Costly Screening and Categorical Inequality*

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Abstract

When characteristics of individuals are imperfectly observed, markers of group membership such as ethnicity and gender can come to be used as proxies. The incentive effects of such stereotypes have previously been explored in the literature on statistical discrimination. This paper endogenizes the degree to which information is imperfect, which itself can be group-contingent. We consider a rationally inattentive screener who evaluates a pool of candidates composed of distinct and observable social categories. There is heterogeneity across categories along multiple dimensions, including the costs of being screened, the degree of bias faced in the screening process, and the manner in which costs of investment in skills is distributed. Candidates choose how much effort to invest in training before being screened, with a payoff in a post-screening market that depends on the screening outcome. We characterize equilibrium in this model, and use it to unify and extend several strands in the literature on categorical inequality, including statistical discrimination, prejudice, and social capital.

Keywords: Categorical Inequality, Statistical Discrimination, Identity, Rational Inattention, Bregman information

JEL: D8, D9, J15, J7

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1 Introduction

When information about relevant characteristics of an individual is unavailable or imprecise, other attributes—such as perceived race, ethnicity, gender, or religion—can come to be used as proxies. The use of such proxies or stereotypes has incentive effects: people adapt their behavior in response to the manner in which they are perceived, and the choices they make affect the actual distribution of characteristics in the groups to which they belong. This is the essence of the idea of statistical discrimination, as formulated in the pioneering work of Phelps (1972) and Arrow (1973).

But the information problem that gives rise to statistical discrimination can be mitigated by screening, such as close scrutiny of application materials, lengthy job interviews, multiple reference letters, and so on. Such screening activities are costly, with greater precision about individual characteristics available at greater expense of time, effort, and money. Furthermore, the expected net benefits of careful screening are contingent on prior beliefs about the distribution of characteristics in the group to which a candidate is perceived to belong. These beliefs may be affected by social realities but also by outright prejudice. And the costs of screening may also be group contingent, for instance if the candidate belongs to a different group than the screener. The implications of this for the nature and extent of statistical discrimination, and for the persistence of categorical inequality more generally, are the topic of this paper.

We consider a rationally inattentive screener facing a pool of candidates who differ along two dimensions. One is the category or social group to which candidates belong, assumed to be observable without cost. The other is their individual qualifications, which we take to be the product of costly and unobserved effort. Qualified candidates have valuable attributes that unqualified candidates lack, and the screener seeks to accept the former while rejecting the latter. When deciding how carefully to examine candidates, the screener balances the benefits of successful sorting against the cost of screening. We thus analyze a screening device in the sense of Stiglitz (1975), fundamental to the vast field of information economics. Where Stiglitz emphasizes the possibility that candidates may strive to signal their qualification at some cost, for example by self-selecting into screening, this paper uses the rational inattention framework to emphasize that the screener may actively choose how much information to acquire about each candidate.

We consider multiple candidate pools, which we call groups or categories, with heterogeneity across and within groups. The cost of investment in becoming qualified can vary across individuals, reflecting idiosyncratic differences in ability or opportunity, and the costs can vary across categories. Also, the unit cost of screening can vary across categories, for example because of greater difficulty in evaluating candidates outside the screener's own cultural or social group. The benefit to the screener of accepting a candidate may depend on not only on the candidate's qualifications but also on the candidate's group identity, reflecting prejudice or favoritism. This generality and flexibility of the model, along with the endogenous treatment of the screener's information acquisition, allows us to explore a number of themes that have been central to the literature on categorical inequality, and to unify and extend this literature in several ways.

One important strand of this literature deals with statistical discrimination, or the unequal treatment of ex ante identical groups via the incentive effects of stereotypes (Arrow, 1973; Spence, 1973; Coate and Loury, 1993; Moro and Norman, 2004). This would correspond to identical distributions of characteristics across categories, including the costs of human capital investment and screening. Unequal treatment can then persist only if there are asymmetric equilibria, which arise generically in our model. In particular, one group may be screened actively and hired conditional on a good signal, while another may be passed over entirely and given no incentive to invest.

A closely related literature, also bracketed under statistical discrimination, allows for the possibility that the productivity characteristics of one group may be observed with greater precision than those of another (Phelps, 1972; Aigner and Cain, 1977; Cornell and Welch, 1996). In our model this corresponds to differential costs of screening. Here one finds unequal treatment even if one focuses on the best or most productive equilibrium for each category. In particular, a group that is costlier to screen will end up being screened less intensively, which will compress the wage difference between accepted and rejected candidates. This blunts the incentive to invest in becoming qualified, resulting in a lower rates of qualification in equilibrium.

Third, there is a literature on preference-based discrimination or prejudice, which involves unequal treatment of individuals even conditional on expected productivity (Becker, 1957; Blinder, 1973; Oaxaca, 1973; Bertrand and Mullainathan, 2004). In our model this corresponds to screener payoffs that depend not just on the qualifications of accepted candidates, but also on the groups to which they belong. We consider two different ways in which a group may face prejudice: lower rewards to the screener from accepting a qualified candidate, and higher penalties to the screener from accepting an unqualified candidate. In both cases, the screening interval—the set of prior beliefs under which active screening occurs—is shifted to the right, which means that some candidates are rejected without screening who would otherwise have been screened, and some are screened who would otherwise have been accepted without screening. However, we show that these two specifications of prejudice can have opposite effects on equilibrium qualification rates in the group facing discrimination.

Finally, persistent categorical inequality has been linked to peer and neighborhood effects in the presence of residential segregation and homophily in social interactions (Loury, 1977; Benabou, 1993; Durlauf, 1996; Lundberg and Startz, 1998; Mookherjee et al., 2010; Bowles et al., 2014). This corresponds to differences across groups in the distributions of investment costs. A group characterized by a less advantageous cost distribution (in the sense of first order stochastic dominance) will have a lower equilibrium qualification rate, as might be expected. But the intensity of screening will adjust in such a manner as to leave unaffected the posterior beliefs about candidates conditional on acceptance or rejection by the screener. That is, the likelihood of being qualified will not be different across groups in equilibrium,

The screening literature typically works with specific information structures that are tailored to the problem at hand. Rational inattention provides a departure from this, allowing us to incorporate information acquisition without the loss of generality associated with having to impose specific structures (Sims, 2003, 2010). Our model for the discrete choice of whether to accept or reject a candidate builds on Matějka and McKay (2015), who allow information acquisition to be endogenously determined according to a certain information cost in relation to prior information. We go further, applying a general Bregman information (Banerjee et al., 2005), which allows us to remain agnostic about the exact form of the information cost.

In general, it can be difficult to formulate hypotheses regarding the prior information in applications of the rational inattention model; in the present case we have the natural option of taking the prior to be the population frequency of qualified candidates. For reasons discussed above, this frequency will be category-contingent in equilibrium, with statistical discrimination, prejudice, and social capital all present and interacting within a single unifying framework.

There are many different environments to which the model can be applied. In the case of labor markets, the screener can be viewed as an educational institution or professional association that offers certification of competence. Alternatively, the candidates could be viewed as firms facing a quality control board, in which case those who receive approval can charge higher prices. Other applications include certification of firms with respect to environmental impact or labor practices, or quality judgments regarding authors, artists, and academic researchers by publishers, reviewers, and funding agencies respectively. Our results should be relevant to all such cases, and indeed to any environment with costly screening and greater compensation for those who achieve a positive screening outcome.

2 Model

We model a heterogeneous population of candidates who face a screener. Candidates choose efforts to pass the screener's test, while the screener chooses the informativeness of the test. Initially, all candidates are unqualified. The more effort a candidate makes, the more likely it is that she will become qualified. Transition to the qualified state depends stochastically on effort, the cost of which may vary across individuals and social groups.

Once candidates have chosen their effort levels and reached their final state, qualified or unqualified, the screener evaluates them and decides whether to accept or reject each one. The screener aims at only accepting qualified candidates, but screening is costly and the screener is only imperfectly able to distinguish qualified from unqualified candidates. Given her expectation about the share of qualified candidates in the applicant pool, the screener decides how much to spend on screening, and how to use the information obtained to decide whom to accept. Spending more resources on screening reduces the probability of mistakes, but mistakes of both types will occur with positive probability.

Candidates accepted by the screener can expect higher earnings. These are determined under competitive conditions in post-screening markets, one for those who were accepted and one for those who were rejected. In equilibrium, candidates adapt their efforts according to how well the screener can distinguish qualified from unqualified candidates, the screener adapts her information acquisition and acceptance decisions to the true state distribution in the pool of candidates, and the post-screening market sets compensation according to the proportions of qualified candidates among those who were accepted or rejected by the screener, respectively.

2.1 The candidates

The population of candidates is treated as a continuum. Each candidate $i \in [0, 1]$ has some individual trait or *characteristic* $\theta \in \Theta$, and belongs to a group or *category*, $\kappa \in K$. Each candidate's *type* $\tau = (\theta, \kappa) \in \Theta \times K$ is fixed and given, and the set of types $\Theta \times K$ is finite. A candidate's characteristic θ is her private information, unobservable to others, while the category κ to which a candidate belongs is costlessly observed without error. Let $\Delta(\Theta)$ be the set of all probability distributions over Θ, and let $\mu_\kappa \in \Delta(\Theta)$ be the distribution of characteristics in category $\kappa \in K$.

One interpretation is that candidates are individuals who engage in education or job training, and categories are defined in terms of some cluster of non-falsifiable and easily observed markers that lead observers to assign candidates to distinct groups, such as gender, race, ethnicity, or religion. Even if all that matters for employers is an individual's personal characteristic θ, the category to which the candidate belongs may affect employers' beliefs, and these may then indirectly influence the individual's training efforts, and thereby the individual's employment outcome. For example, a job candidate who belongs to a negatively perceived category may rationally choose to make less effort (or none at all) than another candidate with identical characteristic θ but belonging to a more positively viewed category, as in Arrow (1973) and Coate and Loury (1993). However, our focus here will not be on asymmetric equilibria with ex ante identical groups, but rather on the implications of differences across groups in primitives, such as the difficulty of screening, the distribution of investment costs, and the rewards and penalties faced by the screener conditional on the accuracy of screening outcomes.

All candidates are initially in the unqualified state but may probabilistically move to the qualified state by investing or making effort. Let $S_i = 1$ if candidate i attains the qualified state, and $S_i = 0$ if she does not. The probability that any candidate i who exerts effort (or makes investment) $x_i \geq 0$, achieves the qualified state is a strictly increasing function F of effort:

$$\Pr\left[S_i = 1\right] = F\left(x_i\right), \tag{1}$$

where $F : \mathbb{R}_+ \to [0, 1]$ is twice differentiable with $F' > 0$, $F(0) = 0$ and $\lim_{x\to\infty} F(x) = 1$. Hence, it is a cumulative probability distribution function with differentiable and positive density $f = F'$. We assume this density to be bounded and unimodal. Let $\mathcal{F}$ denote the class of such cumulative probability functions.

The success function F being taken to be the same for all candidates, irrespective of type, one may interpret a candidate's effort x_i as measured in efficiency units. Candidates (within any given category) may differ with respect to their marginal cost or disutility of effort (per efficiency unit) . We take this marginal cost or disutility to be a positive constant for each candidate, determined by the candidate's individual characteristic θ. For ease of notation, we write θ for the marginal cost or disutility. Thus Θ is a finite set of positive real numbers. A candidate's marginal cost of effort θ may be thought of as representing the combined effects of the candidate's endowments, preferences, technology, and learning opportunities, including social characteristics such as access to quality schooling and networks rich in human capital. Some candidates will have lower costs θ per efficiency unit of effort than others, and the distributions of such costs may differ between categories $\kappa \in K$, as represented by the distributions μ_κ.

