Contracting for Coordination

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Introduction

- Coordination problem: interdependence calls for agents to act consistently but there is strategic risk about what others will do
- Principal contracting for coordination must address strategic risk

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- Principal contracting for coordination must address strategic risk

- Important for both organizations and markets
  - "The key role of management in organizations is to ensure coordination" (Milgrom and Roberts, 1992: 114)
  - Firms coordinate buyers to purchase goods with network externalities

Contracting for coordination

- Principal contracts with set of agents
- Induces game, possibly with multiple equilibria
- What is optimal scheme that guarantees high payoff to principal?

Plan

- Part 1: Contractible actions
  - Exogenous externalities
  - Endogenous externalities
- Part 2: Hidden actions
  - Public contracts
  - Private contracts
- Part 3: Hidden types
  - Monopolist problem

Plan

- Part 1: Contractible actions
  - Exogenous externalities: Segal (2003)
  - Endogenous externalities
- Part 2: Hidden actions
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  - Private contracts
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Setup

- Set $N = \{1, \ldots, N\}$ of agents. Action $a_i \in \{0, 1\}$ for each $i \in N$
- Bilateral contracts: for each i, payment ω_i conditional on $a_i = 1$
- Given $a := (a_1, \ldots, a_N)$, agent i's payoff is

$$U_i(a, \omega_i) = u_i(a) + a_i \omega_i$$

Implementation

- Scheme $\omega = (\omega_i)_i$ induces simultaneous game
- $\widehat{a}$ is NE iff for each i, $\widehat{a}_i \in \operatorname{argmax}_{a_i} U_i(a_i, \widehat{a}_{-i}, \omega_i)$
- Principal wants to guarantee $a^1 := (1, \ldots, 1)$ at least cost
  - Implement a^1 as worst-case ("lowest-action") NE
  - Equivalent to implementing a^1 as unique NE

Principal's problem

- Call $E(\omega)$ the set of NE profiles under ω
- Worst-case implementation constraint (W) is

$$E(\omega + \varepsilon) = \{a^1\} \quad \forall \varepsilon > 0$$

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- Principal solves

$$\min_{\omega} \sum_i \omega_i \text{ subject to } (W)$$

Externalities

- Distinguish between increasing/decreasing externalities
  - Increasing if $\forall i$, $u_i(1, a_{-i}) - u_i(0, a_{-i})$ increasing in a_{-i}
- Implies game with strategic complementarities/substitutabilities

Externalities

- Distinguish between increasing/decreasing externalities
  - Increasing if $\forall i$, $u_i(1, a_{-i}) - u_i(0, a_{-i})$ increasing in a_{-i}
- Implies game with strategic complementarities/substitutabilities
- Many examples with strategic complementarities
  - Investment
  - Teamwork
  - Goods with network externalities
  - Exclusive dealing
  - Bank runs

Decreasing externalities

Proposition

With decreasing externalities, optimal scheme specifies ω^{NE} *s.t.* $\forall i$

$$u_i(1, a^1_{-i}) + \omega^{NE}_i = u_i(0, a^1_{-i})$$

- Worst-case focus has no bite

Increasing externalities

- What if increasing externalities? Supermodular game
- Scheme ω^{NE} induces a^1 as a NE but does not satisfy (W)
- E.g., for $\varepsilon > 0$ small, $(0, \ldots, 0)$ is also NE under $\omega^{NE} + \varepsilon$

Example

- 2 agents. For $i \in \{1, 2\}$, let $u_i(a) = \begin{cases} -1 & : a = a^1 \\ -2 & : a_i = 1, a_{-i} = 0 \\ 0 & : \text{otherwise} \end{cases}$

Example

- 2 agents. For $i \in \{1, 2\}$, let $u_i(a) = \begin{cases} -1 & : a = a^1 \\ -2 & : a_i = 1, a_{-i} = 0 \\ 0 & : \text{otherwise} \end{cases}$
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- To make a^1 an equilibrium at least cost, pay $\omega_L := 1$ to each agent
- To make it unique equilibrium, must make $a_i = 1$ dominant for some i
  - Pay one agent $\omega_H := 2$
  - And then ω_L to the other agent

Ranking schemes

- Given permutation π of N, define $a_{-i}(\pi)$ by $\pi_j < \pi_i \iff a_j = 1$

Definition

ω *is ranking scheme if* $\exists \pi$ *s.t.* $U_i(1, a_{-i}(\pi), \omega_i) = U_i(0, a_{-i}(\pi), \omega_i)\ \forall i$

