

# Uneven Growth: Automation's Impact on Income and Wealth Inequality\*

Benjamin Moll  
Princeton

Lukasz Rachel  
LSE and Bank of England

Pascual Restrepo  
Boston University

August 20, 2019

## Abstract

Over the past forty years, economic growth in the United States has been unevenly distributed: income percentiles corresponding to the lower half of the distribution have stagnated while those at the top have sharply increased. At the same time, the aggregate labor share has fallen and wealth inequality has risen. We study technical change as a candidate cause of these trends. To this end, we develop a tractable theory that links technology to the personal income and wealth distributions, and not just the wage distribution as is commonly done in the existing literature. We use this theory to study the distributional effects of automation, defined as technical change that substitutes labor with capital. We isolate a new theoretical mechanism: automation may increase inequality via increasing returns to wealth. The flip side of this mechanism is that, relative to theories in which returns are unaffected, automation is more likely to lead to stagnant wages and therefore stagnant incomes at the bottom of the income distribution. We confront our model with the data and argue that automation can account for part of the observed trends in the distribution of wages, incomes and wealth as well as macroeconomic aggregates.

---

\*We thank Fabrizio Perri, Matt Rognlie, Richard Rogerson, Stefanie Stantcheva and seminar participants at various institutions for useful comments and Gabriel Zucman for kindly providing some tabulations from the dataset constructed by Piketty, Saez and Zucman (2018). We also thank Benny Kleinman, Nils Lehr and Max Vogler for outstanding research assistance. The views presented here are solely those of the authors and do not represent the views of the Bank of England or its policy committees.

# Introduction

Over the past forty years, economic growth in many advanced economies has been unevenly distributed. In the United States, while the aggregate economy has grown at roughly two percent per year, income percentiles corresponding to the lower half of the distribution have stagnated. At the same time, incomes at the 95th percentile have roughly doubled and top 1 percent incomes have roughly tripled.<sup>1</sup>

One potential driver of these trends that is often cited by pundits and policy makers alike is technical change, and in particular the automation of tasks performed by labor. A large literature in macro and labor economics has studied how technology and automation affect the distribution of labor incomes.<sup>2</sup> But not all income is labor income and capital income is an important income source, particularly at the top of the distribution where incomes have increased the most. Existing theories therefore paint an incomplete picture of technology’s implications for overall income inequality.<sup>3</sup> This shortcoming is particularly acute when it comes to automation, that is, technical change that substitutes labor with capital, thereby increasing the importance of capital in production. To understand how automation shapes inequality, we therefore need to study how the additional capital income is distributed and how it affects capital ownership – that is, wealth inequality.

The objective of our paper is to develop a theory that links technology to the personal income and wealth distributions—and not just that of wages—and to use it to study the distributional effects of automation. Our framework provides a complete and integrated characterization of how technology and, more generally, changes in the economy’s production and market structure affect the personal distribution of income, wages, and capital ownership, as well as macroeconomic aggregates. As we explain in more detail below, we achieve this by adopting a perpetual youth structure with imperfect dynasties as in [Blanchard \(1985\)](#). The theory has two key features relative to the neoclassical growth model. First, the long-run supply of capital is less than perfectly elastic. Second, the theory generates a non-degenerate wealth distribution. It also features heterogeneity in skills and hence an interesting joint distribution of capital and labor income.<sup>4</sup>

Our main argument is that technology affects not only relative wages but also asset returns and this can have substantial distributional effects. This argument has two parts.

---

<sup>1</sup>See for example [Census Bureau \(2015\)](#) and [Piketty, Saez and Zucman \(2018\)](#).

<sup>2</sup>See for example [Katz and Murphy \(1992\)](#), [Krusell et al. \(2000\)](#) and [Autor, Katz and Kearney \(2006\)](#)

<sup>3</sup>Of course, many theories of technical change feature capital in production and therefore also feature capital income. However, these theories then typically assume that this capital is owned by a representative household, which implies either a degenerate or an indeterminate wealth distribution (with symmetric implications for the capital income distribution).

<sup>4</sup>Importantly, ours is a theory of the *personal* income distribution and not just of the *factor* income distribution. The latter type of theory – for example “two class models” with capitalists and workers – cannot speak to a number of empirical regularities in developed countries, for example that individuals at the top of the labor income distribution typically also earn substantial capital incomes (and vice versa).

First, automation directly contributes to income inequality by increasing returns to wealth and the concentration of capital ownership. Second, relative to theories in which returns are unaffected, automation is also more likely to lead to stagnant wages and therefore stagnant incomes at the bottom of the income distribution (even in the long run). The key for understanding both parts of the argument is that automation increases the demand for capital relative to labor. Because the long-run supply of capital is upward-sloping, this demand shift *permanently* increases returns to wealth. Intuitively, equilibrium requires a higher return to wealth in order to incentivize individuals to accumulate and supply the increased amount of capital demanded in production. The first part, that automation directly increases wealth and income inequality, then follows because some individuals receive a higher return on their assets and grow their fortunes more rapidly. The second part, that wages are more likely to stagnate, follows because some of the productivity gains from automation accrue to owners of capital in the form of a higher return to their wealth.

We deviate from a textbook neoclassical growth model in two simple ways. We let individuals differ in their skills so that technology affects relative wages. And we adopt a perpetual youth structure with imperfect dynasties, meaning that new cohorts are born with their labor income but inherit no assets. The perpetual youth structure gives us two features that are crucial to understand how automation affects inequality and macroeconomic aggregates. First, we obtain well-defined steady-state distributions for wealth and income. Second, unlike models that admit a representative household, our model generates a long-run supply of capital that is less than perfectly elastic. This second feature determines the equilibrium impact of automation on returns to wealth (prices) and the amount of capital used in production (quantities). Our model is deliberately simple and abstracts from labor income risk, a more realistic life-cycle and bequests, as well as return heterogeneity—important elements in quantitative theories of the wealth distribution.<sup>5</sup> In exchange, we obtain analytical solutions for steady state aggregates and distributions of wages, incomes and wealth. Furthermore, the economy aggregates, and solving for its transitional dynamics is as easy as in the neoclassical growth model.

We obtain two main analytical results that illuminate how automation affects inequality in our theory. First, the steady-state return to wealth exceeds the discount rate by a premium  $p \times \sigma \times \alpha_{net}$ . Here  $p$  is the death rate,  $\sigma$  is the inverse of the intertemporal elasticity of substitution, and  $\alpha_{net}$  is the net capital share—an object that rises with automation. Because this premium determines the equilibrium gap between the return to wealth and the discount rate it also determines the speed at which *individuals* accumulate wealth during their lives.<sup>6</sup> Second, we show that conditional on wages, individuals' total incomes follow

---

<sup>5</sup>See for example [Krusell and Smith \(1998\)](#); [Castañeda, Díaz-Giménez and Ríos-Rull \(2003\)](#); [Straub \(2019\)](#); [Hubmer, Krusell and Smith \(2016\)](#).

<sup>6</sup>The imperfect dynasties assumption ensures that, in steady state, individuals accumulate wealth during their lives even though the aggregate capital stock is constant.

an exact Pareto distribution in steady state. The scale of this distribution—a measure of its location—is determined by wages. The inverse of its tail parameter—a measure of its thickness—is given by the ratio of the individual accumulation rate and the death rate  $p$ . Our formula for the return premium implies that, in equilibrium, the inverse tail parameter exactly equals the net capital share  $\alpha_{net}$ .

Intuitively, automation affects income inequality via two channels. First, it affects wages, thereby shifting the scales of the conditional income distributions. Second, it increases the net capital share and returns to wealth, thereby resulting in a thicker top tail of these distributions. This second channel, that top tail inequality equals the net capital share, illustrates in a transparent fashion the new mechanism emphasized above that automation contributes to income inequality by permanently increasing returns to wealth and the concentration of capital ownership. It is important to note that this new channel differs from the common argument that a rise in the capital share leads to higher inequality because capital income is more unequally distributed than labor income (Meade, 1964; Piketty, 2014). In fact, we show that such compositional effects are small both relative to the data and to the changes in capital ownership generated by our model. The new channel also operates for other changes in the economy’s market structure that reduce the labor share, such as rising markups, but is absent for changes that leave it unaffected, such as skill-biased technical change affecting only relative wages.

As just discussed, automation also affects inequality through wages. Our theory features the standard mechanism from a large existing literature that automation affects the *relative* wages of different skill types. Its more interesting predictions instead concern wage *levels*. In our theory, automation generates productivity gains but some of these productivity gains accrue to capital owners in the form of a higher return to their wealth. The higher cost of capital permanently limits the expansion of investment and output in response to this technological improvement. As a result, automation can lead to stagnant or falling real wages even in the long run, especially of workers whose skills are more susceptible to automation. This is in contrast to models that admit a representative household.<sup>7</sup> In those models, the supply of capital is perfectly elastic, and automation leads to a substantial increase in capital accumulation and higher average wages in the long run (Acemoglu and Restrepo, 2018b; Caselli and Manning, 2018).

Although our model is intentionally stylized, and some of the assumptions used are stark, we find the mechanisms underlying our results to be quite general. The two key ingredients behind our results are an upward-sloping long-run supply of capital—so that technology persistently affects asset returns—and a nexus between returns to wealth and

---

<sup>7</sup>To be clear, our notion of “models that admit a representative household” allows for skill and wage heterogeneity. In particular, it includes models that make the common assumption that there are different skill types but these are all members of the same representative household, and models where Gorman aggregation theorems hold (see Theorem 5.2 in Acemoglu, 2009)

inequality. An upward-sloping capital supply seems to us a more natural and less extreme starting point than the perfectly-elastic capital supply of the neoclassical growth model and its relatives. It is also a feature of overlapping generations (OLG) models, models with a life-cycle component, and models with labor income risk and precautionary savings (as in Aiyagari-Bewley-Huggett models). A return-inequality nexus emerges naturally in models where the stochastic accumulation of wealth over time generates a skewed distribution of wealth. This includes models with stochastic bequest motives (Benhabib and Bisin, 2007), models with stochastic returns or discount rates (Krusell and Smith, 1998; Benhabib, Bisin and Zhu, 2011), or models with explosive growth and some stabilizing force, like the birth and death process with imperfect dynasties in our model (Wold and Whittle, 1957; Steindl, 1965; Jones, 2015).<sup>8</sup> Also the argument of Piketty (2014, 2015) that top wealth inequality depends on “ $r - g$ ” highlights precisely this return-inequality nexus. Summarizing, we view our model as the core of a more elaborate framework that alters some or all of the specific assumptions we made, but retains the two key features described above.

After presenting the model and our main analytical results, we turn to a numerical evaluation of our model. Our objective here is not to conduct a quantitative exercise designed to judge the match of the model to the data. Instead, we view this exercise as a first step in exploring the range of implications of this class of models for aggregates and distributions under plausible parameterizations. In our numerical exercise, we study how the automation of routine jobs contributed to overall income inequality. To do so, we infer changes in automation by percentile of the wage distribution by computing its exposure to routine jobs, which the literature singles out as jobs that can be easily automated using computer software or other equipment (see Autor, Levy and Murnane, 2003; Autor, Katz and Kearney, 2006). The implicit assumption behind this particular application of our theory is that the automation of routine jobs since 1980 explains the declining share of labor in national income observed during this period.

In line with the literature on wage polarization, we find that the automation of routine jobs explains about sixty percent of the observed changes in relative wages. More novel, we also find that the automation of routine jobs is able to generate a pattern of uneven growth reminiscent of that observed in the US over the last forty years. Two features combine to produce this pattern. First, our model generates a decline in real wages at the bottom and middle of the income distribution, which accounts for part of the income stagnation observed

---

<sup>8</sup>See Benhabib and Bisin (2018) for a review of models capable of generating skewed wealth distributions. Among the models surveyed, only models where the tail of the wealth distribution is induced by the tail of the distribution of labor earnings (“models with skewed earnings”) lack a return inequality nexus. These include models with finite lives and no inheritances, and the simple versions of Aiyagari-Bewley-Huggett models (Stachurski and Toda, 2018). Furthermore, in Aiyagari-Bewley-Huggett models, automation may also affect incentives to engage in precautionary savings (by decreasing the importance of labor income relative to capital income, or by increasing the risk of unemployment as in Rachel, 2019), with implications for the rate at which individuals accumulate wealth.

at these percentiles. Second, our model generates a rising concentration of capital income at the top of the distribution, which accounts for the sharp rise of incomes at the top. Of course, we do not match the observed changes in income because we only focus on the role of automation—one of many technological and secular changes affecting the US economy. We also use our numerical exercise to contrast the predictions of our theory to those of a representative-household model which, for example, predicts that automation results in substantially larger expansions in investment and average wages.

We conclude the paper by contrasting some of the predictions from our model with the data and concurrent trends. First, we discuss how to square our theory with the fact that risk-free interest rates are on a secular decline. Second, we discuss how markups—another factor reducing the share of labor in national income—fits into our story. Third, we use our framework to explore whether concurrent changes in capital taxation, demographics, and the falling price of investment goods—all of which contributed to the rising importance of capital in the economy—also increased inequality. Fourth, we discuss trends in capital accumulation and investment. Fifth, we discuss the historical and contemporary evidence linking the share of capital in national income to inequality and explain why one cannot make sense of this evidence by appealing to a compositional effect alone. Finally, we discuss whether our framework can account for the fast rise in inequality observed in the data and the importance of capital income in this trend.

We view our paper as making contributions to two branches of literature. The first is the literature on routine jobs and automation. Like most theoretical papers in this literature, we use a task-based framework to model automation (Zeira, 1998; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018b). Most of the papers in this literature focus on wage inequality (Autor, Levy and Murnane, 2003; Autor, Katz and Kearney, 2006; Acemoglu and Autor, 2011; Hémous and Olsen, 2018), or study the effect of automation on aggregates and wages using a representative household framework (Acemoglu and Restrepo, 2018b; Caselli and Manning, 2018). One exception is Sachs and Kotlikoff (2012), who study the possibility of immiserizing growth in an OLG model. We contribute to this literature by moving beyond representative-household models and exploring the implications of automation for inequality of total incomes across individuals. By doing so, we show that automation might contribute to rising incomes at the top and stagnant or declining wages at the bottom of the income distribution.

Second, we contribute to the literature exploring the determinants of income inequality. Several papers explore this question quantitatively in general equilibrium models, including Castañeda, Díaz-Giménez and Ríos-Rull (2003); Kaymak and Poschke (2016); Hubmer, Krusell and Smith (2016); Straub (2019). Relative to these papers, our contribution is conceptual and lies in emphasizing how technical change might contribute to inequality by permanently increasing returns to wealth. In contrast, most of these papers explore the

effect of technology through wages. The mechanism generating a Pareto tail in our model is a common feature of random growth processes (see [Gabaix, 2009](#), for a review). In fact, the birth-death process in our model is a simple and tractable example of a random growth process, and the idea that it leads to a Pareto distribution has been used before by many authors. Our main contribution to this literature is in providing an analytical characterization of income distributions in perpetual youth models, taking into account the effect of technology in general equilibrium. Methodologically, our model is close in spirit to models of the Gorman class (see [Chatterjee, 1994](#); [Caselli and Ventura, 2000](#)). In these theories as well as ours, policy functions are linear, and aggregates do not depend on the wealth distribution (“macro matters for inequality, but inequality does not matter for macro”).<sup>9</sup> Unlike in these models, our imperfect dynasties assumption ensures that we have a determinate wealth distribution in steady state and that our model does not admit a representative household.

Section 1 lays out our theory of uneven growth. In Section 2 we take this model to the data with a simple calibration of changes in automation across the wage distribution, and in Section 3 we confront its predictions with observed trends in key macroeconomic and distributional variables. Section 4 concludes.

## 1 A Model of Uneven Growth

The model is cast in continuous time. For expositional clarity, in the main text we outline the model in stationary form. Appendix B provides the exposition of the full model along the transition path.

### 1.1 Economic Environment

**Households.** The economy is populated by individuals with different skills indexed by  $z$ , and we denote by  $\ell_z$  the share of skill  $z$ . Individuals are born, age over time, die at Poisson rate  $p$ , and are then replaced by an offspring with the same skills.

Individuals with a given skill  $z$  maximize the expected discounted stream of utility over

---

<sup>9</sup>The linear policy functions in our theory deliver unrealistic consumption and saving behavior, for example marginal propensities to consume (MPCs) that are independent of household balance sheets. A large recent literature instead argues that models incorporating empirically realistic heterogeneity often deliver strikingly different aggregate implications than do representative agent models, precisely because aggregates depend on distribution. We view linear policy functions and other abstractions with unrealistic implications as costs worth paying in return for our theory’s analytical tractability.

their lifetime subject to a flow budget constraint and the natural debt limit:

$$\begin{aligned} \max_{\{c_z(s), a_z(s)\}_{s \geq 0}} & \int_0^\infty e^{-(\varrho+p)s} \frac{c_z(s)^{1-\sigma}}{1-\sigma} ds \\ \text{s.t. } & \dot{a}_z(s) = w_z + r a_z(s) - c_z(s), \text{ and } a_z(s) \geq -w_z/r \end{aligned} \quad (1)$$

where  $s$  is their age,  $a_z(s)$  denotes their asset holdings and  $c_z(s)$  their consumption at age  $s$ ,  $r$  is the interest rate and  $w_z$  their wage income. Also,  $\varrho$  is the pure rate of time preference, so that the effective discount rate is given by  $\rho := \varrho + p$ .

Our key assumption is that dynasties are imperfect; that is, not all wealth owned by individuals who die is passed to their offspring. In the main text, we operationalize this assumption in the simplest possible way by having newborns start their life with zero assets,  $a_z(0) = 0$  and individuals consume all of their wealth upon death. This is a crude way of capturing the life cycle: people start their lives with little financial assets, accumulate assets over time, and towards the end of their lives, de-accumulate assets to finance their consumption. This is also the case in our model, but here individuals are born with zero financial assets and consume their remaining wealth instantaneously before death. Appendix B shows that this assumption can be derived from individual optimization in combination with an alternative assumption that we term “one day left to live”: at rate  $p$  individuals learn that they will die  $T$  time units from then on so that they optimally spend down their remaining wealth; we then consider the limiting case  $T \rightarrow 0$ . The Appendix shows that also with this “one day left to live” assumption, saving and consumption decisions while alive solve (1).<sup>10</sup>

The assumption that dynasties are imperfect allows us to deviate in a tractable way from representative-household models, and it also ensures that the distribution of wealth is determinate. In contrast, when  $p = 0$  or dynasties are perfect, the model admits a representative household and the wealth distribution is indeterminate (see Caselli and Ventura, 2000). Appendix D discusses other mechanisms that generate imperfect dynasties, including population growth (wealth dissipates as cohorts become larger), annuities (as in Blanchard, 1985), estate taxation, differences in altruism (as in Benhabib and Bisin, 2007), and retirement periods during which individuals consume some of their wealth (as in Castañeda, Díaz-Giménez and Ríos-Rull, 2003).

