Convergence, Financial Development, and Policy Analysis*

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January 18, 2019

Abstract

We study the relationship among inflation, economic growth, and financial development in a Schumpeterian overlapping-generations model with credit constraints. In the baseline case money is super-neutral. When the financial development exceeds some critical level, the economy catches up and then converges to the growth rate of the world technology frontier. Otherwise, the economy converges to a poverty trap with a growth rate lower than the frontier and with inflation decreasing with the level of financial development. We then study efficient allocation and identify the sources of inefficiency in a market equilibrium. We show that a particular combination of monetary and fiscal policies can make a market equilibrium attain the efficient allocation.

JEL Classification: O11, O23, O31, O33, O38, O42

Keywords: Economic Growth, Innovation, Credit Constraints, Convergence, Policy Analysis, Money, Inflation

*We thank Zhengwen Liu for excellent research assistance. We also thank Xiaodong Zhu and participants in many conferences for helpful comments.

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1 Introduction

The industrial revolution marked a dramatic turning point in the economic progress of nations. During the nineteenth century, a number of technological leaders in the Western Europe and North America leapt ahead of the rest of the world, while others lagged behind and became colonies or semi-colonies of the Western powers. After the WWII, most developing countries obtained political independence and started their industrialization and modernization process. One might expect that, with the spread of technology and the advantage of backwardness (Gerschenkron (1962)), the world should have witnessed convergence in income and living standard. Instead, the post WWII was a period of continued and accelerated divergence (Pritchett (1997)). According to Maddison (2008), the per capita GDP in the U.S., the most advanced countries in the 20th century, grew at an average annual growth rate of 2.1% in the period between 1950 and 2008. While some OECD and East Asian economies were able to narrow the per capita GDP gap with an annual growth rate higher than that of the U.S. in the catch up process, most other countries in Latin America, Asia, and Africa failed to achieve so.[1]

Why some countries fail to converge in growth rates despite the possibility of technology transfer has been a puzzle. There are several explanations in the literature.[2] In this paper we focus on the explanation of Aghion, Howitt, and Mayer-Foulkes (henceforth AHM) (2005) and Acemoglu, Aghion, and Zilibotti (2006) based on a Schumpeterian overlapping-generations (OLG) model of economic growth with credit constraints.[3]

[Insert Figure 1 Here.]

We contribute to this literature by analyzing the relationship among growth, inflation, and financial development. Figure 1 presents the cross-sectional evidence on the sample of 71 countries over the period 1960-1995.[4] Panels A and B show that the average inflation rate is negatively related to the average per capita GDP growth rate and positively related to the average money growth rate. Panel C shows that the average inflation rate is negatively related to the average level of financial development and this relationship vanishes at a high level of financial development (about 50%). Panel D displays the countries that fail to converge to the world frontier growth rate, identified by AHM (2005). These countries have a low average level of financial development and their inflation is negatively related to the average level of financial development.

Motivated by the evidence above, we introduce a monetary authority and a government to a closed-economy version of the AHM model. We modify this model in several ways. First, we

[1] In the period of 1950-2008, the average per capita GDP growth rates for the whole Latin America, Asia, and Africa were respectively 1.8%, 1.6%, and 1.2% (Maddison (2008)).

[2] See Banerjee and Duflo (2005) for a survey.

[3] AHM (2005) provide empirical evidence to support the importance of the credit constraints for convergence or divergence.

[4] Appendix B presents data description.

introduce money by assuming money enters utility (Sidrauski (1967)). This money-in-the-utility approach can be microfounded in several ways once one takes into account the role of medium of exchange (McCallum (1983)). Although money can be valued in the OLG model as a store of value (Samuelson (1958)), the equilibrium net nominal interest rate is zero and hence one cannot analyze monetary policy in terms of interest rate rules. Our modeling of money avoids this issue.[5] Second, we introduce intra-generational heterogeneity so that there are savers and borrowers (entrepreneurs) in each period. We can then endogenize the nominal interest rate in a credit market and study how credit market imperfections affect interest rates. Third, we assume savers are risk averse so that we can derive their consumption and portfolio choices. In each period a young saver must choose optimal consumption, money holdings, and saving in terms of nominal bonds.

We show that the market equilibrium in our model can be summarized by a system of four nonlinear difference equations for four sequences of variables: the nominal interest rate, the inflation rate, the normalized R&D investment, and the proximity to the technological frontier. For this equilibrium system, monetary policy is modeled by a money supply rule. If one uses an interest rate rule as in the dynamic new Keynesian literature (Woodford (2003)), then money supply is endogenous and the nominal interest rate is replaced by the money growth rate in the equilibrium system. Due to the complexity of our model, we cannot reduce this system to a scalar one for the proximity variable alone as in the AHM model. However we are still able to provide a full characterization of the steady state along a balanced growth path, which is consistent with the evidence presented in Figure 1.

It turns out that how money supply is introduced to the economy is critical for how money affects the equilibrium allocation and long-run growth. We first show that, if money increments are transferred to the old agents in an amount proportional to their pre-transfer money holdings, then money is super-neutral in the sense that monetary policy does not affect long-run growth and the equilibrium allocation along a balanced growth path.[6] This result dates back to Lucas's (1972) model, in which there is no endogenous growth. The intuition is that the demand for money and saving depends on the ratio of the nominal interest rate and the money growth rate and hence the real interest rate in the long run. Thus only real variables are determined in the steady state.

We show that there are three dynamic patterns as in the AHM model with the difference that our model incorporates inflation:

1. When the credit market is perfect so that the credit constraint does not bind, the economy converges to the world frontier growth rate and there is no marginal effect of financial development.

2. When the credit constraint binds, but is not tight enough, the economy converges to the

[5] Another approach is to introduce a cash-in-advance constraint.
[6] Money growth has a short-run effect on the transition path.

world frontier growth rate with a level effect of financial development.

3. When the credit constraint is sufficiently tight, there is divergence in growth rates with a growth effect of financial development. In this case the economy enters an equilibrium with poverty trap.

We prove that the steady states for all these three cases are saddle points. For any given initial value of the proximity to the frontier, there exists a unique saddle path such that the economy will transition to the steady state. For the first two cases, the transition paths display the feature of the advantage of backwardness (Gerschenkron (1962)). Moreover, the inflation rate rises during the transition. But for the third case, the economy exhibits the feature of the disadvantage of backwardness and falls into the poverty trap with low economic growth, low innovation, and high inflation. The inflation rate declines during the transition. Moreover, the long-run rate of productivity growth increases with financial development and the long-run inflation rate decreases with financial development.

Next we study efficient allocation. Suppose that there is a social planner who maximizes the sum of discounted utilities of all agents in the present and future generations. We derive the efficient allocation and long-run growth rate. By comparing with the efficient allocation, we find there are four sources of inefficiency in a market equilibrium. First, there is monopoly inefficiency in the production of intermediate goods. The resulting price distortion generates an inefficiently low level of final net output when taken the innovation rate as given. Second, the private return to innovation ignores the dynamic externality or spillover effect of technology. Third, the credit market imperfection prevents innovators to obtain necessary funds for R&D. Finally, the OLG framework itself may cause dynamic inefficiency and inefficient within generation consumption allocation.

Can a combination of monetary and fiscal policies correct the preceding inefficiencies and make the market equilibrium attain the efficient allocation? We show that when money increments are transferred to the entrepreneur, money is not super-neutral and there is a particular nominal interest rate such that the market equilibrium can achieve innovation efficiency, but it cannot achieve output and consumption efficiency. The intuition is that money growth is like an inflation tax and there is a wealth effect when the tax is not proportionally distributed to the agents according to their pre-transfer money holdings. Money affects the real economy through the redistribution channel. We then introduce fiscal policies to attain the efficient allocation. We find different policies are needed in different development stages. When the economy faces severe credit market imperfections, the government should try to loosen credit constraints by ensuring better contract enforcements or better monitoring of borrowers. For example, the government can make direct lending to entrepreneurs financed by lump-sum taxes on savers. When the government has better monitoring technologies than private agents, the credit constraints can be overcome. The economy can then avoid the equilibrium with poverty traps.

Our paper is related to several strands of literature. First, it is related to the literature on poverty traps and convergence or divergence in economies with credit market imperfections (e.g., Banerjee and Newman (1993), Galor and Zeira (1993), Howitt (2000), Mookherjee and Ray (2001), Azariadis and Stachurski (2005), Howitt and Mayer-Foulkes (2005), Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006)). As pointed out by Azariadis and Stachurski (2005) in their survey, this literature typically studies models of self-reinforcing mechanisms that cause poverty to persist. In these models there is no technical progress and therefore no positive long-run growth. As discussed earlier, our paper is most closely related to Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006), which incorporate long-run growth. Unlike these two papers, we introduce money, endogenize interest rates, and provide a policy analysis. Howitt and Mayer-Foulkes (2005) also derive three convergence patterns analogous to those in our paper, but the disadvantage of backwardness that prevents convergence in that paper arises from low levels of human capital rather than from credit-market imperfections.

Second, our paper is related to the literature that analyzes the effects of financial constraints or financial intermediation on long-run growth. Early contributions include Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), and King and Levine (1993). None of these papers studies technology transfer and the associated policy issues which are our focus.

Third, our paper is related to the literature on the relation between money and growth. Recent papers include Gomme (1991), Marquis and Reffett (1994), Chu and Cozzi (2014), Jones and Manuelli (1995), Miao and Xie (2013), and Chu et al. (2017), among others. These papers typically introduce money via cash-in-advance constraints in infinite-horizon models, which do not feature poverty traps. By contrast, we follow the money-in-the-utility function approach of McCallum (1983) and Abel (1987) in the OLG framework. Our focus is on how monetary and fiscal policies can attain efficient allocation and avoid poverty traps.

2 The Model

We consider a monetary overlapping generations model of a closed economy based on Aghion, Howitt, and Mayer-Foulkes (2005) and Acemoglu, Aghion, and Zilibotti (2006). Time is discrete and runs forever. Time is denoted by $t = 1, 2, \ldots$. Each generation has a unit measure of identical entrepreneurs and a unit measure of identical savers. Each agent lives for two periods. Only entrepreneurs can conduct innovation, but they face borrowing constraints. Savers lend funds to entrepreneurs, but they cannot innovate. As a benchmark, we follow Lucas (1972) and assume that the government (or central bank) directly transfers money to all agents and the monetary transfer is proportional to each agent's pre-transfer money holdings.

2.1 Production

All agents work for the producers who combine labor and a continuum of specialized intermediate goods to produce a general good according to the production function,

$$Z_t = L_t^{1-\alpha} \int_0^1 A_t(i)^{1-\alpha} x_t(i)^{\alpha} \, di, \tag{1}$$

where L_t is labor demand, $x_t(i)$ is the input of the latest version of intermediate good i, and $A_t(i)$ is the productivity parameter associated with it. The general good is used for consumption, as an input to R&D and also as an input to the production of intermediate goods. The general good is produced under perfect competition. Suppose that the aggregate labor supply is normalized to one and the real price of the general good is also normalized to one. Then the equilibrium real price of each intermediate good equals its marginal product:

$$p_t(i) = \alpha \left(\frac{x_t(i)}{A_t(i)} \right)^{\alpha-1}. \tag{2}$$

For each intermediate good i there is one entrepreneur born each period t who is capable of producing an innovation for the next period. If he succeeds in innovating, then he will be the ith incumbent in period $t+1$. Let $\mu_t(i)$ be the probability that he succeeds. Then the technology evolves according to

$$A_{t+1}(i) = \begin{cases} \bar{A}_{t+1} & \text{with probability } \mu_t(i) \\ A_t(i) & \text{with probability } 1 - \mu_t(i) \end{cases},$$

where $\bar{A}_{t+1}$ is the world frontier technology, which grows at the exogenously given constant rate $g > 0$. That a successful innovator gets to implement $\bar{A}_{t+1}$ is a manifestation of technology transfer in the sense that domestic R&D makes use of ideas developed elsewhere in the world. If an innovation fails, the intermediate good sector i uses the technology in the previous period.

In each intermediate good sector where an innovation has just occurred, the incumbent can produce one unit of the intermediate good using one unit of the general good as the only input. In each intermediate sector there are an unlimited number of people capable of producing copies of the latest generation of that intermediate good at a unit cost of $\chi > 1$. The fact that $\chi > 1$ implies that the fringe is less productive than the incumbent producer. The parameter χ captures technological factors as well as government regulation affecting entry. A higher χ corresponds to a less competitive market. So in sectors where an innovation has just occurred, the incumbent will be the sole producer, at a price equal to the unit cost of the competitive fringe, whereas in noninnovating sectors where the most recent incumbent is dead, production will take place under perfect competition with a price equal to the unit cost of each producer. In either event the price will be χ, and according to the demand function (2) the quantity demanded will be

$$x_t(i) = \left(\frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}} A_t(i). \tag{3}$$

It follows that an unsuccessful innovator will earn zero profits next period, whereas the real profit of a successful incumbent will be

$$\Psi_t(i) = p_t(i)\, x_t(i) - x_t(i) = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \bar{A}_t \equiv \psi \bar{A}_t,$$

where ψ represents the normalized profit:

$$\psi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}.$$

2.2 Entrepreneurs

An entrepreneur born in period $t \geq 1$ is endowed with $\lambda \in (0,1)$ units of labor when young and supplies labor inelasically to the general good producers. He derives utility from consumption c_{t+1}^e when old according to

$$\beta \log\left(E_t c_{t+1}^e\right),$$

where $\beta \in (0,1)$ is the subjective discount factor. This utility function is an increasing transformation of a risk-neutral utility function. We will see the role of the log transformation in Section 4.

An innovation costs N_t units of general good in period t, which represents R&D investment. The young entrepreneur receives labor income λw_t, which may not be sufficient to cover the innovation cost N_t. Suppose that the entrepreneur borrows B_t dollars at the nominal interest rate R_{ft} between periods t and $t+1$ from the savers so that

$$N_t = \frac{B_t}{P_t} + \lambda w_t, \tag{4}$$

where P_t denotes the price level and w_t is the real wage rate.

We follow Aghion, Banerjee, and Piketty (1999) and Aghion, Howitt, and Mayer-Foulkes (2005) to model financial market imperfections. Suppose that the entrepreneur can hide a successful innovation at a real cost κN_t so that he can avoid repaying debt. The parameter $\kappa \in (0,1)$ reflects the degree of financial development. A higher value of κ means that it is more costly for the entrepreneur to misbehave. It measures the degree of creditor protection. To implement the contract without default, the entrepreneur faces an incentive constraint

$$\beta\left(\mu_t \psi \bar{A}_{t+1} - R_{ft}\frac{B_t}{P_{t+1}}\right) \geq \beta \mu_t \psi \bar{A}_{t+1} - \kappa N_t,$$

where the expression on the left-hand side of the inequality is the discounted expected consumption if the entrepreneur behaves and the expression on the right-hand side is the discounted expected consumption if he is dishonest. Simplifying yields the borrowing constraint

$$\frac{B_t}{P_t} \leq \frac{\kappa N_t}{\beta R_{ft}/\Pi_{t+1}}, \tag{5}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate. By (4) this constraint is equivalent to

$$N_t \leq \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa} \lambda w_t \tag{6}$$

for $\beta R_{ft}/\Pi_{t+1} > \kappa$. Thus R&D investment is limited by a multiple of the entrepreneur's net worth λw_t. This multiple is called the credit multiplier by AHM and increases with κ, but decreases with the real interest rate.

Suppose that

$$N_t = \Phi(\mu_t)\bar{A}_{t+1},$$

where the function Φ is twice continuously differentiable and satisfies $\Phi(0) = 0$ and $\Phi' > 0$ and $\Phi'' > 0$. The factor $\bar{A}_{t+1}$ reflects the "fishing-out" effect: the further ahead the frontier moves, the more difficult it is to innovate. This effect is important to have a balanced growth path. We can also rewrite the preceding equation as

$$\mu_t = F\left(N_t/\bar{A}_{t+1}\right), \tag{7}$$

where $F = \Phi^{-1}$ satisfies $F(0) = 0$, $F' > 0$, and $F'' < 0$.