Ranking schemes

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Definition

ω is ranking scheme if $\exists \pi$ s.t. $U_i(1, a_{-i}(\pi), \omega_i) = U_i(0, a_{-i}(\pi), \omega_i)$ $\forall i$

Lemma

With increasing externalities,

1. *Every ranking scheme satisfies (W)*
2. *Any scheme satisfying (W) is dominated by some ranking scheme*

Optimal scheme and discrimination

Proposition

With increasing externalities, optimal scheme specifies π^* *and* ω^* *s.t.* $\forall i$

$$u_i(1, a_{-i}(\pi^*)) + \omega_i^* = u_i(0, a_{-i}(\pi^*))$$

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$$u_i(1, a_{-i}(\pi^*)) + \omega_i^* = u_i(0, a_{-i}(\pi^*))$$

Proposition

With increasing externalities, optimal scheme is discriminatory

That is, if same $u_i(\cdot)$ *for all* i*, then* π^* *is arbitrary and*

$$\omega_i^* > \omega_j^* \iff \pi_i^* < \pi_j^*$$

Plan

- Part 1: Contractible actions
  - Exogenous externalities
  - Endogenous externalities: Halac, Kremer, and Winter (2020)
- Part 2: Hidden actions
  - Public contracts
  - Private contracts
- Part 3: Hidden types
  - Monopolist problem

Investment

- Principal (firm) raises capital from multiple agents (investors)
- Principal's project succeeds or fails
  - $P : \mathbb{R}_+ \to [0, 1]$, strictly increasing
  - Success yields value $V > 0$
- Each agent $i \in N = \{1, \ldots, N\}$ has capital endowment $\overline{x}_i$

Contracts

- For each i, contract specifies investment $x_i \in [0, \overline{x}_i]$, returns (r_i, k_i)
  - r_i if success; k_i if failure
  - $a_i = 1$ means invest x_i in project
  - $a_i = 0$ means invest x_i in safe asset with return $\theta > 0$

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- Given $(a_1, \ldots, a_N)$, agent i's payoff is

$$\left[P\left(\sum_j a_j x_j\right) r_i + \left(1 - P\left(\sum_j a_j x_j\right)\right) k_i\right] a_i x_i + \theta(1 - a_i) x_i$$

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- Principal's budget constraint (BC) is

$$\sum_i r_i a_i x_i \leq V \quad \text{and} \quad \sum_i k_i a_i x_i \leq 0 \quad \forall a = (a_1, \ldots, a_N)$$

Principal's problem

- Two-step approach:
  1. For fixed $(x_i)_i$, find optimal $(r_i, k_i)_i$
  2. Given step 1, find optimal $(x_i)_{i \in N}$

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    1. For fixed $(x_i)_i$, find optimal $(r_i, k_i)_i$
    2. Given step 1, find optimal $(x_i)_{i \in N}$
- (W) requires $E((r_i + \varepsilon, k_i)_i) = \{a^1\}\ \forall \varepsilon > 0$
- Let $X_N := \sum_i x_i$. Principal solves

$$\min_{(r_i,k_i)_i} \sum_i \left[P\left(X_N\right) r_i x_i + \left(1 - P\left(X_N\right)\right) k_i x_i\right]$$

subject to (BC) and (W)

Optimal scheme

- By (BC) and $\theta > 0$, must set $r_i > 0 \geq k_i \ \forall i$
- Implies supermodular game, so ranking lemma applies

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- By (BC) and $\theta > 0$, must set $r_i > 0 \geq k_i \ \forall i$
- Implies supermodular game, so ranking lemma applies
- Optimal scheme specifies π^* and $(r_i^*, k_i^*)_i$ s.t. $\forall i$

$$r_i^* P\left(X_i(\pi^*)\right) + k_i^*\left(1 - P\left(X_i(\pi^*)\right)\right) = \theta$$

where

$$X_i(\pi) := \sum_{j:\pi_j \leq \pi_i} x_j$$

Optimal returns

- First characterize returns, then solve for permutation

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Proposition

Optimal scheme specifies π^* *and* $(r_i^*, k_i^*)_i$ *s.t.* $\forall i$

$$r_i^* = \frac{\theta}{P\left(X_i(\pi^*)\right)} \quad \textit{and} \quad k_i^* = 0$$

Optimal permutation

- Optimal permutation π^* minimizes $\sum_i \frac{\theta}{P\left(X_i(\pi)\right)} x_i$