**Technology.** Our description of the production process emphasizes the role of tasks as the fundamental unit of production. It also emphasizes that different tasks are completed by

---

<sup>10</sup>As we show in Appendix B, the possibility of death has the standard effect of increasing the effective discount rate as in (1), but end-of-life consumption does not alter saving decisions in any other way. Intuitively, under the “one day left to live” assumption, individuals enjoy an infinite end-of-life consumption flow over an infinitesimal time interval  $T \rightarrow 0$ . Because utility is strictly concave, the value of such end-of-life consumption converges to zero as  $T \rightarrow 0$ .

workers of different skills, and that over time, some of the tasks supplied by workers of a given skill might become automated.

Each skill type  $z$  works in a different sector that produces output  $Y_z$ . The economy produces a final good  $Y$  using these sectoral outputs according to a Cobb-Douglas production function

$$Y = A \prod_z Y_z^{\gamma_z} \quad \text{with} \quad \sum_z \gamma_z = 1.$$

Here,  $\gamma_z$  denotes the importance of the sectoral output produced by skill type  $z$  in production. The productivity shifter  $A$  captures the role of factor-neutral technological improvements.

The production of sectoral output  $Y_z$  involves the completion of a unit continuum of tasks  $u$  with a Cobb-Douglas aggregator:

$$\ln Y_z = \int_0^1 \ln \mathcal{Y}_z(u) du.$$

These tasks can be produced using capital and skill- $z$  labor as follows:

$$\mathcal{Y}_z(u) = \begin{cases} \psi_z \ell_z(u) + k_z(u) & \text{if } u \in [0, \alpha_z] \\ \psi_z \ell_z(u) & \text{if } u \in (\alpha_z, 1] \end{cases}$$

The threshold  $\alpha_z$  summarizes the possibilities for the automation of tasks performed by workers of skill  $z$ . Tasks  $u \in [0, \alpha_z]$  are technologically automated and can be produced by capital  $k_z(u)$  or labor  $\ell_z(u)$  at a unit cost of  $R$  and  $w_z/\psi_z$ , respectively. The remaining tasks are not technologically automated and must be produced by labor.

Total capital used in production is given by  $K$ , and its rental rate is given by  $R$ . Capital depreciates at a rate  $\delta \geq 0$  and can be reallocated across tasks at no cost. We normalize total employment in the economy to 1, so that  $\ell_z$  denotes the available supply of labor of skill  $z$ , with a wage given by  $w_z$ .

An increase in  $\alpha_z$  captures the development of automation technologies that expand the range of tasks in which capital is now able to substitute workers of type  $z$ . For example, workers engaged in white-collar office work devote their time to tasks such as accounting, keeping and locating records, and customer support. Workers engaged in blue-collar work devote their time to tasks such as welding, painting, assembling, machining, and supervision. Over time, technological improvements have allowed the automation of some of these tasks, while others, like customer support or supervision, remain the domain of workers.

In the rest of the paper, we view  $A$ ,  $\gamma_z$ , and  $\alpha_z$  as describing the state of technology, and explore the implications of exogenous shifts in  $\alpha_z$  for aggregates and inequality.

**Equilibrium.** Throughout, we assume competitive input and final good markets and define a steady state equilibrium of the economy as follows.

**Definition 1** *The steady state equilibrium is given by aggregate output and capital, factor prices, a set of factor allocations, and consumption and saving policy functions such that:*

- capital and labor,  $\{k_z(u), \ell_z(u)\}_{z,u}$ , are allocated in a cost minimizing way to produce output  $Y$  given factor prices  $\{w_z\}_z, R = r + \delta$ .
- policy functions  $a_z(s), c_z(s)$  maximize individual utility given  $a_z(0) = 0, w_z$  and  $r$ .

- labor markets clear:

$$\int_0^1 \ell_z(u) du = \ell_z \quad \text{for all } z.$$

- the aggregate capital supplied by individuals and used by firms equals  $K$ :

$$K = \sum_z \int_0^{\alpha_z} k_z(u) du = \sum_z \ell_z \int_0^\infty a_z(s) p e^{-ps} ds.$$

## 1.2 Macroeconomic Aggregates

To characterize the steady state equilibrium, we first study individuals' savings and consumption decisions. Optimal asset accumulation by individuals determines the aggregate supply of capital in the economy. We then consider the firms' problem of choosing the cost-minimizing mix of factor inputs, which determines the equilibrium demand for capital. We are able to derive a closed-form solution for both the supply of and demand for capital, and for the interest rate that equilibrates the capital market.

**Lemma 1 (Households policy functions)** *Suppose that  $r > (r - \rho)/\sigma$ . The solution to the individual problem is given by policy functions that are linear in total wealth,  $a_z(s) + w_z/r$ :*

$$\dot{a}_z(s) = \frac{r - \rho}{\sigma} \left( a_z(s) + \frac{w_z}{r} \right), \quad c_z(s) = \left( r - \frac{r - \rho}{\sigma} \right) \left( a_z(s) + \frac{w_z}{r} \right) \quad (2)$$

with  $a_z(0) = 0$ .

**Proof.** Lemma A3 in Appendix B derives individual policy functions outside of the steady state. Lemma 1 is a special case of that more general formulation. ■

The lemma shows that individuals consume a constant share of their effective wealth. As individuals age, their effective wealth grows at a rate equal to  $(r - \rho)/\sigma$ . This rate is identical for all individuals, irrespective of their skill and age. This makes the model

particularly tractable. But it also means that the model abstracts from realistic features, like differences in the propensity to consume and save out of total wealth.

We now turn to the production side of the economy. To simplify the exposition, we make the following assumption for the rest of the paper:

**Assumption 1 (Full adoption of available automation technologies)**

$$\frac{w_z}{\psi_z} > R \quad \text{for all } z.$$

Assumption 1 ensures that in all tasks for which automation technologies are available, the cost of producing them with labor,  $w_z/\psi_z$ , exceeds the cost of producing them with capital,  $R$ . As a result, all automation technologies will be adopted. Assumption 1 involves endogenous factor prices, so it needs to be verified in equilibrium. Lemma A1 in Appendix A shows that a sufficient condition for this assumption is that  $A$  is high, ensuring a high level of wages.

The following lemma characterizes output and factor prices as functions of the stock of capital and technology.

**Lemma 2 (Equilibrium output and factor prices)** *Suppose Assumption 1 is satisfied. Then equilibrium output is*

$$Y = \mathcal{A} K^{\sum_z \gamma_z \alpha_z} \prod_z (\psi_z \ell_z)^{\gamma_z (1 - \alpha_z)}, \quad (3)$$

where  $\mathcal{A}$  is a constant that depends on parameters  $\{A, \alpha_z, \gamma_z\}$ . Factor prices are given by

$$w_z = (1 - \alpha_z) \frac{\gamma_z}{\ell_z} Y, \quad R = \alpha \frac{Y}{K}, \quad (4)$$

where  $\alpha$  is the average degree of automation in the economy:  $\alpha := \sum_z \alpha_z \gamma_z$ .

**Proof.** See Appendix A. ■

Lemma 2 shows that aggregate output is given by a Cobb-Douglas production function. Factor shares are linked to the range of tasks performed by each factor, and the importance of these tasks in final output (the  $\gamma_z$ 's). Automation changes the importance of labor in production. As tasks that were the domain of skill  $z$  get automated, this skill becomes less important in output ( $1 - \alpha_z$  declines) and capital gains importance ( $\alpha$  rises).<sup>11</sup>

<sup>11</sup>Our model abstracts from firms and industries, and so the decline of the labor share following improvements in automation manifests at the aggregate level. In practice, larger and growing firms might be more likely to deploy automation technologies, or firms already operating more automated production processes

Automation not only changes the distribution of income between capital and workers with certain skills, but also increases productivity by allowing the substitution of expensive labor for cheaper capital in more tasks. The contribution of increase in the  $\alpha_z$ 's to changes in total factor productivity is given by

$$d \ln \text{TFP}_\alpha = \sum_z \gamma_z \ln \left( \frac{w_z}{\psi_z R} \right) d\alpha_z > 0, \quad (5)$$

where  $\ln \left( \frac{w_z}{\psi_z R} \right) > 0$  denotes the percent reduction in costs obtained when a task performed by skill type  $z$  is automated.<sup>12</sup> This formula states that the contribution of automation to TFP is a (weighted) sum of the cost-saving associated with automation at the micro-level.

Lemma 1 showed how the return to wealth determines the rate at which individuals accumulate wealth and supply capital, and Lemma 2 characterized how technology shapes the demand for capital. We now characterize the equilibrium of the return to wealth and aggregates. We denote variables in the equilibrium steady state with an asterisk.

**Proposition 1 (Steady state characterization)** *The steady state is unique. The return to wealth,  $r^*$ , is given by the solution to the equation*

$$\frac{1 - \rho/r^*}{p\sigma + \rho - r^*} = \frac{\alpha}{1 - \alpha} \frac{1}{r^* + \delta}. \quad (6)$$

*In turn,  $r^*$  determines the steady state level of the capital-output ratio  $(K/Y)^*$ , output  $(Y^*)$ , and wages  $(w_z^*)$  as the unique solution to equations (3) and (4).*

*In steady state, the return to wealth and the net share of capital income,  $\alpha_{net}^*$ , satisfy:*

$$r^* = \rho + p\sigma\alpha_{net}^*. \quad (7)$$

**Proof.** See Appendix A for the full proof. ■

Equation 6 shows how technology and demographics determine the equilibrium return to wealth in steady state. We can think of the return to wealth as determined by the supply of capital and the demand for capital. To elaborate this argument it is convenient to think in terms of units of capital relative to the total wage bill, so that we are working with the demand and supply of capital *relative* to labor.

---

may expand at the expense of others as automation technologies improve. These dynamics are consistent with the evidence in Autor et al. (2017); Kehrig and Vincent (2018) on the role played by firms in driving the decline in the labor share, and with the theoretical model in Martinez (2019). Changes in  $\alpha_z$  in our model are meant to capture all these potential channels by which improvements in automation affect the aggregate production function.

<sup>12</sup>The contribution of a technology to TFP is given by the resulting increase in output holding factor utilization constant. That is,  $d \ln \text{TFP}_\alpha = d \ln Y|_{K, \ell_z}$ . Given this definition, the total effect on output of automation is given by  $d \ln Y = d \ln \text{TFP}_\alpha + \alpha d \ln K$ , since the  $\ell_z$  are fixed but  $K$  may adjust over time.

To derive the supply of capital, we use the characterization in Lemma 1. Let  $X$  denote the total effective wealth in the economy, where recall that effective wealth is the sum of financial wealth,  $a_z(s)$ , and human wealth,  $w_z/r$ . Integrating equation (2), we obtain

$$0 = \dot{X} = \frac{r - \rho}{\sigma} X - \text{Effective wealth of individuals dying} + \text{Effective wealth of newborns}.$$

Because of market clearing,  $X = K + \bar{w}/r$ , where  $\bar{w} := \sum_z w_z \ell_z$  is the wage bill. Also, the effective wealth of individuals dying and newborns differ by  $pK$ , which is the amount of wealth consumed by individuals upon death and not passed as bequests. The difference of  $pK$  reflects the fact that we have imperfect dynasties: new generations are born with the same human capital as their elders, but with no financial wealth. These observations imply

$$0 = \frac{r - \rho}{\sigma} \left( K + \frac{\bar{w}}{r} \right) - pK,$$

which after rearrangement gives the (relative) supply of capital

$$\left( \frac{K}{\bar{w}} \right)^s = \frac{1 - \rho/r}{p\sigma + \rho - r}. \quad (8)$$

This curves gives an upward-sloping relationship between the return to wealth and the capital supplied by individuals. As the return to wealth rises, individuals increase their accumulation rate and supply more capital relative to their labor income. The upward-sloping line in Figure 1 illustrates this curve.

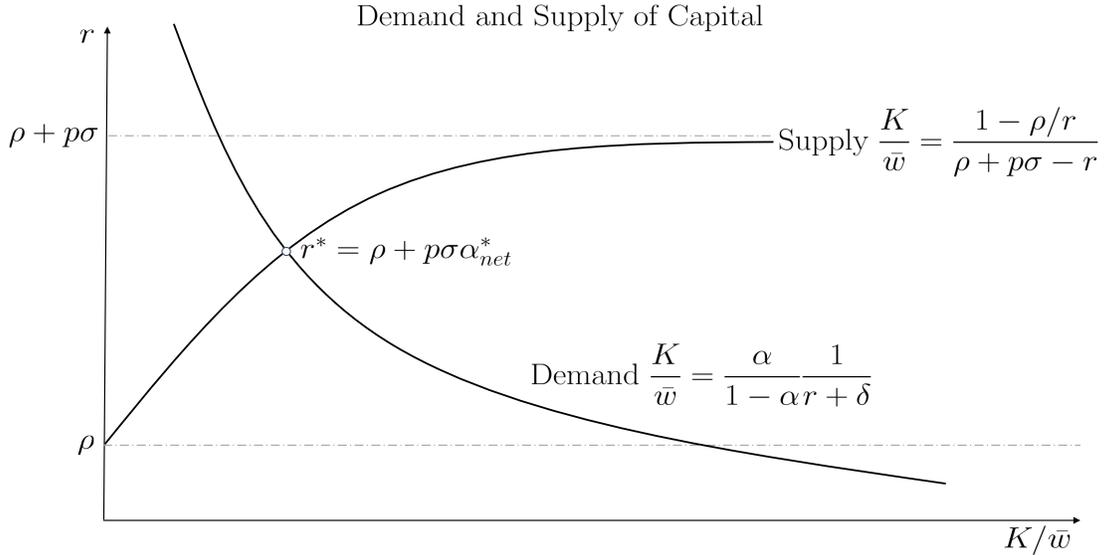


Figure 1: Equilibrium determination of the return to wealth.

On the other hand, from the production side, equation (4) shows that the demand for

capital (normalized by labor income) is given by

$$\left(\frac{K}{\bar{w}}\right)^d = \frac{\alpha}{1 - \alpha} \frac{1}{r + \delta}. \quad (9)$$

This curves gives a downward-sloping relationship between the return to wealth and the capital demanded by firms (normalized by the wage bill). As the return that firms need to pay capital owners rises, they will use less capital relative to labor. Equation (6) gives the return  $r^*$  at which the supply and demand for capital meet.

The main implication of the supply and demand diagram in Figure 1 is that  $r^*$  lies between  $\rho$  and  $\rho + p\sigma$ , which implies that the return to wealth has a premium above  $\rho$ . Equation (7) shows that  $\alpha_{net}^*$  determines exactly where in this range  $r^*$  lies. According to this equation, the premium of  $r^*$  above  $\rho$  is given by  $p\sigma\alpha_{net}^*$ , which implies that in steady state, individuals accumulate wealth at a rate  $p\alpha_{net}^* > 0$ .<sup>13</sup> Note that aggregate wealth  $X$  is constant, but individuals accumulate wealth at a positive rate. This distinction arises due to the imperfect dynasties formulation, which requires individuals to start their lives with zero assets but eventually hold the capital stock of the economy as they age. Thus, in our model the individual rate of wealth accumulation will change with technology and fundamentals; whereas the aggregate rate in steady state will remain at zero.

Figure 1 also illustrates the difference between the equilibrium in our model compared to a model with a representative household (our model with  $p = 0$  or perfect dynasties). With a representative household, the supply of capital is perfectly elastic and the return to wealth is fixed at  $r = \rho$ . In such models, only the quantity of capital adjusts in the long run, ensuring that technology can have at most a short lived impact on asset returns. We view this as a knife edge prediction of representative-household models that our framework is able to relax.

Despite the simplicity of our model, we find the logic behind the upward-sloping supply of capital and the result that the return to wealth has a premium above  $\rho$  linked to the net capital share to be quite general. The driving force behind this result is that the wealth of new cohorts is tightly linked to their human wealth—the net present discounted value of their wages—since younger cohorts hold less financial assets than older ones. A high return to wealth exceeding  $\rho$  is required to get entering cohorts to accumulate and supply a high amount of capital relative to their endowment of labor income. How high above  $\rho$  must

---

<sup>13</sup> The exact formula for  $r^*$  in this equation follows from rearranging equation (8) and holds independently of the production structure of the economy. The reason why the return to wealth exceeds  $\rho$  is different from the known formula  $r = \rho + \sigma g$  which holds along a balanced-growth path where technology grows at a rate  $g > 0$ . In that formula, the growth rate of the economy determines the return to wealth; whereas in ours, changes in factor shares affect the return to wealth even when the economy does not exhibit any growth. By using the fact that  $\dot{X} = gX(t)$ , we can generalize equation (7) to a setting with growth:  $r = \rho + \sigma g + p\sigma\alpha_{net}$ . Here, it is still the case that the return to wealth would exhibit a premium above the return in a representative household economy,  $\rho + \sigma g$ .

the return to wealth be? This will depend on the importance of capital relative to human wealth. In an economy where human wealth is less important than financial wealth (i.e., the net capital share is high), new-born individuals must accumulate assets rapidly and this requires a large premium of the interest rate above  $\rho$ . In an economy where human wealth is more important than financial wealth (i.e., the net capital share is low), new generations already own most of the wealth in the economy in the form of human wealth, and a slow individual accumulation rate is enough to get them to hold all financial assets.

The results in Proposition 1 can be simplified when  $\delta = 0$ . In this case, we obtain simple formulas for aggregates that we will use below to illustrate some of our findings.

**Example 1** *When  $\delta = 0$ , we have that  $\alpha_{net}^* = \alpha$  and*

$$r^* = \rho + p\sigma\alpha, \quad (K/Y)^* = \frac{\alpha}{\rho + p\sigma\alpha}. \quad (10)$$

The following propositions explore the effect of automation on aggregates.

**Proposition 2 (The effects of automation on macroeconomic aggregates)** *The return to wealth,  $r^*$ , the individual accumulation rate  $(r^* - \rho)/\sigma$ , the net capital share  $\alpha_{net}^*$ , and the capital-output ratio,  $(K/Y)^*$ , are all increasing functions of  $\alpha$ . Output increases in all  $\alpha_z$ .*

**Proof.** See Appendix A. ■

The proposition shows that the average extent of automation in the economy,  $\alpha$ , increases the return to wealth, the individual accumulation rate, and the net share of capital. Moreover, these aggregates depend only on the average extent of automation, not the distribution of  $\alpha_z$ . This finding can be seen directly from Figure 2. An increase in  $\alpha$  raises the demand for capital relative to labor. This demand shift increases the return to wealth and the individual accumulation rate,  $(r^* - \rho)/\sigma$ , the capital-wage ratio,  $(K/\bar{w})^*$ , and the net share of capital, which can be written in terms of the objects in Figure 1 as  $\alpha_{net}^* = r^* \times (K/\bar{w})^* / (r^* \times (K/\bar{w})^* + 1)$ . Though it cannot be seen from the figure, the proposition also shows that the capital-output ratio,  $(K/Y)^*$ , expands as automation increases the relative demand for capital.