Having invested effort, each candidate probabilistically either remains in the unqualified state or transits to the qualified state. Thereafter, the candidate is screened. Let $Z_i = 1$ if candidate i is accepted by the screener, and $Z_i = 0$ if rejected. This decision by the screener will affect the future prospects of the candidate. The total utility achieved by an candidate i of type (θ, κ) who exerts effort $x_i \geq 0$ is a random variable that depends on x_i, Z_i and θ as follows:

$$U\left(x_i, Z_i, \theta\right) = Z_i v_1 + (1 - Z_i) v_0 - \theta x_i. \tag{2}$$

Here v_1 and v_0 are the present values to the candidate of the future stream of utilities, wages or profits (depending on the application) obtained after screening by candidates who pass and fail the screener's test, respectively.[1]

[1] We thus exclude the possibility that v_0 and v_1 also depend on the candidate's state S_i. Such dependence

2.2 The screener

The screener does not know candidates' individual characteristics θ nor their individual efforts x_i or personal state S_i obtained. The screener bases her choice of screening intensity on the probabilistic belief $p \in [0,1]$ that she attaches to the event that a randomly drawn applicant is qualified (in state $S = 1$). We call p the screener's *prior*, which in equilibrium will match the endogneously determined share of qualified individuals in the pool from which the candidate is drawn.

The screener receives a bonus $\beta > 0$ for every accepted qualified candidate, and pays a penalty $\gamma > 0$ for every accepted unqualified candidate. Once her screening costs are sunk, her expected additional payoff thus takes the form

$$\pi = \beta \Pr[Z = 1 \wedge S = 1] - \gamma \Pr[Z = 1 \wedge S = 0]. \tag{3}$$

In order to maximize her expected total payoff, the screener thus strives to raise the accept rate of qualified candidates and lower the accept rate of unqualified candidates, net of the cost of screening, where more informative screening is costlier than less informative screening.

Let $y_0 = \Pr[Z = 1 \mid S = 0]$ denote the accept rate of unqualified candidates, and $y_1 = \Pr[Z = 1 \mid S = 1])$ the accept rate of unqualified candidates. The screener can modify these rates by way of costly information acquisition. The two probabilities, y_0 and y_1, are thus (costly) decision variables in the hands of the screener. Writing y for the screener's overall acceptance rate, $\Pr[Z = 1]$, we have

$$y = py_1 + (1 - p)\, y_0. \tag{4}$$

This is the probability that a randomly drawn candidate will be accepted by the screener.

Conditional upon acceptance, the posterior probability that the candidate is qualified is

$$q_1 = \Pr[S = 1 \mid Z = 1] = \frac{py_1}{y}. \tag{5}$$

Likewise, conditional upon rejection, the posterior probability that the candidate is qualified is

$$q_0 = \Pr[S = 1 \mid Z = 0] = \frac{p\,(1 - y_1)}{1 - y}. \tag{6}$$

These conditional probabilities depend on the prior p and on the screener's choice of y_1 and y_0, and

$$\pi = \beta q_1 y - \gamma\,(1 - q_1)\, y$$

can be handled within the present framework but at the cost of more notation and analysis, but without much added insight.

is the screener's expected payoff.[2]

Within limits, and subject to the constraint imposed by the prior p, the screener can choose the accept rates y_0 and y_1. An easy test that most candidates pass would lead both these rates to be close to one, while a very challenging one that most applicants fail would result in both accept rates being close to zero. Tests of intermediate difficulty would better distiniguish between qualified and unqualified candidates, but would also be more costly. Given this trade-off, the screener must decide just how informative to make the screening procedure.

In order to address this problem, we need a model of costly information acquisition. We adopt a specification under which the informativeness of screening is represented by the Bregman information (Banerjee et al., 2005) between screening outcome and the candidate's state, which generalizes the rational inattention approach of Matějka and McKay (2015).

Let $G : [0, 1] \to \mathbb{R}$ be any twice differentiable convex function of Legendre type.[3] Then

$$I = \mathbb{E}\left[G\left(q_Z\right)\right] - G\left(\mathbb{E}\left[q_Z\right]\right) = yG\left(q_1\right) + \left(1-y\right)G\left(q_0\right) - G\left(p\right)$$

is the associated *Bregman information* of the random posterior q_Z, given the prior p (see Banerjee et al., 2005). As shown in Caplin et al. (2019), I defines a uniformly posterior-separable information cost function, measuring the cost of obtaining a conditional posterior from a prior in terms of the expected distance between the two distributions, with distance measured as the convexity gap, $G\left(q_Z\left(p\right)\right) - G\left(\mathbb{E}\left[q_Z\left(p\right)\right]\right) - \left(q_Z\left(p\right) - \mathbb{E}\left[q_Z\left(p\right)\right]\right)G'\left(\mathbb{E}\left[q_Z\left(p\right)\right]\right)$. In particular, if the screener chooses not to screen candidates, then $q_0 = q_1 = p$ and the information cost is zero. A special case is the Shannon mutual information, the most commonly used information measure in the rational inattention literature. Then G is negative entropy (Cover and Thomas, 2006, Ch. 2):[4]

$$G(q) = q\ln q + (1-q)\ln(1-q) \quad \forall q \in [0,1].$$

In the sequel, we will use this as the running example.

The screener's expected total payoff, given her prior p and a Bregman information cost I is her expected benefit of accepting qualified candidates net of the cost of accepting unqualified candidates and net of the cost of information acquisition. This is the objective function that the screener tries to maximize by way of choosing y_0 and y_1, for any given belief $p \in [0, 1]$.

[2] Note that the subscripts on q and y have somewhat different interpretations; in the former case they refer to whether or not the candidate passes the test, and in the latter case to whether or not the candidate is qualified.

[3] Strictly convex, differentiable with a derivative that becomes infinite at the boundary (Bauschke and Borwein, 1997).

[4] With the convention $0 \cdot \ln 0 = 0$.

Recalling that y, q_0 and q_1 are functions of y_0, y_1 and p, the screener's decision problem can be summarized as

$$\max_{(y_0,y_1)\in[0,1]^2} \Pi\left(y_0, y_1, p\right) \tag{7}$$

where

$$\begin{aligned} \Pi\left(y_0, y_1, p\right) &= \pi - I \\ &= \left(\beta+\gamma\right) p y_1 - \gamma y - yG\left(p \cdot \frac{y_1}{y}\right) - \left(1-y\right) G\left(p \cdot \frac{1-y_1}{1-y}\right) + G\left(p\right) \end{aligned} \tag{8}$$

and y is defined in (4). Without loss of generality we have normalized the unit information cost to unity. Comparative statics with respect to the information cost can be carried out by scaling β and γ by the same positive factor.

2.3 Candidates Payoffs

The market, after screening, does not observe a candidate's true qualification, S, but sees only whether the candidate was accepted or rejected by the screener. We assume that the present value of future utilities, wages or profits equals the candidate's expected qualifications, given the screening outcome. Hence, $v_0 = q_0\left(p\right)$ and $v_1 = q_1\left(p\right)$. The utility (or wage or profit) premium from passing the screening is thus $q_1\left(p\right) - q_0\left(p\right)$. In making investment choices, candidates will take into account this payoff differential, along with the likelihood of becoming qualified conditional on effort, and their idiosyncratic cost of effort, in a manner to be described below.

2.4 Equilibrium

We solve for perfect Bayesian equilibrium. First each candidate exerts her chosen level of effort (or makes her investment). Then each candidate's state is realized, according to the probability distribution (1). Since there is a continuum of candidates, none of them has any strategic power. The screener knows each candidate's category, κ, but not the candidates' individual characteristic θ, effort x_i, or achieved state S_i. Based on her prior belief p regarding the proportion qualified in the pool from which the candidate is drawn, the screener optimally chooses the level of information acquisition for each candidate, and her conditional acceptance rule. Finally, the post-screening markets clear. In equilibrium, the screener's expectation about the composition of the candidate pool for each category is correct, and each candidate holds correct expectations about the screener's strategy.

Since there is no capacity constraint for accepting candidates, and no fixed cost, the screener's optimal strategy for each candidate category is independent of her strategy for any other category. Moreover, post-screening markets clear separately for each category. Without loss of generality, we may therefore analyze each candidate category separately.[5] In order to ease notation, we suppress the category index κ when solving the model for each category.

3 Analysis

We solve the model by backward induction. Hence, we first analyze the screener's behavior, given an arbitrary prior p concerning the qualification rate in the applicant pool. We then calculate the associated post-screening outcomes, which motivate the candidates when choosing their effort levels. Given this, we finally turn to the candidates' effort choices.

3.1 The screener

Suppose, thus, that all candidates in the category in question have made their efforts and that their individual states, S_i, have been realized. Let $p \in [0,1]$ be the screener's belief about the population share of qualified candidates in the applicant pool. The screener's optimal strategy entails a choice of how much information to acquire about each candidate, and, once this information is obtained, whether to accept or reject the candidate in question. In order to solve for equilibrium, we need to determine this for any possible prior $p \in [0,1]$.[6]

Proposition 1. *For every $p \in [0,1]$ there exists a unique solution to the screener's decision problem (7). Moreover, there exists a nonempty interval $[p_{\min}, p_{\max}] \subseteq [0,1]$ such that the screener rejects all candidates without screening when $p \leq p_{\min}$ and accepts all candidates without screening when $p \geq p_{\max}$, while for $p \in (p_{\min}, p_{\max})$, the solution is*

$$\hat{y}_0(p) = \frac{1-p_{\max}}{1-p} \cdot \hat{y}(p) \quad \textit{and} \quad \hat{y}_1(p) = \frac{p_{\max}}{p} \cdot \hat{y}(p), \tag{9}$$

where $\hat{y}(p)$ increases linearly from zero to one on the active screening interval $(p_{\min}, p_{\max})$:

$$\hat{y}(p) = \frac{p-p_{\min}}{p_{\max}-p_{\min}}. \tag{10}$$

Furthermore, on this interval $\hat{y}_0(p)$ is strictly convex and $\hat{y}_1(p)$ is strictly concave.

[5] This would evidently not be the case if, for example, there were an overall acceptance capacity, or a desire to keep the shares of admitted candidates from different categories within certain bounds in accordance with representation targets, as in Fryer and Loury (2013).

[6] The screener's information cost belongs to the class of uniformly posterior-separable cost functions discussed by Caplin et al. (2017). The presentation here is self-contained, but Proposition 1 could alternatively be based on the locally invariant posterior property established by Caplin et al. (2017).

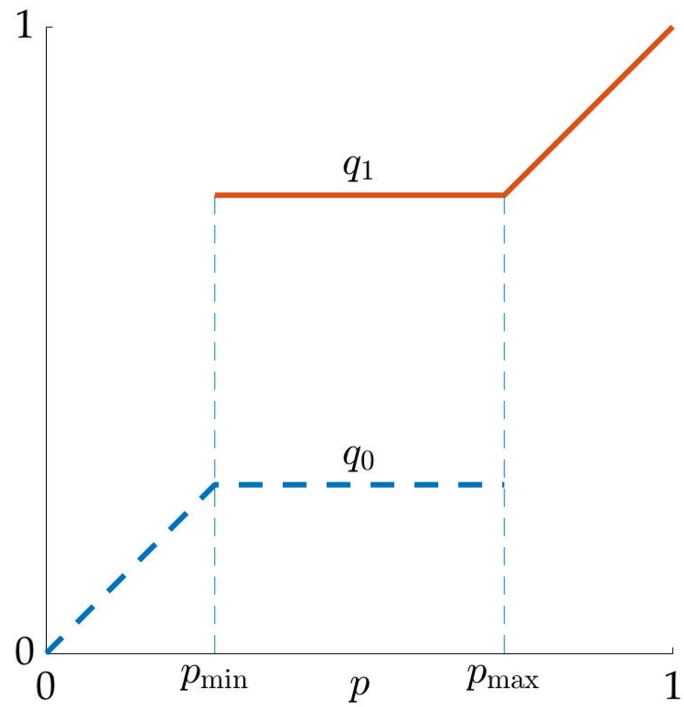

Figure 1: The screener's posterior beliefs about likelihood that a candidate is qualified, conditional on rejection (dashed) and acceptance (solid), as functions of the prior p.