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Proposition

Suppose $1/P(x)$ *convex over* $[0, X]$

For any $(x_i)_i$ *with* $X_N \leq X$*,* π^* *satisfies*

$$\pi_i^* < \pi_j^* \implies x_i \geq x_j$$

Hence, larger investors receive higher net returns than smaller investors

Example

- $\theta = 10\%,\ (x_1, x_2) = (10, 20),\ P(x) = \dfrac{x}{30}$

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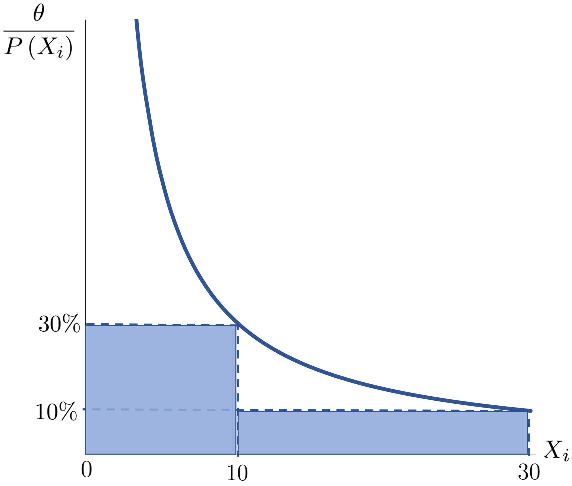

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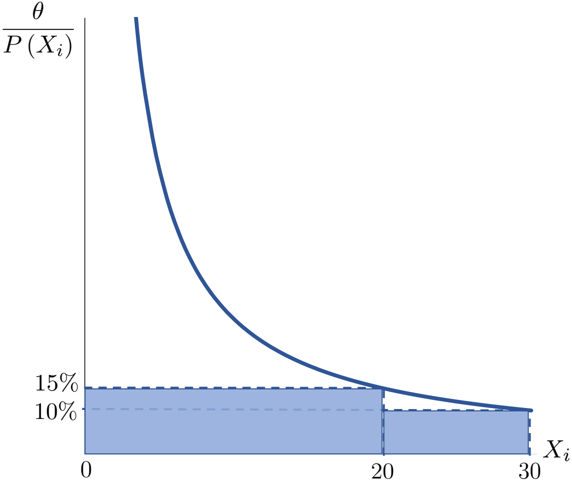
- $\theta = 10\%,\ (x_1, x_2) = (10, 20),\ P(x) = \dfrac{x}{30}$



Optimal investments

- So far $(x_i)_i$ as given. What are the optimal investments?

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Proposition

If $(\widehat{x}_i)_i$ majorizes $(x_i)_i$, principal's cost is lower under $(\widehat{x}_i)_i$

Corollary

Given $(\overline{x}_i)_i$, principal raises capital from agents with largest endowments

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  - Public contracts: Winter (2004)
  - Private contracts
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Teamwork

- Principal induces team of agents to exert effort
  - $a_i \in \{0, 1\}$ is hidden action
  - Effort costs $(c_i)_i$ with $c_i > 0 \ \forall i$

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  - $a_i \in \{0, 1\}$ is hidden action
  - Effort costs $(c_i)_i$ with $c_i > 0 \; \forall i$
- Principal's project succeeds or fails
  - $P : \{0, 1, \ldots, N\} \to [0, 1]$, strictly increasing and convex
- Scheme specifies success-contingent bonuses $b = (b_i)_i$
  - Agents protected by limited liability

Principal's problem

- Given $(a_1, \ldots, a_N)$, agent i's payoff is

$$P(|j : a_j = 1|)b_i - a_i c_i$$

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- Principal solves

$$\min_b P\left(N\right) \sum_i b_i \quad \text{subject to (W)}$$

Example

- 2 agents, $c_i = c,$ project succeeds with prob. $\begin{cases} 1 & \text{: both work} \\ \alpha^2 & \text{: both shirk} \\ \alpha & \text{: one each} \end{cases}$

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- To make work an equilibrium at least cost, pay both agents

$$b_L := \frac{c}{1-\alpha}$$

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- To make work an equilibrium at least cost, pay both agents

$$b_L := \frac{c}{1-\alpha}$$

- To make it unique equilibrium, pay one agent

$$b_H := \frac{c}{\alpha(1-\alpha)}$$

and then b_L to the other agent

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- Supermodular game, so ranking lemma applies

Proposition

Optimal scheme specifies π^* *and* b^* *s.t.* $\forall i$

$$b_i^* = \frac{c_i}{P\left(|j : \pi_j \leq \pi_i|\right) - P\left(|j : \pi_j < \pi_i|\right)}$$