The entrepreneur's expected consumption is given by

$$E_t c_{t+1}^e = \mu_t \psi \bar{A}_{t+1} - \frac{R_{ft} B_t}{P_{t+1}} = F\left(\frac{N_t}{\bar{A}_{t+1}}\right) \psi \bar{A}_{t+1} - \frac{R_{ft} P_t}{P_{t+1}} \left(N_t - \lambda w_t\right).$$

The entrepreneur's objective is to solve the following problem

$$\max_{N_t} F\left(N_t/\bar{A}_{t+1}\right) \psi \bar{A}_{t+1} - \frac{R_{ft} P_t}{P_{t+1}} \left(N_t - \lambda w_t\right)$$

subject to (6). When the credit constraint (6) does not bind, the first-order condition is given by

$$F'\left(\frac{N_t}{\bar{A}_{t+1}}\right) \psi = \frac{R_{ft}}{\Pi_{t+1}}, \tag{8}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate. This condition says that the expected marginal return to R&D is equal to the real interest rate.

The initial old entrepreneur at time t does not have labor income and hence does not conduct innovation. We assume that he simply consumes his money endowment M_0^e and the government proportional transfer $M_0^e z_1$.

2.3 Savers

A saver born at time $t \geq 1$ is endowed with $1 - \lambda$ units of labor when young and supplies labor inelastically to the general good producers. He has the utility function

$$\log(c_t^y) + \beta \log(c_{t+1}^o) + \gamma \log\left(M_t/P_t\right), \ \gamma > 0,$$

where β is the discount factor, c_t^y (c_{t+1}^o) denotes consumption at time t $(t+1)$ when the saver is young (old), M_t denotes money holdings chosen in period t. He faces the following budget constraints

$$c_t^y + \frac{S_t}{P_t} + \frac{M_t}{P_t} = (1-\lambda) w_t,$$

$$c_{t+1}^o = \frac{S_t R_{ft}}{P_{t+1}} + \frac{M_t (1+z_{t+1})}{P_{t+1}},$$

where S_t denotes saving and $z_{t+1} \geq 0$ denotes the proportional rate of the monetary transfer from the government. Note that the above utility specification does not have a satiation level of real balances as in Friedman (1969).

The first-order conditions give

$$\frac{1}{c_t^y} = \beta \frac{1}{c_{t+1}^o} \frac{P_t}{P_{t+1}} R_{ft},$$

and

$$\frac{1}{c_t^y} = \frac{\gamma}{M_t/P_t} + \frac{\beta}{c_{t+1}^o} \frac{P_t(1+z_{t+1})}{P_{t+1}}.$$

Using these conditions and the budget constraints, we can derive that

$$c_t^y = \frac{(1-\lambda) w_t}{1+\beta+\gamma}, \tag{9}$$

$$\frac{M_t}{P_t} = \frac{\gamma(1-\lambda) w_t}{1+\beta+\gamma} \frac{1}{1-(1+z_{t+1})/R_{ft}}, \tag{10}$$

$$\frac{S_t}{P_t} = \frac{(1-\lambda) w_t}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{R_{ft}/(1+z_{t+1})-1} \right]. \tag{11}$$

Thus, consumption, the money demand, and the saving demand are all proportional to the saver's real wealth $(1-\lambda)w_t$. Moreover the money demand decreases with $R_{ft}/(1+z_{t+1})$ and the saving demand increases with $R_{ft}/(1+z_{t+1})$. This property is important for the long-run super-neutrality of money because $R_{ft}/(1+z_{t+1})$ is proportional to the real interest rate in the steady state, which is independent of the inflation rate.

We assume that

$$R_{ft} > (1+z_{t+1}) \left(1 + \frac{\gamma}{\beta}\right). \tag{12}$$

This assumption ensures that the money demand $M_t/P_t > 0$ and the saving demand $S_t/P_t > 0$.

The initial old saver is endowed with money holdings M_0^s and derives utility according $\log(c_1^o)$, where

$$c_1^o = \frac{M_0^s (1+z_1)}{P_1}.$$

2.4 Competitive Equilibrium

Define the aggregate technology as

$$A_t = \int A_t(i)\, di.$$

In equilibrium the probability of innovation will be the same in each sector: $\mu_t(i) = \mu_t$ for all i. Thus average productivity evolves according to

$$A_{t+1} = \mu_t \bar{A}_{t+1} + (1 - \mu_t) A_t. \tag{13}$$

Define the normalized productivity as $a_t = A_t / \bar{A}_t$. Normalized productivity is an inverse measure of the country's distance to the technological frontier, or its technology gap. It describes the proximity to the technological frontier and satisfies the dynamics

$$a_{t+1} = \mu_t + \frac{1 - \mu_t}{1 + g} a_t. \tag{14}$$

Equation (13) implies that

$$\frac{A_{t+1} - A_t}{A_t} = \mu_t \left(\frac{1 + g}{a_t} - 1 \right).$$

Thus there is an advantage of backwardness (Gerschenkron (1962)) in the sense that the further the country is behind the frontier, the faster the country grows (a smaller a_t cause higher growth). On the other hand, the country's growth rate also depends on innovation μ_t. More innovation allows more firms to adopt the frontier technology and hence enhancing growth. Thus the net effect depends on both a_t and μ_t. Here μ_t or R&D investment is like the role of human capital that determines a country's "absorptive capacity" (Nelson and Phelps (1966)).

In equilibrium $L_t = 1$. We then use (1), (2), and $p_t(i) = \chi$ to derive aggregate output of the general good

$$Z_t = \zeta A_t, \text{ where } \zeta \equiv \left(\frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}}.$$

The wage rate is given by

$$w_t = (1 - \alpha) Z_t = (1 - \alpha) \zeta A_t. \tag{15}$$

The equilibrium interest rate R_{ft} and the price level P_t are determined by the market-clearing conditions for credit and money: $B_t = S_t$ and $M_t = (1 + z_t) M_{t-1}$ for $t \geq 1$, where z_t is the money growth rate controlled by the central bank and $M_0 = M_0^s + M_0^e$ is given.

By (4), (11), and the market-clearing condition $B_t = S_t$, we have

$$N_t - \lambda w_t = \frac{(1 - \lambda) w_t}{1 + \beta + \gamma} \left[\beta - \frac{\gamma (1 + z_{t+1})}{R_{ft} - (1 + z_{t+1})} \right]. \tag{16}$$

Value added in the general sector is wage income, whereas value added in the intermediate sectors is profit income. Total GDP is the sum of value added in all sectors:

$$Y_t = w_t + \mu_{t-1} \psi \bar{A}_t = (1 - \alpha) \zeta A_t + \mu_{t-1} \psi \bar{A}_t. \tag{17}$$

3 Equilibrium Balanced Growth Paths

In this section we solve for competitive equilibrium and derive equilibrium balanced growth path and local dynamics.

3.1 Perfect Credit Markets

Suppose that the credit constraint (6) does not bind so that the credit market is perfect. It follows from (8) that the optimal innovation is determined by the condition

$$F'(n_t)\psi = \frac{R_{ft}}{\Pi_{t+1}}, \tag{18}$$

where we define $n_t = N_t/\bar{A}_{t+1}$. We can rewrite (14) as

$$a_{t+1} = F(n_t) + \frac{1 - F(n_t)}{1+g} a_t. \tag{19}$$

Conjecture that the economy will grow at the rate of the world technology frontier along a balanced growth path so that $A_{t+1} = (1+g) A_t$. Using (10) to compute the ratio M_{t+1}/M_t and then imposing the money market-clearing condition $M_{t+1} = M_t (1 + z_{t+1})$, we obtain

$$(1 + z_{t+1}) \frac{P_t}{P_{t+1}} = \frac{M_{t+1}/P_{t+1}}{M_t/P_t} = \frac{w_{t+1} [1 - (1 + z_{t+1}) / R_{ft}]}{w_t [1 - (1 + z_{t+2}) / R_{ft+1}]}.$$

Using (15), $a_t = A_t/\bar{A}_t$, and $A_{t+1} = (1+g) A_t$, we simplify the preceding equation as

$$\Pi_{t+1} = (1 + z_{t+1}) \frac{1 - (1 + z_{t+2}) / R_{ft+1}}{1 - (1 + z_{t+1}) / R_{ft}} \frac{a_t}{a_{t+1} (1+g)}. \tag{20}$$

Thus the inflation rate is determined by money demand and money supply, which in turn are determined by the nominal interest rate, the growth rate of domestic productivity, and the growth rate of money supply. Using $n_t = N_t/\bar{A}_{t+1}$, (15), and (16), we derive that

$$n_t = \frac{(1-\alpha)\zeta a_t}{1+g} \left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma} \left(\beta - \frac{\gamma(1+z_{t+1})}{R_{ft} - (1+z_{t+1})} \right) \right]. \tag{21}$$

Now the competitive equilibrium under perfect credit markets can be summarized by a system of four difference equations (18), (19), (20), and (21) for four sequences $\{R_{ft}\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (12) and (6) are satisfied, given an exogenous sequence of money growth rates $\{z_t\}$. The endogenous predetermined variable is a_t and other equilibrium variables are non-predetermined.

We introduce the following conditions to ensure the existence of the steady-state innovation rate $\mu \in (0,1)$:

$$\Phi'(0) < \frac{\psi}{1+g} \left\{ 1 + \frac{\gamma}{\beta - \left[\frac{\Phi'(0) g}{(1-\alpha)\zeta} - \lambda \right] \frac{1+\beta+\gamma}{1-\lambda}} \right\}^{-1}, \tag{22}$$

and

$$\Phi'(1) > \frac{\psi}{1+g}\left\{1+\frac{\gamma}{\beta-\left[\frac{\Phi(1)(g+1)}{(1-\alpha)\zeta}-\lambda\right]\frac{1+\beta+\gamma}{1-\lambda}}\right\}^{-1}. \tag{23}$$

The following result characterizes the steady state and local dynamics around the steady state. We relegate its proof and the proofs of all other results to the appendix.

Proposition 1 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. Let conditions (22) and (23) hold. There exists a cutoff κ^* such that, if $\kappa \geq \kappa^*$, then the credit constraint does not bind. Moreover there exists a unique steady state $\{\mu^*, n^*, R_f^*, a^*, \Pi^*\}$ with $\mu^*, a^* \in (0,1)$, $n^* > 0$, $\Pi^* = (1+z)/(1+g)$ and the productivity grows at the rate g. In this steady state money is super-neutral in the sense that the steady-state real quantities are independent of money growth rate z. They are also independent of κ. If furthermore*

$$\frac{gn^* F'(n^*)}{F(n^*)} < g + F(n^*), \tag{24}$$

then the steady state is a saddle point and the local equilibrium around the steady state is unique. In a neighborhood of the steady state given $a_1 < a^*$ and $z_t = z$ for all $t \geq 1$, a_t, n_t, μ_t, and Π_t all increase monotonically to the steady state, but A_{t+1}/A_t and R_{ft}/Π_{t+1} decrease monotonically to the steady state.

Proposition 1 states that, if the level of financial development κ is sufficiently high, the credit constraint does not bind. There is a balanced growth path along which output and the productivity grow at the rate g. The inflation rate is constant over time and increases with the money growth rate $1+z$ proportionally and decreases with the productivity growth rate $1+g$ proportionally. All steady-state values are independent of the level of financial development. Since $\mu^* \in (0,1)$, the economy can never reach the world technology frontier in that $a^* \in (0,1)$. For the economy to reach the frontier, we must have condition (23) hold with equality so that $\mu^* = a^* = 1$. This case can happen when innovation profits are sufficiently high, i.e., ψ is sufficiently large.

Proposition 1 also characterizes the local dynamics of the equilibrium system of equations (18), (19), (20), and (21). For simplicity let the exogenous money growth rate $z_t = z$ be constant over time. We impose a technical condition (24), which can be verified in numerical examples and is easily satisfied for small g. When the initial value a_1 is slightly below the steady state value a^*, Proposition 1 shows that there exist unique initial values R_{f1}, Π_1, and n_1 such that $\{a_t, R_{ft}, \Pi_{t+1}, n_t\}_{t=1}^{\infty}$ will converge to the steady state along a saddle path. In particular, a_t, n_t, μ_t, and Π_t all increase monotonically to the steady state, but A_{t+1}/A_t and R_{ft}/Π_{t+1} decrease monotonically to the steady state.

[Insert Figure 2 Here.]

We use a numerical example to illustrate the transition dynamics. As in AHM (2006), we set $\Phi(\mu) = \phi\mu + \frac{\delta}{2}\mu^2$ and $F(n) = \frac{1}{\delta}\left(\sqrt{2n\delta + \phi^2} - \phi\right)$. We choose parameter values as $\alpha = 0.8$, $\chi = 1.15$, $\phi = 0.0134$, $\delta = 0.2604$, $\lambda = 0.01$, $g = 0.04$, $\beta = 0.96$, and $\gamma = 0.017$. Assume that money supply grows at a constant rate $z = 0.06$. Our simple two-period lived OLG model cannot be calibrated to confront with data. We use our numerical example to illustrate the working of our model. We find that the critical value $\kappa^* = 0.678$. We choose an arbitrary $\kappa > \kappa^*$. Then the steady state values are given by $R_f^* = 1.08$, $\Pi^* = 1.0192$, $a^* = 0.5$, $\mu^* = 0.037$, $n^* = 0.0007$. Moreover, the GDP Y_t normalized by $\bar{A}_t$ is equal to 0.024. The steady state is a saddle point. Only a_t is a predetermined variable. Figure 2 illustrates the transition dynamics for the case of perfect credit markets when the economy starts at $a_1 = 0.3$. We find that μ_t, a_t, and Π_t gradually increase to their steady-state values, but R_{ft} decreases to its steady state value. Given that we take the money growth rate fixed, the inflation rate moves inversely with the growth rate of productivity. The transition path illustrates the advantage of backwardness. When the economy initially falls behind the world frontier, both its technology and innovation grow faster. Thus its GDP also grows faster. They eventually catch up with the growth rate of the world frontier.

Notice that the steady-state proximity to frontier a^* depends on the preference and technology parameters. A crucial parameter is the marginal cost of innovation ϕ given the quadratic specification of Φ. A higher ϕ raises the marginal cost and reduces the marginal benefit by reducing the real interest rate, thereby reducing the innovation rate μ^*. This causes the economy's absorptive capacity to be smaller so that a^* is smaller.

3.2 Binding Credit Constraints

Suppose that the credit constraint (6) binds. Using (16) and (6) we obtain

$$w_t \left\{ \lambda + \frac{(1-\lambda)}{1+\beta+\gamma} \left[\beta - \frac{\gamma(1+z_{t+1})}{R_{ft} - (1+z_{t+1})} \right] \right\} = \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa} \lambda w_t. \tag{25}$$

We also require that

$$F'(n_t)\,\psi > \frac{R_{ft}}{\Pi_{t+1}}, \tag{26}$$

which ensures (6) indeed binds in the entrepreneur's optimization problem by the complementary slackness condition. Now the equilibrium system consists of equations (19), (20), (21), and (25) for four sequences $\{R_{ft}\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (12) and (26) hold. The following result characterizes the steady state and the local dynamics.

Proposition 2 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. There exist cutoffs κ^{**} and $\bar{\kappa}$ such that, if $\kappa^{**} < \kappa < \min\{\kappa^*, \bar{\kappa}\}$, then the credit constraint binds and there exists a unique steady state $\{\mu^{**}, n^{**}, R_f^{**}, a^{**}, \Pi^{**}\}$ such that $\mu^{**}, a^{**} \in (0,1)$, $0 < n^{**} < n^*$, $\Pi^{**} = (1+z)/(1+g)$, $R_f^{**} <$*

R_f^*, and the net productivity growth rate is g. Moreover, money is super-neutral, and n^{**}, μ^{**}, and R_f^{**} increase with κ. If in addition

$$\frac{gn^{**}F'(n^{**})}{F(n^{**})} < g + F(n^{**}), \tag{27}$$

then the steady state is a saddle point and the local equilibrium around the steady state is unique. In a neighborhood of the steady state given $a_1 < a^{**}$ and $z_t = z$ for all t, a_t, n_t, μ_t, R_{ft}, and Π_t all increase monotonically to the steady state, but A_{t+1}/A_t and R_{ft}/Π_{t+1} decrease monotonically to the steady state.