These findings are intuitive. At a fundamental level, automation makes human wealth less important than financial wealth. As a result, new cohorts—who start life with their less valuable human wealth—need to accumulate assets at a higher rate so as to supply the capital required for production. A higher return to wealth is required to induce the rapid *individual* accumulation of assets—even though at the aggregate level, the expansion of capital could be small and will be limited by the higher user cost of capital, as shown in Figure 2. The result that returns to wealth and the individual accumulation rate rise with

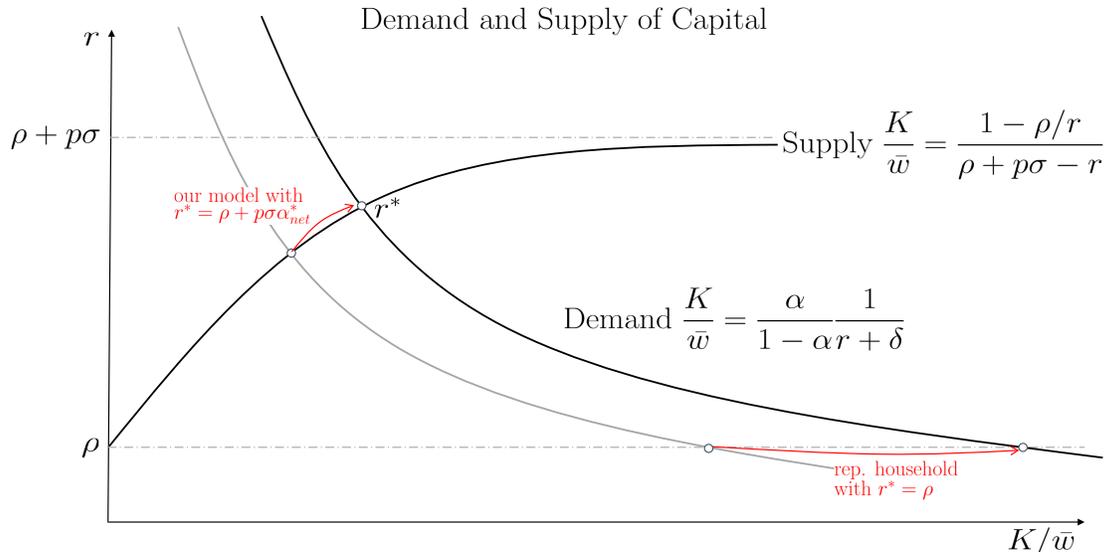


Figure 2: Effect of automation—an increase in  $\alpha$ —on the equilibrium return to wealth,  $r^*$ . The red arrows illustrate how the equilibrium changes in our model and in a model that admits a representative household.

automation is not unique to this model. As we discussed in the introduction, the same result applies in a broader class of models and in other models with a life cycle component and imperfect dynasties.<sup>14</sup>

For comparison, Figure 2 also depicts the effects of automation on returns and capital accumulation in the representative-household version of our model, that is, the special case with a death rate of  $p = 0$ . As in the neoclassical growth model, long-run capital supply is now infinitely elastic at  $r^* = \rho$ . Compared to our model with an upward-sloping long-run capital supply, automation now results in a much larger expansion of investment and has no effect on the return to wealth.

Considering again our model, does the higher demand for capital following an increase in automation primarily result in a higher return to capital or in an expansion of the capital-to-output ratio? Not surprisingly, given the comparison to the representative-household case in the preceding paragraph, the answer depends on the capital-supply elasticity, which is in turn linked to  $p$ . This can be illustrated in the simple case with  $\delta = 0$ , so that the return to wealth and the capital-output ratio are given by the expressions in (10), both of which are increasing in average automation  $\alpha$ . As these equations show,  $p$  determines the extent to which the higher demand for capital results in a higher return to capital (a high  $p$  generates a more inelastic capital supply) or in an expansion of the capital-to-output ratio (a low  $p$  generates a more elastic capital supply).

<sup>14</sup>In models with uninsured labor income risk a-la Aiyagari, a related result applies. In these models, automation reduces the importance of labor income and hence of the precautionary motive for saving. As a result, the interest rate rises (see Proposition 9 in Auclert and Rognlie, 2018). However, the effect on individual accumulation rates is ambiguous, as returns increase but precautionary motives weaken.

Finally, the steady state effect of automation on output is given by

$$d \ln Y^* = \frac{1}{1 - \alpha} d \ln \text{TFP}_\alpha + \frac{\alpha}{1 - \alpha} d \ln (K/Y)^* > 0. \quad (11)$$

Output increases via two channels: because of automation increasing productivity (the first term) and the endogenous capital deepening resulting from the increase in capital accumulation by individuals.<sup>15</sup> For high values of  $p$ , the expansion of output will be more modest as automation will lead to a small expansion of the capital-output ratio.

The main takeaway of Proposition 2 is that, so long as  $p > 0$  and dynasties are imperfect, automation will increase asset returns and the individual accumulation rate permanently. This has significant implications for output and the extent to which capital expands due to automation. As we show next, the same is true for wages and inequality.

**Proposition 3 (The long-run effects of automation on wages)**

- An increase in  $\alpha_z$  reduces the wage  $w_z^*$  relative to other wages  $w_v^*$  for  $v \neq z$ .
- For a given change in the  $\alpha'_z$ s, there exists a threshold  $\bar{p} > 0$  such that, for  $p > \bar{p}$ , the average wage  $\bar{w}^*$  falls; and for  $p < \bar{p}$ ,  $\bar{w}^*$  increases.

**Proof.** See Appendix A. ■

The effect of automation on relative wages is unambiguous and follows from the fact that  $w_z = (1 - \alpha_z) \frac{\gamma_z}{\ell_z} Y$  (see also Hémous and Olsen, 2018; Acemoglu and Restrepo, 2018b).

A more novel implication of the proposition is the possibility that automation may lead to stagnant wages for the average worker. Whether this is the case or not depends on  $p$ , which determines how inelastic the supply of capital is in steady state.

Two complementary intuitions illustrate the importance of the capital supply in determining the behavior of the wage level. First, following a technological improvement that

---

<sup>15</sup>This result contrast the findings by Kotlikoff and Sachs (2014) who study the effects of automation in a two period OLG model. In their setting, only young workers are able to save. By reducing young workers' wages, automation may reduce the capital-output ratio in the long run, potentially causing a decline in output.

raises total factor productivity by  $d \ln \text{TFP}_\alpha > 0$ , we have<sup>16</sup>

$$d \ln \text{TFP}_\alpha = (1 - \alpha)d \ln \bar{w} + \alpha d \ln R, \quad R = r + \delta$$

That is, productivity improvements accrue to capital owners in the form of a higher return to their wealth or to workers in the form of higher average wages. While this expression is very general and also holds outside of steady state and in a much broader class of models, consider now the steady state of our economy. When  $p > \bar{p}$ , the supply of capital becomes so inelastic that all productivity gains from automation accrue to capital in the form of a higher return to wealth. When  $p < \bar{p}$ , the supply of capital becomes so elastic that  $d \ln R^* \approx 0$  and most of the productivity gains from automation will accrue to labor—the inelastic factor.<sup>17</sup>

An alternative intuition comes from studying directly the behavior of the average wage. From Lemma 2, we have that  $\bar{w} = (1 - \alpha)Y$ . From (11) we know that automation results in an output expansion, with the magnitude depending on the productivity increase  $d \ln \text{TFP}_\alpha$  and the expansion in capital supply  $d \ln(K/Y)^*$ . The effect of automation on the average wage is therefore determined by the relative strength of this output expansion and the displacement effect captured by the term  $1 - \alpha$ . With sufficiently inelastic capital supply (i.e.,  $p > \bar{p}$ ), the displacement effect dominates. With sufficiently elastic capital supply (i.e.,  $p < \bar{p}$ ), the output expansion dominates.

### 1.3 Wealth and Income Inequality

We now study the wealth and income distribution. We take advantage of the fact that our model is block recursive: as Proposition 1 shows, the behavior of aggregates is independent of the wealth distribution. In what follows, we take the aggregates in steady state as given and study the resulting wealth distribution.

**Proposition 4 (Automation and the wealth and income distribution)** *Denote individuals’ effective wealth by  $x_z(s) := a_z(s) + w_z^*/r^*$ . The stationary distribution of effective*

---

<sup>16</sup>This result holds in general whenever aggregate output exhibits constant returns to scale and markets are competitive. Under these assumptions, we have  $Y = \bar{w}L + RK$  where we now allow for movements in labor supply  $L$  to underline the argument’s generality so that  $\bar{w}$  denotes the average wage. Differentiating both sides of this identity, we get that, following a technological improvement, we have

$$d \ln Y = d \ln \text{TFP}_\alpha + \alpha d \ln K + (1 - \alpha)d \ln L = (1 - \alpha)d \ln \bar{w} + \alpha d \ln R + \alpha d \ln K + (1 - \alpha)d \ln L, \quad (12)$$

where  $\alpha = RK/Y$ . The expression in the main text follows by canceling the  $d \ln K$  and  $d \ln L$  terms. This derivation shows that the results in the proposition extend beyond our model: any type of technological change will raise wages provided that  $d \ln R = 0$ . See Jaffe et al. (2019, ch.18/19) for a textbook treatment.

<sup>17</sup>This part of the proposition is in line with papers that have studied the impact of automation in settings with a representative household or an infinitely elastic supply of capital, such as Simon (1965); Acemoglu and Restrepo (2018b); Caselli and Manning (2018). These settings correspond to the case with  $p = 0$  in our model.

wealth for skill type  $z$  is

$$g_z(x) = \left(\frac{w_z^*}{r^*}\right)^\zeta \zeta x^{-\zeta-1}, \quad \frac{1}{\zeta} = \frac{1}{p} \frac{r^* - \rho}{\sigma} = \alpha_{net}^*.$$

The conditional and unconditional wealth distributions satisfy

$$\Pr(\text{wealth} \geq a|z) = \left(\frac{a + w_z^*/r^*}{w_z^*/r^*}\right)^{-1/\alpha_{net}^*}, \quad \Pr(\text{wealth} \geq a) = \sum_z \ell_z \left(\frac{a + w_z^*/r^*}{w_z^*/r^*}\right)^{-1/\alpha_{net}^*};$$

and the conditional and unconditional income distributions satisfy

$$\Pr(\text{income} \geq y|z) = \left(\frac{\max\{y, w_z^*\}}{w_z^*}\right)^{-1/\alpha_{net}^*}, \quad \Pr(\text{income} \geq y) = \sum_z \ell_z \left(\frac{\max\{y, w_z^*\}}{w_z^*}\right)^{-1/\alpha_{net}^*}.$$

In the special case with  $\delta = 0$ , we have  $1/\zeta = \alpha$ . In the general case with  $\delta > 0$ ,  $1/\zeta = \alpha_{net}^*$  increases with the average extent of automation in the economy,  $\alpha$ .

**Proof.** See Appendix A. ■

The proposition shows that the distribution of effective wealth for skill type  $z$  is Pareto with scale  $w_z^*/r^*$  and tail parameter  $1/\zeta = \alpha_{net}^*$ . The driving force behind this result is the nature of the process for the accumulation of effective wealth. People start life with effective wealth  $x_z(0) = w_z^*/r^*$  and scale it over time by growing it at a rate  $(r^* - \rho)/\sigma$ . The distribution of wealth is stabilized by deaths that arrive at a rate  $p$ . Figure 3 describes the process of accumulation graphically. This special type of random growth process gives rise to a Pareto distribution (see [Wold and Whittle, 1957](#); [Steindl, 1965](#); [Jones, 2015](#)) with tail parameter

$$\frac{1}{\zeta} = \frac{\text{individual accumulation rate}}{\text{death rate}} = \frac{(r^* - \rho)/\sigma}{p}.$$

As the formula shows, what matters for inequality is the ratio of the rate at which individuals accumulate wealth and the probability with which they die and consume all of it,  $p$ . The formula in the proposition follows from the observation that the steady state return to wealth is given by  $r^* = \rho + p\sigma\alpha_{net}^*$ , which implies an individual accumulation rate of  $(r^* - \rho)/\sigma = p\alpha_{net}^*$ .

The reason why we get a Pareto tail is that some individuals are lucky to live very long lives during which they manage to accumulate wealth exponentially. Instead of interpreting this mechanism literally, we see it as a metaphor for the fact that wealthy people tend to belong to dynasties that have accumulated wealth with no interruption for several generations. For this metaphor to apply,  $p$  should be re-interpreted as the probability that dynastic wealth

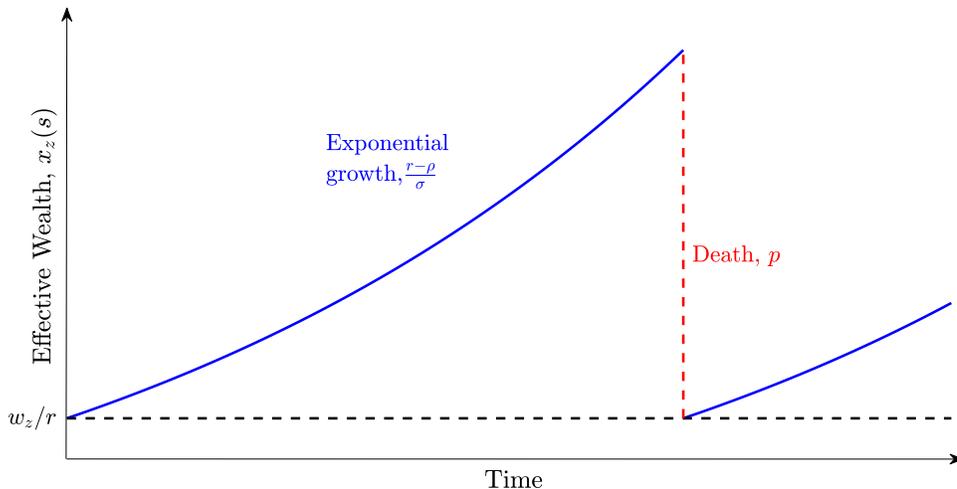


Figure 3: Dynamics of effective wealth accumulation as individuals age.

accumulation stops (say because of changes in the altruism of its members or other shocks) and the dynasty has to start accumulating wealth from scratch. This reinterpretation underscores the importance of bequests and dynastic wealth accumulation as drivers of wealth inequality at the very top. As recognized by several authors (see [Castañeda, Díaz-Giménez and Ríos-Rull, 2003](#); [Benhabib and Bisin, 2018](#)), models with finite lives and no bequests have a hard time generating wealth distributions that are more skewed than earnings.

A complementary view is that our model provides a tractable but unrealistic way of micro-founding a nexus between returns to wealth and inequality present in a broader class of models where the process of individual wealth accumulation results in skewed wealth distributions. This broader class of models includes models with stochastic bequest motives (see [Benhabib and Bisin, 2007](#)), models with stochastic returns or discount rates (see [Krusell and Smith, 1998](#); [Benhabib, Bisin and Zhu, 2011](#)), or models with explosive growth and some stabilizing force, like the birth and death process with imperfect dynasties in our model (see [Wold and Whittle, 1957](#); [Jones, 2015](#)). Despite differences in their details, in all these models a random growth process underlies the dynamics of wealth accumulation at the top of the wealth distribution, creating a natural nexus between return rates and inequality (see [Gabaix et al., 2016](#)). In particular, higher returns shift the rate at which people accumulate wealth, resulting in a larger mass of people populating the upper tail of the wealth distribution over time—those with uninterrupted accumulation at a higher than average rate. It follows that in this broader class of models, technological changes that lead to a higher return to wealth and more rapid individual wealth accumulation will generate a fatter tail in the wealth distribution, as is the case in our model.

The distribution of effective wealth is important in an on itself because it tells us about

inequality in consumption and welfare (a corollary of Lemma 1). But our model also allows us to characterize the conditional and unconditional distributions of wealth and income, which is what we typically measure in the data. The remaining formulas in Proposition 4 provide an exact characterization of the wealth and income distributions. The formulas show that technology affects both distributions via wages—which determine the scale parameters—, but more novel via the net capital share—which determines the thickness of the tail of the wealth and income distribution. Intuitively, technologies like automation that increase the net capital share raise returns to wealth permanently. This allows some individuals to accumulate assets at a consistently higher return during long periods of time, generating more wealth inequality. All the models with a nexus between returns to wealth and inequality mentioned above share the feature that, by increasing returns to wealth, automation will increase wealth inequality.

We now characterize the composition, sources of income, and income shares held by the *top  $q$  income earners*—the *top  $q$*  for short, so that the top 0.1 refers to individuals with the 10% highest incomes. We focus on the tail of the income distribution, where we are able to obtain a clear characterization.

**Proposition 5 (Composition and sources of income at top of income distribution)**

Let  $\bar{q} := \Pr(\text{income} \geq \max_z w_z)$ . For  $q < \bar{q}$ , we have:

- the probability that someone with a wage  $w_z$  is in the top  $q$  is

$$\Pr(\text{skill} = z | \text{top } q) = \frac{\ell_z w_z^{1/\alpha_{net}^*}}{\sum_v \ell_v w_v^{1/\alpha_{net}^*}};$$

- the share of labor income relative to total income held by the top  $q$  is

$$\frac{\mathbb{E}[\text{labor income} | \text{top } q]}{\mathbb{E}[\text{income} | \text{top } q]} = (1 - \alpha_{net}^*) q^{\alpha_{net}^*} \frac{\sum_z \ell_z w_z^{1+1/\alpha_{net}^*}}{\left(\sum_z \ell_z w_z^{1/\alpha_{net}^*}\right)^{1+\alpha_{net}^*}};$$

- the share of national income held by the top  $q$  is

$$S(q) = \Lambda q^{1-\alpha_{net}^*}, \tag{13}$$

where  $\Lambda$  is a constant that depends on the wage distribution.

**Proof.** See Appendix A. ■

The proposition characterizes the skill composition of top income earners and their sources of income. The first part of the proposition shows that the share of individuals

of skill  $z$  among top earners depends on their wage  $w_z$  relative to other wages  $w_v$ . Technology might increase the share of high skill workers among top earners if it is skill biased (that is, if it increases the relative wage of high wage earners). In particular, the automation of tasks performed by workers in the middle and bottom of the skill distribution will increase the share of high skill workers among top earners. On the other hand, an increase in automation (captured here by an increase in the net share of capital  $\alpha_{net}^*$ ) makes relative wages less important in determining the composition of top income earners. Intuitively, many more individuals that managed to accumulate assets for long periods at the higher rate brought by automation will be among top earners independently of their wage.

The second part of the proposition shows that, as we move up in the income distribution, individuals derive more of their income from capital ownership. This reflects the fact that the tail of the income distribution is increasingly made of successful investors for whom labor income represents a small part of their earnings.

The final part of the proposition shows that the share of national income held by the top  $q$  is increasing in  $\alpha_{net}^*$ , and therefore rises with automation. The formula here follows from the fact that for all levels of income above  $\max_v\{w_v^*\}$ , all conditional income distributions have an exact Pareto tail whose thickness depends on the net capital share. The constant  $\Lambda$  adjusts for the different scales of the Pareto distributions below  $\max_v\{w_v^*\}$ . When there is no heterogeneity in wages,  $\Lambda = 1$  and we obtain the usual formula for the top  $q$  percent share in a Pareto distribution (see [Jones, 2015](#)). One implication of this formula is that technology might affect  $S(q)$  through wages (via the  $\Lambda$  term) but this would cause a proportional increase in  $S(q)$  for all  $q < \bar{q}$ . Instead, by raising  $\alpha_{net}^*$ , technology will increase the share of income held by higher percentiles disproportionately.

## 1.4 The Link Between Net Capital Share and Inequality

Propositions 4 and 5 establish a link from the net capital share to income inequality. In this subsection we distinguish our mechanism from previous arguments emphasizing the importance of net capital shares for inequality.

