each country's yield curve. The second and third panels of Table 24 report R^2 values from these regressions. Whereas in the data, variation in two-year bond returns in each country explains around 60% of the variation in 10-year bond returns, in the model the same statistic is nearly 90%.

This motivates the following simple extension of our model. In each country, we add small maturity-specific shocks to the pricing equations for long-term bonds $\{\eta_t^{(\tau)}\}$ and $\{\eta_t^{(\tau)*}\}$, so that the price of a τ-period dollar bond is

$$p_t^{(\tau)} = \mathbb{E}_t\left[\beta_t\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\exp\left(\eta_t^{(\tau)}\right)p_{t+1}^{(\tau-1)}\right],$$

and analogously for Foreign bonds. These shocks may reflect convenience yields that households in each country receive from bonds of different maturities; localized demand shocks in a richer model of the term structure, as in Vayanos and Vila (2021) and Gourinchas et al. (2024); or simply measurement error when mapping the model to the data. To avoid discontinuities in the yield curve (and in keeping with the first two interpretations), we parametrize the shocks across maturities as smooth functions of N underlying structural shocks. In the U.S., we assume

$$\eta_t^{(\tau)} = \sum_{n=1}^{N} f_t^n \exp\left(\frac{-(\tau-4n)^2}{36}\right),$$

where each shock f_t^n is an *iid* normal innovation, and analogously in Foreign. Each of the N shocks in each country therefore has a bell-shaped effect on all maturities centered around maturity $4n$. We consider $N = 10$ shocks (thus centered around maturities $1, 2, \ldots, 10$ years) and set the standard deviation of each shock to $\sigma^f = \sigma^{f*} = 0.004$. Figure 16 plots the effects of one standard deviation shocks on the yield curve in each country. These shocks are so small and transitory that they leave all of the second moments in bond yields in Table 4 unchanged up to reported precision;

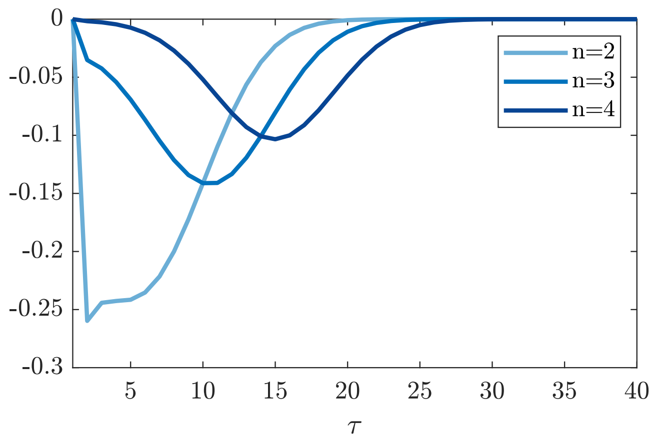

Figure 16: effects of noise shocks $\{f_t^n\}$ along each country's yield curve

Notes: figure depicts effects of one standard deviation shock to $\{f_t^2, f_t^3, f_t^4\}$ on annualized yield curve $-(\tau/4)\log p_t^{(\tau)}$, expressed in *pp*.

since they only affect the pricing of long-term bonds, they also leave the rest of the allocation (including exchange rates) exactly unchanged.

Adding these shocks brings spanning regressions in the model fully in line with the data. We see in the last column of Table 24 that the R^2 values of 10-year bond returns on shorter-maturity bond returns in each country are now more comparable between model and data, and the R^2 values of exchange rates on multiple local currency bond returns falls sharply. The reason is that the small amount of noise in bond returns substantially dampens the portfolio weights in the last column of Table 25, making them comparable to the data. Intuitively, once there exists very small noise in bond returns, one can no longer take massive offsetting positions along the term structure to recover the exchange rate.

D.6 Additional simulation results over 1991-2020

We now present additional simulation results using the shocks recovered from 10-year yields, output per capita, and the excess bond premium series in the data.

Figure 17 depicts relative consumption and U.S. net trade in data and model. The first panel indicates that the model successfully generates the low-frequency movement in relative consumption observed in the data. At the same time, it is more volatile at higher frequencies than the data. The second panel similarly demonstrates that trade flows are more volatile at higher frequencies than the data, and also demonstrates that for extended periods the model-implied net trade series leads that in the data. These observations are all consistent with trade flows responding too quickly to the exchange rate in the model. As discussed earlier, we expect that accounting for investment, dynamic trade, or trade shocks could close the gap. The next subsection explores a model extension with trade shocks to illustrate these ideas.

Figure 18 compares proxies for risk with the unexplained component of the exchange rate using only demand shocks extracted from 10-year yields. The excess bond premium, VIX, and global factor in risky asset prices especially covary with the unexplained component, demonstrating that they complement demand shocks in accounting for the dollar/G10 exchange rate. This mirrors the incremental explanatory power these proxies have in accounting for the exchange rate in Tables 2, 10, and 11.

D.7 Extension to trade shocks

We next present an extension of our model featuring trade shocks. These shocks mitigate the baseline model's counterfactually tight connection between the exchange rate, relative consumption, and net trade, without changing our conclusion that demand shocks account for most of the variation in the exchange rate.

We modify Foreign's consumption aggregator to be

$$c_t^* = \left(\left(\frac{1}{1+\zeta^*}(1-\alpha_t \varsigma) \right)^{\frac{1}{\sigma}} (c_{Ht}^*)^{\frac{\sigma-1}{\sigma}} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{1}{1+\zeta^*}\alpha_t \varsigma \right)^{\frac{1}{\sigma}} (c_{Ft}^*)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Now, α_t is a driving force controlling Foreign's degree of home bias. We assume $\log \alpha_t$ follows an AR(1) process with mean zero, standard deviation of shocks σ^α, and autocorrelation ρ^α. We assume for simplicity that it has a zero correlation with the other shocks in the model, though in a calibration relaxing this assumption we have found it does not change our main findings.