We use Figure 3 to illustrate the determination of the steady-state nominal interest rate presented in Propositions 1 and 2. The curve labeled "Supply" describes the supply of funds for R&D investment normalized by the wage rate, which is given by the expression on the left-hand side of equation (25) without the time subscripts:

$$\frac{N_t}{w_t} = \lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left[\beta - \frac{\gamma(1+z)}{R_f-(1+z)}\right]. \text{ (supply)}$$

The supply is equal to the sum of the entrepreneur's wage and the savers' saving. This curve increases with the nominal interest rate R_f. The curve labeled "Demand" describes the steady-state demand for funds normalized by the wage rate, when the credit constraint does not bind. To derive the demand function, we show that

$$\frac{N_t}{w_t} = \frac{N_t}{(1-\alpha)\zeta A_t} = \frac{n}{(1-\alpha)\zeta a/(1+g)} = \frac{n}{\frac{(1-\alpha)\zeta F(n)}{g+F(n)}}, \text{ (demand)}$$

where the first equality follows from (15), the second from the normalization by $\bar{A}_{t+1}$ and $\bar{A}_{t+1}/\bar{A}_t = 1+g$, and the last from the substitution of a using the steady-state version of equation (19). Using the steady-state version of (18) to substitute for n into the above equation, we obtain the demand for funds as a function of R_f. We can show that this demand function decreases with R_f.

[Insert Figure 3 Here.]

The curves labeled "Limit $\kappa > \kappa^*$" and "Limit $\kappa < \kappa^*$" describe the borrowing limits normalized by the wage rate for $\kappa > \kappa^*$ and $\kappa < \kappa^*$, respectively, which are given by the expression on the right-hand side of the steady-state version of equation (25)

$$\frac{N_t}{w_t} = \frac{\beta R_f/\Pi}{\beta R_f/\Pi - \kappa}\lambda = \frac{\beta R_f(1+g)/(1+z)}{\beta R_f(1+g)/(1+z)-\kappa}\lambda. \text{ (limit)}$$

This expression decreases with R_f.

When $\kappa > \kappa^*$, the equilibrium nominal interest rate R_f^* is determined by the intersection of the demand curve and the supply curve. In this case the credit constraint does not bind and a change in κ does not affect equilibrium as shown in Proposition 1. When $\kappa < \kappa^*$, the credit constraint binds so that the equilibrium nominal interest rate R_f^{**} is determined by the intersection of the supply curve and the borrowing limit curve. From the figure we can see that $R_f^{**} < R_f^*$ and an increase in κ raises R_f^{**}. Moreover the change in κ has a level effect because n^{**} and μ^{**} increase with κ. However the change in κ does not have a growth effect in that the steady-state productivity growth rate is equal to $1 + g$.

Proposition 2 also shows that the steady state is a saddle point as in Proposition 1. When the initial value a_1 is slightly below the steady state value $a^{**} \in (0, 1)$, there exist unique initial values R_{f1}, Π_1, μ_1, and n_1 such that $\{a_t, \mu_t, R_{ft}, \Pi_{t+1}, n_t\}_{t=1}^{\infty}$ will converge to the steady state along a saddle path. In particular, a_t, n_t, μ_t, and Π_t all increase monotonically to the steady state, but A_{t+1}/A_t and R_{ft}/Π_{t+1} decrease monotonically to the steady state.

For a numerical illustration, we choose the same parameter values as in Section 3.1 except that we set $\kappa = 0.5$. Then the credit constraint binds. We find the steady-state values $R_f^{**} = 1.0796$, $\Pi^{**} = 1.0192$, $a^{**} = 0.327$, $\mu^{**} = 0.018$, and $n^{**} = 0.0003$. The normalized GDP is equal to 0.016. Compared to the case of perfect credit markets, credit market imperfections enlarge the distance to the frontier even though the long-run grow rates are the same, in that $a^{**} < a^*$, $\mu^{**} < \mu^*$, $n^{**} < n^*$, and normalized GDP are all smaller. We find that the steady state is also a saddle point. Figure 4 illustrates the transition dynamics, which also display the advantage of backwardness.

[Insert Figure 4 Here.]

3.3 Poverty Trap

When the level of financial development is sufficiently low such that $0 < \kappa < \kappa^{**}$, the credit constraint is too tight so that the economy cannot support a sufficient amount of R&D investment relative to the long-run productivity growth. As a result, the economy enters a poverty trap in which the R&D investment relative to productivity growth approaches zero so that the steady state along a balanced growth path satisfies $n^p = \mu^p = 0$. In the poverty trap steady state the economy still grows but at a rate lower than the technology frontier g. Thus the distance to the frontier approaches zero, $a^p = 0$. The following proposition summarizes the results:

Proposition 3 *Suppose that the monetary transfer is given to the old generation only in a quantity proportional to the pre-transfer money holdings of each. If $0 < \kappa < \kappa^{**}$, then there exists a unique steady-state equilibrium with the inflation rate and nominal interest rate, denoted by Π^p and R_f^p. The economy enters the poverty trap with steady-state values $\mu^p = a^p = n^p = 0$. Money is super-neutral. The steady-state productivity growth rate is given by*

$$\lim_{t\to\infty} \frac{A_{t+1}}{A_t} = F'(0) \frac{(1-\alpha)\,\zeta\lambda\beta R_f^p/\Pi^p}{\beta R_f^p/\Pi^p - \kappa} + 1,$$

which is between 0 and $1+g$ *and increases with* $\kappa \in (0, \kappa^{**})$ *. The steady-state inflation rate satisfies* $\Pi^p > (1+z)/(1+g)$ *and decreases with* $\kappa \in (0, \kappa^{**})$ *. The poverty trap steady state is a saddle point and the local equilibrium around this steady state is unique. In a neighborhood of the steady state given* $a_1 > 0$ *and* $z_t = z$ *for all* t*,* a_t*,* R_{ft}*,* Π_{t+1}, μ_t*, and* n_t *decrease monotonically to the steady state, but* A_{t+1}/A_t *and* R_{ft}/Π_{t+1} *increase monotonically to the steady state.*

It is interesting to compare Propositions 2 and 3. When the credit constraint is not too tight, a_t converges to a positive steady state value $a^{**} \in (0,1)$. The productivity growth A_{t+1}/A_t converges to $1+g$ and the inflation rate Π_t converges to $(1+z)/(1+g)$ by (20). By contrast, when the credit constraint is too tight, a_t converges to zero. We need to use L'Hospital's rule to derive the steady-state productivity growth rate

$$\lim_{t\to\infty} \frac{A_{t+1}}{A_t} = (1+g) \lim_{t\to\infty} \frac{a_{t+1}}{a_t}.$$

We show that this limit is less than $1+g$. Similarly we also use this equation to compute the steady-state inflation rate Π^p by (20), which is higher than $(1+z)/(1+g)$. Unlike in the case with not too tight credit constraints, there is a steady-state growth effect when the level $\kappa \in (0, \kappa^{**})$ of financial development changes.

The transition dynamics are also different, even though both steady states are saddle points. When the credit constraint is not too tight, a_t increases monotonically to the steady state when its initial value a_1 is slightly below the steady state $a^{**} \in (0,1)$. As the economy moves closer to the technological frontier, its productivity growth slows down and gradually decreases to the steady state. By contrast, when the credit constraint is too tight, the economy will fall into the poverty trap with $a^p = 0$ starting from any small positive initial value $a_1 > 0$. The innovation rate and R&D investment also decrease to the steady state. As the economy falls farther away from the technological frontier, its productivity growth will be faster and increase to the steady state, which is lower than the frontier growth rate $1+g$.

[Insert Figure 5 Here.]

To illustrate Proposition 3 numerically, we use the same parameter values as in Section 3.1 except that we set $\kappa = 0.1$. We then find the poverty trap equilibrium with the steady-state values $R_f^p = 1.0792$ and $\Pi^p = 1.0205$. In the steady state, the normalized GDP is equal to 0 and the technology growth rate is 1.0387. The steady-state inflation rate is higher than the two cases studied in Sections 3.1 and 3.2. We also find the poverty-trap steady state is a saddle point. Figure 5 illustrates the transition dynamics when the economy starts at $a_1 = 0.5$. It shows that the economy falls further behind the technological frontier. Both a_t and μ_t decrease to zero. The inflation rate Π_t and the nominal interest rate R_{ft} also decrease to their steady-state values, but the real interest rate R_{ft}/Π_{t+1} increases to its steady-state value. The growth rate of productivity increases to a level lower than the technological frontier. The economy falls into a poverty trap with low economic growth and high inflation. Thus there is a disadvantage of backwardness when the level financial development is extremely low.

4 Efficient Allocation

In this section we study efficient allocation. Following Abel (1987), suppose that a social planner maximizes the sum of discounted utility of all agents in the economy

$$\begin{aligned} & \omega u\left(c_1^e\right)+u\left(c_1^o\right)+\sum_{t=1}^{\infty} \beta^{t-1}\left[u\left(c_t^y\right)+\beta u\left(c_{t+1}^o\right)+\omega \beta u\left(c_{t+1}^e\right)\right] \qquad (28) \\ = & \sum_{t=1}^{\infty} \beta^{t-1}\left[u\left(c_t^y\right)+u\left(c_t^o\right)+\omega u\left(c_t^e\right)\right], \end{aligned}$$

where the planner assigns the utility weight ω to the entrepreneur and discounts utilities of future generations by β. Here we set $u(c)=\log(c)$. As in the dynamic new Keynesian framework, we consider a cashless limit and ignore money in the utility (Woodford (2003) and Gali (2008)). The resource constraint is given by

$$c_t^y+c_t^o+c_t^e+N_t=L_t^{1-\alpha} \int_0^1 A_t(i)^{1-\alpha} x_t(i)^\alpha \, di-\int \chi_t(i) x_t(i) \, di, \tag{29}$$

where $\chi_t(i)=1$, when an innovation occurs in sector i, and $\chi_t(i)=\chi$, otherwise.

Maximizing the expressions on the right-hand side of equation (29) yields the efficient labor input $L_t=1$ and the efficient intermediate goods input

$$x_t(i)=\left\{\begin{array}{ll} \alpha^{\frac{1}{1-\alpha}} A_t(i) & \text{if an innovation occurs} \\ \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(i) & \text{otherwise} \end{array}\right. . \tag{30}$$

We can then compute the GDP (net output):

$$\begin{aligned} Y_t^e & =\int_0^1 A_t(i)^{1-\alpha} x_t(i)^\alpha \, di-\int \chi_t(i) x_t(i) \, di \\ & =\left(\frac{1}{\alpha}-1\right)\left[\alpha^{\frac{1}{1-\alpha}} \mu_{t-1} \bar{A}_t+\left(1-\mu_{t-1}\right)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} \chi A_{t-1}\right]. \qquad (31) \end{aligned}$$

The resource constraint (29) becomes

$$c_t^y+c_t^o+c_t^e+N_t=Y_t^e. \tag{32}$$

where $\mu_0=0$ and A_0 is exogenously given.

Now the planner's problem is to maximize (28) subject to (7), (13), and (32). By the first-order conditions we can immediately derive that

$$c_t^e = \omega c_t^y = \omega c_t^o. \tag{33}$$

Since $\bar{A}_{t+1}/\bar{A}_t = 1+g$, we conjecture that, on the efficient balanced growth path, $a_t = A_t/\bar{A}_t$, μ_t, and $N_t/\bar{A}_{t+1} = n_t$ are constant over time, but c_t^y, c_t^o, and c_t^e all grow at the rate g. In the appendix we show that the efficient steady-state innovation rate μ is determined by the following equation

$$\begin{aligned} \Phi'(\mu) &= \frac{\beta}{1+g}\left(\frac{1}{\alpha}-1\right)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\right] \\ &\quad + \frac{\beta}{1+g-\beta(1-\mu)}\left(\frac{1}{\alpha}-1\right)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\frac{\chi g}{g+\mu}. \end{aligned} \tag{34}$$

The expression on the left-hand side of the equation represents the marginal cost of innovation and the expression on the right-hand side represents the associated present value of marginal benefit. We can easily check that the marginal cost is an increasing function of μ and the marginal benefit is a decreasing function of μ. Given the following assumption, there is a unique solution by the intermediate value theorem, denoted by $\mu_{FB} \in (0,1)$, to the above equation.

Assumption 1 *The parameter values satisfy*

$$\Phi'(0) < \frac{\beta}{1+g}\left(\frac{1}{\alpha}-1\right)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\right] + \frac{\beta}{1+g-\beta}\left(\frac{1}{\alpha}-1\right)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi,$$

and

$$\Phi'(1) > \beta\frac{1}{1+g}\left(\frac{1}{\alpha}-1\right)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\right] + \frac{\beta}{1+g}\left(\frac{1}{\alpha}-1\right)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\frac{\chi g}{g+1}.$$

We then obtain the efficient innovation rate μ_{FB}, the efficient normalized R&D investment $n_{FB} = \Phi(\mu_{FB})$, and the efficient proximity to the frontier a_{FB}. Moreover, the implied real interest rate is given by

$$R_{FB}^r = \frac{u'(c_t^y)}{\beta u'(c_{t+1}^o)} = \frac{1+g}{\beta}. \tag{35}$$

We summarize the preceding analysis below.

Proposition 4 *Under Assumption 1, there exists a unique efficient allocation with $\mu_{FB} \in (0,1)$, $a_{FB} \in (0,1)$, and $n_{FB} > 0$ along the balanced growth path with the productivity growth rate being g.*[7] *Moreover μ_{FB} is independent of ω.*

[7]In the knife-edge case where the second inequality in Assumption 1 holds as an equality, the efficient innovation rate $\mu_{FB} = 1$. In this case $a_{FB} = 1$ and the economy reaches the world frontier technology level.

By the analysis in the previous section, we can immediately see that competitive equilibrium allocation is generally not efficient. There are four sources of inefficiency in the market economy studied in Sections 2 and 3. First, there is monopoly inefficiency in the production of intermediate goods. The resulting price distortion generates an inefficiently low level of final net output when taken the innovation rate as given. Second, entrepreneurs face credit constraints, which distorts innovation investments and within generation consumption allocation. Third, innovators are monopolists. Private innovation does not take into account of the externality effect on future productivity. When choosing innovation investment, entrepreneurs only maximize expected monopoly profits in the next period. Efficient innovation not only causes profits in the next period to rise, but also causes future productivity to rise, which raises future profits. Fourth, there is intertemporal inefficiency in the sense that the equilibrium real interest rate and the implied efficient rate may be different.

In general the market equilibrium innovation may be either higher or lower than the efficient innovation depending on the parameter values. To see this fact we consider the case with a perfect credit market. The equilibrium innovation is determined by equation (18), which can be written as the steady-state form:

$$\Phi'(\mu^*) = \frac{\psi}{R_f^*/\Pi^*}.$$

Comparing this equation with the efficient condition (34), we can see clearly how the market equilibrium generates inefficiency.[8] First, the market real interest rate R_f^*/Π may not be equal to the efficient rate $R_{FB}^r = (1+g)/\beta$. Second, the private return to innovation (the normalized monopoly profit) ψ may not be equal to the one-period social return described by the expression (excluding $\beta/(1+g)$) on the first line of equation (34). Third, the positive externality effect captured by the expression on the second line does not appear in the above equilibrium condition.

In fact we can show that the private return to innovation ψ is smaller than the one-period social return and hence smaller than the total social return.[9] But the market real interest rate may be either higher or lower than the efficient rate R_{FB}^r. When γ is sufficiently large, savers have a sufficiently large preference for money so that his saving is sufficiently low. In this case the market real interest can be higher than the efficient rate and hence the market equilibrium innovation is lower than the efficient level.