Starting with [Meade \(1964\)](#) and more recently with [Piketty \(2014\)](#), several authors have emphasized that a rise in the net capital share might generate inequality via a compositional effect. The argument is that because capital is more unequally distributed than labor income, a rise in the relative importance of capital would generate more income inequality. This argument differs from ours, since we emphasize how technology might increase wealth inequality and wage inequality directly, with major implications for income inequality.

We can use our model to illustrate the differences between compositional effects and our mechanism. As above, denote by  $S(q)$  the top  $q$  percent income share. Also denote by  $\tilde{S}^k(q)$  and  $\tilde{S}^\ell(q)$  the share of aggregate capital income and labor income earned by the top

$q$  percent of the distribution of *total* income. It follows that<sup>18</sup>

$$S(q) = \alpha_{net} \times \tilde{S}^k(q) + (1 - \alpha_{net}) \times \tilde{S}^\ell(q).$$

This simple formula is precisely the one derived by Meade (1964, p.34) and we can use it to decompose changes over time in  $S(q)$  as

$$dS(q) = \underbrace{(\tilde{S}^k(q) - \tilde{S}^\ell(q)) \times d\alpha_{net}}_{\text{Compositional effect at } q} + \underbrace{\alpha_{net} \times d\tilde{S}^k(q) + (1 - \alpha_{net}) \times d\tilde{S}^\ell(q)}_{\text{Changes in within factor distribution at } q}. \quad (14)$$

This decomposition highlights two shortcomings of theories emphasizing compositional effects. First, the difference between the share of capital income and labor income held by the top  $q$ ,  $\tilde{S}^k(q) - \tilde{S}^\ell(q)$  above, is not large enough to generate a substantial rise in income inequality. In the US in 1980, for the top 1%, roughly  $S(q) = 10\%$ ,  $\tilde{S}^\ell(q) = 5\%$  and  $\tilde{S}^k(q) = 20\%$  so that a large increase in the net capital share of ten percentage points would yield an increase in the top 1% income share of only  $(\tilde{S}^k(q) - \tilde{S}^\ell(q)) \times d\alpha_{net} = 0.15 \times 0.1 = 1.5$  percentage points, or a proportional increase of 15%.<sup>19</sup> As we discuss below, this number is small when compared to the effects in our model and in the data. Second, the emphasis on compositional effects misses the possibility that technology might have sizable effects on wage inequality—the term  $d\tilde{S}^\ell(q)$ —and contribute to a more concentrated ownership of capital—the term  $d\tilde{S}^k(q)$ .

Our mechanism amends these shortcomings. First, in our model technology will have sizable effects on inequality, especially at the very top of the income distribution. Equation (13) implies a log-linear relation between  $S(q)$  and  $\alpha_{net}^*$  of the form

$$\ln S(q) = \ln \Lambda - \ln(1/q) + \ln(1/q) \times \alpha_{net}^*, \quad (15)$$

where  $\ln(1/q) > 0$  ( $q \in (0, 1]$  is in percent terms). Our model predicts that a 1 percentage point increase in the net capital share should raise the share of income earned by the top 0.1 percent by about 6.9%(=  $\ln(1000)$ ); the share of income earned by the top 1 percent by about 4.6%(=  $\ln(100)$ ); and the share of income earned by the top 10 percent by about 2.3%(=

<sup>18</sup>Denote by  $y(q)$  the income of individuals at the top  $q$  percentile (the top  $q$  quantile), by  $y_\ell(q)$  their labor income and by  $y_k(q)$  their capital income. Further denote the corresponding aggregates by  $Y := \int_0^1 y(q) dq$ ,  $Y_k = \int_0^1 y_k(q) dq$  and  $Y_\ell = \int_0^1 y_\ell(q) dq$ . We have  $y(q) = y_k(q) + y_\ell(q)$  and  $Y = Y_k + Y_\ell$  and hence  $y(q)/Y = \alpha_{net} \times y_k(q)/Y_k + (1 - \alpha_{net}) \times y_\ell(q)/Y_\ell$  with  $\alpha_{net} := Y_k/Y$ . Hence the top  $q$  percent income share  $S(q) = \int_0^q y(v) dv / Y$  satisfies the equation above.

<sup>19</sup>These are only rough magnitudes to illustrate the quantitative power of this composition effect. We conduct a precise calculations of this kind in Section 3. Meade (1964, Table 2.2) performs similar calculations but obtains much larger effects because he assumes that, for the top 1%,  $\tilde{S}^\ell(q) = 6\%$  and  $\tilde{S}^k(q) = 47\%$ , which he defends as appropriate numbers for the United Kingdom in 1959. With these numbers, the compositional effect is given by  $(\tilde{S}^k(p) - \tilde{S}^\ell(p)) \times d\alpha_{net} = 0.41 \times 0.1 = 4.1$  percentage points.

$\ln(10)$ ). The effects for the top 1 percent share are three times larger than compositional effects.

Second, our mechanism fully accounts for the effect of technology on the distribution of capital income and labor income. It is precisely because our model predicts that automation will generate a thicker tail of wealth that we get a sizable effect of changes in the capital share on income inequality.

The following proposition shows that, in line with this discussion, at the top of the income distribution, changes in wealth inequality—the changes in the  $\tilde{S}^k(p)$  term—dominate compositional effects.

**Proposition 6 (Decomposing changes in inequality)** *Consider a change in the net capital share  $d\alpha_{net}^* > 0$  holding relative wages constant. As  $q \rightarrow 0$ , the share of the total change in  $S(q)$  explained by the composition effect converges to zero.*

**Proof.** See Appendix A ■

The proposition implies that, following an increase in automation, income inequality rises due to a more concentrated ownership of capital at the top (and the usual changes in relative wages); not so much due to a compositional effect.

This result can be illustrated when  $w_z = \bar{w}$ . Because there is no wage inequality, we have

$$S(q) = q^{1-\alpha_{net}^*}, \quad \tilde{S}^\ell(q) = q, \quad \tilde{S}^k(q) = \frac{1}{\alpha_{net}^*} (q^{1-\alpha_{net}^*} - (1 - \alpha_{net}^*)q).$$

The compositional effect is then given by

$$\text{Compositional effect at } q = \frac{1}{\alpha_{net}^*} (q^{1-\alpha_{net}^*} - q) d\alpha_{net}^* > 0,$$

whereas the overall change in the share of income held by the top  $q$  percent is

$$\text{Total change at } q = \ln(1/q) \times q^{1-\alpha_{net}^*} d\alpha_{net}^* > 0.$$

The share of the total change in  $S(q)$  explained by the compositional effect is then given by

$$\frac{1 - q^{\alpha_{net}^*}}{\alpha_{net}^* \ln(1/q)},$$

which converges to zero as  $q \rightarrow 0$ .

## 1.5 The Transition of Aggregates and Distributions

Appendix B presents the full description of the model outside of steady state. The following proposition characterizes the transitional dynamics for the macroeconomic aggregates and the distribution of effective wealth. As for the steady state equilibrium, the transitional dynamics are block recursive: we can first characterize the behavior of macroeconomic aggregates and then use them to trace the evolution of the wealth distribution.

**Proposition 7 (Transitional dynamics)** *The behavior of the macroeconomic aggregates,  $C$  and  $K$  is given by the unique stable solution to the system of differential equations*

$$\begin{aligned}\dot{C} &= \frac{1}{\sigma}(r - \rho)(C - pK) - \mu pK + p\dot{K} \\ \dot{K} &= Y - \delta K - C, \\ \frac{\dot{\mu}}{\mu} &= \mu - r + \frac{1}{\sigma}(r - \rho)\end{aligned}$$

where  $\mu$  denotes the rate at which individuals consume their effective wealth (to simplify notation, we removed the time dependence of aggregates). Also, recall that  $Y$  is given by equation (3) and  $r$  is given by equation (4).

Along the transition path, individuals accumulate effective wealth at a rate  $r(t) - \mu(t)$ , which implies that the distribution of effective wealth for individuals with skill  $z$ ,  $g_z(x, t)$  evolves according to the Kolmogorov Forward Equation

$$\frac{\partial g_z(x, t)}{\partial t} = -\frac{\partial}{\partial x} [(r(t) - \mu(t))xg_z(x, t)] - pg_z(x, t) + p\tilde{\delta}(x - h_z(t)) \quad (16)$$

where  $\tilde{\delta}(\cdot)$  is the Dirac delta function, and  $h_z(t) = \int_t^\infty e^{-\int_t^s r(\tau)d\tau} w_z(s)ds$  is a time-varying reinjection point.

**Proof.** See Appendix B. ■

The proposition shows that the transitional dynamics for aggregates are no more complicated than those in the usual representative-household model. The main difference is that we need to keep track of the extra variable  $\mu(t)$ , which controls the common marginal propensity to consume out of effective wealth. Also, the Euler equation has some extra terms to account for the difference in consumption between new cohorts and the cohorts they replace (the term  $\mu(t)pK(t)$ ), as well as the consumption of the dying (the terms  $pK(t)$  and  $p\dot{K}(t)$ ).

The fact that technology contributes to wealth inequality by permanently raising the return to wealth is the main result of our model. We can use the result in Proposition 7 to explain how this mechanism plays out over time and contrast our findings to an economy that admits a representative household.

Suppose the initial distribution of effective wealth conditional on skills is given by

$$\Pr(x_{z0} > x) = \left( \frac{x}{w_{z0}^*/r_0^*} \right)^{-\zeta_0}, \quad (17)$$

where  $1/\zeta_0 = \alpha_{net0}^*$  as in Proposition 4. Here,  $x_{z0}$  is a random variable denoting the effective wealth of individuals with skill  $z$ .

First consider our model with an upward-sloping long-run supply of capital. Following an increase in automation at time  $t = t_0$ , individuals with skills  $z$  and effective wealth  $x_{z0}$  see a revaluation of their human wealth of  $\Delta_z$  (this could be negative for individuals experiencing a real decline in wages over time). This implies that the distribution of  $x_z$ , denoted by  $g_z(x, t)$ , starts from

$$g_z(x, t_0) = \left( \frac{w_{z0}^*}{r_0^*} \right)^{\zeta_0} \zeta_0 (x - \Delta_z)^{-\zeta_0 - 1} \quad \text{for } x \geq \frac{w_{z0}^*}{r_0^*} + \Delta_z.$$

and from there on evolves according to the Kolgomorov Forward Equation (16). The first term in equation (16) captures the rate at which individuals accumulate assets. This rate equals  $r(t) - \mu(t)$  and converges to  $(r^* - \rho)/\sigma$ , where  $r^* > r_0^*$  since automation increases the return to wealth *permanently*. The remaining terms capture the birth and death process. People die with probability  $p$  and are replaced by their offspring, who start life with an effective wealth equal to  $x_z^0(t)$ . The birth and death process ensures that the distribution  $g_z(x, t)$  converges to a new Pareto distribution which is independent of the starting one.

This discussion implies that technology affects the effective wealth distribution and its evolution in two ways. First, technology will influence the effective wealth distribution via wages, which determine the revaluation effects in the short run, and the reinjection points in the long run. Second, technologies that make capital more important in production will influence the effective wealth distribution by permanently increasing the return to wealth, which causes individuals to accumulate wealth more rapidly during their lives, and generates a Pareto distribution with a thicker tail.<sup>20</sup>

These dynamics can be contrasted to what would happen in a model with individuals who differ in their skills but that admits a representative household, as in Caselli and Ventura (2000). The dynamics of aggregates and the distribution of effective wealth in this class of models corresponds to the special case with  $p = 0$  in Proposition 7 (we will use a superscript h to distinguish the value of aggregates in this case). The wealth distribution is now

---

<sup>20</sup>Other forms of skill-biased technical change that do not involve changes in the share of capital will have a different effect on the income distribution. Such changes will generate a revaluation effect in the short run and affect the steady-state distribution of income only through the reinjection points. As a result, these other forms of skill-biased technical change will affect the scale parameters of the income distribution, but wont generate a thicker tail by raising the returns to wealth.

indeterminate, in the sense that any distribution is consistent with equilibrium in steady state. Despite the indeterminacy, starting from a given initial distribution of wealth and wages, the transitional dynamics of the wealth distribution are uniquely defined. To make things comparable, assume that the initial distribution of effective wealth is given by (17), and coincides with that in our model. Following an increase in automation at time  $t = t_0$ , individuals with skills  $z$  and effective wealth  $x_{z0}$  see a revaluation of their human wealth of  $\Delta_z^h$  (this will differ from the revaluation in our model since wages behave differently in an economy that admits a representative household—see Proposition 3). People then accumulate assets starting from  $x_{z0} + \Delta_z$  at a common rate  $r^h(t) - \mu^h(t)$ , which is temporarily above zero but converges to zero over time (recall that in the representative-household model,  $r^h(t)$  and  $\mu^h(t)$  converge to  $\rho$ , reflecting the fact that the supply of capital is fully elastic). This temporary period of accumulation scales everyone’s effective wealth by the same amount,  $M$ , but does not contribute to thicker tails in effective wealth. The resulting distribution of effective wealth is given by

$$\Pr(x_z > x) = \left( \frac{x/M - \Delta_z^h}{w_{z0}^*/r_0^*} \right)^{-\zeta_0}, \quad \text{for } x \geq M \cdot (\Delta_z^h + w_{z0}^*/r_0^*).$$

This is a shifted Pareto distribution, with the shifts explained by the changes in wages. Unlike in our model, the new steady state distribution has the same tail parameter as the initial distribution.

To summarize, in an economy that admits a representative household, technology only affects inequality through wages, which determine the revaluation effects  $\Delta_z^h$ . The temporary increase in return rates scales everyone’s wealth, but does not contribute to inequality.

## 2 Model Meets Data

### 2.1 Calibration

As discussed in the introduction, we feed changes in  $\alpha_z$  to the model and explore the consequences of this particular type of technological change for aggregates and inequality in wages and wealth. We will focus on changes in automation between 1980 and 2014, a period with a marked shift in technology towards automation (Autor and Salomons, 2018; Acemoglu and Restrepo, 2019), especially of tasks performed by workers in routine jobs, both in manufacturing and in services (Autor, Levy and Murnane, 2003; Acemoglu and Autor, 2011).<sup>21</sup>

---

<sup>21</sup>Our focus on this period does not imply that there was no automation before then. As discussed in Acemoglu and Restrepo (2019), before 1980 jobs were automated in some specific industries and tasks, but automation was counteracted by other technological improvements that raised labor shares in other industries

To bring the model to the data, we interpret  $z$  as indexing the group of workers in a given percentile of the wage distribution, so that we have 100 skill groups. The main ingredient in our calibration is a measure of how automated the tasks being performed by workers in each percentile of the wage distribution have been over time,  $\alpha_z(t)$ . In what follows, we will use a time argument to indicate which variables change over time. We assume that changes in  $\alpha_z(t)$  are driven by the automation of routine jobs, and that all routine jobs have been automated at the same rate over time. To operationalize this assumption, we posit the rule of motion for  $\alpha_z(t)$  (see Appendix C for a derivation of this equation):

$$\frac{1}{1 - \alpha_z(t)} - \frac{1}{1 - \alpha_0} = \omega_{zR} \left( \frac{1}{1 - \alpha(t)} - \frac{1}{1 - \alpha_0} \right). \quad (18)$$

Here,  $\alpha_0$  denotes the extent to which non-routine tasks are automated (assumed invariant over time), and  $\omega_{zR}$  denotes the share of labor income derived by workers in percentile  $z$  from routine jobs relative to the labor income derived by all workers from routine jobs—a measure of the comparative advantage held by these workers in routine jobs. Equation 18 implies that groups of workers who specialize in routine jobs have a bigger share of the tasks they performed being automated over time. The implicit assumption here is that the observed decline in the labor share since 1980 is driven by the automation of routine jobs.

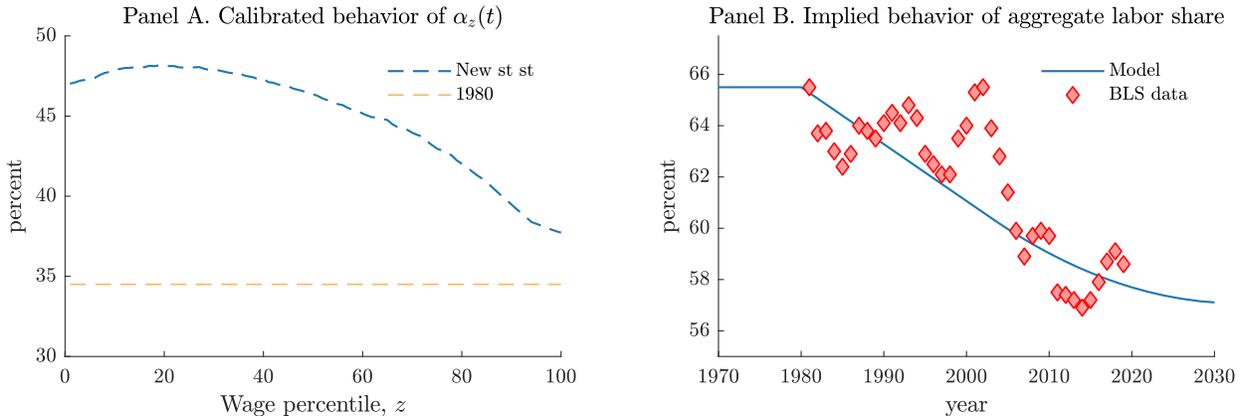


Figure 4: Calibrated  $\alpha_z$  by wage percentile in 1980 and the new steady state (left panel), and the implied behavior of the aggregate labor share compared to the data (right panel).

Using equation (18), we measure  $\alpha_z(t)$  for all percentiles of the wage distribution by computing their  $\omega_{zR}$  using the 2000 Census—a point in the middle of the period we study.<sup>22</sup>

or introduced new labor-intensive roles for labor in production. As a result, the labor share—the key object determining how technology affects wealth inequality—remained stable during this period. Technological change might have contributed to rising *wage* inequality before 1980, but our mechanism did not contribute to rising *wealth* inequality back then.

<sup>22</sup>In our model, the composition of a skill group is assumed invariant. However, in the data, the composition of workers in a given wage percentile might change over time, as the relative ranking of groups of workers with different characteristics changes. In our baseline calibration, we used the 2000 values for  $\omega_{zR}$  as describing

We normalize  $\alpha_z(1980)$  to be equal across all  $z$  (this pins down  $\alpha_0 = \alpha(1980)$ ), and pick the level of  $\alpha(1980)$  and  $\alpha(2014)$  to match the (gross) capital share in these years (0.345 and 0.42, respectively, in the BLS series for the non-financial corporate sector). Finally, we assume a linear increase in  $\alpha_z(t)$  from its value in 1980 to its final value in 2014. This procedure results in the change between 1980 and 2014 in  $\alpha_z(t)$  plotted in the left panel of Figure 4. The average change in  $\alpha_z(t)$  (weighted by  $\gamma_z$ ) is of 8.4 percentage points, which roughly matches the observed decline in the labor share during our period of analysis (8.3 p.p decline).<sup>23</sup> The right panel of Figure 4 plots the implied behavior of the labor share given the change in  $\alpha_z(t)$  over time ( $1 - \alpha(t) = \sum \gamma_z(1 - \alpha_z(t))$  in our model) and the BLS series for the labor share in the corporate non-financial sector.