Figure 19 depicts the impulse responses to a negative innovation to α_t which appreciates the dollar. We use the parameter values described in the next paragraph, but present these impulse responses first to motivate the calibration strategy. The key feature of trade shocks is that they imply that the dollar appreciation is accompanied by an increase in U.S. net trade. This contrasts with both demand and intermediation shocks, for which the expenditure switching mechanism means that a dollar

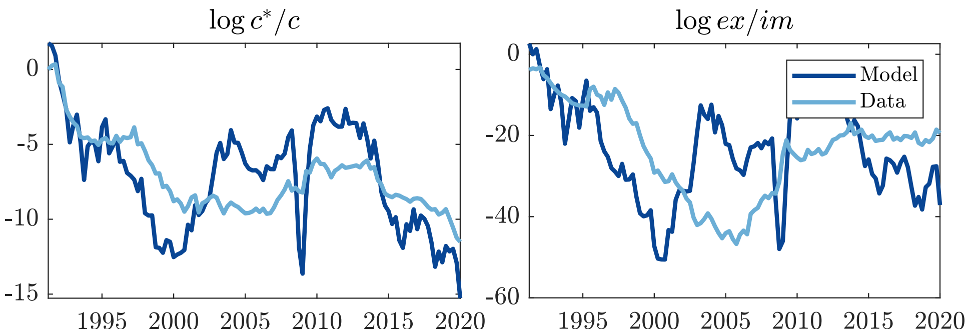

Figure 17: relative consumption and U.S. net trade in data and model

Notes: model-implied exchange rate simulated by inverting $\{\beta_t, \beta_t^*, z_t, z_t^*, \gamma_t\}$ to match 10-year yields and output per capita in U.S. and G10, as well as excess bond premium of Gilchrist and Zakrajsek (2012). All series plotted in *pp*. Constant is added to model-generated series to match same average value as data series.

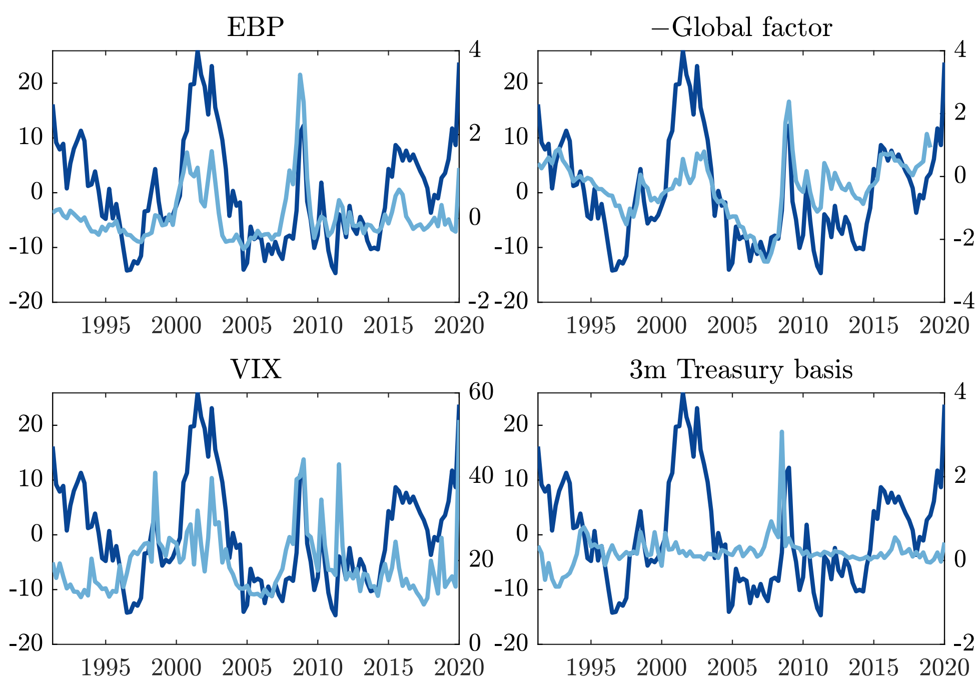

Figure 18: unexplained exchange rate with only demand shocks and proxies for risk

Notes: dark blue line in each panel is data less model with only $\{\epsilon^{\beta}, \epsilon^{\beta^*}\}$ shocks extracted from 10-year yields, plotted on left axis; light blue line in each panel is series in panel title, plotted on right axis. EBP refers to excess bond premium from Gilchrist and Zakrajsek (2012) and kept updated by Favara et al. (2016). Global factor refers to global factor in risky asset prices from Miranda-Agrippino and Rey (2020). Treasury basis is from Du et al. (2018), updated through 2020 using data shared with us by Wenxin Du. All series plotted in *pp*.