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That is, if $c_i = c \; \forall i$, then π^* is arbitrary and

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  - Private contracts: Halac, Lipnowski, and Rappoport (2021)
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Private contracts

- Incentive scheme $\sigma = \langle T, g, B \rangle$:
  - $T = \prod_i T_i$, where each T_i is finite (WLOG $T_i \subseteq \mathbb{N}$)
  - $g \in \Delta T$ (WLOG g_i has full support on T_i)
  - $B = (B_i)_i$, where $B_i : T_i \to \mathbb{R}_+$ is i's bonus from success

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- (W) requires $E(\langle T, g, B + \varepsilon \rangle) = \{a^1\}$ $\forall \varepsilon > 0$
  - Where $E(\sigma)$ is set of BNE under σ, and $a := (a_i(t_i))_{i,t_i}$

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  - Where $E(\sigma)$ is set of BNE under σ, and $a := (a_i(t_i))_{i,t_i}$
- Principal solves

$$\inf_{\sigma} P(N) \sum_i \sum_{t_i} g_i(t_i) B_i(t_i)$$

subject to (W)

Example: Recall public contracts

- 2 agents, $c_i = c,$ project succeeds with prob. $\begin{cases} 1 & \text{: both work} \\ \alpha^2 & \text{: both shirk} \\ \alpha & \text{: one each} \end{cases}$

- To make work unique equilibrium with public contracts, pay one agent

$$b_H := \frac{c}{\alpha(1-\alpha)}$$

  and then pay the other agent

$$b_L := \frac{c}{1-\alpha}$$

- First agent reassures second agent

Example: Introduce private contracts

- Now suppose one agent offered private contract with random bonus:

$$b_H \text{ or } b_L, \text{ each with prob. } \frac{1}{2}$$

- And the other agent is offered

$$b_M := \frac{c}{\frac{1}{2}\alpha(1-\alpha) + \frac{1}{2}(1-\alpha)}$$

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- Agents "reassure" each other $\implies$ both work

Example: Principal's cost and discrimination

- $b_M < \frac{1}{2}b_H + \frac{1}{2}b_L$

  $\Longrightarrow$ Total average payments decrease with private contract

- $b_L < b_M < b_H$

  $\Longrightarrow$ Less transparency can mean less discrimination

- In fact, we show the optimal scheme eliminates discrimination

Ranking schemes

- $\sigma = \langle T, g, B \rangle$ is a ranking scheme if:
  - Every distinct i, j have $g\{t : t_i = t_j\} = 0$
  - Every i and t_i have

$$B_i(t_i)\ \mathbb{E}_g \left[P(|j:\ t_j \leq t_i|) - P(|j:\ t_j < t_i|) \ \middle|\ t_i \right] = c_i$$

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Lemma

1. *Every ranking scheme satisfies (W)*
2. *Any scheme satisfying (W) is dominated by some ranking scheme*

Incentive costs

- Let Π be set of permutations on N
  - Each t (without ties) induces an agent ranking $\pi(t) \in \Pi$
  - Ranking scheme σ induces ranking distribution $\mu^\sigma \in \Delta\Pi$
  - Type t_i has ranking belief $\mu_i^\sigma(\cdot|t_i) \in \Delta\Pi$

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- Given $\mu_i \in \Delta\Pi$, let

$$\frac{c_i}{\mathbb{E}_{\pi\sim\mu_i}\left[P(|j:\ \pi_j \leq \pi_i|) - P(|j:\ \pi_j < \pi_i|)\right]}$$

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- Given $\mu_i \in \Delta\Pi$, let

$$f_i(\mu_i) := \frac{c_i}{\mathbb{E}_{\pi\sim\mu_i}\left[P(|j:\ \pi_j \leq \pi_i|) - P(|j:\ \pi_j < \pi_i|)\right]} \cdot P(N)$$

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- A ranking scheme $\sigma = \langle T, g, B\rangle$ costs the principal

$$\sum_i \mathbb{E}_{t_i\sim g_i}\, f_i\Big(\mu_i^\sigma(\cdot|t_i)\Big)$$

The optimal value

- Principal chooses profile of distributions over ranking beliefs
  - But constrained: if increase an agent's belief, must lower another's
- Interpret as choosing average ranking distribution plus information

