Inequality Traps in Convex and Convexified Economies *

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Abstract

We distinguish "inequality traps" from "poverty traps" in a model of occupational choice with borrowing constraints. In such environments, poverty trap models rely on non-convexities in production or investment, permanently preventing wealth-constrained individuals from escaping poverty. In our analysis, poverty traps are absent, as we either assume convex technologies or allow lotteries; thus credit constraints may impede, but never completely prevent, movement out of poverty. Inequality traps arise instead through general equilibrium effects in the labour market, where a large population of poor individuals depresses wages, hindering upward mobility, while small populations of poor raise wages, facilitating mobility.

To underscore the distinction, we demonstrate, for the first time in the literature, the co-existence in an economy with a convex technology of two steady-state, locally stable wealth distributions, one with a high equilibrium wage, one with a low wage. The former has a higher mean and is less unequal in the generalized Lorenz sense.

We then discuss policy measures. While addressing poverty traps may involve microfinance or asset transfers targeted at the poor, these are unlikely to be effective in the presence of inequality traps, which instead require broader economy-wide measures, such as wealth redistribution. We show that a formal-sector minimum wage may be sufficient to bring an economy out of an inequality trap, even if it is initially too poor to respond to any one-off redistribution.

1 Introduction

How do "inequality traps" differ from "poverty traps" in terms of mechanisms, properties, and policy implications? In this paper, we provide a formal model that allows

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us to distinguish these two types of model and characterise situations when they co-exist and when one exists but not the other. Poverty traps generally refer to individuals or households unable to escape poverty due to wealth constraints that limit opportunities to raise their incomes, thereby reinforcing the tendency to stay poor (the model in Galor and Zeira, 1993 is perhaps the best known example in the context of borrowing constraints and non-convexities in the production technology, but there are many others; see Ghatak, 2015 for a review).

Inequality traps, by contrast, operate at the macro level. They work through aggregate features of the economy, such as the wealth distribution and endogenous returns to occupations, some configurations of which hinder upward mobility for the poor and perpetuate the economic privileges of being wealthy. We show that poverty traps and inequality traps can co-exist: in fact, the earliest models of inequality traps had this property (e.g., Banerjee and Newman, 1993), possibly clouding the distinction. But the two types of traps can also exist independently of one another. We show that the qualitative properties and policy implications of these different environments are quite different.

The persistence of poverty remains a critical policy challenge despite global economic growth. There are, of course, many economy-wide factors that constrain the growth potential of developing countries, such as poor infrastructure, bad policies, imperfect institutions (e.g., relating to property rights, access to markets), and insufficient investment in human capital. But it is also the case that those who are well-off in these countries seem to do well and do not appear to be particularly constrained by these economy-wide factors. Indeed, some of these countries (e.g., India) are well-represented both in the world's richest list as well as countries with still a large absolute number of the extreme poor. Why cannot the poor in developing countries climb out of poverty through hard work, skill, enterprise, and luck eventually climb out of poverty? Models of poverty traps provide one answer: the state of poverty is self-reinforcing due the operation of certain economic forces. For example, capital market frictions and non-convexities in the production or investment (say, in human capital) technology would imply that access to high-return opportunities depends on initial wealth, which in turn leaves the poor with limited options to improve their circumstances.

The problem with an explanation on persistent poverty based on poverty traps along the lines we just described is if we allow for general economic growth and technological progress raising everyone's incomes or institutions that help individuals overcome the lumpiness of investments such as lotteries or informal credit institutions like rotating saving and credit assocations (ROSCAs), a given individual or dynasty would tend to escape it in the long-run – no person or family will stay poor forever. This is inconsistent with the presence and persistence of poverty in general, as that would depend on various factors affecting upward and downward mobility. However, if the presence of many poor people itself makes upward mobility harder, and downward no harder, then we might have an explanation as to why there can be persistent of poverty in economies that have a lot of poor but do not have to anchor it to poverty traps at the individual level.

This is the motivation for studying inequality traps. There is a substantial literature that has looked at the role of capital market frictions and occupational choice and allowed for returns to occupations to be endogenous (e.g., Banerjee and Newman, 1993, Piketty, 1997, Ghatak and Jiang, 2002, and Mookherjee and Ray, 2003) and do display some form of history dependence: multiple locally stable steady states, each with different levels of inequality and aggregate performance, i.e. what we are here calling inequality traps. But in all those papers, they coexist with poverty traps, clouding the key factors at play. The culprit is the presence of a non-convexity in the technology of production, which along with the credit market friction, are also key ingredients of poverty trap models.

We propose a framework where there are no poverty traps at the individual level and yet inequality traps can exist. As in most of the above papers, credit market frictions, as well as insurance market frictions, play key roles. We consider two variants of the model, one with a decreasing-returns, convex technology in which the two factors capital and labor can be arbitrarily finely divided across production units, and another with a "small" production non-convexity that introduces a non-concavity in the value function that rational agents would wish to concavify via lotteries, which we allow. In both settings, individual lineages may move in and out of poverty rather than being stuck there permanently. Nonetheless, the economy as a whole may be stuck in an inequality trap – an equilibrium marked by low aggregate performance and high inequality of income and wealth, when the same set of fundamental preferences and technologies also admit a permanent state of high aggregate performance with less inequality and faster upward mobility.

The problem in both models is that additional wealth accruing to a wealthy agent does no one else any good: in particular the agent has no inducement to use more of a labor-demanding technology that could contribute to bidding up someone else's wage (the so-called trickle down effect). With diminishing returns to capital invested and imperfect credit markets that fail to channel wealth efficiently from those who have it to those who need it, the equalizing forces present in neoclassical models are shut down, and inequality (as well as equality) become self-reinforcing.

While there is evidence that the mechanisms highlighted in poverty trap models may well be present in certain contexts (e.g., Balboni et al., 2022), in our view poverty trap models are best viewed as short-term approximations of forces that could cause poverty to persist, possibly the expression at the individual level of forces operating at the macro level.

The distinction between poverty and inequality traps has significant implications for policy design. Inequality traps point at systemic barriers that hinder mobility as opposed to an absolute threshold that needs to be crossed to escape poverty. Policies such as microfinance or asset transfers can lift individuals out of poverty, one person at a time, but they may have limited impact on systemic poverty because of the unequalizing forces at work. In contrast, addressing inequality traps requires systemic changes, such as wealth redistribution, or, as we will show, minimum wage laws or other structural reforms, to alter the overall distribution of income and opportunities.

One contribution of this paper is to provide the first example of a market inequality trap with a convex technology.[1] This is important because it underscores the conceptual distinction between inequality traps and poverty traps; the latter depend on non-convexities for their existence, and while it has often been asserted/conjectured that inequality traps do not, there has to our knowledge been no formal proof of that claim. Our construction helps fill that gap.

We then turn to a more conventional inequality trap model with technological non-convexities but allow for lotteries (and so, by construction, there is no value-function non-concavity). This proves to be very tractable for global analysis and avoids the logical inconsistencies (or implicit ad hoc assumptions) of non-lotterized non-convex models.

First, we find conditions under which a closed-economy redistribution of wealth can set the economy onto a path toward a desirable steady state, using an explicit characterization of global dynamics. Discussions of the efficacy of wealth redistribution policies have sometimes been imprecise. They begin with the tautologically correct assertion that starting in the basin of attraction (BoA) of a desired steady state is enough to get the economy to converge there. But if the necessary starting condition is to be accomplished by some policy intervention, it must be that points in the BoA are feasibly achieved with the resources available to the policy maker. With inequality traps, those resources are endogenous, and may be so meager that

[1]Piketty (1997) assumes constant returns to scale in capital and labour and everyone is self-employed in his own business supplying his own one unit of labor inelastically and introduces moral hazard in borrowing that generates credit constraints. However, the model implicitly introduces a non-convexity, as those who are credit rationed (in particular, those who cannot borrow at all) are not permitted to sell their excess labour to those who are not constrained. In our model there is no such restriction.

no distribution of the economy's resources lie in the desired steady state's BoA (unless some extra resources are injected from outside).

Again using global analysis, we show that a minimum wage enforced only in the formal sector may be enough to drive the economy to the desired steady state, even if no one-off redistribution can. In effect, controlled regulation of the flow of income among agents in the economy is enough to allow a distribution of wealth in the target steady state's BoA to accumulate over (finite) time, even if such an initial condition cannot be reached via a one-off redistribution.

The plan of the paper is as follows. In section section 2 we introduce a simple model with a production technology involving capital and labour but with no non-convexities, imperfect capital markets, and a labour market with endogenous wages. In section 3 we show that in the presence of capital market frictions and uninsurable risks to wealth that is passed on as bequest, one could have multiple steady states that we interpret as inequality traps but without there being any poverty traps. In section 4 we study a more tractable version of the model that has technological non-convexities, but allows for lotteries, and carry out a number of policy exercises, such as redistribution and minimum wages. In section 5 we discuss empirical evidence on the main economic mechanism that leads to inequality traps, namely, general equilibrium effects through wages. Section 6 concludes.