[8]Notice that $F'(n) = 1/\Phi'(\mu)$.

[9]We need to prove that

$$\psi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} < (\frac{1}{\alpha} - 1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\right].$$

This inequality is equivalent to

$$\frac{\alpha}{1-\alpha} < \frac{\chi\left(\chi^{\frac{\alpha}{1-\alpha}} - 1\right)}{\chi - 1}.$$

The expression on the right-hand side is equal to $\alpha/(1-\alpha)$ when $\chi = 1$ and increases with $\chi > 1$.

Using the parameter values in Section 3.1, we computer the efficient steady-state values: $a_{FB} = 0.858$, $\mu_{FB} = 0.1886$, and $n_{FB} = 0.0072$. The normalized GDP is 0.047. The implied real interest rate is 1.0833. Even though the implied real interest rate is higher than the market real interest rate, it turns out that the market equilibrium gives low levels of innovation and GDP.

5 Monetary and Fiscal Policies

In this section we study monetary and fiscal polices under which a competitive equilibrium can achieve the efficient allocation along the balanced growth path derived in the previous section. For simplicity we set $\omega = 1$. Our analysis can be similarly applied to other values of ω. In Sections 2 and 3 we have shown that monetary policy is super-neutral if money is transferred to savers in an amount proportional to their pre-transfer holdings and entrepreneurs do not hold any money. In this section we relax this assumption. Since entrepreneurs face credit constraints, we naturally assume that the government transfers money increments to young entrepreneurs in a lump-sum manner. Then monetary policy is not super-neutral.

5.1 Perfect Credit Market

We first study the case where the credit market is perfect so that the credit constraint is slack. We consider the following policy tools such that the government's budget balances in each period t. For simplicity we do not consider government spending and government debt.

- The central bank sets a constant nominal interest rate R_f and transfers the money increments τ_{et} to the old entrepreneur.
- The government subsidizes the production of the final good by imposing a tax credit $1 - \tau_{xt}(i)$ on the intermediate input.
- The government subsidizes the old entrepreneur's expected profits from innovation at the constant rate τ_N.
- The government levies a lump-sum tax $T_N \bar{A}_t$ on the old entrepreneur's income.
- The government levies a lump-sum tax $T_w \bar{A}_t$ on the wage income.

When the central bank sets a nominal interest rate, the money growth rate z will be endogenous. Equivalently, we can assume that the money growth rate is an exogenous policy instrument so that the nominal interest rate is endogenous as in Sections 2 and 3. Our focus on the interest rate policy is consistent with the practice in many countries and also with the dynamic new Keynesian literature. Notice that here money increments are transferred to entrepreneurs instead of savers unlike in the model of Section 2. In the appendix we show that savers' money demand decreases with the nominal interest rate R_{ft} and the saving demand increases with R_{ft}, instead of the real interest rate. This property allows money to be not super-neutral.

We first show that monetary policy alone can achieve the efficient innovation and R&D investment, but cannot achieve production and consumption efficiency.

Assumption 2 *The parameter values are such that*

$$\frac{\lambda(1-\alpha)\zeta}{1+g}\frac{(1+g)F(n_{FB})}{g+F(n_{FB})} < n_{FB} < \left[\lambda + (1-\lambda)\frac{\beta+\gamma}{1+\beta+\gamma}\right]\frac{(1-\alpha)\zeta}{1+g}\frac{(1+g)F(n_{FB})}{g+F(n_{FB})}$$

and

$$\frac{\psi F'(n_{FB})}{1+g} > 1 + \frac{\gamma}{\beta}. \tag{36}$$

Assumption 2 ensures that on the balance growth path the efficient level of R&D investment cannot be self-financed by the entrepreneur's wage income alone but can be financed by the entrepreneur's wage income plus savers' total savings. Moreover, the marginal return to the R&D investment must be sufficiently high. Thus monetary transfers and external credit are needed. We are interested in whether a particular monetary policy can ensure savers' savings are efficiently channeled to entrepreneurs through the credit market.

The restriction on γ in (36) is a sufficient condition to ensure that the monetary transfers combined with the entrepreneur's wage income are not sufficient for entrepreneurs to finance the efficient level of the R&D investment. Thus external debt is needed through the credit market. If γ is too large, then the savers' money demand would be large enough so that the government can transfer a sufficient amount of money to the entrepreneurs and the credit market is not needed. In this paper we will not consider this uninteresting case.

Proposition 5 *Suppose that money increments are transferred to young entrepreneurs and there is no fiscal policy. Under Assumption 2, there exists a nominal interest rate $\bar{R}_f > 1 + \gamma/\beta$ such that the market equilibrium under a perfect credit market achieves the efficient R&D investment n_{FB} along the balanced growth path with the productivity growth rate being g.*

The intuition for this result can be seen from the equilibrium optimality condition for innovation or R&D investment, (18). The central bank can choose a particular nominal interest rate (or money growth rate) such that the private optimality condition coincides with the efficient optimality condition for innovation along a balanced growth path.

In the appendix we show that for any given μ_{t-1} the efficient output level is higher than the market equilibrium level because of the monopoly distortion. Monetary policy alone cannot correct this inefficiency. We thus need fiscal policy. To achieve efficient output, the government can subsidize the final good firm's input expenditure by a suitable choice of tax credit $1 - \tau_{xt}(i)$.

Since innovators or entrepreneurs are monopolists and face credit constraints, this causes inefficiency in innovation. To correct this inefficiency, the government can subsidize entrepreneurs' profits at rate τ_N and transfer money increments τ_{et} to young entrepreneurs.

Since there are overlapping generations of agents in the economy, the consumption allocation within and across generations in a market equilibrium may not be efficient. The government can levy lump-sum taxes $T_N \bar{A}_t$ and $T_w \bar{A}_t$ on (or make lump-sum transfers to) entrepreneurs and savers, keeping the government budget balanced in the meantime.

The following result shows that the economy can achieve efficient innovation, production, and consumption by a suitable choice of the above monetary and fiscal policy tools.

Proposition 6 *Suppose that money increments are transferred to young entrepreneurs. Then under Assumption 3 given in the appendix the steady-state efficient innovation and allocation can be implemented by the competitive equilibrium with a perfect credit market along a balanced growth path under the monetary and fiscal policy tools R_f^0, $\tau_{xt}^0(i)$, τ_{et}^0, τ_N^0, T_N^0, and T_w^0 described in the appendix.*

The intuition behind the proposition is that we set the nominal interest rate such that the real interest rate is given by the efficient rate in (35). At this rate we ensure intertemporal efficiency so that $c_t^y = c_t^o$. The implied money growth rate z^0 and inflation rate Π^0 in the steady state satisfy

$$z^0 = \beta R_f^0 - 1, \ \Pi^0 = \frac{1+z^0}{1+g}.$$

Moreover, the subsidy rate τ_N^0 ensures that it is optimal for the entrepreneur to choose the efficient level of innovation. Finally, the taxes or transfers T_N^0 and T_w^0 ensure that within generation consumption allocation is efficient ($c_t^e = c_t^y = c_t^o$) and that the government budget balances. Notice that the signs of T_N^0 and T_w^0 are ambiguous and they may be either interpreted as taxes or subsidies.

We impose Assumption 3, similar to Assumption 2, to ensure that a nontrivial market equilibrium exists given the specific monetary and fiscal policy tools described in Proposition 6. Since this assumption is technical, we present it in the appendix.

5.2 Binding Credit Constraints

When the credit constraint binds so that the credit market is imperfect, the government should improve credit markets to raise κ by imposing better creditor protection and better contract enforcement. If we take κ as given, we can introduce another policy instrument to overcome the credit constraint. Once the credit constraint is slack, we then use the policy tools studied in the previous subsection to achieve the efficient allocation.

Specifically, consider the case of $\kappa < \kappa^0$, where κ^0 is defined in equation (A.62) in the appendix. Then the credit constraint binds and the policies described in Proposition 6 cannot achieve efficiency. Suppose that the government can make direct lending D_t at the nominal interest rate

R_{ft} to the young entrepreneur. Suppose that the government has better monitoring abilities than private agents so that the entrepreneur cannot hide or divert the government funds. Then the credit constraint only applies to $N_t - D_t$. The government finances the loans by levying lump-sum taxes on the young saver and then makes transfer $D_t R_{ft}$ to the old saver. In this case the saver's consumption and portfolio choices are not affected.

To implement the efficient allocation along the balanced growth path, we suppose $D_t = D\bar{A}_{t+1}$ and set D at a value higher than the expression below:

$$\left[\frac{(1+g)\,\eta_{FB}}{1+g-\kappa^0} - \frac{(1+g)\,\eta_{FB}}{1+g-\kappa}\right]\left[\lambda + \frac{\beta R_f^0 - 1}{\beta R_f^0}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma\,(1+g)}{1+g-\beta}\right],$$

where η_{FB} is given in the appendix. Then the credit constraint is slack at the equilibrium allocation and interest rate R_f^0 described in Proposition 6. Now we can apply the analysis in Section 5.1 and Proposition 6 to achieve the efficient allocation.

6 Conclusion

An important feature of developing countries is that credit markets are imperfect due to reasons such as weak contract enforcement, weak creditor protection, and agency issues. We follow AHM (2005) and incorporate this feature into a Schumpeterian overlapping-generations model of economic growth to explain convergence and divergence. Our contribution is to introduce money and study how monetary and fiscal policies can achieve efficient allocation in a market equilibrium. We find that how money increments are transferred to agents is important for their long-run impact on economic growth. When money increments are transferred to agents in an amount proportional to their pre-transfer holdings, money is super-neutral. For a sufficiently low level of financial development, the economy can enter a poverty trap with low economic growth and high inflation. When money increments are transferred to young entrepreneurs, to whom money is most needed, it is not super-neutral. Monetary policy affects the real economy through a redistribution channel. The government should first improve credit market conditions so that entrepreneurs are not credit constrained. Then there is a combination of monetary and fiscal policies such that the economy can avoid the poverty trap and achieve efficient allocation. In this case the economy will grow at a faster rate for some period of time and then gradually converge to the same rate as the world frontier.

One limitation of our model is that we have assumed that the world frontier technology grows at an exogenously given constant rate. In the future research, it is desirable to relax this assumption and treat the technological innovation as endogenously determined in both advanced countries and developing countries. In the new setup, a developing country with well-functioning financial market, appropriate fiscal and monetary policies, and the advantage of backwardness in technological innovation, may achieve absolute convergence and become an advanced country.

Appendix

A Proofs

Proof of Proposition 1: We first study the steady state. Using equations (18), (19), (20), and (21) and setting $z_t = z$, we derive a system of four steady-state equations

$$R_f = \frac{1+z}{1+g} F'(n)\psi, \tag{A.1}$$

$$a = \frac{(1+g)\, F(n)}{g + F(n)}, \tag{A.2}$$

$$n = \frac{(1-\alpha)\,\zeta a}{1+g} \left\{ \lambda + \frac{(1-\lambda)}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{R_f / (1+z) - 1} \right] \right\}, \tag{A.3}$$

and $\Pi = (1+z) / (1+g)$ to determine four steady-state variables R_f, a, n, and Π.

From these equations, we show that n is determined by the equation

$$n = \frac{(1-\alpha)\,\zeta F(n)}{g + F(n)} \left\{ \lambda + \frac{(1-\lambda)}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{F'(n)\psi / (1+g) - 1} \right] \right\}.$$

Equivalently, it follows from $n = \Phi(\mu)$ and $F = \Phi^{-1}$ that μ is determined by the equation

$$\Phi'(\mu) = \frac{\psi}{R_f / \Pi}, \tag{A.4}$$

where

$$\frac{R_f}{\Pi} = (1+g) \left\{ 1 + \frac{\gamma}{\beta - \left[\frac{\Phi(\mu)(g+\mu)}{(1-\alpha)\zeta\mu} - \lambda \right] \frac{1+\beta+\gamma}{1-\lambda}} \right\}$$

and $\Pi = (1+z) / (1+g)$. Since $\Phi'' > 0$, $\Phi' > 0$, and $\Phi(0) = 0$, we can check that $\Phi(\mu)(g+\mu)/\mu$ decreases with μ. Thus the real interest rate R_f/Π is decreasing in μ and $\Phi'(\mu)$ is increasing in μ. Given conditions (22) and (23), it follows from the intermediate value theorem that there is a unique solution $\mu^* \in (0,1)$ to (A.4).[10] The associated R&D investment is given by $n^* = \Phi(\mu^*)$ and hence R_f^* and a^* are determined by (A.1) and (A.2). We also assume that the condition

$$\frac{R_f^*}{1+z} > 1 + \frac{\gamma}{\beta} \tag{A.5}$$

is satisfied so that (12) holds along the balanced growth path. We will verify later that this condition is indeed satisfied in the proof of Proposition 2.

Using (A.3) and (15), we can rewrite the credit constraint (6) along a balanced growth path as

$$n \left(\frac{\beta R_f}{\Pi} - \kappa \right) \leq \frac{\beta R_f}{\Pi} \frac{\lambda (1-\alpha)\,\zeta a}{1+g}.$$

[10] We do not consider the knife-edge case of boundary solutions.

The critical value of κ such that the credit constraint just binds in the steady-state equilibrium is given by

$$\kappa^* = \frac{\beta R_f^*}{\Pi}\left[1 - \frac{\lambda(1-\alpha)\zeta a^*}{n^*(1+g)}\right]. \tag{A.6}$$

When $\kappa > \kappa^*$, the credit constraint does not bind. It follows from (A.4) that money supply does not affect the equilibrium innovation rate μ^*. An increase in the money growth rate raises the nominal interest rate one for one and hence does not affect savings. Thus the supply of funds for innovation does not depend on monetary policy.

Next we study local dynamics. By defining $r_t \equiv R_{ft}/(1+z_{t+1})$ and eliminating R_{ft}/Π_{t+1}, we can reduce the equilibrium system (18), (19), (20), and (21) to three equations

$$\begin{aligned}
a_{t+1} &= F(n_t) + \frac{1-F(n_t)}{1+g}a_t, &\text{(A.7)}\\
r_t - 1 &= F'(n_t)\psi\frac{a_t}{a_{t+1}(1+g)}\frac{r_{t+1}-1}{r_{t+1}}, &\text{(A.8)}\\
n_t &= \frac{(1-\alpha)\zeta a_t}{1+g}\left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left(\beta - \frac{\gamma}{r_t-1}\right)\right] &\text{(A.9)}
\end{aligned}$$

for three variables a_t, n_t, and r_t. Assume that z_t is constant over time.

Log-linearizing the above equations around the steady state yields

$$\begin{aligned}
\hat{a}_{t+1} &= \frac{g}{1+g}\frac{nF'(n)}{F(n)}\hat{n}_t + \frac{1-F(n)}{1+g}\hat{a}_t,\\
\hat{r}_t &= \frac{(r-1)}{r}\left[\frac{F''(n)n}{F'(n)}\hat{n}_t - \frac{g}{1+g}\frac{nF'(n)}{F(n)}\hat{n}_t + \frac{g+F(n)}{1+g}\hat{a}_t\right] + \frac{1}{r}\hat{r}_{t+1},\\
\hat{n}_t &= \hat{a}_t + \frac{\frac{(1-\lambda)}{1+\beta+\gamma}\gamma\frac{r}{(r-1)^2}}{\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left(\beta - \frac{\gamma}{r-1}\right)}\hat{r}_t \equiv \hat{a}_t + \vartheta\hat{r}_t, &\text{(A.10)}
\end{aligned}$$

where a variable without time subscript denotes the steady state value and a variable with a hat denotes the log deviation from the steady state. It follows from condition (A.5) that $\vartheta > 0$.