It may be verified[7] that the screening interval satisfies

$$p_{\min} < \frac{\gamma}{\beta + \gamma} < p_{\max}.$$

As we show below, increasing the cost of information acquisition shrinks the screening interval towards the interior point $\gamma/(\beta+\gamma)$, which is the value of the qualification rate p at which a screener with infinite information cost would switch from blindly rejecting all candidates to blindly accepting all of them.

Using Proposition 1, together with (5) and (6), one readily obtains that the posterior beliefs about qualifications of those accepted and rejected by the screener are constant across the active screening interval $(p_{\min}, p_{\max})$. More precisely:

$$\hat{q}_0(p) = \begin{cases} p & \text{if } p \leq p_{\min} \\ p_{\min} & \text{if } p \in (p_{\min}, p_{\max}) \end{cases} \quad \text{and} \quad \hat{q}_1(p) = \begin{cases} p_{\max} & \text{if } p \in (p_{\min}, p_{\max}) \\ p & \text{if } p \geq p_{\max} \end{cases} \tag{11}$$

This remarkable simplicity is a fruit of the endogeneity of screening and the representation of information costs. As will be shown below, the constancy of the posteriors enables powerful

[7] See the proof of Proposition 7.

comparative statics results that help us better understand categorical inequality. Figure 1 illustrates the posteriors under optimal screening, with the dashed curve representing the posterior qualification rate $\hat{q}_0(p)$ among rejected candidates and the solid curve representing the posterior qualification rate $\hat{q}_1(p)$ among accepted candidates.[8]

If the qualification rate p in the pool of candidates is below $p_{\min}$, then it is not worthwhile for the screener to obtain any information about candidates, so they are all rejected and the posterior qualification rate of the rejected candidates is equal to the prior p. At the other extreme, when $p \geq p_{\max}$, it is also not worthwhile for the screener to obtain any information about candidates, and they are all accepted, so the posterior qualification rate among accepted candidates is then again p. For intermediate values of the prior, however, the screener acquires just enough information to accept or reject candidates such that the posterior qualification rates give the screener no incentive to acquire more information. This happens exactly when the posteriors are equal to the boundary points $p_{\min}$ and $p_{\max}$ of the active screening interval. These boundary points are uniquely determined by the indifference condition that the conditional posterior qualification rates there should equal the prior: $\hat{q}_0(p_{\min}) = p_{\min}$ and $\hat{q}_1(p_{\max}) = p_{\max}$. In the special case of Shannon mutual information, we obtain

$$p_{\min} = \frac{e^{\gamma} - 1}{e^{\beta+\gamma} - 1} \quad \text{and} \quad p_{\max} = \frac{e^{\gamma} - 1}{e^{\gamma} - e^{-\beta}}. \tag{12}$$

For any choice of conditional accept rates, the *screening intensity* is defined as the difference between them: $r = y_1 - y_0$. The optimal screening intensity is positive when the prior p is in the active screening interval and zero otherwise:

$$\hat{r}(p) = \hat{y}_1(p) - \hat{y}_0(p) = \max\left\{0, \frac{p_{\max} - p}{p(1-p)} \cdot \frac{p - p_{\min}}{p_{\max} - p_{\min}}\right\}. \tag{13}$$

Moreover,

Proposition 2. *The optimal screening intensity, $\hat{r}(p)$ is continuous in p and strictly concave on the active screening interval $(p_{\min}, p_{\max})$. Moreover, at any given $p \in (p_{\min}, p_{\max})$, $\hat{r}(p)$ is decreasing in $p_{\min}$ and increasing in $p_{\max}$.*

Figure 2 shows the screening intensity (solid) as well as the conditional accept rates (dashed, for qualified and unqualified candidates) as functions of the qualification rate p in the candidate pool. As can be seen, higher values of the prior p give rise to more lenient tests by the screener, which both qualified and unqualified candidates can more easily pass. This implies that if the screener has more pessimistic beliefs about the qualification rate for those in one group relative to another, the group facing the more pessimistic belief will encounter the more demanding test. Note, however, that as long as both groups are actively screened,

[8]The figure is based on Shannon mutual information with $\beta = \gamma = 1$.

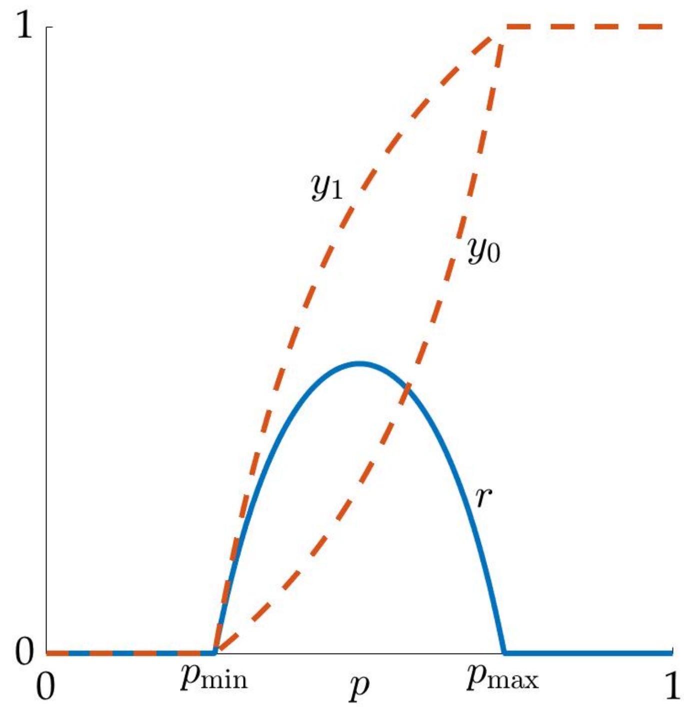

Figure 2: Screening intensity (solid) and conditional acceptance rates (dashed), as functions of the prior p.

posterior beliefs about the qualification rate among those who pass the test will not differ across groups. That is, the pessimistic prior will not correspond to pessimistic posteriors, since screening intensity will adjust to ensure parity across categories in beliefs conditional on test outcomes. This is true as long as the cost of screening doesn't differ across groups, so that they both face the same screening interval. We return to these issues when we consider categorical inequality more explicitly below.

3.2 The value of screening to candidates

When the post-screening markets open, candidates have made their efforts and p and r have already been set. If $p \in [p_{\min}, p_{\max}]$, then the candidates' post-screening utilities (wages or profits) are uniquely determined by (11):

$$v_0 = p_{\min} \quad \text{and} \quad v_1 = p_{\max}. \tag{14}$$

It follows that these utility levels are independent of where the qualification rate p falls inside the active screening interval . Moreover, whenever p falls in the active screening interval, the future values v_0 and v_1 do not depend on the screening intensity r. By contrast, if p lies outside the active screening interval, then $r = 0$. Anticipating this, no candidate makes any effort and all remain unqualified.

We note that since $p_{\max} - p_{\min} < 1$, the premium $v_1 - v_0$ from passing the test is less than the unit productivity difference between qualified ($S = 1$) and unqualified ($S = 0$) candidates. Screening, being costly, is always less than perfect.

In the special case of Shannon information, the following closed-form expression is obtained:

$$v_1 - v_0 = \frac{e^{\beta+\gamma} - e^{\beta} - e^{\gamma} + 1}{e^{\beta+\gamma} - 1}. \tag{15}$$

We note that the premium is increasing both in the screener's bonus β for accepting qualified candidates and in the screener's penalty γ for accepting unqualified candidates. The higher these parameters are in the screener's goal function, the more careful the screening.

3.3 The candidates

Having solved for the screener's behavior, we are now in a position to analyze the candidates' behavior. Each candidate holds some belief about the screener's conditional accept rates, y_0 and y_1, and treat these as fixed and given, along with the ensuing post-screening utilities, v_0 and v_1. The random utility to a candidate i with characteristic θ who makes effort $x_i \geq 0$ is then $U_\theta (Z_i, x_i)$, defined in (2), with v_0 and v_1 given in (14). The candidate's only choice variable is her effort. The *ex ante* expected utility from choosing any effort $x \geq 0$ is thus

$$\mathbb{E}\left[U\left(x, Z_i, \theta\right)\right] = F\left(x\right) \cdot \left[y_1 v_1 + \left(1 - y_1\right) v_0\right] + \left(1 - F\left(x\right)\right) \cdot \left[y_0 v_1 + \left(1 - y_0\right) v_0\right] - \theta x.$$

The first term is the probability that the candidate attains the high state, multiplied by the expected utility in that case. Similarly, the second term is the probability of not attaining the high state, multiplied by the expected utility in that case. The final term is the candidate's cost of effort.

The necessary first-order condition for a positive effort level x to be optimal for a candidate with characteristic θ, which we write as $x = \hat{x}_\theta$, can thus be written in the form

$$f\left(x\right) \cdot r \cdot \left(v_1 - v_0\right) \; = \; \theta, \tag{16}$$

where $r = y_1 - y_0$ is the screening intensity as defined above. If this is zero—which happens if and only if $p \notin (p_{\min}, p_{\max})$— candidates make no effort; $\hat{x}_\theta = 0$ for all $\theta \in \Theta$. In other words, candidates who belong to a category that is thought to have a low qualification rate p rationally choose not to make effort to become qualified, since they will anyhow not be screened. And the same is true for candidates who belong to a category that is thought to have a high qualification rate; they will accepted by the screener without screening. By contrast, if $p \in (p_{\min}, p_{\max})$, and thus $r > 0$, then it may be worthwhile for candidates with low enough cost of effort to make some effort. For each $\theta \in \Theta$, let

$$\lambda_\theta(r) = \frac{\theta}{r \cdot (v_1 - v_0)}. \tag{17}$$

By hypothesis, the probability density function f is unimodal. Hence, the necessary first-order condition (16) holds for at most two values of $x \geq 0$. The necessary second-order condition for a positive effort x to be optimal, $f' \leq 0$, is met by exactly one of the solutions to the first-order condition.

Proposition 3. *Suppose that $p \in (p_{\min}, p_{\max})$. For each $\theta \in \Theta$, every candidate's optimal effort, $\hat{x}_\theta$, is zero if $\max_{x \geq 0} f(x) < \lambda_\theta(r)$. Otherwise, $\hat{x}_\theta \geq 0$ is the maximal solution $x \geq 0$ to the equation $f(x) = \lambda_\theta(r)$.*

In other words, only candidates with $\lambda_\theta(r)$ below the maximum of the density function f make positive effort. Moreover, it follows from (16) that positive optimal efforts are increasing in the screening intensity, r, and in the utility gain, $v_1 - v_0$, and they are decreasing in the type's disutility of effort θ. These qualitative properties are evident in the special case when F is exponential, $F(x) = 1 - e^{-x}$. Then $f(x) = e^{-x}$ and thus $\max_{x \geq 0} f(x) = 1$ and $\hat{x}_\theta = \max\{0, \ln[r(v_1 - v_0)/\theta]\}$.

Returning to the general case $F \in \mathcal{F}$, let

$$r_\theta = \frac{1}{\max_x f(x)} \cdot \frac{\theta}{v_1 - v_0}. \tag{18}$$

This defines $r_\theta > 0$ as the lowest screening intensity r for candidates with characteristic θ to make any effort. It follows from Proposition 3 that the qualification rate, $\hat{p}_\theta = F(\hat{x}_\theta)$, among θ-candidates who make positive efforts, is a continuous and non-increasing function of $\lambda_\theta(r)$, because F is an increasing function and $\hat{x}_\theta$, if positive, is decreasing in $\lambda_\theta(r)$. Hence, $\hat{p}_\theta$, wherever positive, is increasing in r, the screening intensity. Let $\hat{p} : \mathbb{R}_+ \to [0,1]$ be the function that to every possible screening intensity $r \in [0,1]$ assigns the associated qualification rate in the candidate pool:

$$\hat{p}(r) = \sum_{\theta \in \Theta} \mu(\theta)\, \hat{p}_\theta. \tag{19}$$

Define $r_{\min}$ as follows:

$$r_{\min} = \min_{\theta \in \Theta} r_\theta.$$

At or below screening intensity $r_{\min}$ even the lowest cost candidates invest no effort, and hence aggregate investment is zero. Then we have:

Corollary 1. *The function* $\hat{p} : [0,1] \to [0,1]$ *is continuous. It is zero on the interval* $[0, r_{\min}]$ *and positive and increasing on the interval* $(r_{\min}, 1]$.