2 The Basic Model

We start with a convex economy with an imperfect capital market, specifying technology, demographics and preferences. In section 4, we shall relax the convexity assumption by modifying one of the technologies.

2.1 Technology

There is a single consumption good (y) that also serves as the numeraire for capital and wealth that is produced using labour (l) and capital (k). There are three ways to produce the good. The first is the "modern" technology, which is convex, with decreasing returns to scale. Specifically, capital and labour are perfect complements, up to a capacity constraint so that output produced by a single firm is capped at $\overline{y} > 0$:

$$y = \min\{\overline{y}, A \min\{k, l\}\}$$

with A representing a productivity parameter common across all agents who operate this technology. Thus, there is a maximum amount of capital (and labour) $\overline{k} := \overline{y}/A$

that any agent would demand.

There is another, "traditional," technology that requires labour alone and yields $\underline{w} > 0$ per unit of labour applied. Instead of modern and traditional, we may also refer to these two technologies as formal and informal. The way the term "formality" is used refers to whether an organization complies with government regulations, and these typically tend to be larger and more capital-intensive ones. In the policy section where we discuss issues like minimum wage laws, the formal vs. informal sector terminology would be especially relevant.

Finally, there is also a "storage" technology requiring only wealth as an input; it is perfectly divisible as well and returns $r < A$ per unit invested. We can think of this as a deposit account that yields a lower rate of return than operating the modern technology or a simple, risk-free way of holding one's savings.

To avoid trivial cases, we assume the modern technology has strictly higher labour productivity than the traditional one, and that $\overline{k}$ exceeds the per capita endowment of labour (which we normalize to unity):

Assumption 1 *(a)* $\underline{w} < A - r$. *(b)* $\overline{k} > 1$.

Hence efficient allocations entail employing all labour with the modern technology.

2.2 Demographics

Time is discrete, indexed by $t = 0, 1, 2....$ Each period there is a unit measure continuum of risk-neutral agents, each endowed with a unit of labour (thus, $\overline{k} \geq 1$). They may differ in their initial endowments of wealth a. We denote the (endogenous) c.d.f. of wealth by $F_t(a)$, and suppress the time subscript when there is no ambiguity.

Within a period, an agent inherits wealth $a \geq 0$ from its parent, then chooses how to invest its labour and wealth across the three technologies. Once the returns from these activities have accrued, the agent gives birth to one offspring and divides its income between consumption and a bequest (= the offspring's wealth) so as to maximize the "warm glow bequest" utility (Andreoni, 1989)

$$u(c, b) = c^{1-\beta} b^{\beta}. \tag{1}$$

Thus $b = \beta I$, where I is the agent's lifetime income; I also represents the agent's payoff (indirect utility).

The degree of warmth (i.e., bequest propensity β) varies across individuals in the population, independently of wealth, and independently over time within lineages. This assumption, along with assumptions about markets (see next subsection), introduces randomness into wealth transitions over time that allow for downward as well as upward mobility. Other ways to accomplish randomness that have been popular in the literature include production risk (which we have assumed away) with imperfect insurance and mortality risk with imperfect annuity markets. Following Ghatak-Jiang (2002), we use the present formulation for tractability. Specifically, $\beta = \overline{\beta} \in (0, 1)$ with probability q, and $\underline{\beta}$ otherwise, where $\overline{\beta} > \underline{\beta} \in [0, 1)$.

We impose the following condition relating the relative productivity of the modern and traditional technologies and the bequest propensity of the generous:

Assumption 2 *(a)* $\overline{\beta}(A - \underline{w}) > 1$. *(b)* $\overline{\beta}r < 1$.

As advertised, this will help in the construction of multiple steady state distributions; it essentially is a requirement that returns diminish rapidly enough to eventually shut down the trickle-down effect.

2.3 Markets

Labour markets are competitive and subject to no distortions. Since anyone, regardless of initial wealth, has access to the traditional technology, its return $\underline{w}$ is the lower bound for the market wage rate.

Capital markets are imperfect, and in particular, it is costly to enforce credit contracts. A borrower can default on a loan and keep the profits net of wage payments but there is a probability of getting caught (α) and then being subject to a non-pecuniary punishment (F). The incentive constraint of someone who borrows $k - a$ is, therefore:

$$(A - r - w)(k - a) \geq (A - w)(k - a) - \alpha F$$

Note that due to the Leontief technology assumption, if $k-a$ is the loan amount, then that is also the net amount of labour hired, and we are assuming that wage payments would have to made even by defaulting entrepreneurs (here, in equilibrium, there are no defaults). This is the formulation used by Banerjee and Newman (1993), as well as Ghatak and Jiang (2002). We simplify further by setting $\alpha = 0$, so that the capital market does not operate. In this extreme form of capital market imperfections, an agent is constrained by the amount of wealth he or she has, subject to the maximum capital stock per firm that the technology permits, $\overline{k}$:

$$k \leq \min\{a, \overline{k}\}.$$

[[WE SHOULD CHECK that everything goes through straightforwardly if $\alpha > 0$ (in which case everyone can invest a positive amount in modern tech; else we should adopt a multiplicative form of the credit constraint]]

Finally, we assume there is no market in which agents can insure against having stingy parents (i.e., $\beta = \underline{\beta}$). An equivalent formulation would be to let consumption occur in two periods after production with q being the probability of death before the second period in which case the unspent income passes to the child and there being no annuities market with financial products that convert savings into a guaranteed income stream.

2.4 Dependence of payoffs on wealth

We first derive the payoffs of the agents as a function of their wealth. We treat the labour of the entrepreneur as part of the total labour employed, which reflects the opportunity cost of running an enterprise as opposed to joining the labour market.

For $a < \overline{k}$, an agent's payoff is:

$$\begin{aligned} V(a) &= (A - r - w)\, a + w + ar \\ &= (A - w)\, a + w. \end{aligned}$$

For $a \geq \overline{k}$, it is:

$$\begin{aligned} V(a) &= (A - r - w)\, \overline{k} + w + ar \\ &= (A - w)\, \overline{k} + w + \left(a - \overline{k}\right) r. \end{aligned}$$

The marginal return to a for agents with $a < \overline{k}$ is $A - w$, while that of a richer agents is r. Unlike in many of the existing models of occupational choice the payoff function of an entrepreneur is continuous in a, as there is no lumpiness in the production technology.[2]

For any investment in the modern technology to take place at all, it is necessary that

$$A - w \geq r,$$

so that the value function $V(a)$ is not only continuous, but concave. Moreover, there is an upper bound on the wage rate:

$$w \leq \bar{w} := A - r.$$

We already have a lower bound on the wage rate and so equilibrium wages will have

[2] In effect, the "occupational choice" here is not discrete, but continuous; any agent with $a < 1$ would spend at least some of its time in the labour market (or possibly in the traditional sector in case $w = \underline{w}$), and for those with $a \geq 1$ it is a matter of indifference whether labour is supplied to the market or to their own firm.

a lower and an upper bound:

$$w \in [\underline{w}, \bar{w}].$$

By Assumption 1, this is interval is non-degenerate.

2.5 Labour Market Equilibrium

From the utility function (1), everyone inelastically supplies their unit of labour, so total labour supply $L^S \equiv 1$.

For the modern technology, capital and labour are used in fixed proportions, so labour demand L^D is equal to the amount of overall capital investment in the modern sector. As long as $w < \overline{w}$, this is

$$\begin{aligned} L^D &= \left(1 - F\left(\overline{k}\right)\right)\overline{k} + \int_0^{\overline{k}} af\left(a\right)da. \\ &= \overline{k} - \int_0^{\overline{k}} F\left(a\right)da \end{aligned}$$

(in case $w = \overline{w}$, $L^D = [0, \overline{k} - \int_0^{\overline{k}} F\left(a\right)da]$).

Since $\overline{k} > 1$, there can be potentially excess demand as well as excess supply of workers, depending on the wealth distribution (if instead $\overline{k} < 1$, only excess supply would be possible). If

$$L^D > L^S$$

then the equilibrium wage rate is $w^* = \bar{w}$, and if instead

$$L^D < L^S$$

then the equilibrium wage rate is $w^* = \underline{w}$. The case where $L^D = L^S$ is going to be non-generic, but in that case any wage rate $w^* \in [\underline{w}, \bar{w}]$ clears the market. Thus labour-market equilibrium is unique for a.e. F, with either the high wage $\overline{w}$ or the low wage $\underline{w}$ prevailing.

Now we turn to the dynamics of the wealth distribution.