Turning to the remaining parameters, we calibrate  $\gamma_z$  to match the wage distribution in 1980 (obtained from the 1980 Census). Note that the  $\gamma_z$ 's might have changed over time as a result of other forms of skill biased technical change not modeled here, but we do not explore this possibility. We pick  $\psi_z$  to ensure that human labor is 30% more costly than using capital in the production of automated tasks. This number is in line with studies exploring the cost-saving gains from using industrial robots in manufacturing (see Acemoglu and Restrepo, 2019). Because it is not clear that one can extrapolate from these studies, in Appendix C we present results assuming cost saving gains of 15% and 45%. For our baseline value of  $\psi_z$ , the automation of routine jobs explains 15% of the gains in productivity experienced by the US economy during the 1980-2014 period. We view this estimate as plausible given that automation is one of many technological improvements determining productivity during this period.

The remaining parameters are standard and chosen to match aggregates in 1980. We take a capital output ratio of  $K/Y = 3$ , which implies a rental rate of capital  $R = 11.5\%$ . We take a depreciation rate of 5% so that the net capital share equals 23% and the return to wealth equals  $r = 6.5\%$ . We pick  $\rho = p = 3.85\%$  to ensure a 6.5% return to wealth and target a long-run elasticity of capital supply  $d \ln K / dr$  of about 50 (we get an elasticity of 51.28).<sup>24</sup> We view this choice of  $p$  as conservative, in the sense that much of the evidence suggests a more inelastic supply of capital (see Appendix C). The factor-neutral technology term,  $A$ , is chosen to match output per hour in 1980. We do not feed changes in  $A$  to the model, which would capture other sources of technological improvement different from

---

the level of specialization of different groups in routine jobs. We also experimented with measuring  $\omega_{zR}$  using the 1980 Census and obtained similar results. The reason is that  $\omega_{zR}$  is highly correlated over time (the correlation between the 1980 and 2000 measures is of 0.9714).

<sup>23</sup>The average change in  $\alpha_z(t)$  in the model is slightly larger than the one observed in the data. The reason is that over time, we also have changes in  $\gamma_z$  that are not fed to the model. In particular, the observed changes in  $\gamma_z$  imply that other technological changes resulted in a reallocation of value added from highly automated skills to less automated ones at the top of the wage distribution.

<sup>24</sup>As Appendix D shows, in a more general model, the role of  $p$  is replaced by  $p + n + \delta_H$ . here  $n$  is the growth rate of population and  $\delta_H$  the rate at which human capital depreciates as individuals age past their prime. Many plausible combinations of  $p, n$ , and  $\delta_H$  yield a combined value of  $p + n + \delta_H = 3.85\%$ .

Table 1: List of calibrated parameter values

DESCRIPTION	VALUE	TARGET / SOURCE
PREFERENCES		
$\sigma$	Inverse IES	3 Standard calibration
$\varrho$	Pure rate of time preference (p.a.)	0% Target $r = 6.5\%$
$p$	Death rate (p.a.)	3.85% Target capital-supply elasticity $d \ln K/dr \approx 50$
$\rho$	Effective discount rate (p.a.)	3.85% $p + \varrho$
TECHNOLOGY		
$A$	Hicks-neutral productivity term	0.14 $Y/L$ in 1980
$\delta$	Depreciation rate (p.a.)	5% Standard calibration
$Z$	Number of skill types	100 Each type = percentile of wage distribution
$\ell_z$	Labor supply of each type	1% Each type = percentile of wage distribution
$\alpha(1980)$	Capital share in 1980	0.345 BLS labor share in 1980
$\alpha(2014)$	Capital share in 2014	0.428 BLS labor share in 2014
$\alpha_0$	Capital share in non-routine tasks	0.345 Constant $\alpha_z(1980)$ across $z$
$\omega_{zR}$	Routine jobs share in each pctile in 2000	vector Acemoglu-Autor + 2000 Census/ACS
$\gamma_z$	Skill demand shifters in 1980	vector Wage levels in 1980 Census/ACS
$\psi_z$	Productivity of labor relative to capital	vector Automation reduces costs by 30% ( $= \frac{w_z}{\psi_z R}$ )

Notes: The table provides the parameters used in our baseline calibration of the model. For details, see the main text and Appendix C.

automation. Table 1 summarizes the parameters used in our exercise.

## 2.2 Numerical findings

We now present the transitional dynamics and steady state implications of a change in  $\alpha_z$  over time. Because we are not feeding any other shocks to the model, the results here must be interpreted as counter-factual changes relative to the trend that the economy would have experienced in a world with not automation.

Figure 5 presents the transitional dynamics for the labor share (summarizing the technological change fed to the model), output per worker, the net investment rate, the capital-output ratio, the return to wealth, and the average wage per hour. For comparison, we also plotted the transitional dynamics for the representative household case following the same shift in technology.

In our model, automation leads to a modest expansion of output and the capital-output ratio of 11% and 15%, respectively. Despite the increase in the average output per hour of labor, mean wages go down by 4.5% in 2020 and by 2% in the long run. The productivity gains from automation accrue to capital owners in the form of a higher return to their wealth, which goes up by 0.9 percentage points in the long run and 1.3 p.p. by 2020.<sup>25</sup>

<sup>25</sup>Equation (12) shows why a small increase in the return to wealth of roughly 1 percentage point can have a large effect on wages. Using this equation with  $\alpha = 0.4$ —close to the midpoint of our calibration—shows that a small increase in  $R^*$  of 1 percentage point or 10% in percent terms is enough to ensure that the 3% increase in TFP driven by automation results in a decline in real wages,  $\bar{w}$ , of roughly 2%.

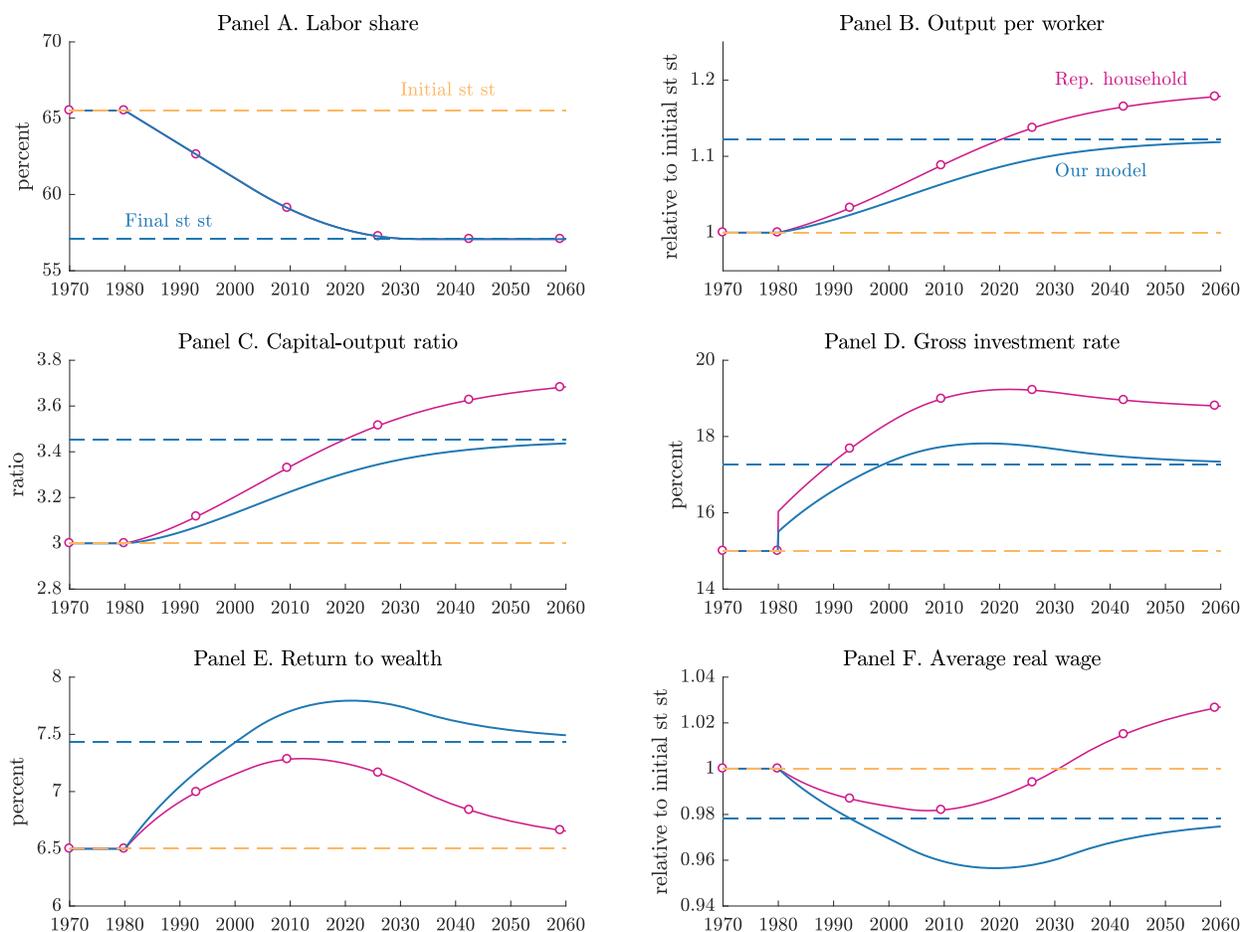


Figure 5: Transitional dynamics for the main aggregates following an improvement in automation technology in 1980. The solid line presents the transitional dynamics in the baseline calibration of our model. The dashed line presents the transitional dynamics when  $p = 0$  and the model admits a representative household.

This is in contrast to what would happen in the representative-household model, where automation leads to a more pronounced economic expansion propelled by a boom in investment, a *temporary* increase in the return to wealth, and higher average wages in the long run. The differences between these models underscore the importance of the capital-supply elasticity. Even though the supply of capital in our model is still fairly elastic, the response of macroeconomic aggregates to automation differs significantly from what one would get in a representative-household model.

The average decline in wages of 0.5 dollars masks substantial heterogeneity. Figure 6 plots the change in wages by percentile from 1980 onwards. In line with the literature emphasizing how the automation of routine jobs affected workers in the middle of the wage distribution, our model generates a polarization of wages over time. The real wage of workers below the 80th percentile of the wage distribution declines over time, but the most pronounced effects are for workers at the 25th percentile of the wage distribution, whose real wages fall by

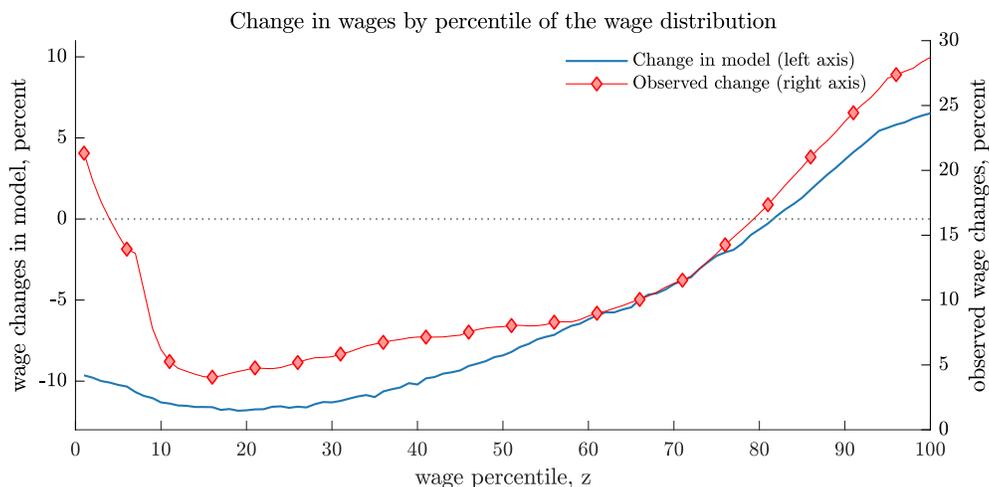


Figure 6: Predicted change in wages by wage percentile (left axis) and observed change in wages by wage percentile (right axis). Observed wage changes computed using the 1980 Census and 2012-2016 ACS. See Appendix C for details.

10%. In contrast, the real wage of workers at the 95th percentile of the wage distribution rises by 5%. For comparison, the figure also plots on a different vertical axis the observed change in wages by percentile between 1980 and 2014 (using data from the US Census and the ACS). The model explains 60% of the observed variation in *relative* wages. Although other technologies not considered here also affected the level of wages and their relative change during this period, our model suggests that automation might have been a significant contributor to wage inequality and to anemic wage growth at the low end of the wage distribution.

We now turn to the implications of our model for income inequality. Figure 7 presents the change in total income at each quantile of the income distribution. The right panel zooms to the top of the income distribution. The figure reveals substantial uneven growth. Below the 50th quantile, individuals experienced declines in total income of 5%. Between the 50th and 80th percentile, individuals experienced a modest income growth of 5%. This is in contrast to the top income quantile, which experienced an increase in income ranging from 30% (for the top 1%) to 50% (for the top 0.1%). Although other technologies not modeled here also shifted incomes, the figure shows that automation is capable of generating substantial rises of income at the very top of the distribution and declines or stagnant incomes at the bottom.

Both wage and wealth inequality combine to produce the pattern observed in Figure 7. The blue-shaded area plots the contribution of changes in labor income. The fall in real wages for individuals at the bottom of the wage distribution contributed to declining incomes for individuals at the middle and bottom of the income distribution. The red-shaded area plots the contribution of changes in capital income. This is uniformly positive, as everyone benefits from a higher return to wealth. But the benefits from a higher return to wealth

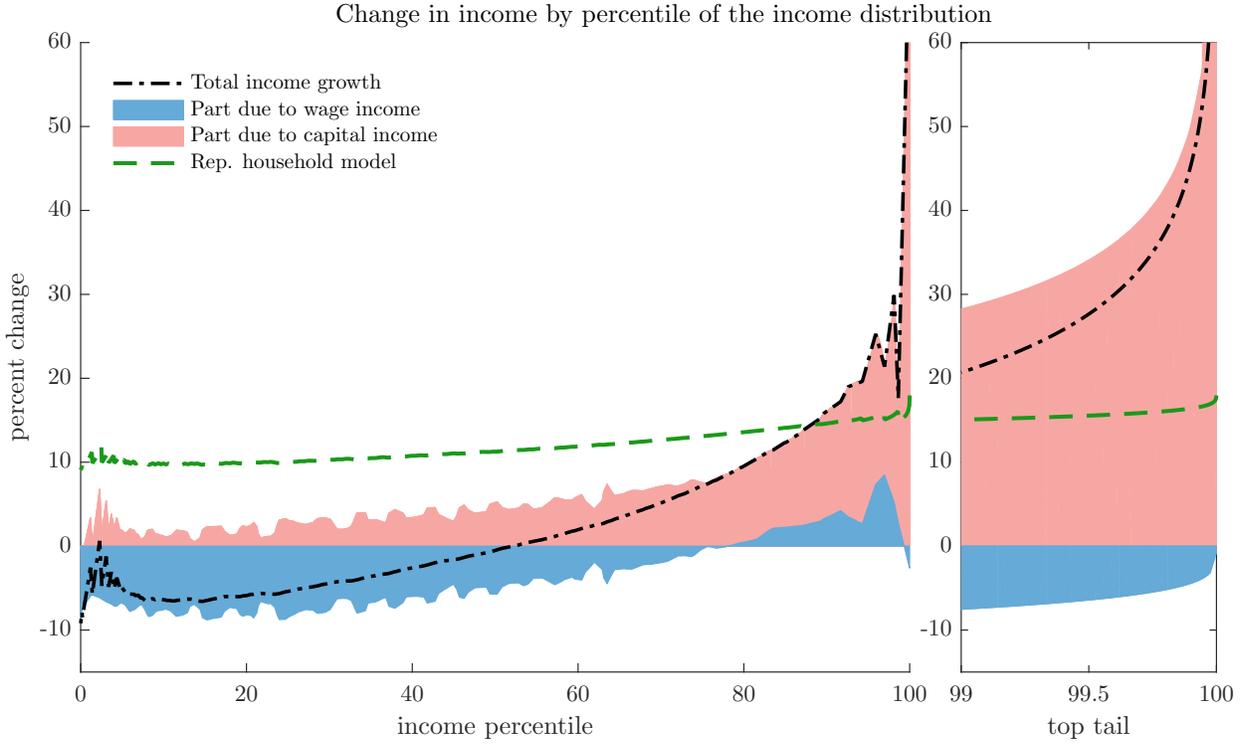


Figure 7: Predicted change in income by income percentile decomposed into the contribution of capital and labor income. The right panel zooms to the top tail of the income distribution. The line with circle dots plots the change in a model that admits a representative household.

are highly dispersed. People at the bottom of the distribution have few assets, and so do not benefit as much from an increase in the return to wealth. In contrast, a higher return to wealth allows some individuals at the top of the income distribution to accumulate large swaths of wealth and earn a high capital income.

Interestingly, although automation is skill biased and raises wages at the top of the wage distribution relative to the middle and bottom (see Figure 6), changes in wage income have a *negative* contribution at the top of the income distribution. Intuitively, there are two effects. On the one hand, Proposition 5 shows that the top of the income distribution is populated by more skilled workers. The skill-biased nature of automation tends to raise the wages of individuals at the top of the distribution more than at other percentiles. However, the permanent increase in returns also means that the top of the income distribution becomes increasingly populated by low wage individuals with very high capital incomes. This shift in the composition of top earners dominates at the top of the income distribution and generates the observed negative contribution of labor income. This negative effect would not be present for other forms of skill biased technical change that do not results in higher returns to wealth.<sup>26</sup>

<sup>26</sup>Formally, the expected wage of individuals at the top of the income distribution is  $\sum_z \ell_z w_z^{1+1/\alpha_{net}^*} / \sum_z \ell_z w_z^{1/\alpha_{net}^*}$ . Skill-biased changes in wages raise this expected wage, but increases in

To illustrate the relevance of having a model where the return to wealth increases permanently, we study the behavior of income inequality in a model with individuals with different skills that admits a representative household (see Caselli and Ventura, 2000). In this model, the return to wealth does not depend on technology, and technology only affects income inequality through wages. The green dashed line in Figure 7 shows the change in income by quantile in this model. Both the representative-household model and ours start from the same conditional distributions of wages and wealth and are subject to the same changes in technology. However, the representative-household model does not generate uneven growth: there is a fairly uniform increase in income between 10% and 18% for all income quantiles. This reflects two differences. First, there is no wage stagnation in the representative-household model, which implies a less pronounced decline in labor income at the middle and bottom of the distribution. Second, the temporary increase in returns to wealth benefits all individuals equally, as they are all able to scale their effective wealth by the same amount. This is in contrast to our model with imperfect dynasties, where individuals who manage to accumulate wealth for longer periods will benefit disproportionately from the higher return to wealth.