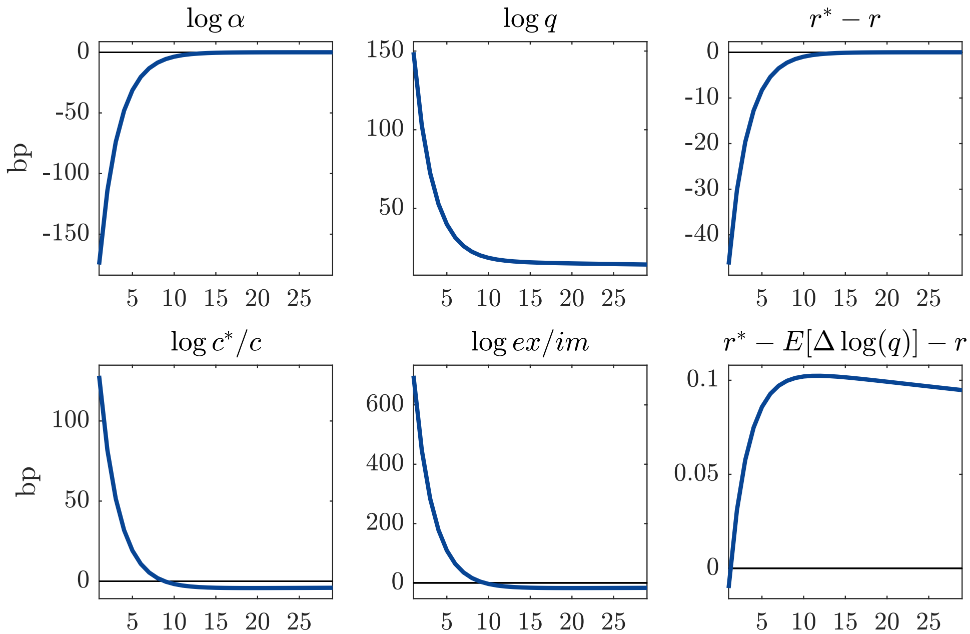

Figure 19: impulse responses to trade shock

appreciation is accompanied by a decline in U.S. net trade.[53] Relatedly, and also unlike demand and intermediation shocks, a dollar appreciation is accompanied by an increase in relative Foreign consumption, since Foreign borrows from the U.S. to finance its higher consumption of imports.[54] In terms of their effects on interest rates, trade shocks are like demand (or supply) shocks: a dollar appreciation is accompanied by an increase in U.S. interest rates versus Foreign interest rates.

Motivated by these effects, we discipline the stochastic properties of trade shocks to target the share of exchange rate variation explained by relative consumption at short horizons and unconditionally. As noted in Table 6 in the main text and Table 22 in this appendix, the R^2 values using relative consumption to explain the exchange rate are higher in the baseline model than in the data. As described in the fourth

[53]As described in footnote 36, the comovement induced by supply shocks depends on parameters.

[54]Since arbitrageurs are thus lending more to Foreign, the risk premium on Foreign bonds also rises, unlike demand shocks. In contemporaneous work, Bodenstein, Cuba-Borda, Goernemann, and Presno (2024) emphasize this effect of trade shocks and argue that it drives most exchange rate variation via endogenous UIP deviations. This channel is small in our calibration because the steady-state price of risk, Γ, is small. We note that while it can be amplified if Γ is higher, this would also imply that trade shocks, like γ_t shocks, induce an unconditional comovement between yield differentials and the exchange rate that is at odds with the data.

	Description	Value	Moment	Target	Model
β	U.S. disc. fact.	0.99758	$r^{(1)}$	0.97%	0.97%
β^*	Foreign disc. fact.	0.99708	$r^{(40)*} - r^{(40)}$	0.20%	0.20%
$\sigma^\beta, \sigma^{\beta^*}$	s.d. β, β^* shocks	0.0019	$\sigma(r^{(40)*} - r^{(40)})$	0.81%	0.80%
$\rho^\beta, \rho^{\beta^*}$	persistence β, β^*	0.98	$\rho_{-1}(r^{(40)*} - r^{(40)})$	0.93	0.92
ρ^{β,β^*}	corr. β, β^* shocks	0.84	$\sigma(\Delta r^{(40)})$	0.53%	0.52%
σ^z, σ^{z^*}	s.d. z, z^* shocks	0.005	$\sigma(\log y^* - \log y)$	1.73%	1.76%
ρ^z, ρ^{z^*}	persistence z, z^*	0.95	$\rho_{-1}(\log y^* - \log y)$	0.91	0.91
ρ^{z,z^*}	corr. z, z^* shocks	0.10	$\sigma(\Delta \log y)$	0.49%	0.51%
σ^γ	s.d. γ shocks	11	$\sigma(\Delta \log q)$	3.91%	4.27%
ρ^γ	persistence γ	0.85	$\rho_{-1}(\log \gamma)$	0.80	0.82
σ^α	s.d. α shocks	0.02	$R^2(\log q, \log c^*/c)$	0.29	0.34
ρ^α	persistence α	0.65	$R^2(\Delta \log q, \Delta \log c^*/c)$	0.03	0.04
ξ	pricing to market	1.3	$\sigma(\log s)/\sigma(\log q)$	0.27	0.28
Γ	arb risk pricing	$6E-4$	$\sigma(\log ex/im)$	11.41%	14.84%
σ	trade elasticity	0.9	$\sigma(\log q)$	11.48%	10.43%
ς	home bias	0.8	$(ex + im)/y$	0.25	0.23
ζ^*	rel. population	1.35	$\zeta^* q^{-1} y^*/y$	1.35	1.34

Table 26: calibrated parameters with trade shocks

Notes: data moments are estimated over 1991 Q2 – 2020 Q1. Model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Externally set parameters in model are $\psi = 1$, $z = 1$, $z^* = 1$, and $\theta = 4$, the latter three all normalizations.

panel of Table 26, we use the volatility σ^α and persistence ρ^α of trade shocks to target these R^2 values for regressions in one-quarter changes and in levels. We reduce the volatilities of demand shocks $\sigma^\beta, \sigma^{\beta^*}$ so that the volatility of the exchange rate is no different than in the baseline model. To facilitate the comparison between this calibration and the baseline model, we leave all other parameters unchanged.