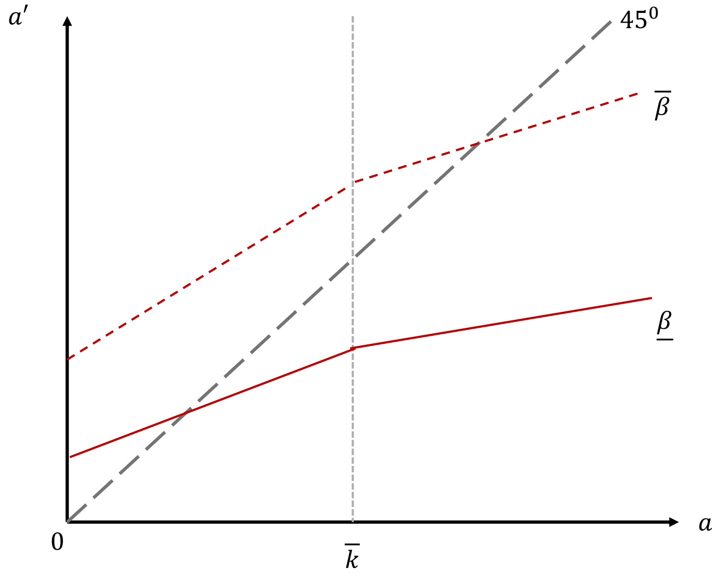

Figure 1: Typical recursion diagram for $w < \overline{w}$

2.6 Dynamics of Wealth Distribution

Let w be the prevailing wage in a particular period. The wealth of a lineage with realized bequest rate β, follows the recursion $a \mapsto a'$:

$$a' = \begin{cases} \beta\left[(A-w)\,a + w\right], & a < \overline{k} \\ \beta\left[(A-w)\,\overline{k} + \left(a - \overline{k}\right) r + w\right], & a \geq \overline{k} \end{cases} \tag{2}$$

Like the value function of wealth, the transition equation is concave in a (linear in case $w = \overline{w}$). But as the wage is endogenous to the population distribution of wealth, the recursion may vary over time with changes in the wage as the distribution evolves. And of course within a period, the transition depends on the realization of β. A typical recursion diagram when the two realizations of β satisfy $\overline{\beta} > \underline{\beta} > 0$ is depicted in Figure 1.

To study the aggregate dynamics, we proceed in the standard way. Since there is a large population whose bequest parameters realize independently across lineages and time, we start with a given w, and follow the corresponding lineage dynamics. These will generically have a globally stable stationary distribution. Then reinterpreting that as the population distribution of wealth, it is simply a matter of checking whether

w is the market clearing wage for the stationary distribution, as described in section 2.5, for it to be stationary for the global dynamical system. Each distribution that is stationary given the wage and clears the market at that wage will be locally stable. As there are only two equilbrium wages to consider ($\underline{w}$ and $\overline{w}$; the others are easily seen to be transitory as well as nongeneric), the task is less daunting than may appear.

3 Decreasing returns and the failure of trickle-down can lead to multiplicity

Our goal here is modest: only to establish the existence of multiple locally stable stationary wealth distributions with different aggregate properties (roughly, first and second moments) for an economy with convex technology. Of course, credit and risk sharing markets are imperfect, but there has never been a question about the contribution of those institutional assumptions to multiplicity.

In the next section we will be more ambitious, performing a global analysis to address how policy might move the economy from one distribution to another in a finite number of periods. There we shall simplify our task by reverting to a "lumpy" technology, though we shall adhere to agent rationality and allow for lotteries. The resulting model has low dimension, so that global analysis is greatly simplified.

We restrict parameters a bit. First, we set $\underline{\beta} = 0$ for the remainder of the paper. This implies that each period, $1-q$ of the population are born with zero, regardless of the distribution of wealth among their parents. Since there is only one other β value to deal with, explicit characterizations of stationary distributions is greatly simplified.

Then, in addition to Assumptions 1 and 2, we impose the further restriction

Assumption 3 $q\overline{\beta}(A-r) > 1 > q\overline{\beta}\underline{w}$.

Assumption 2 in section 2.2 (namely, $\overline{\beta}(A-\underline{w}) > 1 > \overline{\beta}r$) implies the recursion diagram for the low wage will have its $\overline{\beta}$-branch steeper than the 45° line, up to $\overline{k}$, after which its slope is $\overline{\beta}r < 1$, with a fixpoint at $\overline{\overline{a}} = \frac{\overline{\beta}[(A-r-\underline{w})\overline{k}+\underline{w}]}{1-\overline{\beta}r} > \overline{k}$. Meanwhile, with $\underline{\beta} = 0$, the $\underline{\beta}$-branch coincides with the a-axis.

Assumption 3 implies that if the wage is high, any child of a generous parent will inherit wealth greater than $1/q$, even if the parent had 0 to start with, while if the wage is low, it will take at least two generations for the descendent of someone born with zero wealth to inherit more than $1/q$. Finally, note that Assumption 3 permits supposing

$$\overline{k} \in (1/q, \overline{\beta}(A-r)).$$

Under high wage dynamics, there is a globally stable distribution with support in $[0, \overline{a}]$, where $\overline{a} = \frac{\overline{\beta}(A-r)}{1-\overline{\beta}r}$. In fact, the support is contained in the smaller subset $\{0\} \cup [\overline{\beta}(A-r), \overline{a}]$, with $1-q$ at 0, and q in the interval at $[\overline{\beta}(A-r), \overline{a}]$.

At the high wage $\overline{w}$, since $\overline{k} < \overline{\beta}(A-r)$, all q children of generous parents demand $[0, \overline{k}]$ (there is no reason to demand more); since the children of stingy parents inherit zero, they demand zero, so that demand for labour will be $[0, q\overline{k}]$. Thus, with $q\overline{k} > 1$, the unique market clearing wage is $\overline{w} = A - r$.

Now suppose the wage is low. The stationary distribution corresponding to this wage is characterized as follows. Again $1-q$ of the population is at 0. But because $\underline{w}$ is small, it will take a while for a lineage to climb above $\overline{k}$ even if it is fortunate to have a run of generous parents. Specifically, there will be wealth levels $\{a_n\}_{n=0}^{N}$, where $a_0 = 0$ and for $n > 0$, $a_n = \overline{\beta}(\underline{w} + a_{n-1}(A - \underline{w}))$; N is such that $a_N < \overline{k} < a_{N+1}$ (the restriction $1 > q\overline{\beta}\underline{w}$ ensures that $N \geq 1$). The mass at each a_n is $q^n(1-q)$. The remaining $1 - (1-q)\sum_{n=0}^{N} q^n$ of the population will inherit wealth greater than $\overline{k}$. A recursion diagram combining the dynamics for both wages and $N = 1$ is illustrated in Figure 2 (mass points for the high wage case are denoted b_n).

Labour demand will be therefore be

$$\overline{k}[1 - (1-q)\sum_{n=0}^{N} q^n] + (1-q)\sum_{n=0}^{N} q^n a_n =$$

$$q\overline{k} + (1-q)\sum_{n=1}^{N} q^n (a_n - \overline{k}) < q\overline{k} \tag{3}$$

since each $a_n < \overline{k}$. If we have chosen $\overline{k}$ close enough to (but still greater than) $1/q$, then this demand is strictly less than 1 (note the a_n for $n \leq N$ do not depend on $\overline{k}$, so we are free to do so), and the labor market clears at the low wage.

Thus we have established

Proposition 1 *In the convex economy satisfying Assumptions 1, 2, and 3, there exist values of $\overline{k}$ such that two distinct, locally stable wealth distributions exist, one with a high wage and one with a low wage.*

The example is constructed with a small (but positive-measure) part of the parameter space. An important task for future research is to explore how "likely" inequality traps may be.