Note that even when $r > r_{\min}$, some higher cost types may make zero effort. In a sense, these candidate types free ride on the efforts of those with lower costs. If sufficiently numerous, the latter can push the aggregate qualified share in the applicant pool up above $p_{\min}$, thus activating the screener, whereby also some zero-effort candidates are accepted. More generally, there are externalities between different candidate subgroups, defined by their θ characteristics, within the same category κ, whereby candidates in different subgroups suffer or benefit from the presence in their candidate pool of the other subgroups.

3.4 Equilibrium

Write $p_\theta = \mathbb{E}(S|\theta)$ for the probability that a randomly drawn θ-candidate (in the category at hand) is qualified. Likewise, write

$$p = \mathbb{E}(S) = \sum_{\theta \in \Theta} \mu(\theta) \, p_\theta$$

for the population share of qualified candidates in the applicant pool. In equilibrium, this is also the screener's belief. That is, the screener sets the conditional accept rates, y_1 and y_0, optimally in response to the true qualification rate p in the candidate pool, according to Proposition 1. This determines the optimal screening intensity function, $r = \hat{r}(p)$. Likewise, in equilibrium each candidate optimally chooses her effort, in response to the screener's chosen value of $r \geq 0$. The optimal effort choice is unique for all candidates, and results in the qualified share $p = \hat{p}(r)$ in the aggregate candidate pool. Since no candidate can influence this share, a pair $(p, r) \in [0,1] \times \mathbb{R}_+$ is an *equilibrium outcome* if and only if $p = \hat{p}(r)$ and $r = \hat{r}(p)$. Such an equilibrium outcome uniquely determines the screener's and all candidates' choices, along with the candidates post-screening utilities (wages or profits).

Does there always exist at least one equilibrium? Can there be multiple equilibria? We now attack these questions. First, it is easy to see that a *passive* equilibrium always exists, an equilibrium in which no candidate makes any effort and the screener rejects all candidates without screening them. This results in the equilibrium outcome $(p, r) = (0, 0)$. The logic is simple: if there is no screening, then it makes no sense for candidates to make any effort, and if no candidate makes any effort, then none of them are qualified, so then it is optimal for the screener to reject all candidates without screening.[9] Any other equilibrium, with screening and some candidates making effort, requires both p and r to be positive. By Propositions 1 and 2, this implies that $p \in (p_{\min}, p_{\max})$. We call such equilibria *active*.

[9] Recall that acceptance of an unqualified applicant results in negative payoff, $-\gamma$, to the screener.

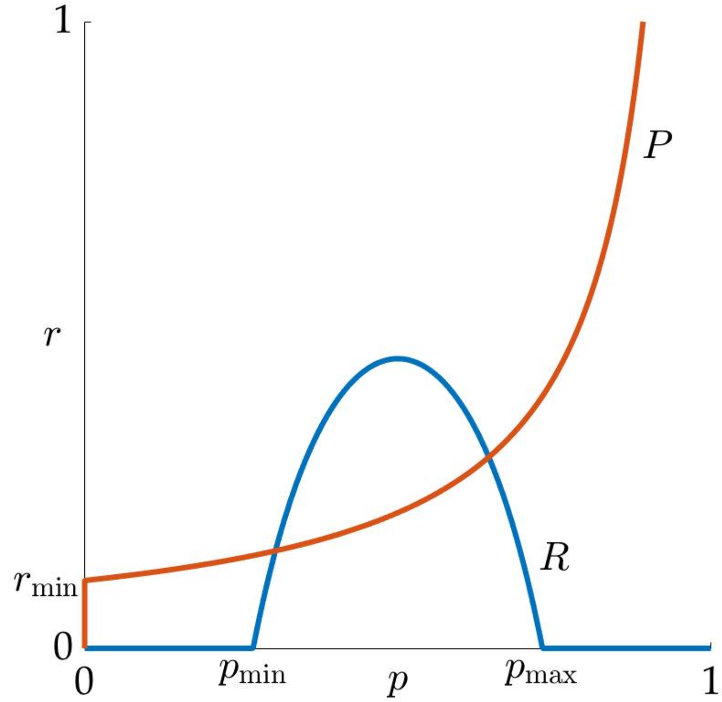

Figure 3: The aggregate candidate response curve P and the screener's response curve R.

All equilibrium outcomes can be conveniently represented as intersections between two continuous curves in the plane, namely, the *aggregate candidate response curve*,

$$P = \{(p, r) \in [0, 1] \times \mathbb{R}_+ : p = \hat{p}(r)\}$$

and the *screener's response curve*,

$$R = \{(p, r) \in [0, 1] \times \mathbb{R}_+ : r = \hat{r}(p)\}.$$

These curves always intersect at the origin (the passive equilibrium), and they may or may not intersect at other points as well. If they do intersect at some other point, then necessarily both p and r are positive, so such equilibria are active. If there are no tangency points between the two curves $p = \hat{p}(r)$ and $r = \hat{r}(p)$, then the number of active equilibria is necessarily even. In sum:

Proposition 4. *There always exists a passive equilibrium in which no candidate invests in skills, screening intensity is zero, and all candidates are rejected. In addition, there may exist active equilibria with positive investment and screening intensity. The number of active equilibria is, for generic parameter values, even.*

This result is illustrated in the following example.

Example 1. *Consider identical and risk-neutral candidates with* $F(x) = 1 - e^{-x}$ *and* $\theta = 0.05$*. Then* $r_{\min} = \theta / (v_1 - v_0)$*, and*

$$\hat{p}(r) = 1 - \frac{\theta}{r} \cdot \frac{1}{v_1 - v_0}$$

for all $r > r_{\min}$*. There are three equilibrium outcomes, two of which are active.*

The example is depicted in Figure 3, where the upward-sloping curve is the aggregate candidate response curve $\hat{p}(r)$, and the quasi-concave curve is the screener's response curve, $\hat{r}(p)$. As is evident from the figure, there are at most two active equilibria if the function $\hat{p}$ is concave wherever positive, that is, on the interval $(r_{\min}, +\infty)$. To obtain a sufficient condition for this, define

$$r_{\max} = \max_{\theta \in \Theta} r_\theta .$$

Above screening intensity $r_{\max}$ all candidates invest effort. We then have:

Proposition 5. *Suppose that the density function* f *is twice differentiable and log concave with* $f' < 0$*. If* $\hat{p}(r_{\max}) > p_{\min}$*, then there exist at most two active equilibria.*

Many commonly used density functions are log-concave. Examples of transition probability functions $F \in \mathcal{F}$ with log concave densities are the exponential and normal, and, for certain parameter ranges, the gamma, chi-square, and Weibull distributions. We note that the claim in this proposition is robust in the sense that it holds also for less than perfectly homogeneous candidate pools. The condition $\hat{p}(r_{\max}) > p_{\min}$ ensures that all candidate types make positive effort.

In sum: for generic parameter values, homogeneous candidate pools, and wide classes of state-transition probability functions, there are either no active equilibria or exactly two.

One may ask which category an individual candidate would like to belong to. The model predicts that the candidate would like to belong to the category with the highest equilibrium qualification rate $\hat{p}$. This follows from the fact that the accept rates, for both qualified and unqualified candidates, are both increasing in $\hat{p}$. Thus the model can be used to explore issues of identity choice along the lines of Kim and Loury (2019). Furthermore, low cost members of a category may wish to expel those with higher costs, in order to raise the equilibrium qualification rate they face. While identity choice and social exclusion are beyond the scope of this paper, they are interesting avenues for future research.

3.5 Stability

The set of equilibria may be refined by considering stability properties with respect to a dynamic process involving gradual behavioral adjustments by both the screener and the candidates. Suppose that the variable pair (p, r), the *state* of the system, is subject to a dynamic

$$\begin{cases} \dot{p} = \phi(p - \hat{p}(r)) \\ \dot{r} = \psi(r - \hat{r}(p)) \end{cases}, \tag{20}$$

where ϕ and ψ are continuously differentiable functions (from $\mathbb{R}$ to $\mathbb{R}$), such that $\phi(0) = \psi(0) = 0$, ϕ', $\psi' \geq 0$ and $\phi'(0), \psi'(0) > 0$. These monotonicity assumptions express the idea that the population share p of qualified candidates increases (decreases) over time when it is below (above) its equilibrium value, at any fixed and given screening intensity r, and that the screening intensity r increases (decreases) over time when below (above) its equilibrium value, at any fixed and given population share p. We call such dynamics *regular*.

A state (p, r) is *stationary* if $\dot{p} = \dot{r} = 0$. A stationary state (p, r) is *Lyapunov stable* if every neighborhood B of (p, r) contains a sub-neighborhood such that the forward solution through every initial state in the sub-neighborhood remains forever in B. A state is *unstable* if it is not Lyapunov stable. A state (p, r) is (locally) *asymptotically stable* if it is Lyapunov stable and it has a neighborhood B' such that the forward solution converges to (p, r) from all initial states in B'. A closed set of states is *forward invariant* if the solution from any initial state in the set remains forever in the set. Without loss of generality we choose as state space for the dynamic any rectangle $C = [0, p_1] \times [0, r_1]$ that is forward invariant and large enough to contain all states $(p, \hat{r}(p))$ where $\hat{r}(p) > 0$.[10]

Now assume that the hypothesis of Proposition 5 holds. Then the system (20) has at least one and at most three stationary states. The passive equilibrium, $(p, r) = (0, 0)$, is always a stationary state. For generic parameter values, there are either two active equilibria or none. Any stationary point of the system (20) is an equilibrium, though not all equilibria will be stable. In the case of two active equilibria, let (p^*, r^*) and (p^{**}, r^{**}), denote the lowest and highest active equilibria as before. Then the former is dynamically unstable, while the latter is asymptotically stable, as is the passive equilibrium:

Proposition 6. *Under the dynamics (20), the passive equilibrium is asymptotically stable, the low active equilibrium is unstable, and the high active equilibrium is asymptotically stable.*

The phase diagram in Figure 4 shows immediately that the passive equilibrium is stable and the lower active equilibrium is a saddle point (the stability of the high active equilibrium requires proof). Proposition 6 allows us to focus on a limited set of equilibria when consid-

[10]Pick $p_1 > 0$ such that $\hat{r}(p) = 0$ for all $p \geq p_1$ and pick $r_1 > 0$ such that $p(r_1) = p_1$.

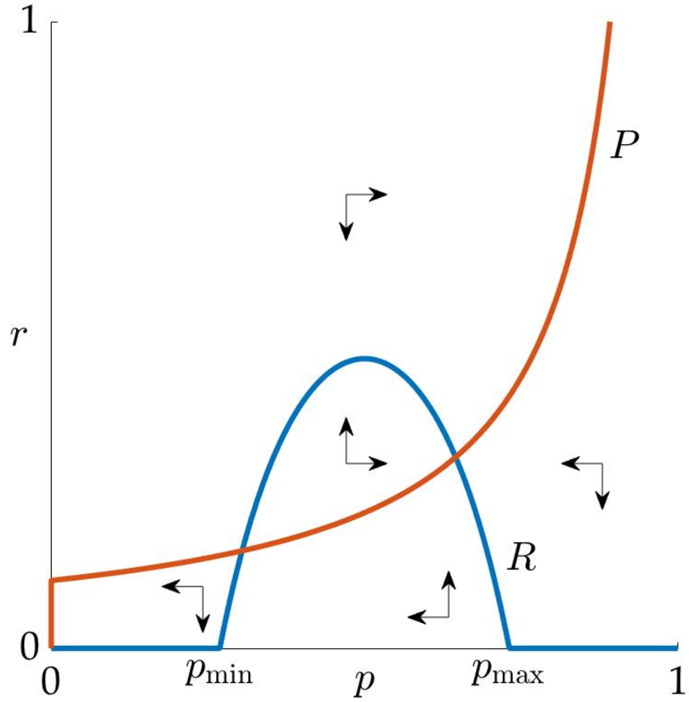

Figure 4: Stability Properties of Equilibria.

ering comparisons across categories. In particular, among active equilibria, we can ignore the unstable equilibrium and focus on the unique stable equilibrium.