The resulting exchange rate comovements are more consistent with the data. Table 27 presents the analog to Table 22 but now with trade shocks added to the model. The signs of the comovements are all unchanged from the baseline model. But now, in the bottom two panels, the share of variation in the exchange rate explained by relative consumption or U.S. net trade is much lower than in the baseline model. At the same time, in the first two panels, the share of variation explained by yield

	1-qtr changes		4-qtr changes		12-qtr changes	
	Data	Model	Data	Model	Data	Model
$\log q$ on $r^{(1)*} - r^{(1)}$						
Coeff	[-4.54,0.19]	-0.62	[-4.03,1.22]	-0.64	[-2.75,1.46]	-0.76
Adj R^2	0.05	0.10	0.03	0.08	-0.00	0.09
$\log q$ on $r^{(40)*} - r^{(40)}$						
Coeff	[-6.42,0.87]	-7.59	[-6.52,-1.89]	-7.93	[-9.28,-2.46]	-8.68
Adj R^2	0.04	0.26	0.07	0.30	0.13	0.38
$\log q$ on $\log c^* - \log c$						
Coeff	[-2.70,-0.80]	-0.43	[-4.23,-1.80]	-0.77	[-6.25,-1.58]	-1.19
Adj R^2	0.03	0.04	0.13	0.11	0.27	0.23
$\log q$ on $\log ex/im$						
Coeff	[-0.33,0.17]	-0.06	[-0.51,0.30]	-0.15	[-0.75,0.24]	-0.30
Adj R^2	-0.01	0.02	-0.01	0.09	0.02	0.24

Table 27: exchange rate comovements in changes in data and model with trade shocks

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

differentials is not too much higher.

This model extension with trade shocks does not change our conclusion that relative demand shocks account for most of the variation in the exchange rate. Table 28 summarizes the share of variance in the exchange rate in levels and quarterly changes accounted for by each set of shocks. Mechanically, these shares are lower for demand shocks than in the baseline calibration because the volatility of these shocks was reduced to target the same exchange rate volatility. But it remains the case that, with trade shocks disciplined to match the explanatory power of relative consumption for the exchange rate, demand shocks account for most of the variation in the exchange rate. Figures 20 and 21 further make this point when we recover the path of $\{\epsilon_t^\alpha\}$ shocks to match the exact path of U.S. net trade over the 1991-2020 period (we continue to use 10-year yield differentials, output per capita, and the excess bond premium to recover the path of the other shocks). By construction, the model fit in Figure 20 is much tighter than in Figure 17 earlier in this appendix. But the resulting exchange rate path in Figure 21 is quite comparable to the baseline path in Figure 5,

	$\{\epsilon_t^{\beta}, \epsilon_t^{\beta^*}\}$	$\{\epsilon_t^{z}, \epsilon_t^{z^*}\}$	$\{\epsilon_t^{\gamma}\}$	$\{\epsilon_t^{\alpha}\}$
$\log q$	73%	3%	21%	3%
$\Delta \log q$	39%	3%	44%	14%

Table 28: variance decomposition in model with trade shocks

Notes: table reports shares of $\sigma^2(\log q)$ (first row) and $\sigma^2(\Delta \log q)$ (second row) due to each set of driving forces alone. Variances are averaged over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

and indeed eliminating $\{\epsilon_t^{\alpha}\}$ shocks from the former does not much change it.

D.8 Present value decompositions

In the main text we explained that small-sample bias can explain why present value decompositions, as in Froot and Ramadorai (2005), suggest that expected interest rate differentials account for little of the variance in the exchange rate. Here we expand on this point.

Consider the standard identity

$$\begin{aligned}\log q_t &= \sum_{\tau=0}^{\infty} \mathbb{E}_t \left(r_{t+\tau} - r^*_{t+\tau} \right) + \sum_{\tau=0}^{\infty} \mathbb{E}_t \left(r^*_{t+\tau} - \Delta \log q_{t+\tau+1} - r_{t+\tau} \right) + \lim_{h\to\infty} \mathbb{E}_t \log q_{t+h}, \\ &\equiv \delta_t^{ir} + \delta_t^{rp} + \delta^q, \end{aligned} \tag{28}$$

where we assume the long-run value of the exchange rate (the last term) is finite and does not depend on t because the real exchange rate is stationary. It follows that the variance of the real exchange rate can be decomposed into the variance of expected interest rate differentials, variance of expected currency risk premia, and their covariance:

$$Var\left(\log q_t\right) = Var\left(\delta_t^{ir}\right) + Var\left(\delta_t^{rp}\right) + 2Cov\left(\delta_t^{ir}, \delta_t^{rp}\right). \tag{29}$$

In the data, it appears that expected risk premia account for most of the variance in the exchange rate. We use a VAR to construct the conditional expectations required to implement decomposition (29). We focus on a two-lag VAR in the log real exchange rate, three- and 10-year nominal yield differentials, and excess bond premium. As in the rest of the paper, we use the three-month nominal yield differential as a measure

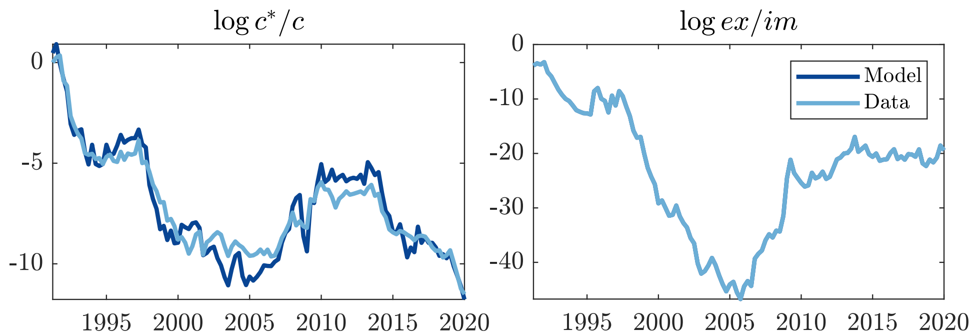

Figure 20: relative consumption and net trade in data and model with trade shocks

Notes: model-implied exchange rate simulated by inverting $\{\beta_t, \beta_t^*, z_t, z_t^*, \gamma_t, \alpha_t\}$ to match 10-year yields and output per capita in U.S. and G10, excess bond premium of Gilchrist and Zakrajsek (2012), and log U.S. net trade. All series plotted in *pp*. Constant is added to model-generated series to match same average value as data series.