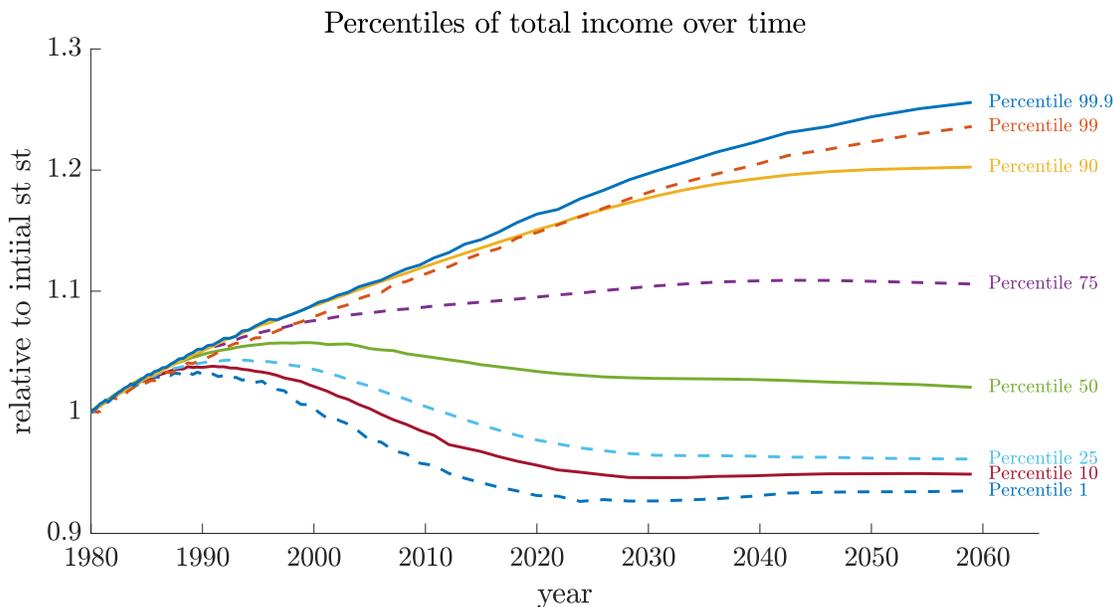


Figure 8: Predicted behavior of income at different percentiles of the income distribution over time. For each percentile, we plot its behavior relative to its steady-state value in 1980.

Figure 8 shows the evolution of several income quantiles over time. In line with our findings above, we see a fanning out of the income distribution, with incomes below the 50th percentile declining over time and incomes above the 50th percentile rising. The fanning

---

$\alpha_{net}^*$  reduce it. See the related discussion of Proposition 5. Note that as we keep moving up the tail, the contribution of labor income converges to zero since individuals' own mostly capital income.

out of top incomes continues over time and takes place slowly, but even by 2020 the model predicts substantial uneven growth.

### 3 Confronting the Model with Past and Current Trends

Our theory has predictions for the evolution of a number of key macroeconomic aggregates as well as the dynamics of the distributions of income and wealth. One purpose of this section is to confront these predictions with the data and to discuss to what extent our model is consistent with observed trends in key variables, both quantitatively and qualitatively. We also use our model as a laboratory to examine a number of other trends that we have not discussed so far but that have been emphasized elsewhere, such as rising markups, falling capital taxes or a rising efficiency of producing investment goods. We do this to showcase the theory’s breadth and its applicability as a framework for thinking through many of the most important trends in modern, advanced economies.

**Secular trends in returns to wealth.** *Ceteris paribus*, our theory predicts that automation increases returns to wealth. This can be most transparently seen in the expression for the steady-state rate of return in Example 1 with  $\delta = 0$ , namely  $r^* = \rho + p\sigma\alpha$  which is increasing in average automation  $\alpha$ . To what extent observed trends in measured returns to wealth are consistent with this prediction?

Before proceeding to discuss these empirical trends, it is important to clarify one key point. In our model, what matters for the evolution of wealth inequality is not the *level* of the return to wealth  $r$  but rather its deviation from the rate of time preference  $r - \rho$ . This gap determines the rate of wealth accumulation  $(r - \rho)/\sigma$  and top tail inequality (see Lemma 1 and Proposition 4). Therefore, for understanding trends in capital income and wealth inequality, secular trends in returns to wealth are, in a sense, not directly relevant and it would be preferable to contrast the model’s prediction for the rate of wealth accumulation with its empirical counterpart. We return to this point below.

With this caveat in mind, Figure 9 plots various measures of returns to capital. To begin, the solid line with diamonds in panel A plots the real return to holding 5-year US treasury bills, that is a risk-free interest rate. As is well-known the last thirty years have seen a strong secular decline in this series: between 1980 and 2014, the treasury rate declined from around eight percentage points per year to zero. Along the lines of the discussion in the preceding paragraph, one possible interpretation of this trend is that the secular decline in safe real interest rates is due to other secular changes captured in a reduced form by a decline in the rate of time preference,  $\rho$ , for example demographics or the “saving glut.” As equation (10) shows, forces that are captured by a decline in  $\rho$  will result in a one-to-one decline in the return to wealth  $r^*$ . These changes would have no effects on individuals’ accumulation rate

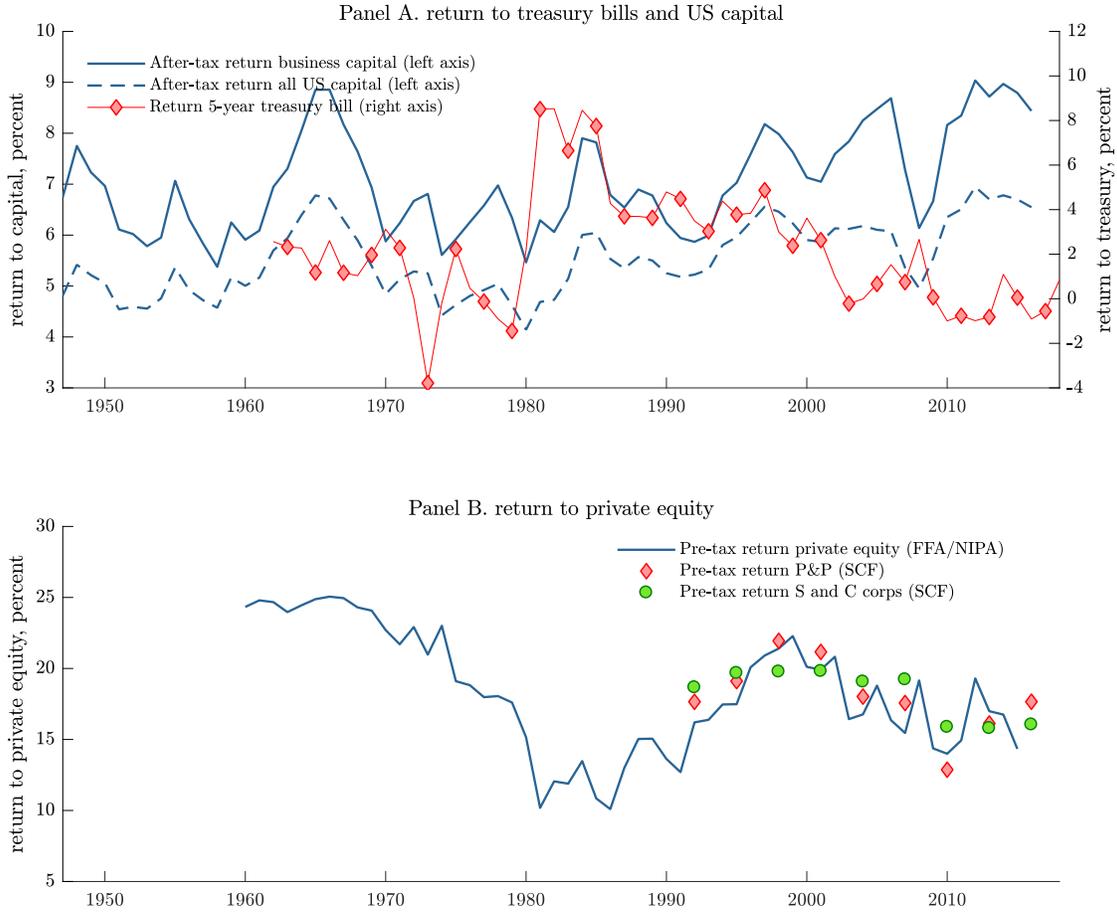


Figure 9: Secular trends in various measures of returns to wealth. The top panel presents after-tax series from [Gomme, Ravikumar and Rupert \(2011\)](#) and the return to a 5-year treasury bill. The bottom panel presents pre-tax series of returns to private equity, computed as in [Moskowitz and Vissing-Jørgensen \(2002\)](#). All series for returns exclude capital gains.

nor wealth inequality, both of which depend on  $\alpha$  but not  $\rho$ . Automation, i.e. a concurrent increase in  $\alpha$  would keep  $r^* = \rho + p\sigma\alpha_{net}^*$  from declining by the same amount as  $\rho$ , thereby generating an increase in  $r^* - \rho$  and wealth inequality.

Risk-free interest rates or rates of return on other safe assets like AAA corporate bonds ( $r_{safe}$  from now on) may provide little information on the importance of capital income and the overall return to wealth obtained by US households, especially those at the top of the wealth distribution. Treasuries and AAA bonds represent a small fraction of the assets in the economy and using risk-free interest rates to infer movements in the average return of all productive assets in the US requires strong assumptions. Also, using risk-free interest rates to impute net capital income as  $r_{safe} \times K$  leads to a large and volatile residual category of “factorless income” whose behavior is hard to justify (see [Karabarbounis and Neiman, 2018](#))

Panel A of Figure 9 therefore plots two alternative return series. Both of these aim to measure the return to the overall stock of capital in the US economy using national accounts

data. The approach defines capital income  $Y_k$  as net output minus wage payments and taxes and computes the return to capital as  $r := Y_k/K$  where  $K$  is the stock of capital measured at replacement cost. That is, the approach computes  $r$  so that  $rK$  matches aggregate capital income from the national accounts. This strategy recognizes that all profits eventually accrue to renters of capital and equity owners, and as such, must be taken into account when computing capital income and the return to capital. This is the approach followed by [Poterba \(1998\)](#); [Mulligan \(2002\)](#); [Gomme, Ravikumar and Rupert \(2011\)](#); [Karabarbounis and Neiman \(2018\)](#) and the BLS multifactor productivity program. Figure 9 plots the after-tax return to US business capital (excluding housing) and to all US capital (including housing) using the data from [Gomme, Ravikumar and Rupert \(2011\)](#). Both measures exhibit an increasing trend between 1980 and 2017, with this measure of the return to wealth rising by 1 to 2 percentage points.<sup>27,28</sup>

Are individuals benefiting from the increasing return to US business capital and overall capital as in our model? Though we do not have direct evidence on the return that people (especially those with high incomes) obtain on their assets, we can use the return to private equity to get a sense of this. The bottom panel of Figure 9 plots a measure of returns from private proprietor and partnerships. Following [Moskowitz and Vissing-Jørgensen \(2002\)](#), we compute these returns using data on private equity from the US Flow of Funds Accounts (FFA) and profits generated by these business from NIPA. The return to private equity shows a marked increase from 13 percentage points per year in 1980 to an average level of 17 percentage points between 1990-2015. Analogous measures constructed from the Survey of Consumer Finances (SCF) confirm that the returns to proprietorships and partnerships (P&P), and to S and C corporations have retained their levels since 1990 and have not followed the sharp decline observed for treasury bills.<sup>29</sup>

Summarizing, since the 1980s the US has seen an upward trend in various measures of returns to wealth, with the exception of risk-free interest rates. To the extent that the increasing return measures are most informative about returns earned by individuals at the

---

<sup>27</sup>We plot the after-tax returns because this is what is relevant for inequality—an issue we discuss in more detail below. The pre-tax series behaves in the same way after 1980 but exhibits a declining trend from 1950 to 1980. This declining trend is partly explained by the sharp decline in capital taxation during this period (see [Poterba, 1998](#)). Finally, we plot a series that excludes capital gains, since capital gains are volatile during this period but do not exhibit any clear trends.

<sup>28</sup>The observed divergence between real risk-free interest rates and the returns to capital shown in Figure 9 has been previously emphasized by [Caballero, Farhi and Gourinchas \(2017\)](#) and [Farhi and Gourio \(2018\)](#) who propose various candidate explanations ranging from rising market power to rising discounts on safe assets. Our baseline model does not feature any elements that drive a wedge between different rates of returns and is therefore silent on this divergence. We briefly discuss an extension of our model with markups below.

<sup>29</sup>As before, we plot a series that excludes capital gains, since capital gains are volatile during this period but do not exhibit any clear trends. Unlike the series for the overall return to capital, this measures of return to private equity are pre-tax, since information on tax rates faced by individuals who own private equity is not available.

top of the income and wealth distributions, these secular trends can qualitatively explain the concurrent increase in income and wealth inequality.

An alternative perspective is that the returns might be on a declining trend due to other forces such as demographic changes (captured by a lower  $\rho$  in our model), which as we discuss below, do not affect top tail inequality. Our mechanism could still increase the rate of wealth accumulation  $(r - \rho)/\sigma$  and top tail inequality, even as these other secular forces operate to reduce  $r^*$ .<sup>30</sup>

**Markups and profits.** The approaches described above to measure  $r$  and returns to private equity assume that all non-labor income is capital income. One potential issue with this approach is the observation that markups might have risen over time (see [De Loecker and Eeckhout, 2017](#)), which implies that a portion of capital income represents economic profits (payments to capital that are above its “normal” return rate). Despite the ongoing debate regarding the rise of markups, the distinction between “normal” returns and economic profits is not consequential for our model. Appendix D provides a generalization of our model including markups and economic profits. The formulas for the return to wealth and the tail parameter of wealth inequality (in the simpler case with  $\delta = 0$ ) generalize to

$$r^* = \rho + p\sigma((1 - \pi)\alpha + \pi) = \rho + p\sigma(1 - \text{labor share})$$

$$\frac{1}{\zeta} = (1 - \pi)\alpha + \pi = 1 - \text{labor share},$$

where  $\pi$  is the profit share of output. The extension shows that the return to wealth and the behavior of wealth inequality depends on the importance of capital income *inclusive* of economic profits. Intuitively, claims on economic profits are also part of the total stock of financial assets that individuals must own, and as such they play the same role as capital.

The above generalization shows that the implications of our model for the return to wealth and the behavior of wealth inequality depend only on the labor share in the economy. Both automation and rising markups will reduce the labor share and increase the return to wealth and top-tail wealth inequality. To the extent that markups are rising, they will also contribute to rising wealth inequality and higher returns to wealth, but would have different implications than automation for aggregates and wage inequality.<sup>31</sup>

---

<sup>30</sup>Another secular force behind the potential decline in  $r^*$  is the deceleration of economic growth (see [Grossman et al., 2017](#)). Suppose that  $\delta = 0$  and  $\psi_z$  grows over time at a rate  $g$ . We have  $r^* = \rho + p\sigma\alpha + \sigma g$  (see also footnote 13). As shown in Appendix D, what matters for inequality in this environment is the individual accumulation rate *relative* to the growth rate of the economy:  $\frac{r-\rho}{\sigma} - g = p\alpha$ . It follows that a deceleration in growth will lower  $r^*$  but won't generate an increase in top tail inequality (with  $\delta > 0$ , it will actually reduce top tail inequality but not by much).

<sup>31</sup>Using a quantitative model, [Boar and Midrigan \(2019\)](#) identify a number of additional channels through which concentration and markups may affect inequality and show that policies that reduce firm concentration may actually have the unintended effect of *increasing* income and wealth inequality.

**Other forces affecting capital markets.** Going back to the diagram in Figure 1, we can think of our theory as explaining how changes in the relative demand for capital lead to a simultaneous increase in the return to wealth, the ratio of capital to wages, and inequality.

We can use our model to study the role of other forces affecting capital markets. On the one hand, we have changes in the supply of capital, driven by demographic trends and by technological changes in the investment sector. On the other hand, we have changes in capital taxation. The main point we wish to make is that all these forces shape the equilibrium of capital markets and contributed to a rise in capital-output or capital-wages ratios since 1950. But so long as capital and labor are gross complements (as they are in our model), such changes cannot account for recent trends in wealth inequality and the rising after-tax return to wealth.

To discuss these forces, we generalize our model slightly so that capital is produced using  $q_K$  units of the final good and capital income is taxed at a rate  $\tau_K \geq 0$  (revenue is used to finance government expenditure). As in our baseline model, wealth inequality depends on the after-tax return to wealth, which we denote by  $r^*$ . Appendix D shows that the formulas for the steady-state return to wealth and the tail parameter of wealth inequality (in the simpler case with  $\delta = 0$ ) generalize to

$$r^* = \rho + p\sigma \frac{\alpha(1 - \tau_K)}{\alpha(1 - \tau_K) + (1 - \alpha)}, \quad \frac{1}{\zeta} = \frac{\alpha(1 - \tau_K)}{\alpha(1 - \tau_K) + (1 - \alpha)} \quad (19)$$

These formulas show that increases in the supply of capital do not contribute to inequality. Demographic changes increasing savings (a reduction in  $\rho$ ) cause an equal decline in  $r^*$ , affect neither the individual accumulation rate nor inequality. Improvements in the investment sector (a rise in  $q_K$ ) reduce the price of a unit of capital, but do not affect the return to wealth nor inequality.

The logic behind these results is the same: although expansions in the supply of capital increase the the quantity of capital, they cause a fully offsetting reduction in the return rate per unit of capital,  $r^*/q_K$ . In our model, this is driven by the fact that the demand for capital has an elasticity of  $-1$  (recall that technology is Cobb-Douglas). As a result, the importance of capital income relative to wages  $(r^*/q_K) \times K/\bar{w}$ —the key object determining the rate of accumulation and inequality—remains constant and equal to  $\alpha(1 - \tau_K)/(1 - \alpha)$ . This line of reasoning shows that, so long as capital and labor are gross complements with an elasticity close to 1, as some of the empirical evidence suggests, the demand for capital will be inelastic and expansions in the supply of capital will bring a small reduction in inequality. Only if capital and labor are highly substitutable, expansions in the supply of capital will increase the net share of capital and result in more inequality.<sup>32</sup>

---

<sup>32</sup>In particular, one would need capital and labor to be net substitutes. More generally, let  $F_{net}(K, \{\ell_z\})$  denote net output and  $F(K, \{\ell_z\})$  denote gross output. A decline in  $\rho$  results in a higher net share of

Regarding taxes, equation (19) shows that, as in a typical market, a reduction in capital taxes will lead to an increase in capital deepening, a lower interest rate faced by firms,  $r^*/(1 - \tau_K)$ , but a higher after-tax return to wealth  $r^*$ . Capital taxes in the US declined sharply between 1950 and 1980 but stabilized since 1980. Our model predicts that the sharp decline in capital taxation contributed to a declining pre-tax return, an increasing after-tax return to wealth, rising wealth inequality, and an expansion of capital, but mostly during 1950-1980.