4 Categorical Inequality

We now turn to a comparison of outcomes across categories, with a focus on mechanisms sustaining inequality among social groups. Consider two categories or groups, $A, B \in K$, with screener bonuses and penalties (β_A, γ_A) and (β_B, γ_B), respectively, and with distributions μ_A and μ_B of the individual characteristic θ, candidates' disutility of effort or cost of investment in human capital. Suppose that the response curves for the two groups are given by $(\hat{p}_A, \hat{r}_A)$ and $(\hat{p}_B, \hat{r}_B)$, and the screening intervals are denoted $(p_{A\min}, p_{A\max})$ and $(p_{B\min}, p_{B\max})$ respectively.

Since there are no capacity constraints and the screener's unit cost of information is constant (though possibly different across groups), the groups can be treated in isolation, and our earlier analysis applies independently for each group. Accordingly, we may use the term stable active equilibrium to mean that each of the groups is at its own stable active equilibrium, and let p_A^* and p_B^* denote the corresponding equilibrium qualification rates.

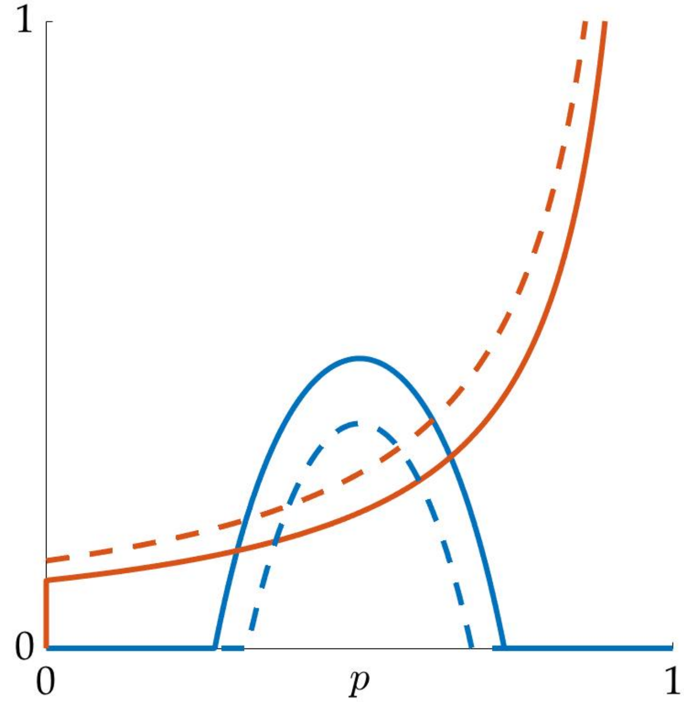

Figure 5: Categorical inequality when one group (shown using dashed lines) is costlier to screen.

4.1 Differential Screening Costs

One branch of the statistical discrimination literature, descended from Phelps (1972), considers *ex ante* identical categories or groups, but allows for the possibility that the attributes of one group may be harder to ascertain that those of another, possibly because the screener herself belongs to the latter group. In the context of our model, the former group would be more costly to screen.

As noted above, an increase in the cost of information acquisition is equivalent to a reduction in the parameters β and γ by the same proportion. Hence, if group $A \in K$ is costlier to screen than group $B \in K$, this is represented in the model by setting $\beta_A = \alpha\beta_B$ and $\gamma_A = \alpha\gamma_B$ for some scaling factor $\alpha \in (0, 1)$. For the disadvantaged group A, this implies a lower qualification rate in equilibrium, more pessimistic beliefs about those who pass the test, and more *optimistic* beliefs about those who fail:

Proposition 7. *Suppose that group A is costlier to screen than group B, while the groups are ex ante identical in all other respects. Then $p_A^* < p_B^*$ in the stable active equilibrium, and $p_{A\,\max} < p_{B\,\max}$ and $p_{A\,\min} > p_{B\,\min}$.*

This result is illustrated in Figure 5, where the dashed lines correspond to the group that is harder to screen. This group faces a lower level of screening intensity at any given qualification rate. Its members will be considered less productive conditional on passing the test, and more productive conditional on failing. This implies a smaller wage premium from passing, and hence a diminished incentive to invest in becoming qualified. Even if both groups have the same distributions of investment costs, the group that is costlier to screen will therefore invest at lower rates at any given value of screening intensity. This further depresses the equilibrium qualification rate.

4.2 Prejudice

Prejudice, in the sense of Becker (1957), refers to a willingness to give up material rewards in order to discriminate against a targeted group. In the present model, screener prejudice of this form against a group A can be represented by reducing the screener's benefit β_A of accepting qualified candidates and/or a higher penalty γ_A for accepting unqualified candidates.

A lower benefit or higher penalty both have the same qualitative effect on the active screening interval, shifting it to the right. However, under a certain condition, they have opposite effects on the incentives of the targeted group to invest in becoming qualified. This condition may be stated as follows. Suppose that the screener's function G is thrice differentiable, and call G *regular* if

$$-\frac{1}{p} < \frac{G'''(p)}{G''(p)} < \frac{1}{1-p} \quad \forall p \in [0,1].$$

These inequalities are satisfied in the special case of Shannon mutual information, and hence in standard rational inattention models.

First consider prejudice in the form of a reduced benefit of accepting qualified candidate. This causes the active screening interval to be shifted to the right, and in the case of regular G, depresses the qualification rate at each level of screening intensity:

Proposition 8. *Suppose that groups A and B are ex ante identical, except that $\beta_A < \beta_B$. In the stable active equilibrium, $p_{A\min} > p_{B\min}$ and $p_{A\max} > p_{B\max}$. If G is regular, then the response functions satisfy $\hat{p}_A(r) < \hat{p}_B(r)$ at every value of r.*

This result is illustrated in Figure 6. The smaller benefit makes the screener more selective, so that candidates who would otherwise have been screened are rejected outright, while those who would have been unconditionally accepted are screened. This has the effect of raising the posterior belief that those who pass the test are qualified. Despite this, candidates' premium from passing the test can fall, because the posterior belief that those who fail the test are qualified also rises. In this case, the incentive to invest is reduced, and the net effect on the qualification rate is ambiguous.

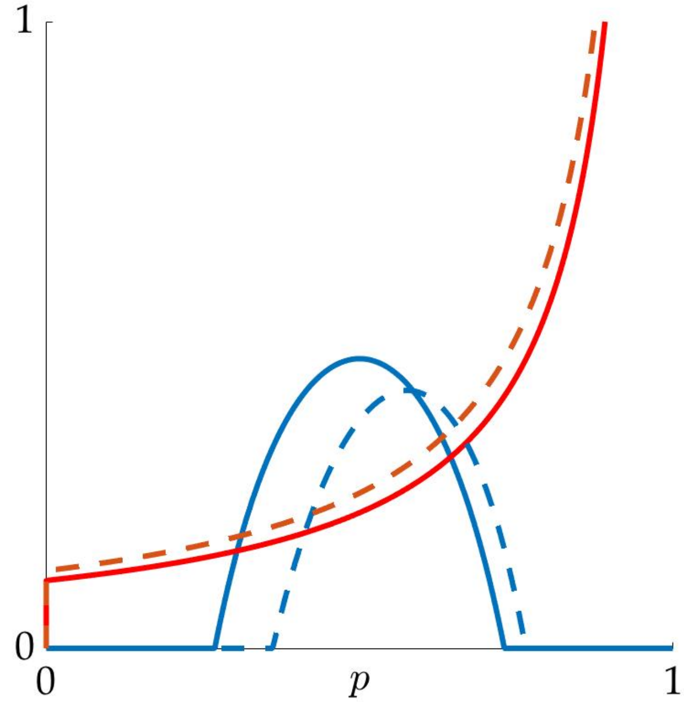

Figure 6: Effects of prejudice: dashed lines correspond to a group with a lower value to the screener conditional on high productivity.

Next consider the case of a larger penalty for accepting an unqualified candidate. Again prejudice shifts the active screening interval to the right but now the incentive effect is reversed:

Proposition 9. *Suppose that groups A and B are identical, except that* $\gamma_A > \gamma_B$. *Then* $p_{A\min} > p_{B\min}$ *and* $p_{A\max} > p_{B\max}$. *If G is regular, then the response functions satisfy* $\hat{p}_A(r) > \hat{p}_B(r)$ *at every value of* r, *and the qualification rates at the stable active equilibrium satisfy* $p^*_A > p^*_B$.

This result is illustrated in Figure 7. As in the case of a lower benefit, the screener becomes more selective and the screening interval is shifted to the right. But this time (in regular cases) the posteriors are adjusted in such a manner as to cause the expected wage premium to rise, and this increases candidates' incentives to make effort or invest. The result is a *higher* qualification rate for the disadvantaged group in the stable active equilibrium.

Regardless of whether prejudice operates through a smaller benefit or greater penalty, a prejudiced screener is more selective when facing the group subject to discriminatory prejudice, rejecting candidates without screening for a greater range of qualification rates, and screening candidates at qualification rates that would result in acceptance without screening for a group not subject to prejudice. As Becker (1993) observed, prejudice results in greater

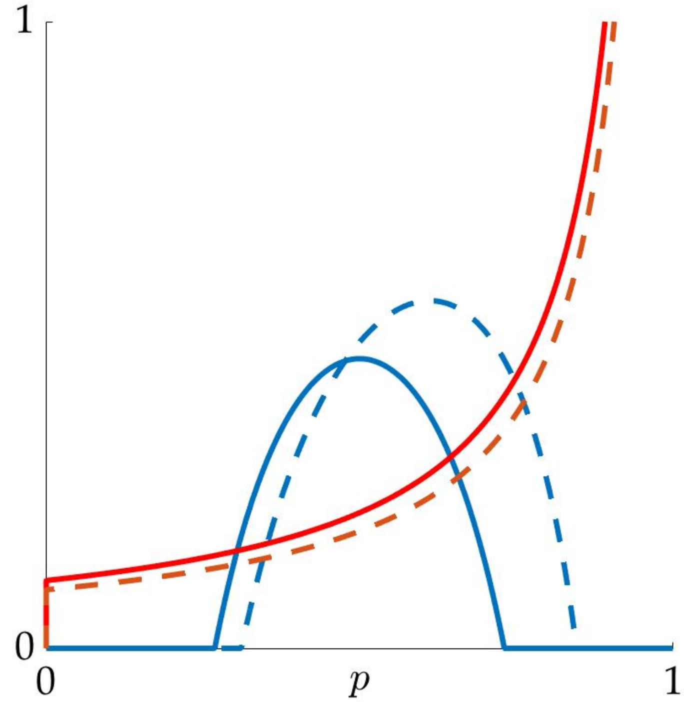

Figure 7: Effects of prejudice: dashed lines correspond to a group facing a higher penalty conditional on low productivity.

expected performance, conditional on acceptance, for the group subject to discriminatory treatment. In fact, this prediction has commonly been used as an outcome-based test for discriminatory treatment.[11]

Prejudice can also take other forms. For instance, there may be a bias in the screener's belief about the distribution μ_A of individual effort costs in group A, and/or in the screener's belief p_A concerning the qualification rate in the group. Such biases, if anticipated by candidates, may remain unfalsified if negative. A negative bias against group A may induce candidates to make no effort and hence the equilibrium qualification rate may be zero, even though there may exist a stable active equilibrium in the absence of bias. In other words, negative biases may remain undetected indefinitely. By contrast, a positive bias towards a group that induces the screener to actively screen candidates who would have been rejected, or to blindly accept candidates who would have been screened, will eventually generate data on qualification rates that would serve to undermine the bias.

[11] In the context of discriminatory policing the outcome test involves a comparison of hit-rates (the proportion of searches that result in contraband recovery) across groups; see Ayres (2002) for a justification, and O'Flaherty and Sethi (2019) for a recent overview.