Eliminating $\hat{n}_t$ yields a system of two linear difference equations

$$\begin{bmatrix} \hat{a}_{t+1} \\ \hat{r}_{t+1} \end{bmatrix} = J \begin{bmatrix} \hat{a}_t \\ \hat{r}_t \end{bmatrix}, \tag{A.11}$$

where

$$J \equiv \begin{bmatrix} \frac{g}{1+g}\frac{nF'(n)}{F(n)} + \frac{1-F(n)}{1+g} & \vartheta\frac{g}{1+g}\frac{nF'(n)}{F(n)} \\ (1-r)\left[1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g} + \frac{F''(n)n}{F'(n)}\right] & r + (r-1)\vartheta\left[\frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{F''(n)n}{F'(n)}\right] \end{bmatrix}.$$

We now study the eigenvalues of J to determine the local stability of the equilibrium system. Consider the quadratic characteristic equation

$$G(\nu) = |J - \nu I| = 0.$$

After a tedious calculation we obtain

$$\begin{aligned} G(0) &= \left[\frac{g}{1+g}\frac{nF'(n)}{F(n)}+\frac{1-F(n)}{1+g}\right]r-\frac{1-F(n)}{1+g}(r-1)\vartheta\frac{F''(n)n}{F'(n)} \\ &\quad +\vartheta\frac{g}{1+g}\frac{nF'(n)}{F(n)}(r-1), \end{aligned} \tag{A.12}$$

and

$$\begin{aligned} G(1) &= \left[\frac{g}{1+g}\frac{nF'(n)}{F(n)}+\frac{1-F(n)}{1+g}-1\right](r-1)\left[1-\frac{F''(n)n}{F'(n)}\right] \\ &\quad +\vartheta\frac{g}{1+g}\frac{nF'(n)}{F(n)}(r-1)\vartheta\frac{F''(n)n}{F'(n)}. \end{aligned} \tag{A.13}$$

Under conditions (24), (A.5), $F'>0$, and $F''<0$, we can check that $G(0)>0$ and $G(1)<0$. It follows from the intermediate value theorem that there exists an eigenvalue $\nu_1\in(0,1)$ such that $G(\nu_1)=0$. Since $\lim_{x\to\infty}G(x)=\infty$, it follows from the intermediate value theorem that there exists another eigenvalue $\nu_2>1$ such that $G(\nu_2)=0$. We conclude that the steady state is a saddle point.

[Insert Figure 6 Here.]

Finally we study transition dynamics. We set $z_t=z$ for all t for simplicity. Then we can write the log-linearized equilibrium solution as

$$\hat{a}_{t+1}=\phi_a\hat{a}_t,\ \hat{r}_t=\hat{R}_{ft}=\phi_r\hat{a}_t,\ \hat{n}_t=\phi_n\hat{a}_t,\ \hat{r}_{ft}=\phi_{rr}\hat{a}_t,\ \text{and}\ \hat{\Pi}_{t+1}=\phi_\Pi\hat{a}_t,$$

where $r_{ft}\equiv R_{ft}/\Pi_{t+1}$ denotes the real interest rate. We want to determine the signs of all the coefficients. We first use the phase diagram in Figure 6 to determine the signs of ϕ_a and ϕ_r. By (**??**) the locus $\hat{a}_{t+1}=\hat{a}_t$ represents the equation

$$\hat{r}_t=\frac{1-\frac{g}{1+g}\frac{nF'(n)}{F(n)}-\frac{1-F(n)}{1+g}}{\vartheta\frac{g}{1+g}\frac{nF'(n)}{F(n)}}\hat{a}_t, \tag{A.14}$$

which has a positive slope by condition (24). The locus $\hat{r}_{t+1}=\hat{r}_t$ represents the equation

$$r_t=\frac{1-\frac{g}{1+g}\frac{nF'(n)}{F(n)}-\frac{1-F(n)}{1+g}+\frac{F''(n)n}{F'(n)}}{1+\vartheta[\frac{g}{1+g}\frac{nF'(n)}{F(n)}-\frac{F''(n)n}{F'(n)}]}\hat{a}_t. \tag{A.15}$$

Since $F''<0$ and $F'>0$, the slope of this line may be either positive or negative. The left panel of Figure 6 plots the case in which the locus $\hat{r}_{t+1}=\hat{r}_t$ is negative. We can see that, if $\hat{a}_1<0$, namely if the initial value a_1 is below the steady state, the interest rate $\hat{r}_t$ declines but a_t increases over time to their steady-state values along the saddle path. We now turn to case in which the slope of the locus $\hat{r}_{t+1}=\hat{r}_t$ is positive, illustrated in the right panel of Figure 6. We can show that the slope of the locus $\hat{a}_{t+1} = \hat{a}_t$ is greater than the slope of the locus $\hat{r}_{t+1} = \hat{r}_t$. We can see that, if the initial value a_1 is below the steady state, r_t and a_t both increase over time, approaching their steady-state values.

In summary we have shown that $\phi_a \in (0,1)$ and ϕ_r can be either positive or negative. Up to the first order approximation, the productivity growth satisfies

$$\log A_{t+1} - \log A_t = \hat{a}_{t+1} - \hat{a}_t + \log(1+g) = (\phi_a - 1)\hat{a}_t + \log(1+g).$$

Thus, when a_1 is slightly below the steady state, the productivity growth is positive and decreases to the steady state. By $\hat{n}_t = \hat{a}_t + \vartheta \hat{r}_t$, we have $\phi_n = 1 + \vartheta \phi_r$. If $\phi_r > 0$, then we have $\phi_n > 0$ since $\vartheta > 0$. In the case of $\phi_r < 0$ as in the left panel of Figure 6, we see that the saddle path is flatter than the locus $r_{t+1} = r_t$. Namely we must have

$$\phi_r > \frac{1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g} + \frac{F''(n)n}{F'(n)}}{1 + \vartheta[\frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{F''(n)n}{F'(n)}]}.$$

This implies that

$$\begin{aligned}
\phi_n &= 1 + \vartheta\phi_r > 1 + \vartheta \frac{1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g} + \frac{F''(n)n}{F'(n)}}{1 + \vartheta[\frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{F''(n)n}{F'(n)}]} \\
&= \frac{1 + \vartheta\frac{g+F(n)}{1+g}}{1 + \vartheta\left[\frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{F''(n)n}{F'(n)}\right]} > 0,
\end{aligned}$$

where the inequality follows from $F' > 0$, $F'' < 0$, and $\vartheta > 0$. Thus, as a_t increases to the steady state, n_t will do so too. Since $\mu_t = F(n_t)$ and $F' > 0$, μ_t also increases to the steady state.

Log-linearizing equation (18) yields

$$\widehat{r}_{ft} = \frac{nF''(n)}{F'(n)}\hat{n}_t = \frac{nF''(n)}{F'(n)}\phi_n \hat{a}_t \equiv \phi_{rr}\hat{a}_t.$$

It follows from $F'' < 0$, $F' > 0$, and $\phi_n > 0$ that $\phi_{rr} < 0$. Thus as a_t increases to the steady state, the real interest rate r_{ft} decreases to the steady state. Finally, log-linearizing equation (20) given $z_t = z$ for all t yields

$$\begin{aligned}
\hat{\Pi}_{t+1} &= \hat{a}_t - \hat{a}_{t+1} - \frac{1}{r-1}(\hat{r}_t - \hat{r}_{t+1}) = (1-\phi_a)\hat{a}_t - \frac{1}{r-1}(1-\phi_a)\phi_r\hat{a}_t \\
&= (1-\phi_a)\left(1 - \frac{1}{r-1}\phi_r\right)\hat{a}_t \equiv \phi_\Pi \hat{a}_t.
\end{aligned}$$

If $\phi_r < 0$, then we have $\phi_\Pi > 0$. If $\phi_r > 0$, we use the equation

$$\hat{\Pi}_{t+1} = \hat{r}_t - \widehat{r}_{ft} = (\phi_r - \phi_{rr})\hat{a}_t \equiv \phi_\Pi \hat{a}_t$$

to deduce $\phi_\Pi > 0$. In both cases, Π_t increases with a_t to the steady state. Q.E.D.

Proof of Proposition 2: The equilibrium system consists of equations (19), (20), (21), and (25). First we study the steady state, in which the equilibrium system becomes (A.2), (A.3), and the following equation

$$\underbrace{\lambda + \frac{1-\lambda}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{R_f/(1+z)-1}\right]}_{\text{supply}} = \underbrace{\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa}}_{\text{limit}}, \tag{A.16}$$

where $\Pi = (1+z)/(1+g)$. The expression on the left-hand side of equation (A.16) is increasing in R_f and the expression on the right-hand side is decreasing in R_f. When $R_f/(1+z) = 1+\gamma/\beta$, the credit supply takes the value λ, which is below the borrowing limit when $\beta R_f/\Pi - \kappa > 0$. When $R_f \to \infty$, the credit supply approaches $\lambda + \frac{(1-\lambda)\beta}{1+\beta+\gamma}$, which is higher than the credit limit λ. Thus by the intermediate value theorem there is a unique solution for R_f to equation (A.16) such that

$$\frac{R_f}{1+z} > \max\left\{1+\frac{\gamma}{\beta}, \frac{\kappa\Pi}{\beta(1+z)}\right\}. \tag{A.17}$$

Let R_f^{**} denote the solution. Using equations (A.3), (A.2) and (A.16), we derive that

$$n = \frac{\lambda(1-\alpha)\zeta F(n)}{g+F(n)} \frac{\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa}.$$

We can equivalently rewrite this equation in terms of μ as

$$\Phi(\mu)(g+\mu) = \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa}\mu. \tag{A.18}$$

Notice that there is a trivial solution $\mu = 0$ to the above equation since $\Phi(0) = 0$. We rule out this solution by the following condition:

$$\Phi'(0)\, g < \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa} < \Phi(1)(g+1). \tag{A.19}$$

Then it follows from the intermediate value theorem that there is a unique solution, denoted by $\mu^{**} \in (0,1)$, to equation (A.18). The corresponding R&D investment level is denoted by $n^{**} = \Phi(\mu^{**}) > 0$.

Define the critical values κ^{**} and $\bar{\kappa}$ for κ such that

$$\Phi'(0)\, g = \frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \kappa^{**}},$$

$$\frac{(1-\alpha)\zeta\lambda\beta R_f^{**}/\Pi}{\beta R_f^{**}/\Pi - \bar{\kappa}} = \Phi(1)(g+1).$$

where R_f^{**} is the solution to equation (A.16) and is a function of κ. We can verify that the expression

$$\frac{R_f^{**}/\Pi}{R_f^{**}/\Pi - \kappa}$$

increases with κ along the supply curve in Figure 3. Thus the values κ^{**} and $\bar{\kappa}$ are unique. When $\kappa^{**} < \kappa < \bar{\kappa}$, condition (A.19) holds.

From Figure 3, we can see that the unconstrained equilibrium interest rate under perfect credit market is higher than that under binding credit constraint. Thus (A.17) and hence (22) and (A.5) must hold for the unconstrained equilibrium. If $\kappa^{**} < \kappa < \min\{\kappa^*, \bar{\kappa}\}$, then the unconstraint equilibrium derived in Proposition 1 violates the credit constraint and condition (A.19) is satisfied. For (26) to hold, we need

$$\psi F'(n^{**}) > \frac{R_f^{**}}{\Pi}.$$

Since (A.1) holds at n^* and R_f^* and since $n^{**} < n^*$ and $R_f^{**} < R_f^*$, the above condition follows from the concavity of F. The rest of the proof follows from the analysis in the main text using Figure 3. In particular, an increase in κ raises the nominal interest rate R_f^{**} and hence raises n^{**} by combining equations (A.3) and (A.2). It also raises the corresponding a^{**} by (A.2). But there is no growth effect because the economy still grows at the rate g on the balanced growth path.

Next we study the local stability. As in the proof of Proposition 1, we rewrite the equilibrium system (19), (20), (21), and (25) as

$$a_{t+1} = F(n_t) + \frac{1 - F(n_t)}{1+g} a_t, \tag{A.20}$$

$$r_t - 1 = \frac{R_{ft}}{\Pi_{t+1}} \frac{(r_{t+1} - 1)}{r_{t+1}} \frac{a_t}{a_{t+1}(1+g)}, \tag{A.21}$$

$$n_t = \frac{(1-\alpha)\zeta a_t}{1+g}\left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left(\beta - \frac{\gamma}{r_t - 1}\right)\right], \tag{A.22}$$

$$\left\{\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\left[\beta - \frac{\gamma}{r_t - 1}\right]\right\} = \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa}\lambda. \tag{A.23}$$

Define $r_{ft} \equiv R_{ft}/\Pi_{t+1}$. Log-linearizing this system yields

$$\begin{aligned}
\hat{a}_{t+1} &= \frac{g}{1+g}\frac{nF'(n)}{F(n)}\hat{n}_t + \frac{1 - F(n)}{1+g}\hat{a}_t, \\
\frac{r}{r-1}\hat{r}_t &= \hat{r}_{ft} + \frac{1}{r-1}\hat{r}_{t+1} + \hat{a}_t - \hat{a}_{t+1}, \\
\hat{n}_t &= \hat{a}_t + \vartheta \hat{r}_t, \\
\hat{r}_{ft} &= -\frac{\vartheta(\beta R_f/\Pi - \kappa)}{\kappa}\hat{r}_t \equiv -\varrho \hat{r}_t,
\end{aligned} \tag{A.24}$$

where ϑ is given in (A.10). Notice that $\varrho, \vartheta > 0$ by condition (A.17).

Simplifying yields a system of two equations for $\hat{a}_t$ and $\hat{r}_t$:

$$\begin{aligned}
\begin{bmatrix} \hat{a}_{t+1} \\ \hat{r}_{t+1} \end{bmatrix} &= \begin{bmatrix} \frac{g}{1+g}\frac{nF'(n)}{F(n)} + \frac{1-F(n)}{1+g} & \vartheta \frac{g}{1+g}\frac{nF'(n)}{F(n)} \\ (1-r)\left[1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g}\right] & r + (r-1)[\vartheta \frac{g}{1+g}\frac{nF'(n)}{F(n)} + \varrho] \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{r}_t \end{bmatrix} \\
&\equiv J \begin{bmatrix} \hat{a}_t \\ \hat{r}_t \end{bmatrix}
\end{aligned} \tag{A.25}$$

Consider the quadratic characteristic equation $G(\nu) \equiv |J - \nu I| = 0$. We can check that $G(0) > 0$ and $G(1) < 0$ by condition (27). Moreover $\lim_{x\to\infty} G(x) = \infty$ As in the proof of Proposition 1, we deduce that there is an eigenvalue inside the unit circle and an eigenvalue outside the unit circle. Thus the steady state is a saddle point.

[Insert Figure 7 Here.]

Finally we study transition dynamics using the phase diagram in Figure 7. The locus $a_{t+1} = a_t$ represents the line

$$\hat{r}_t = \frac{1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g}}{\vartheta \frac{g}{1+g}\frac{nF'(n)}{F(n)}}\hat{a}_t,$$

and the locus $r_{t+1} = r_t$ represents the line

$$\hat{r}_t = \frac{1 - \frac{g}{1+g}\frac{nF'(n)}{F(n)} - \frac{1-F(n)}{1+g}}{1 + \vartheta \frac{g}{1+g}\frac{nF'(n)}{F(n)} + \varrho}\hat{a}_t.$$

Notice that both lines have a positive slope and the locus $\hat{r}_{t+1} = \hat{r}_t$ is flatter than the locus $\hat{a}_{t+1} = \hat{a}_t$. Thus, if the initial value $\hat{a}_1 < 0$, then both $\hat{r}_t$ and $\hat{a}_t$ will increase over time to their steady-state values.