The optimal value

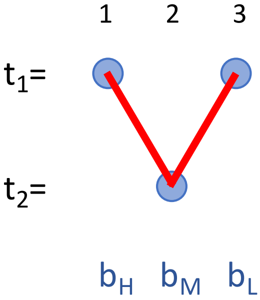
- Principal chooses profile of distributions over ranking beliefs
  - But constrained: if increase an agent's belief, must lower another's
- Interpret as choosing average ranking distribution plus information
- Show problem reduces to optimizing over average ranking distribution:

Theorem

Principal's optimal value is

$$\min_{\mu \in \Delta\Pi} \sum_i f_i(\mu)$$

Back to example



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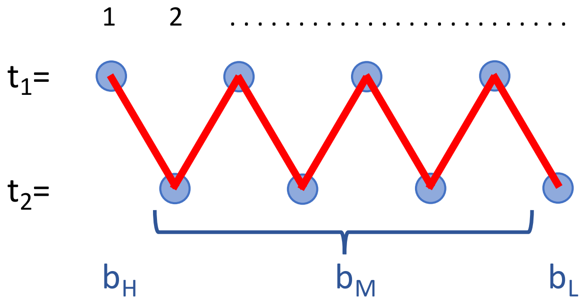

Optimal scheme

- Auxiliary program characterizes optimal incentives:

Theorem

There is unique bonus profile b^* which minimizes $\sum_i b_i$ among all

$$b \in \left\{ \frac{1}{P(N)} \left(f_1(\mu), \ldots, f_N(\mu) \right) : \; \mu \in \Delta\Pi \right\}$$

A sequence $(\sigma^m)_m$ that satisfies (W) is optimal iff the limit bonus distribution under σ^m (exists and) is degenerate on b^*

No discrimination

Corollary

If $c_i = c_j$, then $b_i^* = b_j^*$ and every optimal $(\sigma^m)_m$ has

$$\mathbb{P}^m\{|b_i - b_j| < \varepsilon\} \to 1 \ \forall \varepsilon > 0$$

$\Longrightarrow$ No discrimination between identical agents; little between similar

$\Longrightarrow$ Rank uncertainty strictly optimal for similar agents

Plan

- Part 1: Contractible actions
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- Part 3: Hidden types
  - Monopolist problem: Halac, Lipnowski, and Rappoport (2024)

Monopolist problem

- Monopolist sells good to set of buyers
- Externalities: Buyer's benefit from good increases with # other buyers
- Hidden types: Buyer's benefit from good depends on private info

Setup

- Unit population of buyers. Seller offers personalized $p_i \in \mathbb{R}_+$ to each
  - Buyers have private types $\theta_i \in [\underline{\theta}, \overline{\theta}]$
- Given total purchased quantity $q \in [0, 1]$, buyer of type θ_i gets payoff

$$u(\theta_i, q) - p_i$$

if he buys at p_i, and zero if he does not buy

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- Today's presentation: take $\theta_i \sim U[0, 1]$ and $u(\theta_i, q) = \theta_i \overline{v}(q)$
  - With $\overline{v}(0) = 0$ and $1/\overline{v}(\cdot)$ convex

Seller's problem

- Quantity demanded and revenue from price distribution $\Pi \in \Delta(\mathbb{R}_+)$:

$$D_q(\Pi) := \int D_q(p)\ \mathrm{d}\Pi(p) \ \text{ where } D_q(p) := 1 - \frac{p}{\overline{v}(q)}$$

$$R_q(\Pi) := \int R_q(p)\ \mathrm{d}\Pi(p) \ \text{ where } R_q(p) := pD_q(p)$$

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- Seller's optimal value is

$$\sup_{\Pi \in \Delta(\mathbb{R}_+)} \quad \min_{q^* \in [0,1]} \quad R_{q^*}(\Pi)$$

$$\text{subject to} \quad D_{q^*}(\Pi) = q^*$$

Benchmark 1: Complete information

- Suppose no hidden types: θ_i's are observable

Benchmark 1: Complete information

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- Suppose no hidden types: θ_i's are observable
- Monopolist sells to everyone using ranking scheme
  - Offer each buyer price that makes him indifferent if only preceding buy
- Cannot apply same methodology under incomplete information
  - Seller cannot control order of deletion of dominated strategies
  - New approach: work with anticipated q rather than buyer types

Benchmark 2: Best-case implementation

- Suppose seller can select her preferred equilibrium. Then problem is

$$\sup_{\Pi \in \Delta(\mathbb{R}_+)} \quad \max_{q^* \in [0,1]} \quad R_{q^*}(\Pi)$$

$$\text{subject to} \quad D_{q^*}(\Pi) = q^*$$

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Proposition

Under best-case implementation, every optimum has degenerate Π

Worst-case implementation

- Externalities mean other equilibria under any posted $p > 0$
  - Worst equilibrium has zero revenue
- Optimal Π under worst-equilibrium selection must be non-degenerate
- What is the optimal form of price dispersion?