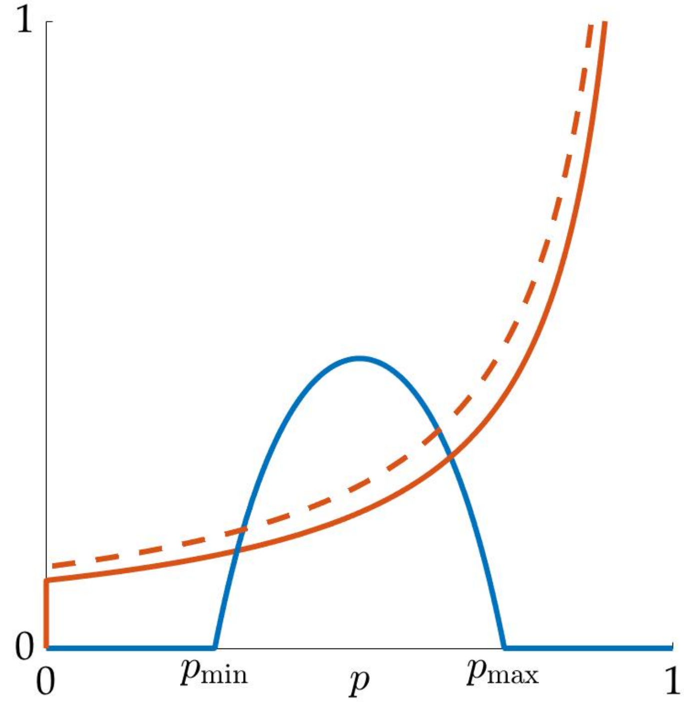

Figure 8: Categorical inequality when opportunities differ; the dashed line corresponds to the category with the less favorable distribution of investment costs.

4.3 Social capital

An important mechanism through which categorical inequality is sustained over time is differential access to quality schooling and social networks rich in human capital, perpetuated through residential and social segregation (Loury, 1977; Bowles et al., 2014). In the present model this may be represented by differences in the distributions of effort or investment costs, μ_κ in different groups $\kappa \in K$. We say that a group $A \in K$ is *disadvantaged* in comparison with group $B \in K$ if the probability distribution μ_A first-order stochastically dominates the distribution μ_B. This implies a lower level of effort or investment in group A, at any given screening intensity, and hence a lower qualification rate at the group's stable active equilibrium. However, since the screener's response function is independent of the group's distribution μ, equilibrium posterior beliefs, conditional on the screening outcome, do not differ across groups:

Proposition 10. *Suppose that group A is disadvantaged in comparison with group B. At the stable active equilibrium, $(p_{A\min}, p_{A\max}) = (p_{B\min}, p_{B\max})$, but $p^*_A < p^*_B$.*

This result is illustrated in Figure 8, where the dashed curve corresponds to the disadvantaged group. In the stable active equilibrium, the disadvantaged group will have a smaller qualification rate, reflecting the diminished opportunities its members face. However, the screening intensity will adjust in such a manner as to leave unchanged the posterior beliefs about accepted and rejected candidates. That is, even though a randomly selected individual in the disadvantaged population will have a lower likelihood of being qualified, this would not be true of candidates in the post-screening market.

If the differences in opportunity are sufficiently large, there may not exist any equilibrium in which the disadvantaged group is screened and selected. In this case any initial disparity in opportunity will be further entrenched and perpetuated. And even if equalization of opportunity results in the emergence of a stable active equilibrium, transition to such a state may be blocked because the passive equilibrium is also dynamically stable.

5 Discussion

We have proposed a canonical model describing interactions between heterogeneous categories of candidates and a rationally inattentive screener seeking to accept those who are likely to be qualified, while rejecting those unqualified. Candidates choose how much effort to invest, and the screener chooses how much costly information to acquire. The payoff to candidates depends on a post-screening market. This basic structure can be found in a variety of environments. While the effort of candidates increases monotonically with screening intensity, the screener is only active when facing a candidate category with a qualification rate believed to lie within a certain interval. Outside this interval, the screener is passive and accepts or rejects everybody depending only on whether the perceived qualification rate is high or low. The location and size of the screening interval is completely determined by the screener's payoff function.

For candidates that are actively screened, the posterior qualification rates of accepted and rejected candidates correspond exactly to the endpoints of the screening interval and do not depend on the qualification rate among the candidates. This is intuitive: the screener would maintain her decision if she were to screen the same candidates again. In a generic setting, there is one passive and two active equilibria, of which the passive and the high active equilibria are stable.

This relatively simple model has allowed us to reconsider and extend the literature on categorical inequality within a unified framework that allows for endogenous information acquisition. When disparities in social capital exist, a disadvantaged group will invest at lower rates, but the intensity of screening adjusts in such a manner as to result in posterior beliefs about productivity that are uniform across categories. That is, the screening process is calibrated to eliminate stereotypes among those accepted. However, when disparities across groups exist in the costs of screening, posterior beliefs about productivities are no longer uniform across groups: accepted candidates drawn from a group facing higher screening costs will be thought to be of lower productivity, on average, than accepted candidates from categories that are easier to screen. This blunts the incentive to invest in the first place, resulting in lower equilibrium qualification rates in the disadvantaged group.

The model also allows for an examination of the implications of prejudice in the sense of Becker (1957). In this case posterior beliefs conditional on selection are more favorable for the group subject to bias, consistent with outcome tests for discrimination in the empirical literature. That is, the expected productivity conditional on selection by the screener is greater for members of the category facing prejudice.

Several useful extensions are possible, we mention two. Individuals who are identical but belong to different categories will face different degrees of screening and different outcomes, so there is an incentive to switch categories. This is possible in some situations, though costly, and the effects of this on categorical inequality could be explored within our framework. Additionally, one could explore the effects of affirmative action on the intensity of screening, equilibrium investment rates, and posterior beliefs about candidates drawn from various categories. Such affirmative action policies may be sighted or color-blind, and the consequences of one or the other policy on the level of screening intensity and the resulting outcomes would be worth examining within the parsimonious framework developed here.

Appendix

Lemma 1. *The screener's objective (8) has a unique maximum. When the maximum is interior, then the screener is actively acquiring information, the conditional posteriors are independent of p and satisfy $\hat{q}_0 < p < \hat{q}_1$. Otherwise the posterior is equal to the prior.*

Proof. We solve the screener's problem in two steps. First, we fix y and carry out the maximization with respect to q_0, q_1.[12] This problem has a concave objective and convex constraints. The solution is a function of y, which turns out to be strictly concave and hence has a unique maximum.

Fix y and solve for q_0, q_1. Let $0 < y < 1$. The Lagrangian is

$$\Lambda\left(q_0, q_1, \lambda\right) := \beta y q_1 - \gamma y\left(1 - q_1\right) - I\left(q_0, q_1\right) + \lambda\left(y q_1 + (1-y)\, q_0 - p\right).$$

The objective is seen to be concave and corner solutions are ruled out by the properties of G. The first-order conditions for the maximization of Π are

$$\begin{aligned}
0 &= \frac{\partial \Lambda}{\partial q_1}\left(\hat{q}_0, \hat{q}_1, \hat{\lambda}\right) = (\beta + \gamma)\, y - y G'\left(\hat{q}_1\right) + \hat{\lambda} y \\
0 &= \frac{\partial \Lambda}{\partial q_0}\left(\hat{q}_0, \hat{q}_1, \hat{\lambda}\right) = -\left(1 - y\right) G'\left(\hat{q}_0\right) + \hat{\lambda}\left(1 - y\right) \\
0 &= \frac{\partial \Lambda}{\partial \lambda}\left(\hat{q}_0, \hat{q}_1, \hat{\lambda}\right) = y \hat{q}_1 + (1-y)\, \hat{q}_0 - p.
\end{aligned}$$

Solving these leads to

$$\begin{aligned}
G'\left(\hat{q}_1\right) - G'\left(\hat{q}_0\right) &= \beta + \gamma && (21) \\
\hat{\lambda} &= G'\left(\hat{q}_0\right) && (22) \\
\hat{q}_1 &= \frac{p}{y} - \frac{1-y}{y}\hat{q}_0. && (23)
\end{aligned}$$

Now, G' is increasing and $\beta + \gamma > 0$ and then (21) implies that $\hat{q}_0 < \hat{q}_1$. From (23) and $\hat{q}_0 < \hat{q}_1$ we obtain that $\hat{q}_0 < p < \hat{q}_1$. We note also for later use that

$$y = \frac{p - \hat{q}_0}{\hat{q}_1 - \hat{q}_0}. \tag{24}$$

We now view $\hat{q}_0, \hat{q}_1$ as functions of y and insert these into the objective to obtain a

[12] Perhaps things can be made easier by using the notion of infimal convolution.

concentrated objective.

$$\Pi\left(y\right) = \beta y \hat{q}_1 - \gamma y \left(1 - \hat{q}_1\right) - yG\left(\hat{q}_1\right) - \left(1 - y\right) G\left(\hat{q}_0\right) + G\left(p\right).$$

The constraint (23) is automatically satisfied at each value of y. We denote derivatives with respect to y as $\hat{q}_0'$ and so on. In order to compute the first- and second-order derivatives of $\Pi\left(y\right)$, we derive from (21) and (23) that

$$\begin{aligned} G''\left(\hat{q}_0\right)\hat{q}_0' &= G''\left(\hat{q}_1\right)\hat{q}_1', \\ \hat{q}_1' &= -\frac{p - \hat{q}_0}{y^2} - \frac{1-y}{y}\hat{q}_0'. \end{aligned}$$

Combining these equations and solving for $\hat{q}_1'$ shows that

$$\hat{q}_1' = -\frac{p - \hat{q}_0}{y^2} \frac{1}{1 + \frac{1-y}{y}\frac{G''\left(\hat{q}_1\right)}{G''\left(\hat{q}_0\right)}} < 0.$$

Then, using (21) and $0 = \hat{q}_1 - \hat{q}_0 + y\hat{q}_1' + \left(1 - y\right)\hat{q}_0'$,

$$\begin{aligned} \Pi'\left(y\right) &= \beta\hat{q}_1 - \gamma\left(1 - \hat{q}_1\right) - G\left(\hat{q}_1\right) + G\left(\hat{q}_0\right) + \left(\beta + \gamma\right) y\hat{q}_1' - yG'\left(\hat{q}_1\right)\hat{q}_1' - \left(1 - y\right) G'\left(\hat{q}_0\right)\hat{q}_0' \\ &= \beta\hat{q}_1 - \gamma\left(1 - \hat{q}_1\right) - G\left(\hat{q}_1\right) + G\left(\hat{q}_0\right) - G'\left(\hat{q}_0\right)\left(y\hat{q}_1' + \left(1 - y\right)\hat{q}_0'\right) \\ &= \beta\hat{q}_1 - \gamma\left(1 - \hat{q}_1\right) - G\left(\hat{q}_1\right) + G\left(\hat{q}_0\right) + G'\left(\hat{q}_0\right)\left(\hat{q}_1 - \hat{q}_0\right). \end{aligned} \tag{25}$$

Differentiating again leads to

$$\begin{aligned} \Pi''\left(y\right) &= \left(\beta + \gamma\right)\hat{q}_1' - G'\left(\hat{q}_1\right)\hat{q}_1' + G'\left(\hat{q}_0\right)\hat{q}_0' + G''\left(\hat{q}_0\right)\left(\hat{q}_1 - \hat{q}_0\right)\hat{q}_0' + G'\left(\hat{q}_0\right)\left(\hat{q}_1' - \hat{q}_0'\right) \\ &= \left(\beta + \gamma\right)\hat{q}_1' + \left(G'\left(\hat{q}_0\right) - G'\left(\hat{q}_1\right)\right)\hat{q}_1' + G''\left(\hat{q}_0\right)\left(\hat{q}_1 - \hat{q}_0\right)\hat{q}_0' \\ &= G''\left(\hat{q}_1\right)\left(\hat{q}_1 - \hat{q}_0\right)\hat{q}_1' < 0. \end{aligned}$$

This means that Π has a unique maximum, either given by the first-order condition or at the boundary.