We show below (Corollary 3) that the low wage steady state (inequality trap) is more unequal than the high wage steady state in the most common sense of the term (second order stochastic dominance or generalized Lorenz dominance). But

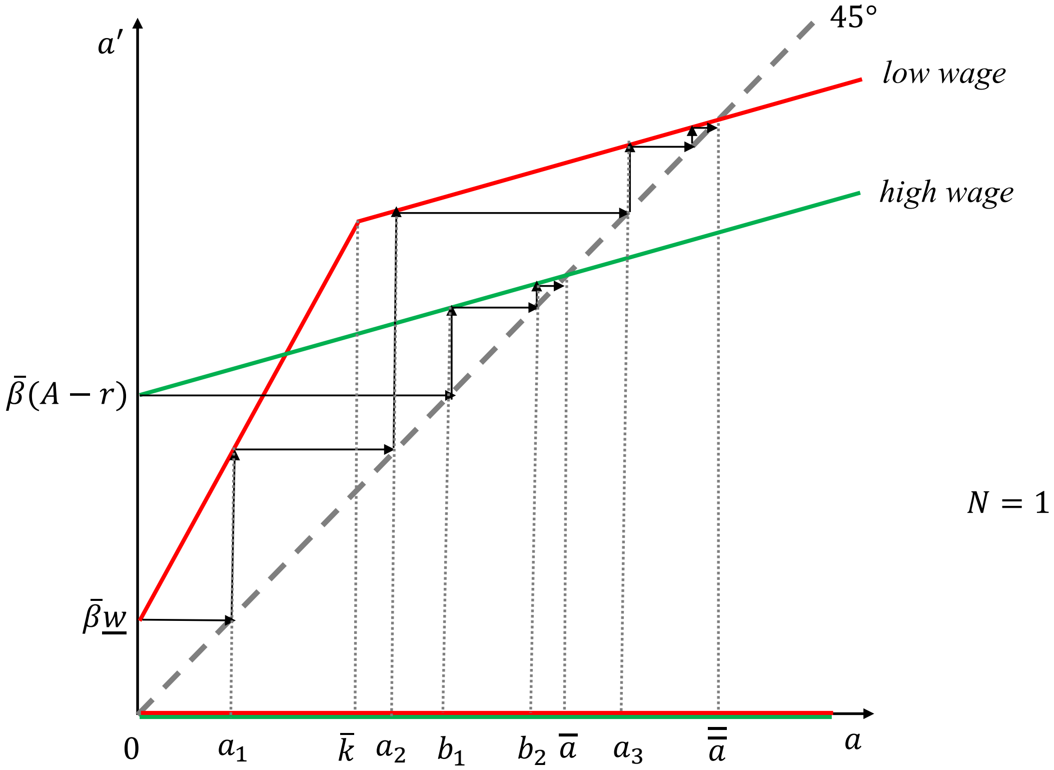

Figure 2: Recursions for $\overline{w}$ (green/light) and $\underline{w}$ (red/dark) with $N = 1$

before doing so, we establish the following proposition regarding means of income and wealth, which holds whenever the two types of steady state coexist:

Proposition 2 *Mean wealth and income are lower in the low-wage steady state than in the high-wage steady state.*

This is because in the former case, the imperfect credit market is not channeling all wealth into the most productive activities, so that aggregate production is lower. Whereas when there is less inequality, there is less need for the market to channel wealth into production (producers are already in possession of it), so more of it is employed productively.

To establish the proposition, note that with high wages, the entire unit population is employed with productive technology, so that labor produces A at capital expense r. Any remaining wealth is invested at return r; thus in steady state if aggregate wealth is $\mathbb{E}[a|\overline{w}]$, income is

$$A + (\mathbb{E}[a|\overline{w}] - 1)r,$$

(recall $\overline{w} = A - r$, so this is just $\overline{w} + r\mathbb{E}[a|\overline{w}]$); since $q\overline{\beta}$ of that is passed on to become the wealth of the next generation, in steady state

$$\mathbb{E}[a|\overline{w}] = q\overline{\beta}[A + (\mathbb{E}[a|\overline{w}] - 1)r],$$

or

$$\mathbb{E}[a|\overline{w}] = q\overline{\beta}\frac{\overline{w}}{1 - q\overline{\beta}r}.$$

Whereas in the inequality trap, only some fraction $\underline{D} < 1$ of the population, along with $\underline{D}$ units of wealth, are employed with the productive technology; remaining wealth is invested at r and remaining labor produces $\underline{w}$, so

$$\mathbb{E}[a|\underline{w}] = q\overline{\beta}[\underline{D}A + (\mathbb{E}[a|\underline{w}] - \underline{D})r + (1 - \underline{D})\underline{w}]$$

or

$$\mathbb{E}[a|\underline{w}] = q\overline{\beta}\frac{\underline{D}\overline{w} + (1 - \underline{D})\underline{w}}{1 - q\overline{\beta}r} < \mathbb{E}[a|\overline{w}].$$

Note that this argument does not rely on the specific stochastic specification of the bequest propensity β (for instance, there could be more than two values of β, they could all be strictly positive, etc. – simply replace $q\overline{\beta}$ with $\mathbb{E}\beta$). It depends only on the fact that labour and capital are "underemployed" (misallocated away from the most productive technology) in the inequality trap and so aggregate production and income are commensurately lower. Thus the inequality trap produces lower aggregate income and, because the bequest propensities are independent of wealth, aggregate wealth as well.[3]

It is well known (e.g., Hanoch and Levy, 1969) that if one c.d.f. F crosses another G once from below, where the mean of F weakly exceeds that of G, then F second-order stochastically dominates (SSD) G: every risk averter would prefer F to G. Equivalently, the generalized (mean-corrected) Lorenz curve for F lies above that for G (Thistle, 1989).[4] This is a widely accepted definition of what it means for G to be "more unequal" than F. Armed with Proposition 2 we can now state

Corollary 3 *The low-wage steady state wealth distribution in Proposition 1 is more unequal than the high-wage steady state wealth distribution.*

The proof, deferred to the Appendix, involves showing that the c.d.f. for the high-wage distribution $F_{\overline{w}}$ does indeed cross that of the low-wage distribution $F_{\underline{w}}$ once from below; the conclusion then follows from Proposition 2. The key ingredients of the argument are first that the two distributions have identical ranges, and second that the supports $\{a_n\}$ of $F_{\underline{w}}$ and $\{b_n\}$ of $F_{\overline{w}}$ also have a single-crossing property.

Like the "size" distributions of income and wealth, "functional" income inequality (profits $A - r$ minus the wage w) is larger in the low-wage regime than in the high-wage one. Since aggregate income and wealth are lower as well, the unequal economy is also the less prosperous one, even if some of its individuals are wealthier than the best-off individuals in the equal economy.

We have not relied on technological non-convexities, only on limited liability leading to a malfunctioning credit market as well as imperfect insurance against stingy parents. Moreover, agents' value functions are all concave, unlike in some models where "non-convexities are snuck in the back door" via non-homothetic preferences or increasing returns to wealth in the credit enforcement technology.

Establishing the existence of two distinct stationary distributions, each with a different wage and mean wealth, does not fully characterise the dynamics of the economy. Based on general arguments about the behaviour of linear dynamical systems, Markov chains in particular, each distribution is locally stable, i.e., starting from wealth distributions that are "close enough" to the stationary one, the economy will converge to it. But this does not rule out the possibility of more complicated behaviour, where the economy might head toward one distribution, then turn tail and

[3]Of course, the bequest propensity distribution does potentially affect the *existence* of an inequality trap, as illustrated by the simple case in which the bequest is deterministic. Then it is not hard to show that only one (trivial) stationary distribution can exist. Details are in the Appendix.

[4]We take the two distributions to have supports in a common interval $[0, M]$. SSD is defined as $\int_0^x F(t)dt \leq \int_0^x G(t)dt$ for all $x \in [0, M]$, with strict inequality for a non-null set of x, or equivalently, as $\int_0^M U(t)dF(t)dt \geq \int_0^M U(t)dG(t)dt$ for all increasing concave $U(\cdot)$.

head toward the other, possibly "orbiting" in this way forever (see Lloyd-Ellis and Bernhardt, 2000 for an example). Below, we will discuss some policy implications, which require global analysis, so we shall use a slightly different and more tractable model of inequality traps.

4 "Small" Non-convexities: Indivisibilities, Poverty Traps, and Lotteries

Frequently inequality traps are confounded with poverty traps, even though the latter is an individual (lineage) phenomenon, while the former operates at the general equilibrium level. Poverty traps are often modeled by combining a non-convexity (say, an indivisible project) with a credit constraint. Unfortunately, early investigations of inequality traps almost always depended on models with the same technology and assumptions about the credit market, possibly giving the impression that the non-convexity is necessary for the inequality trap to be operative. As we have already seen, this is patently untrue.

To help underscore the difference between poverty traps and inequality traps, we introduce a non-convexity in the production technology, while maintaining the credit constraint. As has been noted in the literature (Gall, 2008), the non-convexity creates an incentive for rational agents to accept lotteries, which like convex technologies, concavifies their value functions. More saliently, it causes the poverty trap to disappear, along with most of its macro implications. By contrast, inequality traps may remain intact, as we will show. Moreover, the "lotterized" version of the inequality trap proves to be tractable, and we shall use it to carry out policy analysis.

The credit market is imperfect as before, but we modify the productive technology in two ways. First and most important, it yields A only if $\bar{k}$ units of capital and $n \geq 1$ units of labor are invested; as before, no further returns are generated by investing more of either factor, but now returns are zero if investment is less than these thresholds. Second, we assume that the agent operating the productive technology does not hire out its own endowment of labor on the market but rather uses it to operate its own enterprise in some other set of "entrepreneurial" tasks than what workers perform – this is not essential for our conclusions, but greatly simplifies the algebra of the policy thought experiments that we shall consider.