We now examine the dynamics of other variables. As in the proof of Proposition 1, we only need to study the signs of coefficients ϕ_{rr}, ϕ_n, and ϕ_Π. Since $R_{ft} = r_t(1+z)$ and z is constant, R_{ft} increases over time as a_t increases over time given initial $\hat{a}_1 < 0$. Since $\hat{n}_t = \hat{a}_t + \vartheta\hat{r}_t \equiv \phi_n \hat{a}_t$ and $\phi_n > 0$, as both $\hat{a}_t$ and $\hat{r}_t$ increase overtime, so does $\hat{n}_t$. Since

$$\hat{r}_{ft} = -\frac{\vartheta(\beta R_f/\Pi - \kappa)}{\kappa}\hat{r}_t \equiv -\varrho\hat{r}_t = -\varrho\phi_r\hat{a}_t \equiv \phi_{rr}\hat{a}_t,$$

we have $\phi_{rr} < 0$. Thus the real interest rate R_{ft}/Π_{t+1} decreases with a_t to the steady state. The growth rate of the economy up to the first-order approximation is given by

$$\log A_{t+1} - \log A_t = \hat{a}_{t+1} - \hat{a}_t + \log(1+g) = (\phi_a - 1)\hat{a}_t + \log(1+g).$$

It follows from $\phi_a \in (0,1)$ that the growth rate of the economy declines as a_t increases. It follows from

$$\hat{\Pi}_{t+1} = \hat{r}_t - \hat{r}_{ft} = (\phi_r - \phi_{rr})\hat{a}_t \equiv \phi_\Pi \hat{a}_t,$$

$\phi_r > 0$, and $\phi_{rr} < 0$ that the inflation rate Π_t increases with a_t to the steady state. Q.E.D.

Proof of Proposition 3: The equilibrium system still consists of equations (19), (20), (21), and (25). We first study the steady state which is characterized by equations (A.3), (A.2), (A.16), and

$$\Pi = \frac{1+z}{1+g}\lim_{t\to\infty}\frac{a_t}{a_{t+1}}. \tag{A.26}$$

Notice that a_t may not converge to a positive constant when $0 < \kappa < \kappa^{**}$. As in the proof of Proposition 2, equation (A.18) still holds. But when $\kappa \in (0, \kappa^{**})$, the first inequality in assumption (A.19) is violated so that the following inequality holds in the steady state:

$$\Phi'(0)\, g > \frac{(1-\alpha)\,\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa}. \tag{A.27}$$

Thus the only solution to equation (A.18) is $\mu^p = 0$. As a result, we have $n^p = a^p = 0$ in the steady state.

The following algebra shows that the productivity growth will converge to a rate between 0 and $1+g$ for $\kappa < \kappa^{**}$:

$$\begin{aligned}
\lim_{t\to\infty} \frac{A_{t+1}}{A_t} &= (1+g) \lim_{t\to\infty} \frac{a_{t+1}}{a_t} = (1+g) \lim_{t\to\infty} \left(\frac{\mu_t}{a_t} + \frac{1-\mu_t}{1+g} \right) \\
&= (1+g) \lim_{t\to\infty} \frac{F(n_t)}{a_t} + 1 = (1+g) \lim_{a\to 0} \frac{F(n)}{a} + 1 \\
&= (1+g)\, F'(0) \frac{\partial n}{\partial a}|_{a\to 0} + 1 = F'(0) \frac{(1-\alpha)\,\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa} + 1 \\
&< F'(0)\, \Phi'(0)\, g + 1 = g + 1,
\end{aligned}$$

where we have used equations (A.3) and (A.16) and $\mu_t \to 0$ to derive the second last equality. The last inequality holds because (A.27) holds. By (A.26) and the above expression for $\lim_{t\to\infty} \frac{a_{t+1}}{a_t}$, the steady-state inflation rate satisfies

$$\Pi = \frac{1+z}{F'(0) \frac{(1-\alpha)\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa} + 1} > \frac{1+z}{1+g}. \tag{A.28}$$

The poverty-trap steady state is characterized by a system of two equations (A.16) and (A.28) for two variables R_f and Π.

We modify Figure 3 to show the existence of a unique solution denoted by Π^p and R_f^p. Now the horizontal axis shows the real interest rate R_f/Π instead of the nominal interest rate R_f. The borrowing-limit curve still describes the expression on the right-hand side of equation (A.16) as a decreasing function of R_f/Π. The supply curve describes the expression on the left-hand side of (A.16), which is written as a function R_f/Π :

$$\lambda + \frac{1-\lambda}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{\frac{R_f}{\Pi} \frac{\Pi}{1+z} - 1} \right] = \lambda + \frac{1-\lambda}{1+\beta+\gamma} \left[\beta - \frac{\gamma}{\frac{R_f}{\Pi} \left(F'(0) \frac{(1-\alpha)\zeta\lambda\beta R_f/\Pi}{\beta R_f/\Pi - \kappa} + 1 \right)^{-1} - 1} \right],$$

where we have used (A.28) to substitute for Π. We can check that the above expression increases with R_f/Π. As in the proof of Proposition 2, there is a unique intersection point between the borrowing-limit and supply curves such that (A.17) holds, which determines the equilibrium real interest rate R_f/Π. Then Π^p and R_f^p are determined.

It follows from (A.28) that $\Pi^p > (1+z)/(1+g)$ and Π^p decreases with κ as $\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}$ increases with κ (see Figure 3). Similarly, $\lim_{t\to\infty} A_{t+1}/A_t$ increases with κ.

Next we study the local stability of the poverty trap steady state. The equilibrium system is still given by equations (A.20) through (A.23). Since the steady-state values of n_t and a_t are zero, we cannot use log-linearization. Instead we use linearization in levels to derive

$$F(n_t) = F(0) + F'(0)n_t = F'(0)n_t.$$

Substituting this equation into (19) yields

$$a_{t+1} = F'(0)n_t + \frac{1-F'(0)n_t}{1+g}a_t \tag{A.29}$$

Combining equations (21) and (25) yields

$$n_t = \frac{(1-\alpha)\zeta a_t}{1+g}\frac{\lambda\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1}-\kappa}.$$

Linearizing around the steady state yields

$$n_t = \frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}a_t. \tag{A.30}$$

Substituting this equation into (A.29), we obtain the approximate law of motion for a_t :

$$a_{t+1} = \left\{F'(0)\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}+\frac{1}{1+g}\right\}a_t - \frac{F'(0)}{1+g}\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}a_t^2. \tag{A.31}$$

We have shown earlier that

$$0 < F'(0)\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}+\frac{1}{1+g} < 1,$$

when $\kappa \in (0,\kappa^{**})$. It follows from (A.31) that a_t decreases monotonically to the steady state $a^p = 0$ whenever it starts at any small $a_1 > 0$. Thus the steady state is a saddle point. It follows from (A.30) that n_t also decreases monotonically to the steady state $n^p = 0$. Since $\mu_t = F(n_t)$, μ_t also decreases monotonically to the steady state $\mu^p = 0$.

We can derive the approximate productivity growth rate around the steady state

$$\begin{aligned}\frac{A_{t+1}}{A_t} - 1 &= \frac{a_{t+1}}{a_t}(1+g) - 1 \\ &= F'(0)(1-\alpha)\zeta\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa} - F'(0)\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}a_t,\end{aligned} \tag{A.32}$$

where the second equality follows from substitution of (A.31). Since $F'(0) > 0$ and $\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa} > 0$, the productivity growth rate increases to the steady state when a_t decreases to the steady state.

We log-linearize equation (A.21) to derive

$$\frac{r}{r-1}\hat{r}_t = \hat{r}_{ft} + \frac{1}{r-1}\hat{r}_{t+1} + \hat{x}_t \tag{A.33}$$

where we define $r_{ft} = R_{ft}/\Pi_{t+1}$ and

$$x_t \equiv \frac{a_t}{a_{t+1}} = \frac{1}{F'(0)\frac{(1-\alpha)\zeta}{1+g}\left[\frac{\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}\lambda\right] + \frac{1}{1+g} - \frac{F'(0)}{1+g}\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}a_t}.$$

Here the second equality follows from equation (A.31). We then obtain the log-linearized equation

$$\hat{x}_t = \frac{F'(0)\frac{(1-\alpha)\zeta}{1+g}\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}}{F'(0)\,(1-\alpha)\,\zeta\frac{\lambda\beta R_f/\Pi}{\beta R_f/\Pi-\kappa}+1}a_t \equiv \phi_x a_t.$$

Substituting (A.24) into (A.33) yields

$$\hat{r}_t = \frac{1}{r+(r-1)\varrho}\hat{r}_{t+1} + \frac{r-1}{r+(r-1)\varrho}\hat{x}_t. \tag{A.34}$$

We now drop the quadratic term in (A.31) and write the first-order approximation to the law of motion as $a_{t+1} = \phi_a a_t$, where $\phi_a \in (0,1)$. Since $r = R_f/(1+z) > 1$ and $\varrho > 0$ by (A.17), we iterate (A.34) forward to derive

$$\hat{r}_t = \frac{(r-1)\,\phi_x}{r+(r-1)\varrho-\phi_a}a_t.$$

Thus, as a_t decreases to the steady state, $\hat{r}_t$ also decreases to the steady state and so does R_{ft}. It follows from (A.24) and $\varrho > 0$ that the real interest rate $r_{ft} = R_{ft}/\Pi_{t+1}$ increases to the steady state when $\hat{r}_t$ or R_{ft} decreases to the steady state. Finally, since

$$\hat{\Pi}_{t+1} = \hat{r}_t - \hat{r}_{ft} = (1+\varrho)\hat{r}_t,$$

the inflation rate Π_t decreases to the steady state when $\hat{r}_t$ decreases to the steady state. Q.E.D.

Proof of Proposition 4: Let $\beta^t\Lambda_t$ and $\beta^t\Lambda_t q_t$ be the Lagrange multipliers associated with (32) and (13), respectively. The variable q_t represents the shadow value of the technology A_{t+1}. The first-order conditions are given by

$$\begin{aligned}
\omega u'\left(c_t^e\right) &= u'\left(c_t^o\right) = u'\left(c_t^y\right) = \Lambda_t, \\
\Lambda_t &= \beta\Lambda_{t+1}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}}\bar{A}_{t+1} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi A_t\right]\frac{F'\left(N_t/\bar{A}_{t+1}\right)}{\bar{A}_{t+1}} \\
&\quad +\Lambda_t q_t[\bar{A}_{t+1}-A_t]\frac{F'\left(N_t/\bar{A}_{t+1}\right)}{\bar{A}_{t+1}}, \\
\Lambda_t q_t &= \beta(1-\mu_{t+1})\Lambda_{t+1}q_{t+1} + \beta^2\Lambda_{t+2}(1-\mu_{t+1})(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi.
\end{aligned}$$

In the steady state, we have

$$q = \frac{\beta}{1+g}(1-\mu)q + \left(\frac{\beta}{1+g}\right)^2 (1-\mu)(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi,$$

and

$$\frac{1}{F'(n)} = \beta\frac{1}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\frac{a}{1+g}\right] + q\left[1-\frac{a}{1+g}\right].$$

Equation (13) implies

$$a = \frac{F(n)(1+g)}{g+F(n)}. \tag{A.35}$$

Using the above three equations we can derive

$$\begin{aligned}\frac{1}{F'(n)} &= \frac{\beta}{1+g}(\frac{1}{\alpha}-1)\left[\alpha^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\right] \\ &\quad + \frac{\beta}{1+g-\beta(1-F(n))}(\frac{1}{\alpha}-1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\chi\frac{g}{g+F(n)}.\end{aligned}$$

This equation is equivalent to equation (34). We can easily check that the expression on the left-hand side of the equation is an increasing function of μ and the expression on the right-hand side is a decreasing function of μ. Given Assumption 1, it follows from the intermediate value theorem that there is a unique solution, denoted by $\mu_{FB} \in (0,1)$, to the above equation. Then we obtain the efficient investment level $n_F = \Phi(\mu_{FB})$. Plugging it into (A.35) gives a_{FB}. The efficient consumption and production allocation is derived in the main text. Q.E.D.

Proof of Proposition 5: Suppose that there is no fiscal policy. When money increments are transferred to entrepreneurs instead of savers, the saver's consumption and portfolio choices are given by

$$c_t^y = \frac{(1-\lambda)w_t}{1+\beta+\gamma}, \tag{A.36}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta+\gamma}\frac{R_{ft}}{\Pi_{t+1}}(1-\lambda)w_t, \tag{A.37}$$

$$\frac{M_t}{P_t} = \frac{\gamma}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}(1-\lambda)w_t, \tag{A.38}$$

$$\frac{S_t}{P_t} = \frac{1}{1+\beta+\gamma}\frac{\beta R_{ft}-\beta-\gamma}{R_{ft}-1}(1-\lambda)w_t, \tag{A.39}$$

where we assume that

$$R_{ft} > 1 + \frac{\gamma}{\beta},$$

so that savings and money demand are positive. Notice that the demand for money and savings depends on the nominal interest rate instead of the real interest rate. The monetary transfer is given by

$$\frac{M_t - M_{t-1}}{P_t} = \frac{z_t}{z_t+1}\frac{M_t}{P_t} = \frac{z_t}{z_t+1}\frac{\gamma}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}(1-\lambda)w_t,$$

where the second equality follows from (A.38).

The competitive equilibrium for a given interest rate sequence $\{R_{ft}\}$ under perfect credit markets can be summarized by a system of four difference equations, (18), (19), (20), and

$$n_t = \frac{(1-\alpha)\zeta a_t}{1+g}\left[\lambda + \frac{z_t}{1+z_t}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_{ft}}{R_{ft}-1} + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{\beta R_{ft}-\beta-\gamma}{R_{ft}-1}\right]. \tag{A.40}$$

for four sequences $\{z_t\}, \{a_t\}, \{\Pi_{t+1}\}$, and $\{n_t\}$ such that (6) and $R_{ft} > 1+\gamma/\beta$ are satisfied. Equation (A.40) says that R&D investment is financed by the entrepreneur's wage income, monetary transfer, and external credit.

In the steady state the system becomes three equations (A.1), (A.2), and

$$n = \frac{(1-\alpha)\zeta a}{1+g}\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f}{R_f-1} + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{\beta R_f-\beta-\gamma}{R_f-1}\right], \tag{A.41}$$

for three unknowns n, z, and a, when the nominal interest rate R_f is set by the monetary authority. Given the efficient innovation rate μ_{FB}, we have $n_{FB} = \Phi(\mu_{FB})$, and

$$a_{FB} = \frac{(1+g)F(n_{FB})}{g+F(n_{FB})}.$$

We now show that the monetary authority can set a specific nominal interest rate such that the efficient innovation can be implemented in a market equilibrium with a perfect credit market on the balanced growth path. Specifically, using the steady-state system, we can derive one equation for one unknown R_f :

$$n_{FB} = \frac{(1-\alpha)\zeta a_{FB}}{1+g}\left[\lambda + \frac{(1-\lambda)}{1+\beta+\gamma}\frac{(\gamma+\beta)(R_f-1) - \gamma\psi F'(n_{FB})/(1+g)}{R_f-1}\right],$$

The expression on the right-hand side of the equation is an increasing function of R_f. Under Assumption 2, this function takes a value lower than n_{FB} at $R_f = 1+\gamma/\beta$ and a value higher than n_{FB} when $R_f \to \infty$. It follows from the intermediate value theorem that there exists a unique solution, denoted by $\bar{R}_f > 1+\gamma/\beta$, to the above equation. Given $R_f = \bar{R}_f$, we can also easily show that $n = n_{FB}$ is the only equilibrium solution.

Once $\bar{R}_f$ is determined, we can solve for the money growth rate z using (A.1):

$$1+z = \frac{\bar{R}_f(1+g)}{F'(n_{FB})\psi}.$$

Other equilibrium variables can also be easily determined.

Finally we show that there exists a cutoff κ_0 such that when $\kappa \geq \kappa_0$ the credit constraint does not bind in the market equilibrium described above. The credit constraint is given by

$$\left(\frac{\beta R_{ft}}{\Pi_{t+1}} - \kappa\right) N_t \leq \frac{\beta R_{ft}}{\Pi_{t+1}}\left[\lambda w_t + \frac{z_t}{1+z_t}\frac{\gamma(1-\lambda)}{1+\beta+\gamma}\frac{R_{ft}}{R_{ft}-1}w_t\right],$$

where the second term in the square bracket is the monetary transfer. In the steady state this constraint becomes

$$\left(\frac{\beta R_f}{\Pi}-\kappa\right)n\leq\frac{\beta R_f}{\Pi}\frac{(1-\alpha)\,\zeta a}{1+g}\left[\lambda+\frac{z}{1+z}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma R_f}{R_f-1}\right],$$

where we have used equation (15). The desired cutoff κ_0 is defined by the following equation

$$n_{FB}=\frac{\beta\bar{R}_f\,(1+g)\,/\,(1+z)}{\beta\bar{R}_f\,(1+g)\,/\,(1+z)-\kappa_0}\frac{(1-\alpha)\,\zeta a_{FB}}{1+g}\left[\lambda+\frac{z}{1+z}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma\bar{R}_f}{\bar{R}_f-1}\right].$$

The proof is completed. Q.E.D.