We note that the first-order condition for $\hat{y}$ does not involve p. The same is true for the first-order conditions (21) and (23) that determine $\hat{q}_0, \hat{q}_1$. We conclude (as expected, see Caplin et al., 2017) that the conditional posteriors are independent of p when the screener is active. ∎

Lemma 2. *Since $\hat{q}_0(0) = p$, we have from (21)*

$$\begin{aligned}
\hat{q}_1(0) &= (G')^{-1}(\beta + \gamma + G'(p)) &(26)\\
\frac{\partial \hat{q}_1(0)}{\partial p} &= \frac{G''(p)}{G''(\hat{q}_1(0))} > 0 &(27)\\
\frac{\partial \hat{q}_1(0)}{\partial \gamma} &= \frac{\partial \hat{q}_1(0)}{\partial \beta} = \frac{1}{G''(\hat{q}_1(0))} > 0. &(28)
\end{aligned}$$

Similarly $\hat{q}_1(1) = p$ and

$$\begin{aligned}
\hat{q}_0(1) &= (G')^{-1}(G'(p) - (\beta + \gamma)) &(29)\\
\frac{\partial \hat{q}_0(1)}{\partial p} &= \frac{G''(p)}{G''(\hat{q}_0(1))} &(30)\\
\frac{\partial \hat{q}_0(1)}{\partial \gamma} &= \frac{\partial \hat{q}_0(1)}{\partial \beta} = -\frac{1}{G''(\hat{q}_0(1))} < 0. &(31)
\end{aligned}$$

Proof of Proposition 1. Lemma 1 establishes that the amount of information acquired will be such that the conditional posteriors are independent of the prior. Define for convenience $H(y) = G(y) - yG'(y)$ and note that $H'(y) = -yG''(y) < 0$. Use this and (21) to rewrite (25) as

$$\Pi'(y) = -\gamma + H(\hat{q}_0(y)) - H(\hat{q}_1(y)).$$

Note that $\hat{q}_0(0) = \hat{q}_1(1) = p$, since the posterior equals the prior when the screener either accepts or rejects all candidates. Optimality at an interior value of y requires that $\Pi'(0) > 0$ and $\Pi'(1) < 0$. which is equivalent to

$$\begin{aligned}
0 &< \Pi'(0) = -\gamma + H(p) - H(\hat{q}_1(0)), &(32)\\
0 &> \Pi'(1) = -\gamma + H(\hat{q}_0(1)) - H(p). &(33)
\end{aligned}$$

Differentiating the bounds of the screening interval with respect to p and using Lemma 2 eqs. (27) and (30) yields

$$\begin{aligned}
\frac{\partial \Pi'(0)}{\partial p} &= (\hat{q}_1(0) - p)\, G''(p) > 0,\\
\frac{\partial \Pi'(1)}{\partial p} &= (p - \hat{q}_0(1))\, G''(p) > 0.
\end{aligned}$$

We find that both $\Pi'(0)$ and $\Pi'(1)$ are monotone functions of p such that the equations $\Pi'(0) = 0$ and $\Pi'(1) = 0$ have unique solutions. Then (32) determines the lower bound of the screening interval while (33) determines the upper bound. The screening interval is nonempty due to the strict concavity of $\Pi(y)$.

Define $p_{\min}$ as value of p that achieves equality in (32) and similarly $p_{\max}$ as the value of p that achieves equality in (33). The screening interval is then $[p_{\min}, p_{\max}]$. We have noted that the conditional posteriors are independent of p. It follows that $p_{\min} = \hat{q}_0(\hat{y})$ and $p_{\max} = \hat{q}_1(\hat{y})$.

The assertion that $\hat{y}$ varies linearly as a function of p follows from (24). The expressions for the conditional accept rates follow from Bayes' law.

$$\begin{aligned}
\hat{y}_0 &= \frac{p - p_{\min}}{p_{\max} - p_{\min}} \frac{1 - p_{\max}}{1 - p}, \\
\hat{y}_1 &= \frac{p - p_{\min}}{p_{\max} - p_{\min}} \frac{p_{\max}}{p}.
\end{aligned}$$

It is straightforward to verify that these are strictly convex and concave, respectively, on the screening interval.

■

Proof of eq. (12). In the case of the Shannon mutual information:

$$\begin{aligned}
G(y) &= y \ln y + (1 - y) \ln(1 - y) \\
G'(y) &= \ln \frac{y}{1 - y} \\
H(y) &= \ln(1 - y)
\end{aligned}$$

We know from the first-order conditions that

$$G'(q_1(y)) - G'(q_0(y)) = \beta + \gamma.$$

The screening interval is determined by the equations

$$\begin{aligned}
\gamma &= H(q_0(0)) - H(q_1(0)), q_0(0) = p_{\min} \\
\gamma &= H(q_0(1)) - H(q_1(1)), q_1(1) = p_{\max}.
\end{aligned}$$

Solving for $p_{\min}$ leads to two equations:

$$\begin{aligned}
\gamma &= \ln(1 - p_{\min}) - \ln(1 - q_1(0)) \\
\gamma + \beta &= \ln \frac{q_1(0)}{1 - q_1(0)} - \ln \frac{p_{\min}}{1 - p_{\min}}.
\end{aligned}$$

Solve the first equation for

$$q_1(0) = 1 - (1 - p_{\min})\, e^{-\gamma}$$

and insert into the second:

$$\begin{aligned} \frac{p_{\min}}{1 - p_{\min}} &= \frac{1 - (1 - p_{\min})\, e^{-\gamma}}{(1 - p_{\min})\, e^{-\gamma}} e^{-(\gamma+\beta)} \Leftrightarrow \\ p_{\min} &= \frac{e^{\gamma} - 1}{e^{\beta+\gamma} - 1}. \end{aligned}$$

Similarly, solving for $p_{\max}$, yields

$$q_0(1) = 1 - (1 - p_{\max})\, e^{\gamma}$$

and

$$\begin{aligned} \frac{p_{\max}}{1 - p_{\max}} &= \frac{1 - (1 - p_{\max})\, e^{\gamma}}{(1 - p_{\max})\, e^{\gamma}} e^{\gamma+\beta} \Leftrightarrow \\ p_{\max} &= \frac{1 - e^{-\gamma}}{1 - e^{-(\beta+\gamma)}}. \end{aligned}$$

■

Proof of Proposition 2. It is easily verified that the screening intensity (13) is a strictly concave function of p. Treating $(p_{\min}, p_{\max})$ as parameters, we find as required that

$$\begin{aligned} \frac{\partial \hat{r}}{\partial p_{\min}} &= -\frac{(p - p_{\max})^2}{(p_{\max} - p_{\min})^2} \frac{1}{p(1-p)} < 0, \\ \frac{\partial \hat{r}}{\partial p_{\max}} &= \frac{(p - p_{\min})^2}{(p_{\max} - p_{\min})^2} \frac{1}{p(1-p)} > 0. \end{aligned}$$

■

Lemma 3. *Increasing the reward β for accepting a qualified candidate shifts the screening interval to the left. Increasing the penalty γ for accepting an unqualified candidate shifts the screening interval to the right.*

$$\begin{aligned} \frac{\partial p_{\min}}{\partial \beta} &< 0, \frac{\partial p_{\max}}{\partial \beta} < 0 \\ \frac{\partial p_{\min}}{\partial \gamma} &> 0, \frac{\partial p_{\max}}{\partial \gamma} > 0. \end{aligned}$$

Proof. By the proof of Proposition 1, the conditional posterior for rejected candidates is determined by the equation

$$0 = \Pi'(0) = -\gamma + H(p_{\min}) - H(p_{\max}).$$

Differentiating this equation, we find that

$$\begin{aligned}
-p_{\min} G''(p_{\min}) \frac{\partial p_{\min}}{\partial \beta} &= -p_{\max} G''(p_{\max}) \frac{\partial p_{\max}}{\partial \beta}, \\
-p_{\min} G''(p_{\min}) \frac{\partial p_{\min}}{\partial \gamma} &= 1 - p_{\max} G''(p_{\max}) \frac{\partial p_{\max}}{\partial \gamma}.
\end{aligned}$$

From (21), we obtain similarly that

$$\begin{aligned}
G''(p_{\max}) \frac{\partial p_{\max}}{\partial \beta} - G''(p_{\min}) \frac{\partial p_{\min}}{\partial \beta} &= 1, \\
G''(p_{\max}) \frac{\partial p_{\max}}{\partial \gamma} - G''(p_{\min}) \frac{\partial p_{\min}}{\partial \gamma} &= 1.
\end{aligned}$$

Solving the first set of equations yields

$$\begin{aligned}
\frac{\partial p_{\min}}{\partial \beta} &= \frac{p_{\max}}{p_{\min}} \frac{G''(p_{\max})}{G''(p_{\min})} \frac{\partial p_{\max}}{\partial \beta}, \\
\frac{\partial p_{\min}}{\partial \gamma} &= -\frac{1}{p_{\min} G''(p_{\min})} + \frac{p_{\max}}{p_{\min}} \frac{G''(p_{\max})}{G''(p_{\min})} \frac{\partial p_{\max}}{\partial \gamma}.
\end{aligned}$$

Inserting into the second set and solving yields

$$\begin{aligned}
\frac{\partial p_{\min}}{\partial \beta} &= -\frac{1}{G''(p_{\min})} \frac{p_{\max}}{p_{\max} - p_{\min}} < 0, \\
\frac{\partial p_{\max}}{\partial \beta} &= -\frac{1}{G''(p_{\max})} \frac{p_{\min}}{p_{\max} - p_{\min}} < 0, \\
\frac{\partial p_{\min}}{\partial \gamma} &= \frac{1}{G''(p_{\min})} \frac{1 - p_{\max}}{p_{\max} - p_{\min}} > 0, \\
\frac{\partial p_{\max}}{\partial \gamma} &= \frac{1}{G''(p_{\max})} \frac{1 - p_{\min}}{p_{\max} - p_{\min}} > 0.
\end{aligned}$$

■

Proof of Proposition 5. By Proposition 2, the screening intensity function, $\hat{r}$ is concave on the screening interval. We will show that the aggregate candidate response function $\hat{p}$ is concave. In that case, there can be at most two equilibria.

Suppose that $r > 0$. The condition that $\hat{p}(r_{\max}) > p_{\min}$ implies that $\hat{x}_\theta(r) > 0$ for all p in the screening interval and for all $\theta \in \Theta$. We then have

$$\hat{p}(r) = \sum_{\theta \in \Theta} \mu(\theta) F_\theta(\hat{x}_\theta(r)),$$

where the first-order condition (16) holds for each type θ, and where the wages are determined by $p_{\min}$ and $p_{\max}$. It is then sufficient to show that $\partial^2 F_\theta(\hat{x}_\theta(r))/\partial r^2 < 0$ for all $\theta \in \Theta$.