We shall set $\bar{k} = 1$ for the remainder of this section. Subsistence (or informal) income remains at $\underline{w}$, which is the lowest equilibrium value for w, and the yield on wealth invested in the (divisible) storage technology is still r. Our credit market is still imperfect, exactly as before, so entrepreneurship is only accessible to those with wealth ≥ 1. We assume

$$A - n\underline{w} - r > \underline{w},$$

so that efficient production will still have all agents operating full-time in the modern sector)

Indifference between entrepreneurship and working (for $a > 1$, $\overline{w} + ar = A - r - n\overline{w} + ar$) again generates an upper bound for the equilibrium wage: is $\overline{w} = \frac{A-r}{n+1}$ (the factor $n + 1$ results from the capital:labor ratio n plus the "loss" of labor income for the entrepreneur).

Generically, the market-clearing wage in the entrepreneurial/formal sector is either $\overline{w}$ or $\underline{w}$. The former obtains if the fraction of the population with wealth less than 1 ("poor") is less than n times that with wealth at least 1 (rich), i.e., the fraction of poor is less than $1/(n + 1)$. If the fraction poor is greater than that, the wage is $\underline{w}$. Random bequests work as before: the parent passes β of its income on to its child, with the rate realisation $\beta \in \{0, \overline{\beta}\}$ independent across individuals, time, and wealth. Let q be the probability that $\beta = \overline{\beta}$.

For simplicity, we impose

Assumption 4 $\overline{\beta}\overline{w} > 1$.

This is less restrictive than may appear at first blush: first note that it already weakens Assumption 3; by tweaking the credit market enforcement parameters, the threshold for investing could be reduced to some $\underline{a} < 1$; we are requiring that the high wage is enough for generous parents to provide the minimum seed capital to their children. This could be relaxed even further at the cost of some additional computation.

The value function faced by an individual as a function of its inherited wealth a is:

$$dw + (1 - d)\underline{w} + ar, a < 1$$

$$A - r - nw + ar, a \geq 1,$$

where d is the probability of getting a "good" (formal sector) job ($d < 1$ only if $w = \underline{w}$, else $d = 1$). When $w < \overline{w}$, there is a discontinuity in the value at $a = 1$, which introduces a non-concavity in the value function, and consequently rational agents would be happy to engage in lotteries.

When the wage is $\overline{w}$ (in which case $d = 1$), there is no incentive to take a lottery, as the expected utilities of working and entrepreneurship are equal (indeed, the high wage is defined by equating these two payoffs). When it is low ($\underline{w}$), $d < 1$, but income is the same regardless of whether an agent gets a good or bad job, and any agent with $a \in [0,1)]$) would accept a (fair) lottery in exchange for his inheritance that pays 1 with probability a, and 0 with probability $1-a$. We assume that all agents in $(0,1)$ play such lotteries before choosing the best occupation that is feasible given the lottery outcome (working at the low wage or subsistence if 0, entrepreneurship if 1). The new lotterized value functions is then

$$\hat{V}(a) = \begin{cases} (1-a)w + a(A-nw), & a < 1 \\ A - nw + (a-1)r, & a \geq 1 \end{cases},$$

which is concave.

4.1 Dynamics

For the high wage, we can study lineage dynamics by reducing the number of states to just two: treat the interval $[0,1)$ as one state, and $[1,\overline{\overline{a}}]$ as the other, where $\overline{\overline{a}}$ is a finite upper bound on the wealth any agent can possess in the long run.[5] An agent born with a in the in the first state leaves its offspring there (in fact at 0) with probability $1-q$, and in the other state (at $\beta(a+\overline{w}) > 1$) with probability q. Those born in the second state leave their offspring in the first and second states respectively with the same probabilities. Thus a (unique) stationary distribution $\overline{\mathbf{p}} = (1-q, q)$ is reached immediately from any initial condition. Since demand in the labor market is nq and supply is $1-q$, this distribution is an equilibrium steady state provided $nq > 1-q$, or

Assumption 5 $q > \frac{1}{n+1}$.

For the low wage, lineage dynamics can be characterised by a three-state Markov chain. After one period, all agents with wealth $a \in [0,1)$ will either leave their offspring 0 (if they are stingy), $x := \beta\underline{w}$ (if they lose the lottery but are generous), or in $[1,\overline{\overline{a}}]$ (if they win and are generous). Thus we can reduce the state space to $\{0, x, [1,\overline{\overline{a}}]\}$. The ensuing transition probabilities are depicted in the matrix:

$$\begin{bmatrix} 1-q & 1-q & 1-q \\ q & q(1-x) & 0 \\ 0 & qx & q \end{bmatrix} \tag{4}$$

By way of explanation, note that 0's have no chance of winning the lottery, so their dynamics are governed by the bequest rate alone, since they are confined to

[5] We can take $\overline{\overline{a}}$ to be equal to $\frac{\overline{\beta}(A-r-\underline{w})}{1-\overline{\beta}r}$, the fixpoint of the function representing generous dynamics of unconstrained agents under the low-wage regime.

subsistence/low-wage formal sector jobs. The x's who lose the lottery (probability $1-x$) and are generous (probability q) leave their offspring with x, the same state they started in; those who win and are generous leave them in the third state $[1, \overline{\overline{a}}]$. The remaining $(1-q)$ x's are stingy and leave their offspring destitute. Finally, those born in $[1, \overline{\overline{a}}]$ either leave their kids there or send them back to 0.

This matrix has a unique stationary distribution $\underline{\mathbf{p}} = (1-q, \frac{q(1-q)}{1-q+qx}, \frac{q^2x}{1-q+qx})$. As long as

$$\frac{n(q^2x + q(1-q)x)}{1-q+qx} < 1 - q + \frac{q(1-q)(1-x)}{1-q+qx}, \tag{5}$$

or

Assumption 6 $x < \frac{1-q}{nq}$,

formal sector demand is inadequate to bid the wage up beyond $\underline{w}$, and this steady state sustains the low-wage labor market equilibrium.[6] Thus, depending on whether the economy starts out near enough to $\underline{\mathbf{p}}$ or $\overline{\mathbf{p}}$, it will converge to one or the other steady state, with corresponding differences in the income and wealth distributions, including in the first moments.

Notice that both distributions display mobility among the states. But compared to the high-wage steady state, income and wealth in the low-wage steady state are more unequally distributed, a true "inequality trap." It is the low wages resulting from competition among the large number of poor that make it difficult (albeit not impossible) for any of them to move up the wealth distribution. They are replaced by others moving down the ladder after experiencing bad luck.

Contrast with poverty traps. A poverty trap model would exogenously fix the wage at $\underline{w}$ and have to rule out lotteries. It would also have to rule out the random bequest we have specified in order to stave off total collapse of the economy: with every lineage eventually born with 0, the (unique) distribution would be supported in $[0, x/(1-\overline{\beta}r)]$.

Only then would we have the (extreme) path dependence at the lineage and aggregate levels, where any initial distribution of wealth supported on the two states (low wealth, high wealth) would be stationary. Convexifying such a model would result in a unique steady state distribution, with either everyone at high wealth (without the random bequests) or at $\underline{\mathbf{p}}$ (with random bequests). Poverty trap models, while descriptive of individual experience in an economy with persistent poverty, seem rather unsatisfactory as tools for analysing its aggregate performance.

[6] Demanders at $\underline{\mathbf{p}}$, each of whom demand n workers, comprise the rich plus the x of the x's who are lottery winners; supply consists of all the $0's$ plus the lottery losers.

4.2 Policies to Escape Inequality Traps

Inequality traps are aggregate phenomena, and thus need aggregate responses. The literature has tended to focus on redistribution of initial wealth to accomplish this task, or on the role of outside interventions (such as land reforms in southeast Asia imposed by the Japanese), but as we will show, such policies may be ineffective if the economy initially has too little wealth. We shall discuss what remedies might be effective otherwise. One possibility is redistribution coupled with external injections of wealth from Bill Gates (or if the economy is on Mars, Elon Musk). But an alternative policy that is "closed," in the sense of not requiring injections from outside the economy, is a formal sector minimum wage.

A standard way of studying the effects of policy with nonlinear systems is so-called local analysis, linearising around a steady state and checking for stability of that steady state. We can't do this here because the questions we are asking – can wealth redistribution or a minimum wage pull a whole economy out of an inequality trap, i.e., away from one steady state and toward another? – require global analysis, where we ensure that the new path following the policy leads the system into the basin of attraction and therefore converges to the desired steady state.

Wealth Redistribution. We continue to use the model with lotteries and indivisible technology, as it is particularly tractable. Indeed, it can be reduced to a single-dimensional model: since in every period after the first, $p_0 = 1-q$, $p_x + p_R = q$, where p_R is the fraction of the population with wealth in $[1, \overline{\overline{a}}]$. We can therefore reduce the three-dimensional dynamics involving (p_0, p_x, p_R) to ones involving just p_R.