Proof of Proposition 6: First we show that for any given μ_{t-1} the efficient GDP is higher than the market GDP because of the monopoly distortion. To show this result, we observe that the efficient GDP Y_t^e is given by (31). By (17), the equilibrium GDP in the market economy is given by

$$\begin{aligned}
Y_t &= w_t+\mu_{t-1}\psi\bar{A}_t\\
&= (1-\alpha)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}\left[\mu_{t-1}\bar{A}_t+(1-\mu_{t-1})A_{t-1}\right]+(\chi-1)\,\mu_{t-1}\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}\bar{A}_t,
\end{aligned}$$

where we have substituted equations (13) and (15) and the expressions for ζ and ψ. We can easily verify that $Y_t^e>Y_t$.

To achieve the efficient GDP, the government can subsidize the final good firm's input expenditure. Let $\tau_{xt}(i)$ be the subsidy to input i in period t. Then the final good producer's problem is given by

$$\max\ L_t^{1-\alpha}\int_0^1 A_t\,(i)^{1-\alpha}\,x_t\,(i)^{\alpha}\,di-\int_0^1\tau_{xt}(i)p_t(i)x_t(i)di-w_tL_t.$$

This leads to

$$x_t(i)=\left(\frac{\tau_{xt}(i)p_t\,(i)}{\alpha}\right)^{\frac{1}{\alpha-1}}A_t\,(i)\,. \tag{A.42}$$

Since $p_t\,(i)=\chi$, it follows from (30) and (A.42) that setting

$$\tau_{xt}(i)=\tau_{xt}^0(i)\equiv\begin{cases}\frac{1}{\chi} & \text{if an innovation occurs}\\ 1 & \text{otherwise}\end{cases}$$

achieves the efficient intermediate input level and final GDP Y_t^e.

In this case a successful innovator produces intermediate good $x_t\,(i)=\alpha^{\frac{1}{1-\alpha}}\bar{A}_t$ and earns monopoly profits

$$p_t\,(i)\,x_t\,(i)-x_t\,(i)=\chi\alpha^{\frac{1}{1-\alpha}}\bar{A}_t-\alpha^{\frac{1}{1-\alpha}}\bar{A}_t=\psi^*\bar{A}_t,$$

where $\psi^*\equiv\alpha^{\frac{1}{1-\alpha}}(\chi-1)>\psi$. Since the final good firm earns zero profit, the real wage under the government policies is given by

$$w_t=(1-\alpha)\left[\alpha^{\frac{\alpha}{1-\alpha}}\mu_{t-1}\bar{A}_t+(1-\mu_{t-1})\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}A_{t-1}\right]. \tag{A.43}$$

The total subsidy is given by

$$\int_0^1 (1-\tau_{xt}(i))\, p_t(i)\, x_t(i)\, di = \alpha^{\frac{1}{1-\alpha}} \mu_{t-1} \bar{A}_t (\chi - 1).$$

Let $w_{Dt} = w_t - T_w \bar{A}_t$ denote the after-tax wage. Since money increments are transferred to entrepreneurs instead of savers, we rederive the saver's decision rules as

$$c_t^y = \frac{(1-\lambda) w_{Dt}}{1+\beta+\gamma}, \tag{A.44}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta+\gamma} \frac{R_{ft}}{\Pi_{t+1}} (1-\lambda) w_{Dt}, \tag{A.45}$$

$$\frac{M_t}{P_t} = \frac{\gamma}{1+\beta+\gamma} \frac{R_{ft}}{R_{ft}-1} (1-\lambda) w_{Dt}, \tag{A.46}$$

$$\frac{S_t}{P_t} = \frac{1}{1+\beta+\gamma} \frac{\beta R_{ft} - \beta - \gamma}{R_{ft}-1} (1-\lambda) w_{Dt}, \tag{A.47}$$

where we assume that

$$R_{ft} > 1 + \frac{\gamma}{\beta},$$

so that savings and money demand are positive.

The entrepreneur's budget constraint (4) when young becomes

$$N_t = \frac{B_t}{P_t} + \lambda w_{Dt} + \tau_{et}, \tag{A.48}$$

where τ_{et} is the monetary transfer

$$\tau_{et} = \frac{M_t - M_{t-1}}{P_t}. \tag{A.49}$$

The entrepreneur's problem is to maximize his expected consumption when old:

$$\max\, (1+\tau_N) F\left(N_t / \bar{A}_{t+1}\right) \psi^* \bar{A}_{t+1} - T_N \bar{A}_{t+1} - R_{ft} \frac{P_t}{P_{t+1}} [N_t - \lambda w_{Dt} - \tau_t^e].$$

Suppose that the credit constraint is slack. The first-order condition implies that

$$(1+\tau_N) F'(n_t) \psi^* = R_{ft} \frac{P_t}{P_{t+1}}. \tag{A.50}$$

By the market-clearing condition for loans and equations (A.46), (A.47), (A.48), and (A.49), we derive that

$$N_t = \lambda w_{Dt} + \frac{z_t}{1+z_t} \frac{(1-\lambda)\gamma}{1+\beta+\gamma} \frac{R_{ft}}{R_{ft}-1} w_{Dt} + \frac{1-\lambda}{1+\beta+\gamma} \frac{\beta R_{ft} - \beta - \gamma}{R_{ft}-1} w_{Dt}. \tag{A.51}$$

The three terms on the right-hand side of this equation give three sources of funds for the R&D investment: internal funds (wage), government monetary transfers, and external debt.

In the steady state equation (A.2) still holds and (A.50) becomes

$$(1+\tau_N) F'(n) \psi^* = \frac{R_f}{\Pi}, \tag{A.52}$$

where $\Pi = (1+z)/(1+g)$. Using (15) and (A.2), we rewrite (A.51) as

$$n = \eta \left[\lambda + \frac{z}{1+z} \frac{(1-\lambda)}{1+\beta+\gamma} \frac{\gamma R_f}{R_f - 1} + \frac{1-\lambda}{1+\beta+\gamma} \frac{\beta R_f - \beta - \gamma}{R_f - 1} \right], \tag{A.53}$$

where it follows from (A.43) that

$$\eta \equiv \frac{w_{Dt}}{\bar{A}_{t+1}} = (1-\alpha) \left[\frac{\alpha^{\frac{\alpha}{1-\alpha}} \mu}{1+g} + (1-\mu) \left(\frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}} \frac{a}{(1+g)^2} \right] - \frac{T_w}{1+g} \tag{A.54}$$

is constant along a balanced growth path. The variable η represents the normalized after-tax wage on the balanced growth path.

By (A.48), and (A.49), the credit constraint (5) becomes

$$N_t \leq \frac{\beta R_{ft}/\Pi_{t+1}}{\beta R_{ft}/\Pi_{t+1} - \kappa} \left[\lambda w_{Dt} + \frac{z_t}{1+z_t} \frac{\gamma (1-\lambda)}{1+\beta+\gamma} \frac{R_{ft}}{R_{ft} - 1} w_{Dt} \right]. \tag{A.55}$$

In the steady state this constraint becomes

$$n \leq \frac{\eta \beta R_f / \Pi}{\beta R_f / \Pi - \kappa} \left[\lambda + \frac{z}{1+z} \frac{1-\lambda}{1+\beta+\gamma} \frac{\gamma R_f}{R_f - 1} \right], \tag{A.56}$$

where we have used equation (**??**).

Since we do not consider government spending and government debt, the following government budget constraint must be satisfied:

$$\tau_N F\left(N_{t-1}/\bar{A}_t\right) \psi^* \bar{A}_t + \alpha^{\frac{1}{1-\alpha}} \mu_{t-1} \bar{A}_t (\chi - 1) + \tau_{et} = T_N \bar{A}_t + T_w \bar{A}_t + \frac{M_t - M_{t-1}}{P_t},$$

where the second term represents the total subsidy to intermediate inputs. By (A.49), this constraint along a balanced growth path becomes

$$\tau_N F(n) \psi^* + \alpha^{\frac{1}{1-\alpha}} \mu (\chi - 1) = T_N + T_w. \tag{A.57}$$

The steady-state competitive equilibrium under fiscal and monetary policy instruments $\{R_f, \tau_x(i), T_w, \tau_N, T_N\}$ consists of four equations (A.2), (A.52), (A.53), and $\Pi = (1+z)/(1+g)$ for four variables n, a, z, and Π such that (A.56) and (A.57) hold.[11]

We use (33) and $\omega = 1$ to derive the efficient consumption for the young saver

$$c_t^y = \frac{1}{3} \left(Y_t^e - N_t \right).$$

To implement this efficient consumption in a market equilibrium, we use (A.44) to set the labor income tax as

$$T_w \bar{A}_t = w_t - \frac{1+\beta+\gamma}{3(1-\lambda)} \left(Y_t^e - N_t \right).$$

[11]During the transition path, we may use the interest rate rule

$$R_{ft} = R_f \left(\frac{\Pi_t}{\Pi} \right)^{\theta}.$$

Using equations (15) and (31), we derive that

$$\begin{aligned} T_w &= \left[1-\frac{1+\beta+\gamma}{3(1-\lambda)}\right](1-\alpha)\left[\alpha^{\frac{\alpha}{1-\alpha}}\mu+(1-\mu)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}\frac{a}{(1+g)}\right] \\ &\quad +\frac{1+\beta+\gamma}{3(1-\lambda)}(1+g)\,n, \end{aligned} \tag{A.58}$$

on the balanced growth path. We set T_N such that the government budget constraint (A.57) is satisfied.

It remains to choose τ_N and R_f such that the competitive equilibrium implies efficient production and innovation such that $a=a_{FB}$, $n=n_{FB}$, and $\mu=F(n_{FB})$. We maintain the following assumption similar to Assumption 2 such that the efficient R&D investment cannot be self-financed by the entrepreneur's wage income and monetary transfers and external credit are needed.

Assumption 3 *Parameter values are such that* $0<\gamma<1-\beta$ *and*

$$\lambda\eta_{FB}<n_{FB}<\eta_{FB}\left[\lambda+\frac{(1-\lambda)(\beta+\gamma)}{1+\beta+\gamma}\right],$$

where η_{FB} *is defined in (A.54) and where* T_w *is defined in (A.58) with* $\mu=\mu_{FB}$ *and* $a=a_{FB}$*.*

We now set policy variables R_f^0, τ_N^0, T_N^0, and T_w^0 such that they satisfy

$$n_{FB}=\eta_{FB}\left[\lambda+\frac{1-\lambda}{1+\beta+\gamma}\frac{\beta(\gamma+\beta)\left(R_f^0-1\right)-\gamma}{\beta\left(R_f^0-1\right)}\right], \tag{A.59}$$

$$\tau_N^0=\frac{(1+g)/\beta}{F'(n_{FB})\psi^*}-1, \tag{A.60}$$

$$T_N^0=\tau_N^0F(n_{FB})\psi^*+\alpha^{\frac{1}{1-\alpha}}F(n_{FB})(\chi-1)-T_w^0,$$

$$\begin{aligned} T_w^0 &= \left[1-\frac{1+\beta+\gamma}{3(1-\lambda)}\right](1-\alpha)\left[\alpha^{\frac{\alpha}{1-\alpha}}\mu_{FB}+(1-\mu)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}\frac{a_{FB}}{(1+g)}\right] \\ &\quad +\frac{1+\beta+\gamma}{3(1-\lambda)}(1+g)\,n_{FB}. \end{aligned} \tag{A.61}$$

As in the proof of Proposition 5, we use the intermediate value theorem to show that under Assumption 3 there exists a unique solution for $R_f^0>1+\gamma/\beta$ to equation (A.59).

Define the cutoff κ^0 by the equation

$$n_{FB}=\frac{(1+g)\,\eta_{FB}}{1+g-\kappa^0}\left[\lambda+\frac{\beta R_f^0-1}{\beta R_f^0}\frac{1-\lambda}{1+\beta+\gamma}\frac{\gamma(1+g)}{1+g-\beta}\right]. \tag{A.62}$$

Then when $\kappa\geq\kappa^0$ the credit constraint does not bind on the balanced growth path.

Given the above monetary and fiscal policy variables, the steady-state system (A.2), (A.52), (A.53), and $\Pi = (1+z)/(1+g)$ becomes

$$a = \frac{F(n)(1+g)}{g+F(n)}, \ \Pi = \frac{1+z}{1+g}, \ \frac{F'(n)}{F'(n_{FB})} = \frac{R_f^0}{\Pi}\frac{\beta}{1+g},$$

$$n = \eta\left[\lambda + \frac{z}{1+z}\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f^0}{R_f^0-1} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_f^0-\beta-\gamma}{R_f^0-1}\right],$$

for four variables n, a, μ, and z.

We can simplify this system to one equation for n :

$$\frac{n}{\eta} = \lambda + \left[1 - \frac{F'(n)}{\beta R_f^0 F'(n_{FB})}\right]\frac{(1-\lambda)}{1+\beta+\gamma}\frac{\gamma R_f^0}{R_f^0-1} + \frac{1-\lambda}{1+\beta+\gamma}\frac{\beta R_f^0-\beta-\gamma}{R_f^0-1}, \tag{A.63}$$

where

$$\eta = \frac{1+\beta+\gamma}{3(1-\lambda)}\left[(1-\alpha)\left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}F(n)}{1+g} + (1-F(n))\frac{\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}}F(n)}{(1+g)(g+F(n))}\right] - (1+g)n\right].$$

We can check that $n = n_{FB}$ is a solution to equation (A.63). We next show that this is only solution. Since $F(n)$ is concave and $F(0) = 0$, we can show that $F(n)/n$ decreases with n. Thus η/n decreases with n or n/η increases with n. We also know that the expression on the right-hand side of (A.63) increases with n. Two monotonic curves can only have one intersection point if there is any. Thus there is a unique solution $n = n_{FB}$ to equation (A.63).

We can then verify that the solution to the above system is given by

$$a = a_{FB}, \ n = n_{FB}, \ z = \beta R_f^0 - 1, \ \Pi = \frac{\beta R_f^0}{1+g}.$$

Since the market real interest rate $R_f^0/\Pi = \beta(1+g)$ is the same as the efficient rate in (35), the old saver consumption satisfies $c_{t+1}^o = c_t^y(1+g)$. Thus $c_t^o = c_t^y$ on the balanced growth path. We then attain consumption efficiency by (33) with $\omega = 1$. Since the above system has a unique solution, the preceding solution is the only steady-state equilibrium that attains the efficient innovation, production, and consumption allocation. Q.E.D.

B Data Description

For Figure 1 we follow Levine, Loayza, and Beck (2000) and AHM (2005) and consider cross-sectional data on 71 countries over the period 1960–1995. As in their papers, we use private credit, defined as the value of credits by financial intermediaries to the private sector, divided by GDP, as our preferred measure of financial development. We construct this measure using the updated 2017 version of the Financial Development and Structure Database. We have also used other measures of financial development and the pattern in Figure 1 does not change. We construct the average per capita GDP growth rates using the Penn World Table and construct the average inflation rates and the average (broad) money growth rates using the World Bank WDI database. We delete outliers with average inflation rates higher than 40%, but the pattern in Figure 1 still holds for the full sample. The outliers are Argentina, Bolivia, Brazil, Chile, Israel, Peru, and Uruguay. The non-convergence countries used in Panel D of Figure 1 are identified according to Table II of AHM (2005).