From the first-order condition we obtain (with simplified notation) that for all $\theta \in \Theta$:

$$0 = f' \cdot \hat{x}' \cdot r \cdot (v_1 - v_0) + f \cdot (v_1 - v_0) \Leftrightarrow \hat{x}' = -\frac{f}{f' \cdot r}$$

and

$$0 \;=\; f'' \cdot \left(\hat{x}'\right)^2 \cdot r + f' \cdot \hat{x}'' \cdot r + f' \cdot \hat{x}' \Leftrightarrow \hat{x}'' = -\frac{f'' \cdot (\hat{x}')^2 \cdot r + f' \cdot \hat{x}'}{f' \cdot r}.$$

Then

$$\frac{\partial F(\hat{x}(r))}{\partial r} = f \cdot \hat{x}'$$

and

$$\begin{aligned} \frac{\partial^2 F(\hat{x}(r))}{\partial r^2} &= f' \cdot (\hat{x}')^2 + f \cdot \hat{x}'' \\ &= f' \cdot (\hat{x}')^2 - f \cdot \frac{f'' \cdot (\hat{x}')^2 \cdot r + f' \cdot \hat{x}'}{f' \cdot r} \\ &= \left[(f')^2 - f \cdot f''\right] \cdot \frac{(\hat{x}')^2}{f'} - f \cdot \frac{\hat{x}'}{r}. \end{aligned}$$

But this is negative since $\ln f$ is concave iff $(f')^2 > f'' f$, $f' < 0$ and $\hat{x}' > 0$. ■

Proof of Proposition 6. In order to pin down the stability properties of the three stationary states, we linearize the vector field (ϕ, ψ) around each of them. The Jacobian of the vector field at any point $(p, r) \in C$ where both $\hat{p}$ and $\hat{r}$ are differentiable is

$$\begin{pmatrix} \phi'(0) & \phi'(0)\hat{p}'(r) \\ \psi'(0)\hat{r}'(p) & \psi'(0) \end{pmatrix}.$$

The associated eigenvalues are the algebraic roots λ to the equation

$$(\lambda - \phi'(0))(\lambda - \psi'(0)) \;-\; \phi'(0)\psi'(0)\hat{p}'(r)\hat{r}'(p) \;=\; 0.$$

The eigenvalues of the vector field's Jacobian are thus

$$\lambda = \frac{\phi'(0) + \psi'(0)}{2} \pm \sqrt{\left(\frac{\phi'(0) + \psi'(0)}{2}\right)^2 + [\hat{p}'(r)\,\hat{r}'(p) - 1]\,\phi'(0)\,\psi'(0)}.$$

By assumption, $\phi'(0), \psi'(0) < 0$. Hence, $\phi'(0) + \psi'(0) < 0$ and $\phi'(0)\,\psi'(0) > 0$. We now examine the three stationary states in turn, using standard techniques(see e.g. Theorem 4.2 in Hale, 1969):

1. At the passive equilibrium, $\hat{p}'(r) = 0$. Thus, both eigenvalues have negative real parts. Hence, this stationary state is asymptotically stable.

2. At the low active equilibrium, the $\hat{p}$-curve intersects the $\hat{r}$ curve from above: $[\hat{p}'(r)]^{-1} < \hat{r}'(p)$. Hence, $\hat{p}'(r)\,\hat{r}'(p) > 1$. Thus one eigenvalue (the one with the plus sign in the equation) has a positive real part. Hence, this stationary state is unstable (a saddle point).

3. At the high active equilibrium, the $\hat{p}$-curve intersects the $\hat{r}$ curve from below: $[\hat{p}'(r)]^{-1} > \hat{r}'(p)$. Hence, $\hat{p}'(r)\,\hat{r}'(p) < 1$. Thus, again both eigenvalues have negative real parts, so also this stationary state is asymptotically stable.

■

Proof of Proposition 7. Replace (β, γ) by $(\tilde{\beta}, \tilde{\gamma}) = (\beta/\alpha, \gamma/\alpha)$. Increasing α corresponds to a higher screening cost. Find from (21) that

$$G''(\tilde{p}_{\max})\frac{\partial \tilde{p}_{\max}}{\partial \alpha} - G''(\tilde{p}_{\min})\frac{\partial \tilde{p}_{\min}}{\partial \alpha} \quad = \quad -\frac{\beta + \gamma}{\alpha^2}, \tag{34}$$

which is equivalent to

$$\frac{\partial \tilde{p}_{\max}}{\partial \alpha} = \frac{G''(\tilde{p}_{\min})}{G''(\tilde{p}_{\max})}\frac{\partial \tilde{p}_{\min}}{\partial \alpha} - \frac{1}{G''(\tilde{p}_{\max})}\frac{\beta + \gamma}{\alpha^2}. \tag{35}$$

Write (25) as

$$0 = -\frac{\gamma}{\alpha} + H(\tilde{p}_{\min}) - H(\tilde{p}_{\max}),$$

such that differentiating and solving leads to

$$\begin{aligned}
0 &= \frac{\gamma}{\alpha^2} + H'(\tilde{p}_{\min})\frac{\partial \tilde{p}_{\min}}{\partial \alpha} - H'(\tilde{p}_{\max})\frac{\partial \tilde{p}_{\max}}{\partial \alpha} \\
&= \frac{\gamma}{\alpha^2} - \tilde{p}_{\min} G''(\tilde{p}_{\min})\frac{\partial \tilde{p}_{\min}}{\partial \alpha} + \tilde{p}_{\max} G''(\tilde{p}_{\max})\left(\frac{G''(\tilde{p}_{\min})}{G''(\tilde{p}_{\max})}\frac{\partial \tilde{p}_{\min}}{\partial \alpha} - \frac{1}{G''(\tilde{p}_{\max})}\frac{\beta+\gamma}{\alpha^2}\right) \\
&= \frac{\gamma}{\alpha^2} - \tilde{p}_{\min} G''(\tilde{p}_{\min})\frac{\partial \tilde{p}_{\min}}{\partial \alpha} + \tilde{p}_{\max} G''(\tilde{p}_{\min})\frac{\partial \tilde{p}_{\min}}{\partial \alpha} - \tilde{p}_{\max}\frac{\beta+\gamma}{\alpha^2} \Leftrightarrow \\
\frac{\partial \tilde{p}_{\min}}{\partial \alpha} &= -\frac{1}{\alpha^2}\frac{\gamma - \tilde{p}_{\max}(\beta+\gamma)}{\tilde{p}_{\max} - \tilde{p}_{\min}}\frac{1}{G''(\tilde{p}_{\min})}.
\end{aligned}$$

Inserting back into (35) leads to

$$\begin{aligned}
\frac{\partial \tilde{p}_{\max}}{\partial \alpha} &= \frac{G''(\tilde{p}_{\min})}{G''(\tilde{p}_{\max})}\frac{\partial \tilde{p}_{\min}}{\partial \alpha} - \frac{\beta+\gamma}{\alpha^2 G''(\tilde{p}_{\max})} \\
&= -\frac{1}{\alpha^2}\frac{1}{G''(\tilde{p}_{\max})}\frac{\gamma - \tilde{p}_{\max}(\beta+\gamma)}{\tilde{p}_{\max} - \tilde{p}_{\min}} - \frac{\beta+\gamma}{\alpha^2 G''(\tilde{p}_{\max})} \\
&= \frac{1}{\alpha^2 G''(\tilde{p}_{\max})}\frac{(\beta+\gamma)\tilde{p}_{\min} - \gamma}{\tilde{p}_{\max} - \tilde{p}_{\min}}.
\end{aligned}$$

This implies that

$$\begin{aligned}
\frac{\partial \tilde{p}_{\min}}{\partial \alpha} &> 0 \Leftrightarrow \frac{\gamma}{\beta+\gamma} < \tilde{p}_{\max} \\
\frac{\partial \tilde{p}_{\max}}{\partial \alpha} &> 0 \Leftrightarrow \tilde{p}_{\min} < \frac{\gamma}{\beta+\gamma}.
\end{aligned}$$

If $\tilde{p}_{\min} > \frac{\gamma}{\beta+\gamma}$ then also $\tilde{p}_{\max} > \frac{\gamma}{\beta+\gamma}$, which implies that $\frac{\partial \tilde{p}_{\min}}{\partial \alpha} < 0$ for all $\alpha > 0$.We conclude that $\tilde{p}_{\min} \leq \frac{\gamma}{\beta+\gamma}$. Similar reasoning leads to the conclusion that $\frac{\gamma}{\beta+\gamma} \leq \tilde{p}_{\max}$. Altogether we conclude that increasing the scale of the information cost makes the screening interval narrower. ■

Proof of Proposition 8. We recall Lemma 3. The screener's chosen screening intensity $\hat{r}$ changes as

$$\begin{aligned}
\frac{\partial \hat{r}(p)}{\partial \beta} &= \frac{\partial \hat{r}(p)}{\partial p_{\min}} \frac{\partial p_{\min}}{\partial \beta} + \frac{\partial \hat{r}(p)}{\partial p_{\max}} \frac{\partial p_{\max}}{\partial \beta} \\
&= -\frac{(p-p_{\max})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \frac{\partial p_{\min}}{\partial \beta} + \frac{(p-p_{\min})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \frac{\partial p_{\max}}{\partial \beta} \\
&= -\frac{(p-p_{\max})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \left(-\frac{1}{G''(p_{\min})} \frac{p_{\max}}{p_{\max}-p_{\min}} \right) \\
&\quad + \frac{(p-p_{\min})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \left(-\frac{1}{G''(p_{\max})} \frac{p_{\min}}{p_{\max}-p_{\min}} \right) \\
&= \frac{1}{p(1-p)} \frac{1}{(p_{\max}-p_{\min})^3} \left(\frac{(p-p_{\max})^2 p_{\max}}{G''(p_{\min})} - \frac{(p-p_{\min})^2 p_{\min}}{G''(p_{\max})} \right),
\end{aligned}$$

which is positive iff

$$\frac{(p-p_{\min})^2}{(p-p_{\max})^2} < \frac{p_{\max} G''(p_{\max})}{p_{\min} G''(p_{\min})}.$$

Note that LHS increases from 0 to ∞ as a function of p, while the RHS is constant. There is thus a unique threshold value of p at which $\frac{\partial \hat{r}(p)}{\partial \beta}$ changes from positive to negative.

Consider the wage differential between accepted and rejected candidates as β increases:

$$\begin{aligned}
\frac{\partial}{\partial \beta}(p_{\max}-p_{\min}) &= -\frac{1}{G''(p_{\max})} \frac{p_{\min}}{p_{\max}-p_{\min}} + \frac{1}{G''(p_{\min})} \frac{p_{\max}}{p_{\max}-p_{\min}} \\
&= \frac{1}{p_{\max}-p_{\min}} \left(\frac{p_{\max}}{G''(p_{\min})} - \frac{p_{\min}}{G''(p_{\max})} \right).
\end{aligned}$$

this is positive iff

$$p_{\min} G''(p_{\min}) < p_{\max} G''(p_{\max}).$$

It is straightforward to show that if G is regular then $pG''(p)$ is increasing, which implies the desired inequality.

Similarly,

$$\begin{aligned}
\frac{\partial \hat{r}(p)}{\partial \gamma} &= \frac{\partial \hat{r}(p)}{\partial p_{\min}} \frac{\partial p_{\min}}{\partial \gamma} + \frac{\partial \hat{r}(p)}{\partial p_{\max}} \frac{\partial p_{\max}}{\partial \gamma} \\
&= -\frac{(p-p_{\max})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \frac{\partial p_{\min}}{\partial \gamma} + \frac{(p-p_{\min})^2}{(p_{\max}-p_{\min})^2} \frac{1}{p(1-p)} \frac{\partial p_{\max}}{\partial \gamma} \\
&= \frac{1}{p(1-p)} \frac{1}{(p_{\max}-p_{\min})^3} \left(\frac{(p-p_{\min})^2 (1-p_{\min})}{G''(p_{\max})} - \frac{(p-p_{\max})^2 (1-p_{\max})}{G''(p_{\min})} \right),
\end{aligned}$$

which is positive iff

$$\frac{(p - p_{\min})^2}{(p - p_{\max})^2} > \frac{(1 - p_{\max}) G'' (p_{\max})}{(1 - p_{\min}) G'' (p_{\min})},$$

and we note again that there is a unique value of p where the sign changes. The wage differential changes according to

$$\begin{aligned}
\frac{\partial}{\partial \gamma} (p_{\max} - p_{\min}) &= \frac{1}{G'' (p_{\max})} \frac{1 - p_{\min}}{p_{\max} - p_{\min}} - \frac{1}{G'' (p_{\min})} \frac{1 - p_{\max}}{p_{\max} - p_{\min}} \\
&= \frac{1}{p_{\max} - p_{\min}} \left(\frac{1 - p_{\min}}{G'' (p_{\max})} - \frac{1 - p_{\max}}{G'' (p_{\min})} \right),
\end{aligned}$$

which is positive iff

$$(1 - p_{\max}) G'' (p_{\max}) < (1 - p_{\min}) G'' (p_{\min}).$$

We note that regularity of G implies that $(1 - p) G'' (p)$ is decreasing in p, which in turn implies the desired inequality. ■

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