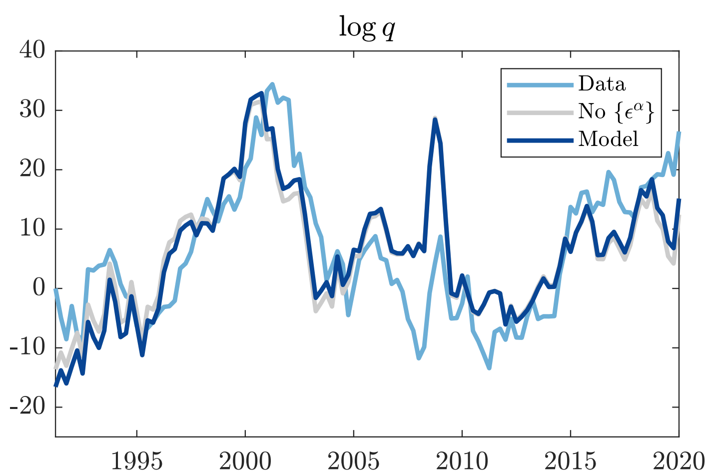

Figure 21: dollar/G10 exchange rate in data and model with trade shocks

Notes: model-implied exchange rate simulated by inverting $\{\beta_t, \beta_t^*, z_t, z_t^*, \gamma_t, \alpha_t\}$ to match 10-year yields and output per capita in U.S. and G10, excess bond premium of Gilchrist and Zakrajsek (2012), and log U.S. net trade. All series plotted in *pp*. Constant is added to model-generated series to match same average value as data series.

	Data	Model	Model	Model	Only $\{\epsilon_t^{\beta}, \epsilon_t^{\beta*}\}$	Only $\{\epsilon_t^{\gamma}\}$
Sample	91Q2-20Q1	91Q2-20Q1	116 quarters*	116 quarters	116 quarters	116 quarters
VAR	yes	yes	yes	no	no	no
$\% \, Var\left(\delta_t^{ir}\right)$	11	33	50	193	246	8
	[2,47]					
$\% \, Var\left(\delta_t^{rp}\right)$	110	125	98	94	65	165
	[73,186]					
$\% \, 2Cov\left(\delta_t^{ir}, \delta_t^{rp}\right)$	-21	-58	-39	-187	-211	-74
	[-119,15]					

Table 29: decomposing variance of real exchange rate

Notes: each cell reports percentage points of $Var\left(\log q_t\right)$. In data, VAR to compute conditional expectations include 2 lags in log real exchange rate, 3m yield differential, 10y yield differential, and excess bond premium, and 90% confidence intervals using nonparametric bootstrap reported in brackets. In model, analogous VAR uses 2 lags in $\log q_t$, $r_t^{(1)*} - r_t^{(1)}$, $r_t^{(40)*} - r_t^{(40)}$, and $\log \gamma_t$. Second column uses shocks recovered from 1991-2020, and remaining columns are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.
* This column reports medians instead of means because of large outliers in statistics implied by VAR in samples of only 116 quarters each.

of the expected real interest rate differential.[55] The first column of Table 29 reports that 110% of the variance in the dollar/G10 exchange rate over our maintained sample period is due to the variance in expected risk premia.

Our model reproduces the result that most of the variation in the exchange rate appears due to expected risk premia. In the second column of Table 29, we estimate a VAR on the data generated by the model using shocks recovered from 1991-2020 in the prior subsection. As in the data, the VAR features two lags and includes $\log q_t$, $r_t^{(1)*} - r_t^{(1)}$, $r_t^{(40)*} - r_t^{(40)}$, and $\log \gamma_t$. The VAR run on the model implies that 125% of the variance in the exchange rate over this period is due to the variance in expected risk premia, comparable to the data. The third column of Table 29 averages the results from decompositions applied to many model-generated simulations of 116 quarters each, similarly finding that most of the variation in the exchange rate is due

[55] We obtain very similar results constructing expected real interest rates using nominal yields and expected inflation differentials. We also obtain similar results in specifications dropping the 10-year yield differential and excess bond premium, or adding more lags to the VAR.

to expected risk premia. This underscores that, from the perspective of the model, the shocks recovered over the 1991-2020 period are not unusual.

The reason for the dominant role of risk premia is that the VAR underestimates the persistence of interest rate differentials in small samples.[56] The fourth column of Table 29 uses actual, model-consistent expectations rather than those by implied by the VAR. We see that now, interest rate differentials account for most of the variance in the exchange rate.[57] The last two columns perform this decomposition given only demand or intermediation shocks (recalling that supply shocks contribute to little of the variance in the exchange rate). We see that the dominant role of interest rate differentials is driven by demand shocks. That said, our model does not imply that risk premium fluctuations are irrelevant: both due to intermediation shocks, and through the changes in arbitrageurs' exposure to exchange rate risk induced by demand shocks, time-varying currency risk premia also affect the exchange rate.

D.9 Comparison to Itskhoki and Mukhin (2021)

We next compare our results to those in Itskhoki and Mukhin (2021), on which we build. That paper concludes that currency intermediation shocks drive nearly all the volatility in the exchange rate. Here we describe why we reach a different conclusion.