Define $p_w := \frac{1-xq(n+1)}{(1-x)(n+1)}$ as the size of p_R below which the market-clearing wage is low, and above which it is high (p_w is the value of p_R that satisfies demand = supply, i.e., $n(xp_x + p_R) = p_0 + (1-x)p_x$, where $p_x = q - p_R$ and $p_0 = 1-q$; it is straightforward to verify that Assumptions 5 and 6 imply $\underline{p}_R < p_w < q$). Then from the above discussion about high and low wage dynamics, and using the last row of (4),

$$p'_R = \begin{cases} q, & p'_R \geq p_w \\ xq^2 + q(1-x)p_R, & p'_R < p_w \end{cases}$$

Thus the economy converges to $\underline{\mathbf{p}}$ whenever $p_R < p_w$, and $\overline{\mathbf{p}}$ whenever $p_R \geq p_w$ (we break the ambiguity in the market clearing wage when $p_R = p_w$ in the "hopeful" direction of convergence to $\overline{\mathbf{p}}$).

Any "closed" redistribution of wealth to a distribution inducing a state $\hat{\mathbf{p}}$ must ensure that $\hat{p}_R \geq \hat{p}_w$ if it is to break the inequality trap, i.e., converge to $\overline{\mathbf{p}}$. Maximizing $\hat{p}_R$ for a given aggregate wealth $\mathbb{E}a < 1$ entails giving 1 unit of wealth (the minimum needed to become an entrepreneur) to a portion of the population and 0 to the rest until all wealth is exhausted: leaving wealth distributed among the "poor" (wealth less than 1) or not taking it from those with wealth greater than 1 only serves to reduce $\hat{p}_R$. This implies that the best chance for exiting the inequality trap occurs when $\hat{p}_R = \mathbb{E}a$.[7]

Assuming that the economy was at (or close to) $\underline{\mathbf{p}}$ before the redistribution, we can state

Proposition 4 *A closed one-time redistribution can succeed in pulling the economy from the low-wage steady state* $\underline{\mathbf{p}}$ *to the high wage steady state* $\overline{\mathbf{p}}$ *if and only if*

$$\frac{xq(1-q+q\beta(A-r)-nqx)}{(1-q+xq)(1-q\beta r)} \geq \frac{1-xq(n+1)}{(1-x)(n+1)}. \tag{6}$$

The expression on the left side of (6) is the mean wealth at the low wage steady state $\underline{\mathbf{p}}$.[8] The right side is p_w. In particular if x is low enough (equivalently, the informal sector is sufficiently unproductive) so that condition (6) fails, no closed one-off redistribution can succeed (to see this, let $x \to 0$; then $\mathbb{E}_{\underline{\mathbf{p}}}a \to 0$, while $p_w \to \frac{1}{n+1}$). On the other hand, at the upper bound $\frac{1-q}{nq}$ for x, condition (6) reduces to

$$\frac{q\beta(A-r)}{(1-q\beta r)(n+1)} = \frac{q\beta\overline{w}}{1-q\beta r} > \frac{q}{n+1},$$

which is satisfied from Assumption 4. Thus properly designed one-off redistributions can be effective for larger values of x.[9]

Raising informal productivity x or formal sector productivity A increases the likelihood that (6) is satisfied. So redistribution may be complementary to productivity enhancement in eliminating inequality traps. (Recall that if x is raised beyond $\frac{1-q}{nq}$, the inequality trap disappears without redistribution.)

Another possibility is to redistribute wealth over a longer time horizon. Starting at $\underline{\mathbf{p}}$, the planner could implement a policy that redistributes all inherited wealth equally across the population at the beginning of every period. As noted in footnote

[7] Observe that if the planner allocates $\mathbb{E}a$ to every agent, then the agents will subsequently accomplish an optimal distribution themselves, via lotteries: $\mathbb{E}a$ of them will end up with 1, the rest with 0. They have strict incentives to engage in such lotteries, since they correctly anticipate the equilibrium wage to be less than $\overline{w}$ if $\mathbb{E}a < \frac{1}{n+1}$; even if $\mathbb{E}a \geq \frac{1}{n+1}$, then equilibrium entails that enough agents participate in the lottery to bring the market clearing wage to $\overline{w}$, i.e., $p_R \geq \frac{1}{n+1}$. We shall return to this point below.

[8] Calculation available upon request.

[9] It is straightforward to check that $\mathbb{E}_{\underline{\mathbf{p}}}a$ is increasing in x, while p_w is decreasing, so that such redistributions are effective above, and ineffective below, a unique cutoff value of x.

7, agents will then "re-redistribute" the wealth over $\{0, 1\}$ themselves, via lotteries. The demand for labor is then $n\mathbb{E}a$ (corresponding to the $\mathbb{E}a$ of the population who win 1 unit of wealth), which results in the equilibrium wage $\underline{w}$ whenever $\mathbb{E}a < \frac{1}{n+1}$. In this range, average wealth follows the dynamic

$$[\mathbb{E}a]' = q\overline{\beta}\left(A[\mathbb{E}a] + (1 - (n+1)[\mathbb{E}a])\underline{w}\right),$$

which converges to

$$[\mathbb{E}a]^* = \frac{q\overline{\beta}\underline{w}}{1 - q\overline{\beta}[A - (n+1)\underline{w}]}.$$

Note that Assumptions 4 and 5 jointly imply that

$$[\mathbb{E}a]^* > \frac{1}{n+1}$$

(after some manipulation, the condition reduces to $q\overline{\beta}A > 1$, which follows from the two Assumptions), so that under this policy, in finite time the economy switches to the high-wage dynamic and converges to a high-wage steady state.[10]

While the length of time needed for this policy to reach the high-wage steady state depends on parameters (once in its basin of attraction, the policy could be shut down if there were no concern about future aggregate or redistributive shocks), one particular case is worth noting. If x is near its upper bound $\frac{1-q}{nq}$, the mean wealth given by the left-hand side of (6) already exceeds $\frac{1}{n+1}$ (substituting $x = \frac{1-q}{nq}$, the left hand side of (6) exceeds $\frac{1}{n+1}$ whenever $q\overline{\beta}A > 1$, which has already been verified), so that the policy could be one-off. This underscores the nature of inequality traps: though aggregate wealth may already be high enough to bring the economy to a high wage-steady state, where the economy performs at its potential, the way wealth is distributed under laissez-faire precludes it from doing so.

Minimum Wages. Political barriers to the policies just discussed may be formidable, not least because they may be difficult to dismantle even after they have served their purpose. An alternative policy would be a minimum wage. We assume this could only be enforced in the formal sector. Apart from political opposition, the policy is feasible as it only involves redistributing *income* from wealthy to poor, not seeking cash injections from outside or raising the productivity of the subsistence sector.

Assume the economy is at or near $\underline{\mathbf{p}}$ to begin with. Let a minimum wage $w_m > \underline{w}$

[10] This steady state would not be identical to the one reached under laissez-faire, as wealth would be equalized by the policy rather than spread out over $[1, \overline{a}]$ and 0, but the aggregate income and wealth would be the same.

be imposed on the formal sector. To make things interesting, let's not make it so high that children of generous formal sector workers automatically are born wealthy enough to be entrepreneurs:

$$y := \overline{\beta} w_m < 1. \tag{7}$$

Of course, this implies $w_m < \overline{w}$. Because there is excess supply in the labor market, only some agents will get formal sector jobs. We assume uniform rationing in the labor market, i.e., the probability d of securing a formal sector job is independent of one's initial wealth. ("Diversity" policies, where d is decreasing in wealth, or "legacy" policies, where it is increasing, would be interesting to study but are beyond the scope of this paper.) Since hiring workers is voluntary, we must have $d = D/S$, where D is labor market demand and S is supply. So now an agent born with $a < 1$ will either win a wealth lottery and become an entrepreneur, or lose that and enter the "labor market lottery," landing a formal sector job at wage w_m with probability d or an informal job paying $\underline{w}$ with probability $1 - d$.