Which constraints matter?

Proposition

Under worst-case selection, (q^*, Π^*) *is optimal iff it solves*

$$\max_{q\in[0,1],\ \Pi\in\Delta(\mathbb{R}_+)} \quad R_q(\Pi)$$

$$\textit{subject to} \qquad D_{\hat{q}}(\Pi) \geq \hat{q} \quad \forall \hat{q} \in (0, q)$$

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- Let $\Gamma : \mathbb{R}_+ \to \mathbb{R}_+$ right-continuous, nondecreasing. Say Γ is *greedy* if

$$D_{\hat{q}}(\Gamma) = \hat{q} \quad \forall \hat{q} \in (0, 1)$$

Optimal price distribution

Theorem

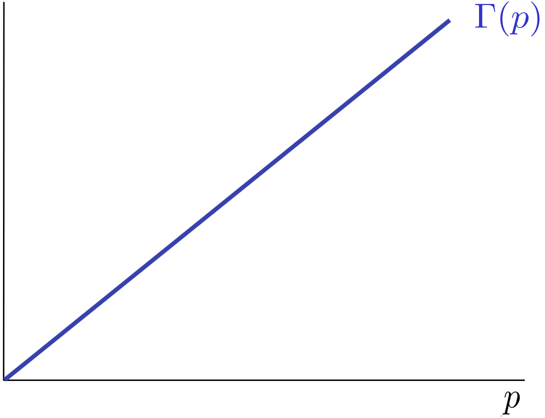
Any optimal Π^* *is greedy up to*

$$p^* := \max \ \mathrm{Supp}(\Pi^*) < \overline{v}(q^*),$$

with mass point at p^*

Example 1

- Take $u(\theta, q) = \theta q$ and $\theta \sim U[0,1]$
  - $\Gamma(p) = p/\mathbb{E}[\theta]$ satisfies $D_q(\Gamma) = q$ for all $q \in [0,1]$



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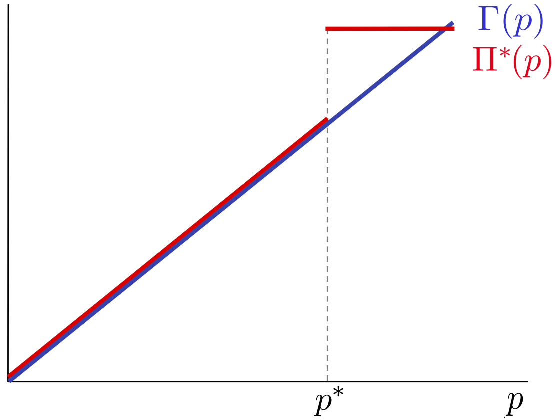
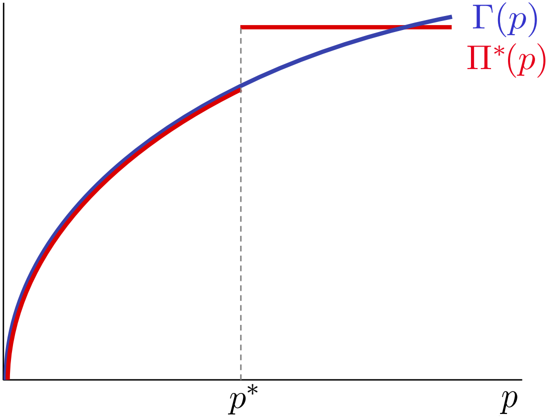


Example 2

- Take $\theta \sim U[0,1]$ and $u(\theta, q) = \theta \overline{v}(q)$ with $\overline{v}(q) = q^2$
  - $\Gamma(p) = (3/2)\sqrt{p}$ satisfies $D_q(\Gamma) = q$ for all $q \in [0,1]$



Effects of externalities

- Seller induces higher max price and higher quantity than in best-case
- If stronger externalities, higher quantity and lower weight on low p's
- If groups of heterogeneous externalities, build demand weak to strong

Concluding remarks

- Contracting for coordination arises in many applications
- Possibility of multiple equilibria calls for robust approach
- We studied principal's optimal worst-case implementation scheme
  - Implications for contracts and outcomes in organizations and markets
  - And still many open questions!

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Thank you!