There are two key differences between our calibration and that in Itskhoki and Mukhin (2021). First, the model in Itskhoki and Mukhin (2021) does not include demand (discount factor) shocks. Second and relatedly, intermediation shocks are calibrated differently than in our model. In particular, in Itskhoki and Mukhin (2021), intermediation shocks have quarterly autocorrelation of 0.97 (higher than our 0.85), and the volatility of intermediation shocks is calibrated to target a correlation between changes in the real exchange rate and relative consumption growth of -0.4.

Because of these differences, the calibration in Itskhoki and Mukhin (2021) features a larger role for intermediation shocks in driving exchange rate variation. In the absence of demand shocks, intermediation shocks are the only ones which can deliver a negative correlation between the exchange rate and consumption growth,

[56]This generalizes the standard result that there is downward bias in estimating the autocorrelation of a persistent process in small samples. See, for instance, Kendall (1954).

[57]It is small sample bias which explains these results, rather than any other misspecification in the VAR, because if we hypothetically had a much larger sample of data, we have verified that the VAR and model-consistent expectations would imply very similar variance decompositions.

consistent with our Proposition 1. The calibration thus features more volatile and, by assumption, persistent intermediation shocks.

Because intermediation shocks play a more important role, the model in Itskhoki and Mukhin (2021) implies a counterfactual comovement between the exchange rate and yield differentials. Table 30 reports key moments of interest regarding the exchange rate. The first three columns compare moments from the data, our model, and Itskhoki and Mukhin (2021). Both models generate comparable moments to the data in regards to exchange rate volatility, autocorrelation, predictability (or lack thereof), and comovement with relative consumption. However, the model in Itskhoki and Mukhin (2021) counterfactually implies that the dollar is strong when U.S. yields are relatively low in the last two rows. Figure 4 in the main text makes evident that this difference between models is clear even accounting for sampling uncertainty. The fourth column of Table 30 reports moments from our model without demand shocks, dropping the excess bond premium as a proxy for intermediation shocks, and instead disciplining the latter to target the volatility and persistence of the exchange rate directly.[58] The resulting moments are closer to those in Itskhoki and Mukhin (2021), including the comovement of the exchange rate with yield differentials. This clarifies that it is primarily the shocks included in the model and their calibration, rather than any other differences between models (such as endogenous production and nominal rigidities in Itskhoki and Mukhin (2021)), which account for the differences.[59] To visualize the implications of the counterfactual comovement with yield differentials, Figure 22 depicts the implied time-series of the 10-year yield differential when we recover intermediation shocks to match the observed exchange rate from 1991-2020. The implied 10-year yield differential is nearly the reverse of that in the data.

By adding demand shocks to the model and disciplining them using observed yield differentials, we are able to preserve the successes of Itskhoki and Mukhin (2021) while also matching the comovement between the exchange rate and yield differentials in the data. The last column of Table 30 reports moments in our baseline calibration with only demand shocks. Demand shocks disciplined by observed yield differentials already deliver a quite volatile exchange rate and negative correlation between the

[58] The calibration is detailed in Table 31.

[59] Relatedly, the working paper version of Itskhoki and Mukhin (2021) adds an additional shock which affects the exchange rate (expenditure share shocks) and the comovement between the exchange rate and short-term yield differential is flipped (see Table A2 in Itskhoki and Mukhin (2017)). The comovement with longer-term yield differentials is not reported.

	Data	Model	IM (21)	$\sigma^{\beta}, \sigma^{\beta^*} =$ 0, γ targets q	Only $\{\epsilon_t^{\beta}, \epsilon_t^{\beta^*}\}$
Volatility and autocorrelation of real exchange rate					
$\sigma(\Delta \log q_t)/\sigma(\Delta \log c_t)$	7.82	4.95	5.67	4.97	6.15
$\rho(\log q_t)$	0.94	0.91	0.91	0.92	0.94
Exchange rate predictability					
$\Delta \log q_{t+1}$ on $r_t^* - r_t$	-0.74	-0.29	-2.73	-4.04	2.14
R^2 of $\Delta \log q_{t+1}$ on $r_t^* - r_t$	-0.00	-0.00	0.02	0.03	0.03
Exchange rate comovements					
$\rho(\Delta \log q_t, \Delta(\log c_t^* - \log c_t))$	-0.20	-0.77	-0.39	-0.76	-1.00
$\log q_t$ on $r_t^{(1)*} - r_t^{(1)}$	-3.28	-2.59	3.94	10.06	-7.36
$\log q_t$ on $r_t^{(40)*} - r_t^{(40)}$	-8.48	-9.40	31.20	23.62	-11.06

Table 30: moments in data, our model, and Itskhoki and Mukhin (2021)

Notes: model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Column entitled "IM (21)" reports moments from Itskhoki and Mukhin (2021) obtained using online replication package and simulating calibration "IRBC+" (column 7 of Table 1 in that paper) in same way. In predictability regression, we regress change in nominal exchange rate on nominal interest rate differential, as we do in data. We price long bonds using the expectations hypothesis and we use nominal yield differential for $r_t^{(\tau)} - r_t^{(\tau)*}$, as we do in data. Column entitled "$\sigma^{\beta}, \sigma^{\beta^*} = 0$, γ targets q" reports results from calibration summarized in Table 31.

exchange rate and relative consumption. In this sense, observed yield differentials leave little room for intermediation shocks to account for much more exchange rate volatility, whether they have small volatility and high persistence, or larger volatility and lower persistence. We choose to discipline intermediation shocks with lower persistence than Itskhoki and Mukhin (2021) only because plausible proxies for these shocks from financial markets, such as the excess bond premium, imply low persistence. When these shocks have low persistence, they can account for deviations from UIP at high frequencies which a model with only demand shocks would miss.