This version of the model can be analyzed by reducing the state space to just four points: $\{0, x, y, [1, \overline{\overline{a}}]\}$. After one period, depending on the outcomes of the various lotteries (wealth, labor market, parental), agents will inherit $0, x, y$ or $a \geq 1$. The transition matrix is

$$\begin{bmatrix} 1-q & 1-q & 1-q & 1-q \\ (1-d)q & (1-x)(1-d)q & (1-y)(1-d)q & 0 \\ dq & (1-x)dq & (1-y)dq & 0 \\ 0 & xq & yq & q \end{bmatrix} \tag{8}$$

Now clearly if we make w_m so low that y also satisfies Assumption 6 (i.e., $y < \frac{1-q}{nq}$), then the minimum wage economy will retain the inequality trap, though this one will be "softer" with some workers getting good jobs, and the wealthy being somewhat less wealthy. The interesting question is whether with a suitably chosen minimum wage satisfying (7) we can pull the economy out of the trap altogether, so that eventually the labor market clears at the high wage $\overline{w}$ and and the minimum wage would not even bind. There is an affirmative answer:

Proposition 5 *If $y > \frac{1-q}{nq}$, $\overline{\mathbf{p}}$ is the unique, globally stable distribution.*

In other words, with a suitably high minimum wage, even if enforced in the formal sector alone, the inequality trap vanishes: any economy starting in the original basin of attraction of $\underline{\mathbf{p}}$ will converge to $\overline{\mathbf{p}}$. The minimum wage allows the economy to eventually accumulate enough agents with wealth at least 1 that there will be excess demand in the labour market and the high wage steady state will prevail. Just as in the convex technology case, letting rich agents instead accumulate additional wealth beyond the minimum efficient scale is wasteful of resources when there are agents below that scale, because they do not use their wealth to hire additional workers. So while the minimum wage diminishes the accumulation of wealth by wealthy lineages, it does so with the social benefit of allowing *more* wealthy lineages to emerge and bid up wages.

To establish the proposition, first note that for $\mathbf{p} = (p_0, p_x, p_y, p_R)$ (p_R again being the fraction with $a \in [1, \overline{a}]$), and $w < \overline{w}$, the demand for labor is $D = n(p_R + xp_x + yp_y)$, the last two terms coming from the wealth lottery winners. Supply $S = 1 - q + (1 - x)p_x + (1 - y)p_y$. Also, $p_0 \equiv 1 - q$, as that fraction of the population always receives 0, no matter how wealth is distributed in the previous generation. Thus $p_x + p_y + p_R \equiv q$, and we are effectively down to a two-dimensional model. Observe that $d < 1$ (i.e., $D < S$) if and only if

$$(1 - x)p_R + (y - x)p_y < \frac{1}{n+1} - xq.$$

From the last two rows of (8), and using $d = D/S$, we have[11]

$$p'_y = qD = qn(p_R + xp_x + yp_y) = qn(qx + (1 - x)p_R + (y - x)p_y),$$

and

$$p'_R = q(p_R + xp_x + yp_y) = q(qx + (1 - x)p_R + (y - x)p_y).$$

Thus beginning at (p_y, p_R) where $D < S$ (in particular at the low wage steady state $\underline{\mathbf{p}}$), upon introduction of w_m, the system will move to the ray $p_y = np_R$ in the next period, and remain on it as long as $D < S$. Thus, we achieve further dimensional reduction, with the p_R-dynamic now becoming

$$p'_R = q^2 x + q[1 - x + n(y - x)]p_R.$$

There are two cases:

Case 1: $q[1 - x + n(y - x)] < 1$. Here p_R follows the linear recursion to its unique

[11] This is where the assumption that entrepreneurs don't supply to the labour market comes in. If we were to replicate what we did for the convex model but for the indivisibility in project scale, then for the base case we would obtain similar results, but for this policy thought experiment, the expressions would involve $d(1-d/n))$, a good deal messier. The additional occupational indivisibility results in canceling terms that retain the piecewise linear dynamics.

fixpoint $\frac{q^2x}{1-q[1-x+n(y-x)]}$. But at this fixpoint,

$$\begin{aligned}
&(1-x)p_R + (y-x)p_y \\
&= [1-x+n(y-x)]p_R \\
&= \frac{q^2x[1-x+n(y-x)]}{1-q[1-x+n(y-x)]}.
\end{aligned}$$

The last expression strictly exceeds $\frac{1}{n+1} - xq$ whenever $y > \frac{1-q}{nq}$.[12] Thus, after a finite number of periods, demand exceeds supply, and the wage rises to $\overline{w}$, and following the high wage dynamics the economy converges to the high wage steady state.

Case 2: $q[1-x+n(y-x)] \geq 1$. The dynamic is "explosive" here, leading p_R to reach its boundary value $q/(n+1)$ in finite time (since $p_y = np_R$ and $p_y + p_R \leq q$, p_R cannot exceed $q/(n+1)$ as long as $D < S$). But again, with $p_R = q/(n+1)$, $\frac{q[1-x+n(y-x)]}{n+1} \geq \frac{1}{n+1} > \frac{1}{n+1} - xq$, so $D > S$.

Thus, in either case, in finite time, the minimum-wage economy switches to the high-wage regime, where it remains forever after, since the dynamics there immediately take the economy to $(p_0, p_x, p_y, p_R) = (1-q, 0, 0, q)$.

Observe that once the new steady state is reached, the minimum wage is no longer binding (if $w_m < \overline{w}$). Nevertheless, keeping it in place may be desirable if the economy is subject to aggregate or redistributive shocks.

5 Discussion

A key mechanism that the policy implications of our model highlights is that policies that are aimed at the poor can have significant general equilibrium effects that influence the labour market and wages. In this section we discuss some recent empirical work that provides strong suggestive evidence for this channel.

[12]By hypothesis, $q[1-x+n(y-x)] < 1$, so

$$\begin{aligned}
\frac{q^2x[1-x+n(y-x)]}{1-q[1-x+n(y-x)]} &> \frac{1}{n+1} - xq \iff \\
q^2x[1-x+n(y-x)] &> (\frac{1}{n+1} - xq)(1-q[1-x+n(y-x)]) \\
= \frac{1}{n+1} - xq - \frac{1}{n+1}q[1-x+n(y-x)] &+ q^2x[1-x+n(y-x)] \iff \\
(n+1)xq + q[1-x+n(y-x)] > 1 &\iff q + qny > 1 \iff y > \frac{1-q}{nq}.
\end{aligned}$$

A number of papers show that policies that raise the outside option of workers have a positive effect on private sector wages. For example, Muralidharan et al (2022) show that India's National Rural Employment Guarantee Scheme (NREGS) led to a 14% increase in earnings for beneficiary households and a 26% reduction in poverty, but notably, 86% of the income gains came from non-programme earnings, driven by higher private-sector wages and employment. The scheme provides a guaranteed employment option that raises the minimum wage at which they are willing to work, i.e., their reservation wages, and reduces the likelihood of accepting low-paying private-sector jobs. This puts positive pressure on private sector wages, especially in regions where such programmes have a strong presence. In a related paper on the same programme, Imbert and Papp (2015) show that the introduction of the employment guarantee scheme is associated with a rise in the daily wages for casual labourers working privately (i.e., not directly participating in the scheme) in early-implementing districts compared to later ones, with an estimated 4.7% wage rise and a 1.5% drop in private employment.

Similarly, a number of recent papers show that directly redistributive programmes aimed at the poor also put upward pressure on wages in the labour market. Bandiera et al (2017) study an asset transfer programme in Bangladesh that provided livestock assets and skills to ultra-poor women. They find that this significantly reduced their participation in casual wage labour activities such as agricultural work and maid services. With fewer ultra-poor women participating in these labour markets, there is reduction in labour supply to casual labour activities and this is seen to be leading to significant wage increases: for example, women in those sectors who did not receive transfers in the treated villagesi saw their wages rise by roughly 10% compared to those in control villages. The paper concludes that the impact of redistributive programmes like the one they study should include these general equilibrium effect on wages. Another paper studying a large-scale cash transfer program in rural Kenya (Egger et al, 2022) similarly finds that wages increased as a result of the programme, particularly for non-recipient households

Relatedly, there is also strong evidence that negative economic shocks can have general equilibrium effects on wages. Breza and Kinnan (2021) study the microfinance crisis in the Indian state of Andhra Pradesh (AP), in 2010 as a natural experiment to understand the general equilibrium effects of a credit squeeze on rural labor markets. This event was triggered by a government ordinance that severely curtailed microfinance activities in the state, leading to a sharp contraction in credit supply both locally and nationally. Districts with greater exposure to the crisis experienced larger declines in weekly household earnings and casual daily wages. Reduced credit access both hindered business investments reducing labor demand, and the resulting falling incomes curtailed household consumption, thereby reducing demand for local goods and services, putting further downward pressure on wages. Notably, the reduction in wages extended beyond microfinance borrowers, affecting non-borrowing laborers as well, highlighting the interconnectedness of rural labor markets and the significance of general equilibrium effects.

6 Concluding Remarks

This paper explores the distinction between inequality traps and poverty traps, showing that inequality traps can exist even in the absence of poverty traps. Inequality traps arise from the interplay between the wealth distribution and endogenous returns to occupations that can lead to multiple steady states - a high-wage, low-inequality one and a low-wage, high-inequality one - even without the existence any individual-level poverty traps. We rule out poverty traps by assuming convex technology, or allowing for lotteries in case there are non-convexities.

The paper also examines the impact of policies, and shows that inequality traps require systemic changes like wealth redistribution or minimum wage laws unlike policies that could help individuals overcome poverty traps. We show that a minimum wage in the formal sector can move the economy toward the more desirable steady-state, even when redistribution fails to do this for lack of adequate aggregate wealth.