**Investment and capital.** Our model predicts that automation should lead to an expansion in investment and the capital-output ratio, though less so than in a representative-household model (Figure 2). Figure 10 plots various measures of the empirical counterparts of these variables. The left panel plots two series from the BLS capturing how the value of capital services used by the private sector has evolved relative to GDP, and a series from the BEA giving the value of the US stock of private non-residential fixed assets relative to GDP. Since the 1970s the US capital-to-GDP ratio has increased somewhat according to these measures, with the increase being more pronounced for the BLS measure of capital services (see Gouros and Klier, 2015). The right panel shows that since the 1970s, the ratio of private non-residential fixed investment to GDP (from the BEA) also increased somewhat. Because the price of investment goods declined dramatically during this period, we also find it useful to look at the behavior of the quantity of private non-residential fixed investment relative to the quantity of GDP. The right panel shows that in terms of quantities, investment grew faster than GDP through the postwar period. The prediction in our model of an expansion in the capital-output and investment-output ratios of 15% (see Figure 5) is at the low end of what these series show.

Our point here is not that the effects of automation predicted by our model quantitatively account for the observed trends in investment and capital accumulation (clearly, many other factors affect these variables). Rather, it is that our model with an upward-sloping long-run capital supply provides a partial answer to the question “if the decline in the US labor share was driven by automation, shouldn’t investment and capital have increased?” The answer is “yes, but only slightly, precisely because capital supply is upward-sloping.” To see this in more detail, recall from above that the (gross) capital share increased from 0.345 to 0.43 percent between 1980 and 2014, a 25 percent increase. How much would we expect the capital-output ratio to increase? The answer is simple and can be gauged from manipulating

---

capital if and only if capital is a net substitute for labor (that is  $\partial \ln F_{net} / \partial \ln K$  is increasing in  $K$ ). A decline in  $\rho$  results in a higher net share of capital if and only if capital is a net substitute for labor (that is  $\partial \ln F_{net} / \partial \ln K$  is increasing in  $K$ ). A increase in  $q_K$  results in a higher net share of capital if and only if capital is a gross substitute for labor (that is  $\partial \ln F / \partial \ln K$  is increasing in  $K$ ). See Rognlie (2015) for a related discussion of models capable of explaining the recent rise in the net capital share. .

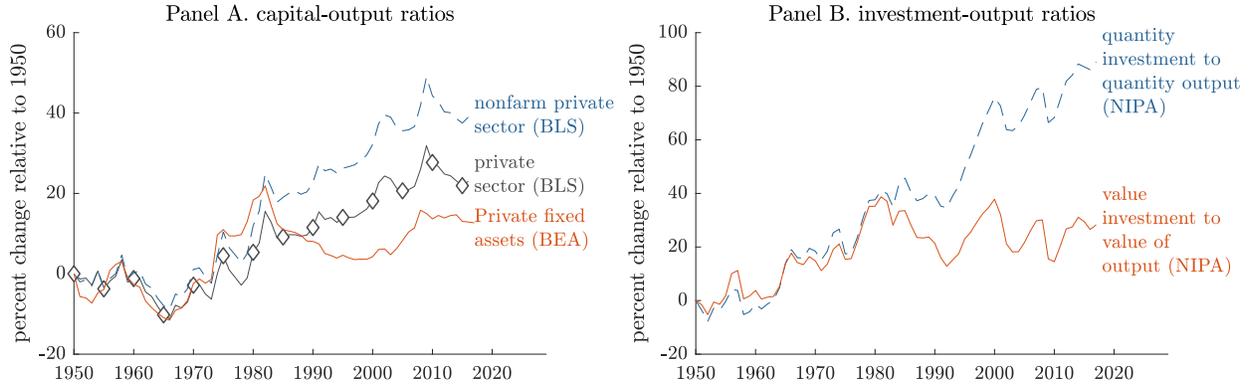


Figure 10: Percent change since 1950 in capital and investment to GDP ratios in the US. Panel A: ratio of capital services to value added for private and nonfarm sectors (BLS Multifactor Productivity series); and ratio of private nonresidential fixed assets (BEA fixed-assets Table 4.1 Line 1 plus Table 4.4 Line 1) to GDP (NIPA Table 1.1.5 Line 1). Panel B: ratio of private nonresidential fixed investment (NIPA Table 1.1.5 Line 9) to GDP (NIPA Table 1.1.5 Line 1); and ratio of quantity of private nonresidential fixed investment (NIPA Table 1.1.3 Line 9) to real GDP (NIPA Table 1.1.3 Line 1).

the definition of the capital share  $\alpha = RK/Y$ :

$$\frac{(K/Y)_{2014}}{(K/Y)_{1980}} = \frac{\alpha_{2014}/\alpha_{1980}}{R_{2014}/R_{1980}} \quad (20)$$

Now consider two scenarios, and assume in both that  $R_{1980} = 11.5\%$  as in our calibration. Consider first the case where the rental rate  $R$  is unchanged so that  $R_{2014}/R_{1980} = 1$ , as in the representative-household model. In that case, the capital-output ratio must increase by 25 percent. But suppose instead that the rental rate  $R$  rises by one percentage point, as it is the case in our model. Then  $\frac{(K/Y)_{2014}}{(K/Y)_{1980}} = \frac{0.43/0.345}{12.5\%/11.5\%} = 1.15$ . Rather than increasing by 25%, the capital-output ratio increases by only 15%. Summarizing, it does not follow that automation predicts massive expansions in investment and capital accumulation. While automation leads to an increase in the gross capital share  $\alpha = RK/Y$ , a substantial fraction of this increase may show up in  $R$  rather than  $K/Y$ .

**The link between capital shares and inequality in the data.** Does the prediction that rises in the net capital share are accompanied by large increases in top income inequality receive support from the data? Bengtsson and Waldenström (2018) explore this link in a long panel with data for 21 countries going back to at least the 1930s. Their analysis shows that a 1 percentage point increase in the net capital share is associated with: a 6.67% increase in the top 0.1 percent share of income; a 3.84% increase in the top 1 percent share of income; and a 1.55% increase in the top 10 percent share of income.<sup>33</sup> Figure 11 illustrates this link

<sup>33</sup>In the published paper, Bengtsson and Waldenström (2018) use a log-log specification, whereas our theory suggests estimating a log-linear specification. We used their data and the same methodology behind their estimates in Table 2 of their paper to estimate the log-linear specifications reported here. Interestingly,

for the US during the postwar period using the data in [Bengtsson and Waldenström \(2018\)](#) as well as the most recent update of [Piketty and Zucman \(2014\)](#) data. Since 1980, the 8 percentage point increase in the net capital share was accompanied by a 60% increase (in log points) in the top 1% share (a 7.5% increase for every one percentage point increase in the net capital share). All these estimates are of the same order of magnitude to what the mechanism in our model would predict—recall from (15) that in our model, a 1 percentage point increase in the capital share is associated with a  $\ln(1/q)$  increase in the top  $q$  share. The 8 percentage point increase in the US net capital share should result in a 37% increase in the top 1% share of income.

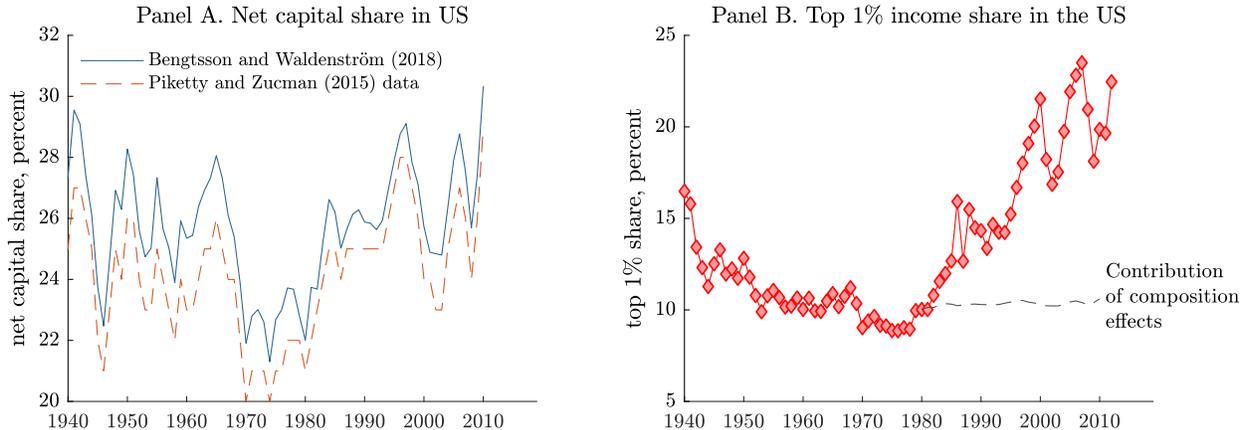


Figure 11: Net capital share and top 1% income share in the US. Data from [Bengtsson and Waldenström \(2018\)](#) and [Piketty and Zucman \(2014\)](#). The figure superimposes the contribution of compositional effects to the rise in top 1% income share.

In contrast, the standard compositional effect emphasized in the literature and defined more precisely in Section 1.4 cannot account for a quantitatively meaningful increase in top income inequality. This is easy to see from a back-of-the-envelope calculation based on equation (14) using US data. In 1980, the top one percent income share was 10% and about 35% of the income of that group was capital income (see [Piketty and Saez, 2003](#)). The fact that  $\alpha_{net} = 23\%$  implies that the shares of capital and labor incomes going to the top 1% of the total income distribution were  $\tilde{S}^k(q) = 15.3\%$  and  $\tilde{S}^\ell(q) = 8.3\%$ .<sup>34</sup> Therefore, if there had only been a composition effect, the 8 percentage point increase in the US net capital share would have increased the top 1% income share by  $dS(q) = (\tilde{S}_k(q) - \tilde{S}_\ell(q)) \times d\alpha_{net} = (0.153 - 0.083) \times 0.08 = 0.56$  percentage points. This is equivalent to a 5.6% increase, a tenth

their data show an increase in the share of income held by the top 0.1% relative to the top 1% and the top 10%, as well as an increase in the share of income held by the top 1% relative to the top 10%, which is consistent with capital shares being associated with a thicker Pareto tail of the income distribution.

<sup>34</sup>As in footnote 18, denote by  $y(q)$ ,  $y_k(q)$  and  $y_\ell(q)$  the income, capital income and labor income of the top  $q$  percent, the corresponding aggregates by  $Y$ ,  $Y_k$ ,  $Y_\ell$  and  $S(q) = y(q)/Y$ ,  $\tilde{S}_k(q) = y_k(q)/Y_k$ . Then, using that  $y_k(q) = \alpha_{net}(q)y(q)$  where  $\alpha_{net}(q) = 35\%$  is that quantile's share of capital income, and  $y_k(q) = \tilde{S}_k(q)Y_k = \tilde{S}_k(q)\alpha_{net}Y$ , we have  $\tilde{S}_k(q) = (\alpha_{net}(q)/\alpha_{net})S(q) = (35\%/23\%) \times 10\% = 15.3\%$ .

of what we observe in the data and a fifth of what our mechanism generates. The dashed line in Figure 11 shows the small contribution of compositional effects to the evolution of the top 1% income share.<sup>35</sup>

The link between capital shares and inequality can be also seen at other historical periods during which automation (or mechanization) was a dominant force, like the onset of the industrial revolution in Britain. As documented in Allen (2009) and reproduced here in Figure 12, from 1760 to 1840, the capital share (excluding land) rose from 20% to 40% in Britain and the labor share declined from 60% to 50%. In line with our model, the return to wealth (what Allen terms the profit rate) doubled from 10 percentage points to 20 percentage points at the same time as average wages stagnated. Data from Lindert (2000) show a sharp rise in income inequality starting exactly at this period, as can be seen from the evolution of the top 5% income share in Britain, plotted in the right panel of the figure.

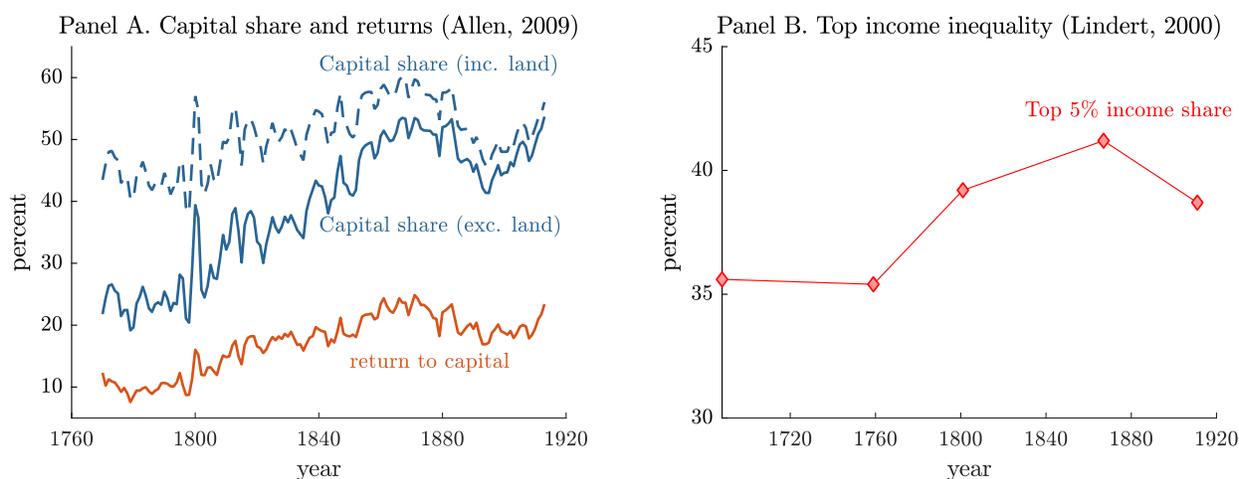


Figure 12: Gross capital share, the return to capital, and the top 5% income share during the British industrial revolution. Data from Allen (2009) and Lindert (2000).

**Capital income and the rise in top inequality.** One of this paper’s main premises is that not all income is labor income and that, at any given point in time, capital income is important, particularly at the top of the income distribution. Along the same lines, our theory predicts that capital income is important for generating a large increase in top

<sup>35</sup>Writing fifty years ago, Meade (1964) painted a bleak picture of automation’s impact, explicitly linking it to capital income and wealth inequality: “But what of the future? Suppose that automation should drastically reduce [the labor share...] There would be a limited number of exceedingly wealthy property owners; the proportion of the working population required to man the extremely profitable automated industries would be small; wage-rates would thus be depressed; [...] we would be back in a super-world of an immiserised proletariat and of butlers, footmen, kitchen maids, and other hangers-on.” He then goes on to justify the connection between the capital share and income inequality with the composition-effect logic explained above. Our analysis suggests that Meade was right to emphasize a connection between the capital share and inequality but that he had in mind the wrong mechanism (see Figure 11).

inequality – see Figure 7. A natural question is therefore to what extent empirical trends are consistent with this prediction.

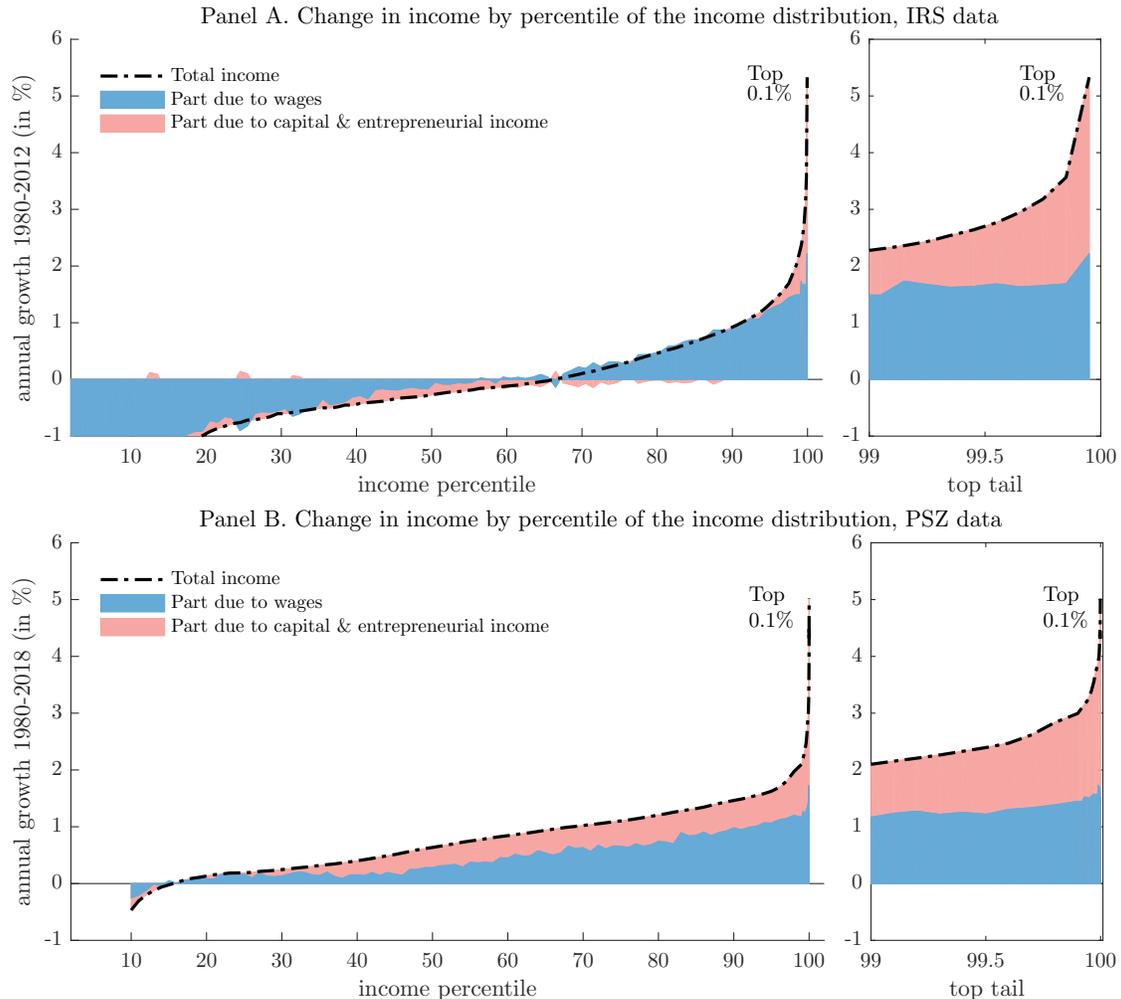


Figure 13: Capital income and the rise in top inequality (IRS and PSZ data)

To this end, we use US data to examine to what extent uneven growth in the US economy is accounted for by capital income and wage income. We use two different data sources: the NBER IRS public use sample from 1980 to 2012 and the data of [Piketty, Saez and Zucman \(2018\)](#) from 1980 to 2018 (PSZ, hereafter).<sup>36</sup> The IRS dataset is based on administrative tax records and therefore yields reliable information for the very top of the distribution. However, it only records *fiscal* income and therefore omits tax-exempt income such as pension income. To remedy this, PSZ try to impute this “missing income” by making various adjustments to

<sup>36</sup>The NBER IRS public use sample is available through the NBER. See <https://users.nber.org/~taxsim/gdb/> and <https://www.nber.org/taxsim-notes.html>. This IRS data is also used in [Piketty and Saez \(2003\)](#). For the PSZ data, we do not have micro data at the individual or household level (in contrast to the IRS data). The figure instead uses tabulations by income percentiles from the [Piketty, Saez and Zucman \(2018\)](#) dataset that were kindly computed by Gabriel Zucman.

the fiscal income series and bringing to the table other data sets as well as making various assumptions. Given this, the main advantage of the PSZ data are the more comprehensive income concept being used that also adds up to national income but this comes up at the cost of strong assumptions and numerous imputations. Conversely, the main advantage of the IRS data is the transparency and simplicity of the data construction but this comes up at the cost of substantial amounts of “missing income.”