D.10 Alternative calibrations

We finally detail the robustness of our results to alternative calibrations summarized in the main text.

Table 32 reports calibrated parameters when we allow relative demand, relative

	Description	Value	Moment	Target	Model
β	U.S. disc. fact.	0.99758	$r^{(1)}$	0.97%	0.97%
β^*	Foreign disc. fact.	0.99708	$r^{(40)*} - r^{(40)}$	0.20%	0.20%
σ^z, σ^{z^*}	s.d. z, z^* shocks	0.005	$\sigma(\log y^* - \log y)$	1.73%	1.74%
ρ^z, ρ^{z^*}	persistence z, z^*	0.95	$\rho_{-1}(\log y^* - \log y)$	0.91	0.91
ρ^{z,z^*}	corr. z, z^* shocks	0.10	$\sigma(\Delta \log y)$	0.49%	0.50%
σ^γ	s.d. γ shocks	6	$\sigma(\Delta \log q)$	3.91%	3.96%
ρ^γ	persistence γ	0.98	$\rho_{-1}(\log q)$	0.94	0.92
ξ	pricing to market	1.3	$\sigma(\log s)/\sigma(\log q)$	0.27	0.27
Γ	arb risk pricing	$2E-2$	$\sigma(\log ex/im)$	11.41%	12.02%
σ	trade elasticity	0.9	$\sigma(\log q)$	11.48%	10.59%
ς	home bias	0.8	$(ex+im)/y$	0.25	0.23
ζ^*	rel. population	1.35	$\zeta^* q^{-1} y^*/y$	1.35	1.35

Table 31: calibrated parameters given $\sigma^\beta = \sigma^{\beta^*} = 0$ and γ targeting q

Notes: data moments are estimated over 1991 Q2 – 2020 Q1. Model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Externally set parameters in model are $\psi = 1$, $z = 1$, $z^* = 1$, and $\theta = 4$, the latter three all normalizations.

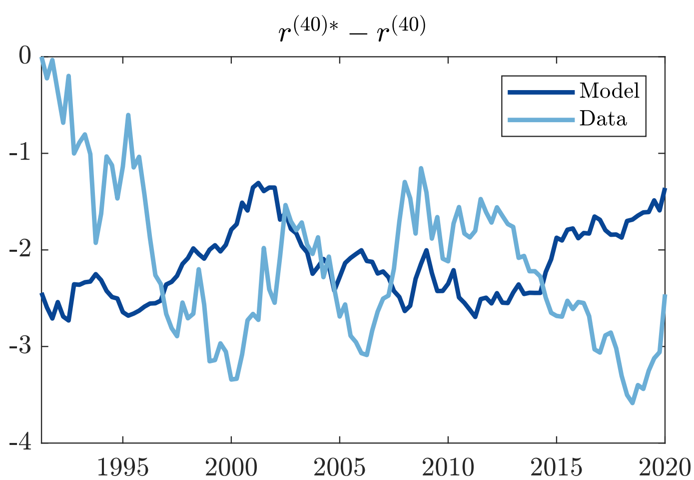

Figure 22: yield differential in calibration with $\sigma^\beta = \sigma^{\beta^*} = 0$ and γ targeting q

Notes: model-implied yield differential simulated by inverting $\{z_t, z_t^*, \gamma_t\}$ to match output per capita in U.S. and G10 and real exchange rate. All series plotted in *pp*. All yields are annualized.

supply, and intermediation shocks to be correlated with one another. These correlations are disciplined by the correlations between the 10-year bond yield differential, output per capita, and the excess bond premium. Table 33 demonstrates that even in this calibration we find that the demand wedges account for most of the variance in the exchange rate: the implied correlations between these wedges are not large enough to change our results.

Table 34 reports calibrated parameters when we target stochastic properties of the three-month rather than 10-year yield differential.[60] Since the three-month yield differential is even more persistent than the 10-year yield differential, this calibration implies an even more persistent process for relative demand than our baseline model. At the same time, the calibration implies slightly less volatile demand shocks than our baseline model, so that the ultimate variance decomposition of the exchange rate, reported in Table 35, remains quite comparable to our baseline. We find our baseline calibration more plausible since exchange rates depend on the expected future path of interest rates and long yields may better reflect this information than shorter-dated yields, given the interest rate smoothing of central banks.

[60] To facilitate comparison with the baseline calibration, we maintain the same average gap between β_t and β_t^*, disciplined by the average 10-year yield differential.