There are several avenues for future research. For example, it is worth characterizing conditions under which inequality traps exist with more general production technologies. Another interesting question is whether investing in improvements in formal or informal sector technology (A or x in the model) would be more effective at eliminating an inequality trap. We have also yet to consider the relative efficacy of minimum wages (an ex-ante income distribution) and conventional income taxes and transfers (ex-post redistibution), or a (finite) sequence of wealth redistributions in cases where one-off distributions are ineffective. Moreover, while our model admits both upward and downward mobility in both the equal and unequal steady states, we have not drawn a tight connection between quantitative mobility and inequality. These questions, which are certainly at the heart of much public discussion are also ripe for further analysis with models such as ours.

Appendix

Uniqueness of the steady state with deterministic bequests. All notation and assumptions are as in the text, except that we dispense with Assumptions 2(a) and 3, and assume $q = 1$, so there is a unique bequest propensity $\beta > 0$.

Lineage wealth continues to follow the concave recursion (2), where now only the wage varies. Steady-state distribution corresponding to each wage must be trivial, i.e., every lineage has the same wealth, since they all face the same wage. The unique fix-point and common steady-state wealth of any w-dynamic is $a^*(w) \in \{a_1^*(w), a_2^*(w)\}$, where $a_1^*(w) := \frac{\beta w}{1-\beta(A-w)}$ is the fixpoint of the $(a < \overline{k})$-dynamic (only if $1-\beta(A-w) > 0$), and $a_2^*(w) := \frac{\beta[(A-r-w)\overline{k}+w]}{1-\beta r}$ is the fixpoint of $(a \geq \overline{k})$-dynamic.

However, for $w < \overline{w}$, there can never be a steady state $a_2^*(w)$, for such w can only clear the labor market if aggregate demand is no greater than the supply 1, that is, the common wealth must not exceed 1, while having everyone at $a_2^*(w)$ means everyone has wealth at least $\overline{k} > 1$, a contradiction. (For the remaining case, $a_1^*(\overline{w}) = a_2^*(\overline{w})$.) Similarly, no wage for which $1 - \beta(A - w) \leq 0$ can be part of a steady state, all lineages would be carried above $\overline{k}$ in finite time, and the wage would be bid up to $\overline{w}$. Thus, there is no ambiguity in referring to a hypothetical steady-state common wealth as $a^*(w)$ and assuming this is well defined, with $1 - \beta(A - w) > 0$ (such wages exist, since $1 - \beta(A - \overline{w}) > 0$).

It is straightforward to check that whenever $1-\beta(A-w) > 0$, $a^*(w) > 1$ if and only if $\beta A > 1$. Thus, if $\beta A > 1$, the market clearing condition is $L^D = \min\{a^*(w), \overline{k}\} > 1 = L^S$, which implies the equilibrium wage is $\overline{w}$. There can be no other steady state, because the required common wealth $a^*(w) > 1$ would not clear the market at w. Correspondingly, if $\beta A < 1$, then $a^*(w) < 1$, the unique market clearing wage is $\underline{w}$, and therefore the unique steady state common wealth is $a^*(\underline{w})$. Only if $\beta A = 1$, in which case $a^*(w) = 1$, all w, is there indeterminacy in the wage, but not in income or wealth, which are A and 1 respectively, regardless of w.

Proof of Corollary 3. As discussed in the text, it is enough to show that $F_{\overline{w}}$ crosses $F_{\underline{w}}$ once from below (that is, there is some a^* with $F_{\overline{w}}(a) \leq F_{\underline{w}}(a)$, all $a < a^*$, and $F_{\overline{w}}(a) \geq F_{\underline{w}}(a)$, all $a > a^*$, with each inequality strict on non-null sets.

It may help to refer to Figure 2. As described in the text, the distribution $F_{\underline{w}}$ has support $\{a_n\}_{n=0}^{\infty}$, where a_n is the nth iterate starting at 0 of the recursion (2) for $\beta = \overline{\beta}$ and $w = \underline{w}$ (thus, $a_0 = 0$). Its value on $[a_n, a_{n+1})$ is $(1-q)\sum_{i=0}^{n} q^i$. Similarly, the support $\{b_n\}_{n=0}^{\infty}$ of $F_{\overline{w}}$, $b_0 = 0$, is generated from (2) with $\beta = \overline{\beta}$ and $w = \overline{w}$; $F_{\overline{w}}(b) = (1-q)\sum_{i=0}^{n} q^i, b \in [b_n, b_{n+1})$. Both sequences are strictly monotone, with

$a_n \to \overline{\overline{a}}$ and $b_n \to \overline{a}$, where $\overline{\overline{a}} > \overline{a} > \overline{k}$.

Observe that (1) $F_{\overline{w}}$ and $F_{\underline{w}}$ have identical ranges; and (2) while $a_0 = b_0, a_1 < b_1$ from Assumption 3 (in fact we don't need $a_1 < \overline{k} < b_1$, as is assumed in the text, for this proof, only the weaker $a_1 < b_1$; here we take for granted that $F_{\underline{w}} \neq F_{\overline{w}}$). Thus, $F_{\overline{w}}(a) \leq F_{\underline{w}}(a)$ on $[0, b_1)$, strictly so on (a_1, b_1).

Since $\overline{\overline{a}} > \overline{a}$, there exist (infinitely many) n such that $a_n > b_n$. Let M be the smallest such n (from above, $M \geq 2$, and $a_n \leq b_n$ for $n < M$). We now assert that $a_M > b_M \Rightarrow a_n > b_n$ all $n \geq M$. It follows that the sequence $\{a_n\}$ crosses $\{b_n\}$ once from below, and as a consequence $F_{\overline{w}}(a) \geq F_{\underline{w}}(a)$, all $a \in [b_M, \overline{\overline{a}}]$ (with strict inequality on $[\overline{a}, \overline{\overline{a}})$, at least). Thus $F_{\overline{w}}$ crosses $F_{\underline{w}}$ once from below, as desired (a^* can be chosen arbitrarily from $[b_{M-1}, b_M]$).

To establish the assertion, note that the affine recursion with initial condition $x = \underline{x}$

$$x' = \beta[\chi_0 + \chi_1 x]$$

results, for $t > 0$, in

$$x_t = (\beta\chi_1)^t \underline{x} + \beta\chi_0 \sum_{i=0}^{t-1} (\beta\chi_1)^i. \tag{9}$$

Now consider the subsequences $\{a_n\}_{n=M}^{\infty}$ and $\{b_n\}_{n=M}^{\infty}$, and suppose first that $a_M \geq \overline{k}$. Then using (2) and (9), for $n > M$,

$$a_n = (\overline{\beta} r)^{n-M} a_M + \overline{\beta} A_0 \sum_{i=0}^{n-M-1} (\overline{\beta} r)^i$$

and

$$b_n = (\overline{\beta} r)^{n-M} b_M + \overline{\beta} B_0 \sum_{i=0}^{n-M-1} (\overline{\beta} r)^i$$

where $A_0 := (A - \underline{w} - r)\overline{k} + \underline{w}, B_0 := \overline{w} = A - r$. Since $\overline{k} > 1$, $A_0 > B_0$. As $a_M > b_M$ as well (by definition), $a_n > b_n$, as claimed.

Now suppose instead $a_M < \overline{k}$, the initial conditions $a_0 = b_0 = 0$ yield

$$a_M = \overline{\beta}\underline{w} \sum_{i=0}^{M-1} (\overline{\beta}(A - \underline{w}))^i, \text{ and } b_M = \overline{\beta}\overline{w} \sum_{i=0}^{M-1} (\overline{\beta} r)^i = b_M;$$

$a_M > b_M$ implies $\frac{\underline{w}}{\overline{w}} > \frac{\sum_{i=0}^{M-1}(\overline{\beta} r)^i}{\sum_{i=0}^{M-1}(\overline{\beta}(A-\underline{w}))^i}$. But $\frac{\sum_{i=0}^{I}(\gamma)^i}{\sum_{i=0}^{I}(\delta)^i}$ is decreasing in I whenever $0 < \gamma < \delta$, so $\frac{\underline{w}}{\overline{w}} > \frac{\sum_{i=0}^{M}(\overline{\beta} r)^i}{\sum_{i=0}^{M}(\overline{\beta}(A-\underline{w}))^i}$, and therefore $a_{M+1} > b_{M+1}$. By induction, $a_n > b_n$ for all $n > M$ whenever $a_n \leq \overline{k}$. Once the iterates a_n exceed $\overline{k}$, the argument in the previous paragraph for $a_M \geq \overline{k}$ applies, using a_{N+1} as defined in the text (the smallest iterate of the $\underline{w}$-recursion greater than $\overline{k}$) and b_{N+1} as the initial conditions ($a_{N+1} > b_{N+1}$ from the argument just concluded).

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