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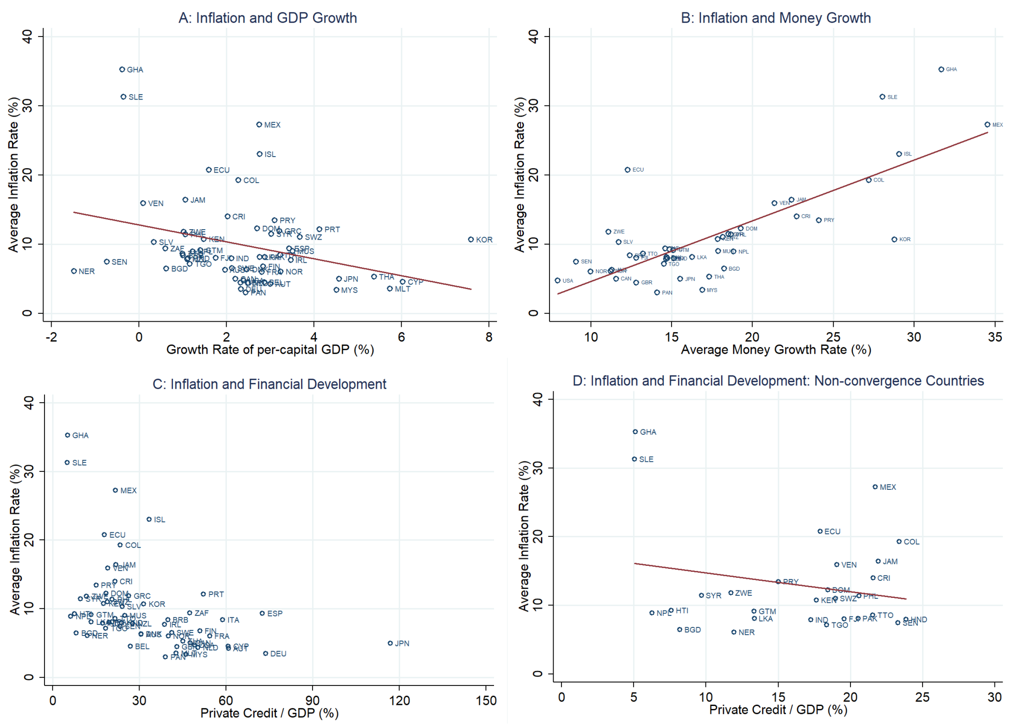

Figure 1: The average inflation rate, the average per capita GDP growth rate, the average money growth rate, and the average level of financial development, 1960-1995.

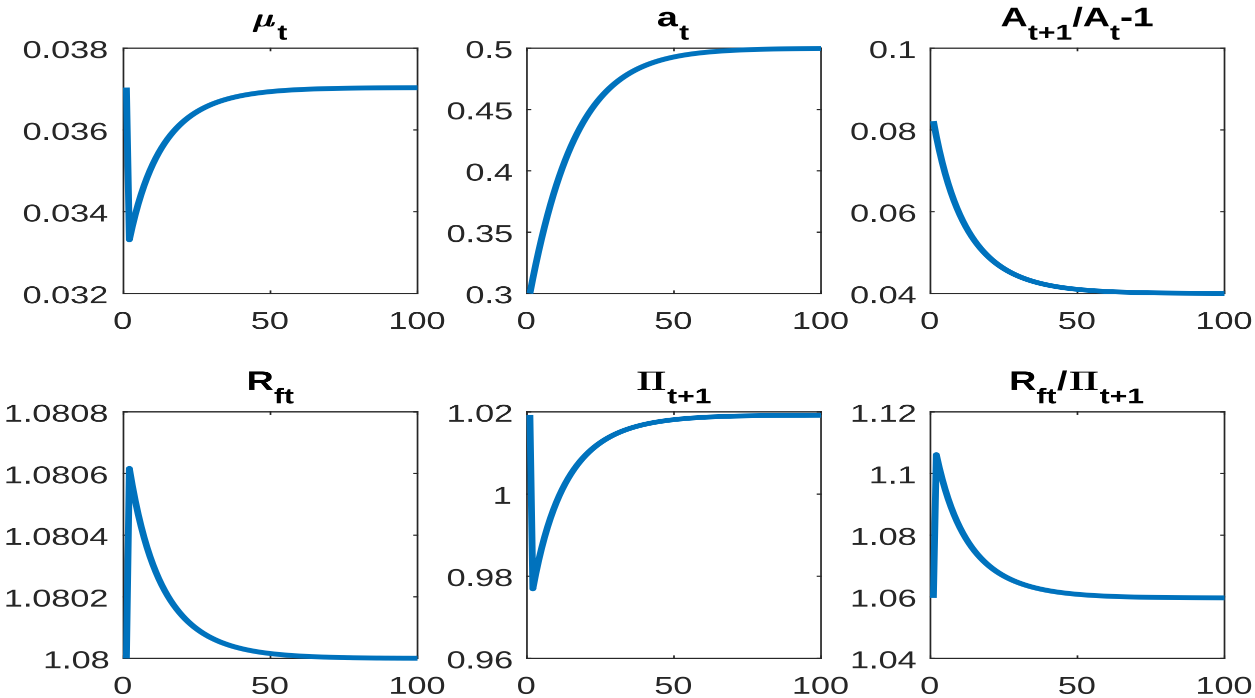

Figure 2: Transition dynamics for the case of perfect credit markets.

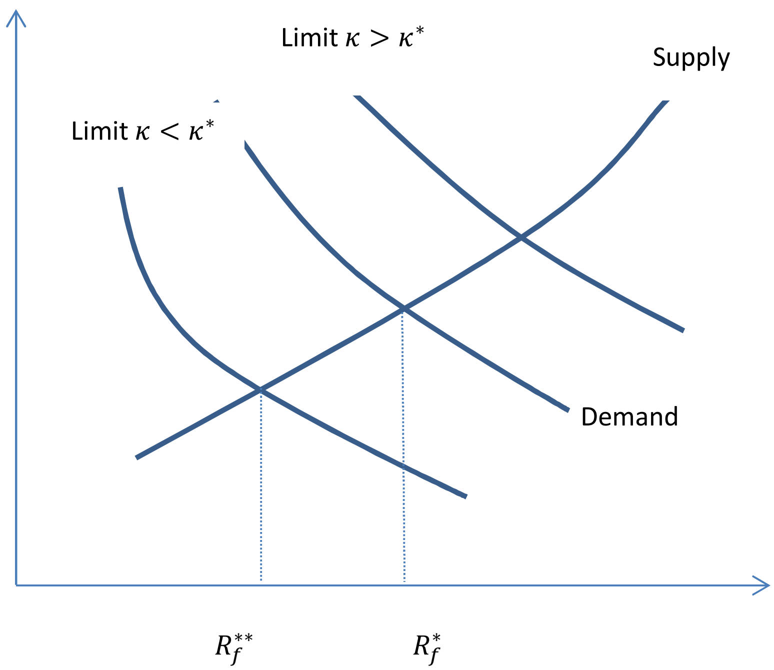

Figure 3: Determination of the steady-state equilibrium nominal interest rates. The curves labeled "Supply" and "Demand" describe the supply of and demand for funds normalized by the wage rate, respectively. The curves labeled "Limit $\kappa > \kappa^*$" and "Limit $\kappa > \kappa^*$" describe the borrowing limits normalized by the wage rate for $\kappa > \kappa^*$ and $\kappa < \kappa^*$, respectively.

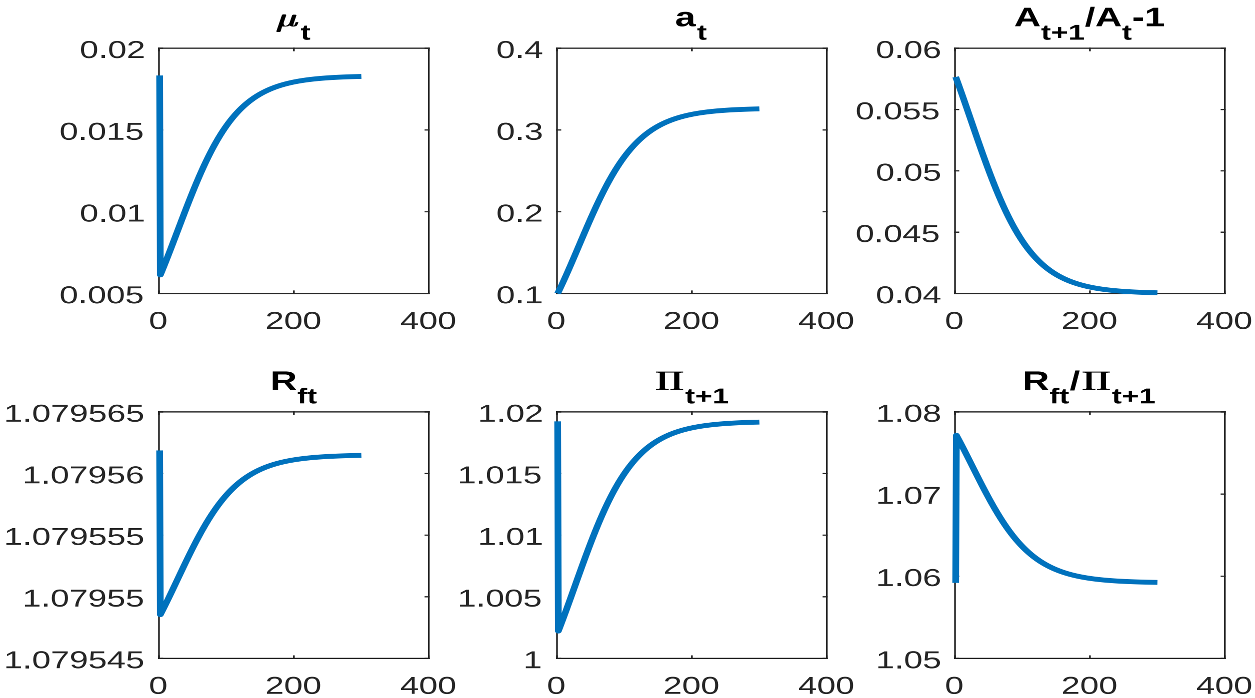

Figure 4: Transition dynamics for the case of not too tight credit constraints.

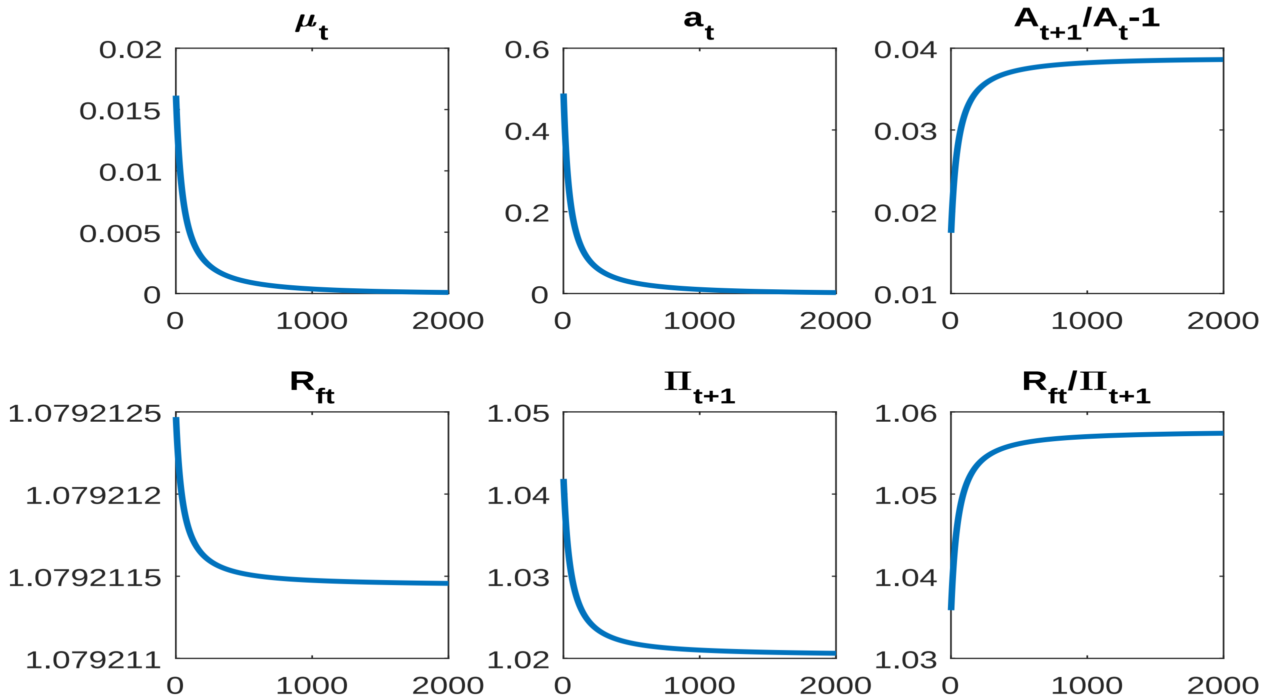

Figure 5: Transition dynamics for the case of poverty trap.

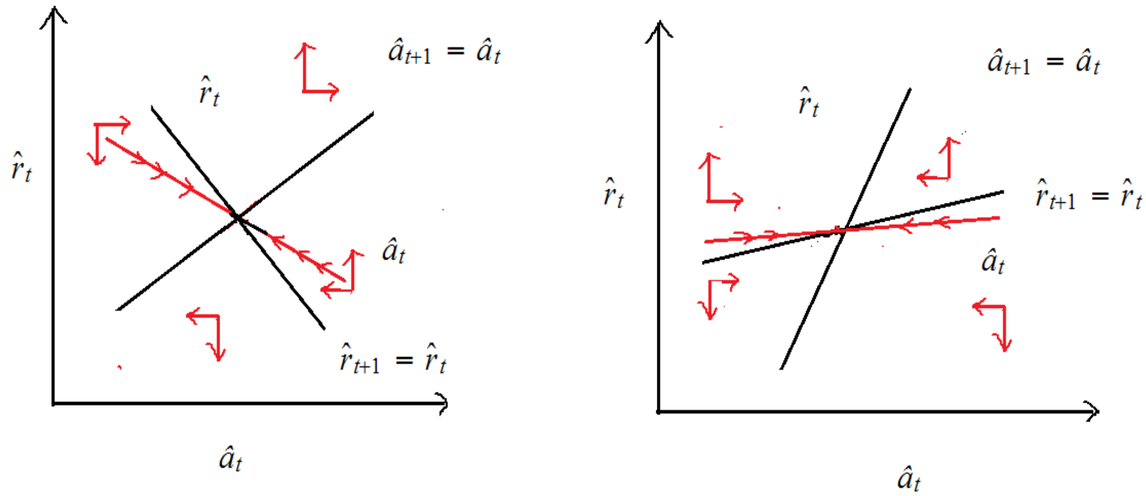

Figure 6: Phase diagram for the case with perfect credit market.

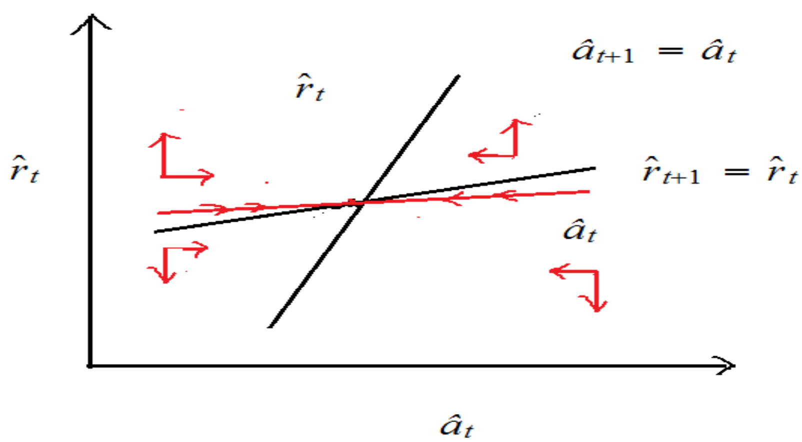

Figure 7: Phase diagram for the case with not too tight credit constraint.