	Description	Value	Moment	Target	Model
β	U.S. disc. fact.	0.99758	$r^{(1)}$	0.97%	0.97%
β^*	Foreign disc. fact.	0.99708	$r^{(40)*} - r^{(40)}$	0.20%	0.20%
$\sigma^{\beta}, \sigma^{\beta^*}$	s.d. β, β^* shocks	0.002	$\sigma(r^{(40)*} - r^{(40)})$	0.81%	0.74%
$\rho^{\beta}, \rho^{\beta^*}$	persistence β, β^*	0.98	$\rho_{-1}(r^{(40)*} - r^{(40)})$	0.93	0.94
ρ^{β,β^*}	corr. β, β^* shocks	0.85	$\sigma(\Delta r^{(40)})$	0.53%	0.53%
σ^{z}, σ^{z^*}	s.d. z, z^* shocks	0.005	$\sigma(\log y^* - \log y)$	1.73%	1.71%
ρ^{z}, ρ^{z^*}	persistence z, z^*	0.95	$\rho_{-1}(\log y^* - \log y)$	0.91	0.92
ρ^{z,z^*}	corr. z, z^* shocks	0.20	$\sigma(\Delta \log y)$	0.49%	0.48%
σ^{γ}	s.d. γ shocks	11.5	$\sigma(\Delta \log q)$	3.91%	3.92%
ρ^{γ}	persistence γ	0.85	$\rho_{-1}(\log \gamma)$	0.80	0.82
$\rho^{\beta,z}$	corr. β and z	-0.30	$\rho(\Delta r^{(40)}, \Delta \log y)$	0.23	0.25
$\rho^{\beta^*,z}$	corr. β^* and z	-0.25	$\rho(\Delta r^{(40)*}, \Delta \log y)$	0.16	0.17
ρ^{β,z^*}	corr. β and z^*	-0.35	$\rho(\Delta r^{(40)}, \Delta \log y^*)$	0.42	0.36
ρ^{β^*,z^*}	corr. β^* and z^*	-0.45	$\rho(\Delta r^{(40)*}, \Delta \log y^*)$	0.38	0.40
$\rho^{\beta,\gamma}$	corr. β and γ	0.40	$\rho(\Delta r^{(40)}, \Delta \log \gamma)$	-0.52	-0.45
$\rho^{\beta^*,\gamma}$	corr. β^* and γ	0.40	$\rho(\Delta r^{(40)*}, \Delta \log \gamma)$	-0.35	-0.32
$\rho^{z,\gamma}$	corr. z and γ	-0.45	$\rho(\Delta \log y, \Delta \log \gamma)$	-0.25	-0.28
$\rho^{z^*,\gamma}$	corr. z^* and γ	-0.20	$\rho(\Delta \log y^*, \Delta \log \gamma)$	-0.33	-0.32
ξ	pricing to market	1.3	$\sigma(\log s)/\sigma(\log q)$	0.27	0.27
Γ	arb risk pricing	$6E-4$	$\sigma(\log ex/im)$	11.41%	11.61%
σ	trade elasticity	0.9	$\sigma(\log q)$	11.48%	10.01%
ς	home bias	0.8	$(ex + im)/y$	0.25	0.23
ζ^*	rel. population	1.35	$\zeta^* q^{-1} y^*/y$	1.35	1.34

Table 32: calibrated parameters with fully correlated shocks

Notes: data moments are estimated over 1991 Q2 – 2020 Q1. Model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Externally set parameters in model are $\psi = 1$, $z = 1$, $z^* = 1$, and $\theta = 4$, the latter three all normalizations.

	$\{\epsilon_t^{\beta}, \epsilon_t^{\beta^*}\}$	$\{\epsilon_t^{z}, \epsilon_t^{z^*}\}$	$\{\epsilon_t^{\gamma}\}$
$\log q$	78%	3%	25%
$\Delta \log q$	46%	3%	57%

Table 33: variance decomposition in calibration with fully correlated shocks

Notes: table reports shares of $\sigma^2(\log q)$ (first row) and $\sigma^2(\Delta \log q)$ (second row) due to each set of driving forces alone. Columns do not sum to 100% because shocks are correlated. Variances are averaged over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.

	Description	Value	Moment	Target	Model
β	U.S. disc. fact.	0.99758	$r^{(1)}$	0.97%	0.97%
β^*	Foreign disc. fact.	0.99708	$r^{(40)*} - r^{(40)}$	0.20%	0.20%
$\sigma^{\beta}, \sigma^{\beta^*}$	s.d. β, β^* shocks	0.0012	$\sigma(r^{(1)*} - r^{(1)})$	1.58%	1.62%
$\rho^{\beta}, \rho^{\beta^*}$	persistence β, β^*	0.99	$\rho_{-1}(r^{(1)*} - r^{(1)})$	0.96	0.90
ρ^{β,β^*}	corr. β, β^* shocks	0.60	$\sigma(\Delta r^{(1)})$	0.51%	0.56%
σ^{z}, σ^{z^*}	s.d. z, z^* shocks	0.005	$\sigma(\log y^* - \log y)$	1.73%	1.74%
ρ^{z}, ρ^{z^*}	persistence z, z^*	0.95	$\rho_{-1}(\log y^* - \log y)$	0.91	0.91
ρ^{z,z^*}	corr. z, z^* shocks	0.10	$\sigma(\Delta \log y)$	0.49%	0.51%
σ^{γ}	s.d. γ shocks	11.5	$\sigma(\Delta \log q)$	3.91%	3.89%
ρ^{γ}	persistence γ	0.85	$\rho_{-1}(\log \gamma)$	0.80	0.82
ξ	pricing to market	1.3	$\sigma(\log s)/\sigma(\log q)$	0.27	0.27
Γ	arb risk pricing	$5E-3$	$\sigma(\log ex/im)$	11.41%	11.63%
σ	trade elasticity	0.9	$\sigma(\log q)$	11.48%	10.22%
ς	home bias	0.8	$(ex+im)/y$	0.25	0.23
ζ^*	rel. population	1.35	$\zeta^* q^{-1} y^*/y$	1.35	1.35

Table 34: calibrated parameters using three-month yields

Notes: data moments are estimated over 1991 Q2 – 2020 Q1. Model moments are averages over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period. Externally set parameters in model are $\psi = 1$, $z = 1$, $z^* = 1$, and $\theta = 4$, the latter three all normalizations.

	$\{\epsilon_t^\beta, \epsilon_t^{\beta^*}\}$	$\{\epsilon_t^z, \epsilon_t^{z^*}\}$	$\{\epsilon_t^\gamma\}$
$\log q$	77%	4%	20%
$\Delta \log q$	47%	4%	50%

Table 35: variance decomposition in calibration using three-month yields

Notes: table reports shares of $\sigma^2(\log q)$ (first row) and $\sigma^2(\Delta \log q)$ (second row) due to each set of driving forces alone. Variances are averaged over 1,000 simulations of 116 quarters each using a 1,000 quarter burn-in period.