Figure 13 provides our analysis of the sources of uneven growth in the United States. Panel (a) uses the IRS data and panel (b) the PSZ data. The dash-dotted black line labelled “total income” in both panels plots the average annual income growth between 1980 and 2012 for the 100 percentiles of the income distribution. The right-hand side of each panel provides a “zoom” of the top tail of the distribution where much of the action is. As this figure shows, the income percentiles corresponding to the lower half of the distribution have stagnated or declined between 1980 and 2012. In contrast, top income percentiles have grown rapidly. For example, in both data sets, the top 0.1 percentile increased at a yearly rate of more than 5 percent. The hockey stick shape of income growth at different percentiles in the figure is qualitatively similar to that predicted by our model (see Figure 7).<sup>37</sup>

The blue and red shaded areas in both panels of Figure 13 decompose the growth in total income into a part due to labor income and a part due to capital and entrepreneurial income.<sup>38</sup> In both datasets, capital income is unimportant for income growth at the bottom of the distribution. As in our model, the stagnant incomes at the bottom of the distribution are driven by declines in real wages. However, further up in the distribution and especially within the top 1 percent, capital and entrepreneurial income becomes more and more important. The extent to which capital income accounts for the observed uneven growth and at which percentiles it starts to be important differs between the two datasets. The IRS data in panel (a) suggest that up to the 90th percentile, essentially all growth in total income is accounted for by labor income and that capital and entrepreneurial income are mostly important within the top 1% and especially at the top 0.1% where capital and entrepreneurial income account

---

<sup>37</sup>The change in total incomes in panels (a) and (b) is qualitatively similar but there are some quantitative differences in terms of both the average growth and the relative growth across different income percentiles. The blue line in panel (b) labelled “total income” is the same as that in Figure II(a) of [Piketty, Saez and Zucman \(2018\)](#) and they discuss this discrepancy. In particular, they note that their series yields “more growth for the bottom 90% since 1980 than suggested by the fiscal data studied by [Piketty and Saez \(2003\)](#)” which is essentially our IRS series. They go on to say that “the main reason for this discrepancy is that the tax-exempt income of bottom 90% earners – which fiscal data miss – has grown since 1980” and that “all of this increase derives from the rise of imputed capital income earned on tax-exempt pension plans.” [Piketty, Saez and Zucman’s](#) Appendix C.2 discusses in more detail the importance of capital income for the rise of the top 1% income share and compares their results to those of [Smith et al. \(2019\)](#).

<sup>38</sup>Denoting by  $y_t(q)$  the  $q$ th income percentile at time  $t$  and by  $y_{\ell,t}(q)$  and  $y_{k,t}(q)$  that percentile’s labor and capital income, we decompose the  $T$ -year growth rate from a base year  $t = 0$ ,  $\frac{y_T(q) - y_0(q)}{y_0(q)}$ , into a part due to labor income  $\frac{y_{\ell,T}(q) - y_{\ell,0}(q)}{y_0(q)}$  and a part due to capital income  $\frac{y_{k,T}(q) - y_{k,0}(q)}{y_0(q)}$ . Figure 13 plots an *annualized* version of this decomposition in which we scale the annualized  $T$ -year growth rate,  $(y_T(q)/y_0(q))^{1/T} - 1$  by the fractions of  $T$ -year growth  $(y_T(q)/y_0(q)) - 1$  due to each income type. See Appendix A.2 for more detail.

for around 60% of the cumulative growth from 1980-2012. In contrast, the PSZ data in panel (b) suggest that capital income plays an important role already around the median of the income distribution, accounts for around half of cumulative growth of the top 1% and is twice as important as labor income for the rise in top 0.01% incomes. The empirical pattern in both data sets is qualitatively consistent with that generated by our model – see Figure 7. The fact that wage income plays a more prominent and positive role at the tail of the income distribution in the data than in our model suggest there are forms of skill-biased technical change other than automation affecting relative wages (see the discussion of Figure 7 in Section 2.2).

**Dynamics of top income and wealth inequality.** The past forty years have seen a rapid rise in top income inequality in the United States (see e.g. [Piketty and Saez, 2003](#); [Piketty, Saez and Zucman, 2018](#)). There is also an ongoing debate about the dynamics of top wealth inequality,<sup>39</sup> with some observers arguing that top wealth inequality has also increased rapidly. While our theory can account for large changes in inequality between steady states – see for example Figure 7 – it cannot generate the rapid transition dynamics of top income inequality observed in the data. Figure 8 shows that the model-implied transition dynamics at the top of the distribution are very slow and, for example, the 99.9 percentile of the income distribution still has not converged 80 years after the initial shock.

That theories like ours cannot generate fast transition dynamics is a known result: [Gabaix et al. \(2016\)](#) have shown that standard theories of the Pareto tails of the income and wealth distributions, which build on a random growth mechanism, generate transition dynamics that are too slow relative to those observed in the data. Our theory is exactly a special case of such a theory – see Proposition 4 – and is therefore subject to the same criticism.

The good news is that we know how to “fix” random growth theories to deliver fast transition dynamics like those observed in the data. [Gabaix et al. \(2016\)](#) show that what is needed are particular deviations from Gibrat’s law, what they call “type- and scale-dependence.” For the case of wealth dynamics, heterogeneous and persistent rates of return to wealth are one candidate for generating such type- and scale-dependence and seem to be a prevalent feature of the data ([Fagereng et al., 2016](#)). Future work should build more quantitatively serious theories of the general-equilibrium interaction between technology and income and wealth distribution that feature these model elements.<sup>40</sup>

---

<sup>39</sup>See for example [Kopczuk \(2015\)](#), [Saez and Zucman \(2016\)](#), and [Bricker et al. \(2016\)](#).

<sup>40</sup>Alternatively, theories with changing asset prices – another model feature of the data that we do not model – are promising for generating fast wealth inequality dynamics.

## 4 Conclusion

In this paper, we developed a tractable framework to study the effects of technology on income inequality. Our theory allowed us to go beyond wages and to explore how technology affects wealth inequality and overall income inequality. We used our framework to study the effects of automation and identified a new channel by which technology may affect inequality. Technology affects not only wages but also asset returns to wealth and this can have substantial distributional effects. Because the productivity gains from automation accrue to owners of capital in the form of higher returns, automation does not necessarily lead to a large expansion of investment, output, and wages. Instead, the higher returns raise the cost of capital and moderate the response of investment. Automation may therefore result in stagnant or declining real wages, especially at the bottom and middle of the wage distribution.

There are two fruitful avenues for future work. First, one could use our tractable framework to study the distributional consequences of other types of technical change, changes in market structure and markups, and government policies, like the taxation of capital or estates, or redistributive policies. For example, it could be worthwhile to integrate our model with a theory of international trade so as to examine the effects of globalization on income and wealth distribution, and not just that of wages as is common in the trade literature. Similarly, our analytically tractable theory featuring a less-than-perfectly-elastic capital supply and non-degenerate wealth distribution may serve as a useful laboratory for exploring the optimal taxation of capital income and wealth.

Second, one could think of building more elaborate quantitative models to study the effect of technologies on inequality. As explained in the introduction, these more elaborate models should retain the two key features underscored by our analysis: an upward-sloping supply of capital and a return inequality nexus. More elaborate versions of our model could include realistic life-cycle structures, a careful treatment of bequests and intergenerational transfers, and heterogeneity in portfolio and return rates. Successful quantitative extensions should also include some form of scale-dependence and type-dependence to account for the rapid rise of inequality observed in the data.

In connecting our model to the data, one crucial question is how important is the rise of capital income at the top of the distribution. This is still an open and interesting question. Some researchers assert that capital income plays a more prominent role in European countries. If so, perhaps our theory applies more forcefully to the European context. After all, Western Europe is ahead of the US when it comes to industrial automation (see [Acemoglu and Restrepo, 2018a](#)), and the decline in the labor share appears common to all these countries ([Karabarbounis and Neiman, 2013](#)). In the US, wages play a more prominent role, pointing to the importance of other forms of skill-biased technical change different from the

automation of routine jobs. Besides wages, entrepreneurial income seems to play an important role at the top of the income distribution (see [Smith et al., 2019](#)). One key question going forward is to understand how much of the rise in entrepreneurial income is the result of technologies that allow entrepreneurs to grow their firms without paying a high share of the generated income in wages, or if rising entrepreneurial incomes at the top reflect other changes in regulation, market structure, and the way we reward talent. That is, we need to understand whether rising entrepreneurial incomes reflect the higher returns brought by automation or these other factors.

## References

- ACEMOGLU, D. (2009): *Modern Economic Growth*. Princeton University Press, New York.
- ACEMOGLU, D., AND D. AUTOR (2011): *Skills, Tasks and Technologies: Implications for Employment and Earnings* vol. 4 of *Handbook of Labor Economics*, chap. 12, pp. 1043–1171. Elsevier.
- ACEMOGLU, D., AND P. RESTREPO (2018a): “Demographics and Automation,” Working Paper 24421, National Bureau of Economic Research.
- (2018b): “The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment,” *American Economic Review*, 108(6), 1488–1542.
- (2019): “Automation and New Tasks: How Technology Displaces and Reinstates Labor,” *Journal of Economic Perspectives*, 33(2), 3–30.
- ALLEN, R. C. (2009): “Engels? pause: Technical change, capital accumulation, and inequality in the british industrial revolution,” *Explorations in Economic History*, 46(4), 418–435.
- AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. V. REENEN (2017): “The Fall of the Labor Share and the Rise of Superstar Firms,” NBER Working Papers 23396, National Bureau of Economic Research, Inc.
- AUTOR, D. H., AND D. DORN (2013): “The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market,” *American Economic Review*, 103(5), 1553–1597.
- AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2006): “The Polarization of the U.S. Labor Market,” *American Economic Review*, 96(2), 189–194.
- AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): “The Skill Content of Recent Technological Change: An Empirical Exploration,” *The Quarterly Journal of Economics*, 118(4), 1279–1333.
- BENGTSSON, E., AND D. WALDENSTRÖM (2018): “Capital Shares and Income Inequality: Evidence from the Long Run,” *The Journal of Economic History*, 78(03), 712–743.
- BENHABIB, J., AND A. BISIN (2007): “The distribution of wealth: Intergenerational transmission and redistributive policies,” Discussion paper, New York University.
- (2018): “Skewed Wealth Distributions: Theory and Empirics,” *Journal of Economic Literature*, 56(4), 1261–91.
- BENHABIB, J., A. BISIN, AND S. ZHU (2011): “The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents,” *Econometrica*, 79(1), 123–157.
- BLANCHARD, O. J. (1985): “Debt, Deficits, and Finite Horizons,” *Journal of Political Economy*, 93(2), 223–247.
- BOAR, C., AND V. MIDRIGAN (2019): “Markups and Inequality,” Working paper, NYU.
- BRICKER, J., A. HENRIQUES, J. KRIMMEL, AND J. SABELHAUS (2016): “Measuring Income and Wealth at the Top Using Administrative and Survey Data,” *Brookings Papers on Economic*

- Activity*, 47(1 (Spring), 261–331.
- CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2017): “Rents, Technical Change, and Risk Premia Accounting for Secular Trends in Interest Rates, Returns on Capital, Earning Yields, and Factor Shares,” *American Economic Review*, 107(5), 614–620.
- CASELLI, F., AND A. MANNING (2018): “Robot arithmetic: new technology and wages,” LSE Research Online Documents on Economics 87371, London School of Economics and Political Science, LSE Library.
- CASELLI, F., AND J. VENTURA (2000): “A Representative Consumer Theory of Distribution,” *American Economic Review*, 90(4), 909–926.
- CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J.-V. RÍOS-RULL (2003): “Accounting for the U.S. Earnings and Wealth Inequality,” *Journal of Political Economy*, 111(4), 818–857.
- CENSUS BUREAU, U. (2015): “Real Household Income at Selected Percentiles: 1967 to 2014,” Discussion paper, U.S. Census Bureau, Current Population Survey, 1968 to 2015 Annual Social and Economic Supplements.
- CHATTERJEE, S. (1994): “Transitional dynamics and the distribution of wealth in a neoclassical growth model,” *Journal of Public Economics*, 54(1), 97–119.
- DE LOECKER, J., AND J. EECKHOUT (2017): “The Rise of Market Power and the Macroeconomic Implications,” NBER Working Papers 23687, National Bureau of Economic Research, Inc.
- FAGERENG, A., L. GUISO, D. MALACRINO, AND L. PISTAFERRI (2016): “Heterogeneity and Persistence in Returns to Wealth,” NBER Working Papers 22822, National Bureau of Economic Research, Inc.
- FARHI, E., AND F. GOURIO (2018): “Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia,” NBER Working Papers 25282, National Bureau of Economic Research, Inc.
- GABAIX, X. (2009): “Power Laws in Economics and Finance,” *Annual Review of Economics*, 1(1), 255–293.
- GABAIX, X., J.-M. LASRY, P.-L. LIONS, AND B. MOLL (2016): “The Dynamics of Inequality,” *Econometrica*, 84, 2071–2111.
- GOMME, P., B. RAVIKUMAR, AND P. RUPERT (2011): “The Return to Capital and the Business Cycle,” *Review of Economic Dynamics*, 14(2), 262–278.
- GOURIO, F., AND T. KLIER (2015): “Recent trends in capital accumulation and implications for investment,” Chicago Fed Letters 344, Chicago Fed.
- GROSSMAN, G. M., E. HELPMAN, E. OBERFIELD, AND T. SAMPSON (2017): “The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration,” NBER Working Papers 23853, National Bureau of Economic Research, Inc.
- HÉMOUS, D., AND M. OLSEN (2018): “The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality,” Working paper, University of Zurich.
- HUBMER, J., P. KRUSELL, AND A. SMITH (2016): “The Historical Evolution of the Wealth Distribution: A Quantitative-Theoretic Investigation,” NBER Working Papers 23011, National Bureau of Economic Research, Inc.
- JAFFE, S., R. MINTON, C. MULLIGAN, AND K. MURPHY (2019): *Chicago Price Theory*. Princeton University Press.
- JONES, C. I. (2015): “Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality,” *Journal of Economic Perspectives*, 29(1), 29–46.
- KARABARBOUNIS, L., AND B. NEIMAN (2013): “The Global Decline of the Labor Share\*,” *The Quarterly Journal of Economics*, 129(1), 61–103.
- (2018): “Accounting for Factorless Income,” NBER Working Papers 24404, National

- Bureau of Economic Research, Inc.
- KATZ, L., AND K. M. MURPHY (1992): “Changes in Relative Wages, 1963?1987: Supply and Demand Factors,” *The Quarterly Journal of Economics*, 107(1), 35–78.
- KAYMAK, B., AND M. POSCHKE (2016): “The evolution of wealth inequality over half a century: The role of taxes, transfers and technology,” *Journal of Monetary Economics*, 77(C), 1–25.
- KEHRIG, M., AND N. VINCENT (2018): “The Micro-Level Anatomy of the Labor Share Decline,” NBER Working Papers 25275, National Bureau of Economic Research, Inc.
- KOPCZUK, W. (2015): “What Do We Know about the Evolution of Top Wealth Shares in the United States?,” *Journal of Economic Perspectives*, 29(1), 47–66.
- KRUSELL, P., L. E. OHANIAN, J.-V. RÍOS-RULL, AND G. L. VIOLANTE (2000): “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis,” *Econometrica*, 68(5), 1029–1053.
- KRUSELL, P., AND A. A. SMITH, JR. (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, 106(5), 867–896.
- LINDERT, P. (2000): “Three Centuries of Inequality in Britain and America,” in *Handbook of Income Distribution*, ed. by A. Atkinson, and F. Bourguignon, vol. 1, chap. 03, pp. 167–216. Elsevier, 1 edn.
- MARTINEZ, J. (2019): “Automation, Growth and Factor Shares,” Discussion paper, London Business School.
- MEADE, J. E. (1964): *Efficiency, Equality and the Ownership of Property*. George Allen & Unwin. Available at <http://www.princeton.edu/~moll/meade.pdf>.
- MOSKOWITZ, T. J., AND A. VISSING-JØRGENSEN (2002): “The returns to entrepreneurial investment: A private equity premium puzzle?,” *American Economic Review*, 92(4), 745–778.
- MULLIGAN, C. B. (2002): “Capital, Interest, and Aggregate Intertemporal Substitution,” NBER Working Papers 9373, National Bureau of Economic Research, Inc.
- PIKETTY, T. (2014): *Capital in the Twenty-First Century*. Harvard University Press, Cambridge.
- (2015): “About Capital in the Twenty-First Century,” *American Economic Review*, 105(5), 48–53.
- PIKETTY, T., AND E. SAEZ (2003): “Income Inequality in the United States, 1913-1998,” *The Quarterly Journal of Economics*, 118(1), 1–41.
- PIKETTY, T., E. SAEZ, AND G. ZUCMAN (2018): “Distributional National Accounts: Methods and Estimates for the United States,” *The Quarterly Journal of Economics*, 133(2), 553–609.
- PIKETTY, T., AND G. ZUCMAN (2014): “Capital is Back: Wealth-Income Ratios in Rich Countries 1700?2010,” *The Quarterly Journal of Economics*, 129(3), 1255–1310.
- POTERBA, J. M. (1998): “The rate of return to corporate capital and factor shares: new estimates using revised national income accounts and capital stock data,” *Carnegie-Rochester Conference Series on Public Policy*, 48(1), 211–246.
- RACHEL, L. (2019): “Automation, Unemployment and Inequality,” Discussion paper, London School of Economics.
- ROGNLIE, M. (2015): “Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?,” *Brookings Papers on Economic Activity*, 46(1 (Spring)), 1–69.
- SACHS, J. D., AND L. J. KOTLIKOFF (2012): “Smart Machines and Long-Term Misery,” Working Paper 18629, National Bureau of Economic Research.
- SAEZ, E., AND G. ZUCMAN (2016): “Editor’s Choice Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data,” *The Quarterly Journal of Economics*, 131(2), 519–578.
- SIMON, H. A. (1965): *The shape of automation for men and management*, vol. 13. Harper & Row, New York.

- SMITH, M., D. YAGAN, O. M. ZIDAR, AND E. ZWICK (2019): “Capitalists in the Twenty-First Century,” NBER Working Papers 25442, National Bureau of Economic Research, Inc.
- STACHURSKI, J., AND A. A. TODA (2018): “An Impossibility Theorem for Wealth in Heterogeneous-agent Models with Limited Heterogeneity,” Papers 1807.08404, arXiv.org.
- STEINDL, J. (1965): *Random Processes and the Growth of Firms: a Study of the Pareto Law*. Griffin, London.
- STRAUB, L. (2019): “Consumption, Savings, and the Distribution of Permanent Income,” Discussion paper, Harvard University.
- WOLD, H. O. A., AND P. WHITTLE (1957): “A Model Explaining the Pareto Distribution of Wealth,” *Econometrica*, 25(4), 591–595.
- ZEIRA, J. (1998): “Workers, Machines, and Economic Growth,” *The Quarterly Journal of Economics*, 113(4), 1091–1117.