Robots Are Us: Some Economics of Human Replacement*

Seth G. Benzell[1], Laurence J. Kotlikoff[1], Guillermo LaGarda[1], and Jeffrey D. Sachs[2]

[1]Department of Economics, Boston University
[2]The Earth Institute, Columbia University

March 29, 2015

Abstract

Will smart machines replace humans like the internal combustion engine replaced horses? If so, can putting people out of work, or at least out of good work, also put the economy out of business? Our model says yes. Under the right conditions, more supply produces, over time, less demand as the smart machines undermine their customer base. Highly tailored skill- and generation-specific redistribution policies can keep smart machines from immiserating humanity. But blunt policies, such as mandating open-source technology, can make matters worse.

*We would like to thank Dilip Mookherjee, Dietrich Vollrath, and Kevin Cooke for their useful comments.

1 Introduction

Whether it's bombing our enemies, steering our planes, fielding our calls, rubbing our backs, vacuuming our floors, driving our taxis, or beating us at Jeopardy, it's hard to think of hitherto human tasks that smart machines can't do or won't soon do. Few smart machines look even remotely human. But they all combine brains and brawn, namely sophisticated code and physical capital. And they all have one ultimate creator – us.

Will human replacement - the production by ourselves of ever better substitutes for ourselves - deliver an economic utopia with smart machines satisfying our every material need? Or will our self-induced redundancy leave us earning too little to purchase the products our smart machines can make?

Ironically, smart machines are invaluable for considering what they might do to us and when they might do it. This paper uses the most versatile of smart machines – a run-of-the-mill computer – to simulate one particular vision of human replacement. Our simulated economy – an overlapping generations model – is bare bones. It features two types of workers consuming two goods for two periods. Yet it admits a large range of dynamic outcomes, some of which are quite unpleasant.

The model's two types of agents are called high-tech workers and low-tech workers. The first group has a comparative advantage at analytical tasks, the second in empathetic and interpersonal tasks. Both work full time, but only when young. High-tech workers produce new software code, which adds to the existing stock of code. They are compensated by licensing their newly produced code for immediate use and by selling rights to its future use. The stock of code – new plus old – is combined with the stock of capital to produce automatable goods and services (hereafter referred to as 'goods'). Goods can be consumed or used as capital. Unlike high-tech workers, low-tech workers are right brainers – artists, musicians, priests, astrologers, psychologists, etc. They produce the model's other good, human services (hereafter referred to as 'services'). The service sector does not use capital as an input, just the labor of high and low-tech workers.

Code references not just software but, more generally, rules and instructions for generating output from capital. Because of this, code is both created by and is a substitute for the analytical labor provided by high-tech workers in the good (autmomatable) sector. Code is not to be thought of as accumulating in a quantitative way (anyone who has worked on a large software project can testify that fewer lines of code often mean a better program) but rather in efficiency units. Code accumulation may be a result of programmers typing out code directly, of machine learning systems getting better at a task under the supervision of human trainers[1], or of innovation in designing learning algorithms

[1] Astro Teller, Google's 'Director of Moonshots', discusses in Madrigal (2014) the importance of this work to Google's current projects:

> *Many of Google's famously computation driven projects—like the creation of Google Maps—employed literally thousands of people to supervise and correct automatic systems. It is one of Google's open secrets that they deploy human*

themselves. In the United States, more than 5 percent of total wages is paid to those engaged in computer or mathematical occupations[2]; a much larger share of compensation is being paid to those engaged in creating code broadly defined.

Code needs to be maintained, retained, and updated. If the cost of doing so declines via, for example, the invention of the silicon chip, the model delivers a tech boom, which raises the demand for new code. The higher compensation received by high-tech workers to produce this new code engenders more national saving and capital formation, reinforcing the boom. But over time, as the stock of legacy code grows, the demand for new code and, thus for high-tech workers, falls.

The resulting tech bust reflects past humans obsolescing current humans. This process explains the choice of our title, *Robots Are **Us***. The combination of code and capital that produce goods constitutes, in effect, smart machines, aka robots. And these robots contain the stuff of humans – accumulated brain and saving power. Take Junior – 2013's World Computer Chess Champion. Junior can beat every current and, possibly, every future human on the planet. Consequently, his old code has largely put new chess programmers out of business.

Once begun, the boom-bust tech cycle can continue if good producers switch technologies à la Zeira (1998) in response to changes over time in the relative costs of code and capital. But whether or not such Kondratieff waves materialize, tech busts can be tough on high-tech workers. In fact, high-tech workers can start out earning far more than low-tech workers, but end up earning far less.

Furthermore, robots, captured in the model by more code-intensive good production, can leave all future high-tech workers and, potentially, all future low-tech workers worse off. In other words, technological progress can be immiserating. This finding echoes that of Sachs and Kotlikoff (2012). Although our paper includes different features from those in Sachs and Kotlikoff (2012), including two sectors, accumulating code stocks, endogenous technological change, property rights to code, and boom-bust cycle(s), the mechanism by which better technology can undermine the economy is the same. The eventual decline in high-tech worker and, potentially, low-tech worker compensation limits what the young can save and invest. This means less physical capital available for next period's use. It also means that good production can fall over time even though the technological capacity to produce goods expands.

The long run in such cases is no techno-utopia. Yes, code is abundant. But capital is dear. And yes, everyone is fully employed. But no one is earning very much. Consequently, there is too little capacity to buy one of the two things,

intelligence as a catalyst. Instead of programming in that last little bit of reliability, the final 1 or 0.1 or 0.01 percent, they can deploy a bit of cheap human brainpower. And over time, the humans work themselves out of jobs by teaching the machines how to act. "When the human says, 'Here's the right thing to do,' that becomes something we can bake into the system and that will happen slightly less often in the future," Teller said.

[2]This figure is the share of wages paid to workers in Computer or Mathematical Occupations in the May 2013 NAICS Occupational Employment and Wage Estimates.

in addition to current consumption, that today's smart machines (our model's non-human dependent good production process) produce, namely next period's capital stock. In short, when smart machines replace people, they eventually bite the hands of those that finance them.

These findings assume that code is excludable and rival in its use. But we also consider cases in which code is non-excludable, non-rival, or both. Doing so requires additional assumptions but lets us consider the requirement that all code be open source, i.e., non-excludable. Surprisingly, such freeware policies can worsen long-run outcomes.

Our paper proceeds with some economic history – Ned Ludd's quixotic war on machines and the subsequent Luddite movement. As section 2 indicates, Ludd's instinctive fear of technology, ridiculed for over a century, is now the object of a serious economic literature. Section 3 places our model within a broader framework of human competition with robots to indicate what we, for parsimony's sake, exclude. Section 4 presents our model and its solution method. Section 5 illustrates the surprising range of outcomes that even this simple framework can generate. Section 6 considers how the nature of code ownership and rivalry affects outcomes. Section 7 follows Zeira (1998) in letting the choice of production technique respond to relative scarcity of inputs, in our case capital and code. Section 8 considers potential extensions of the model to encompass the broader range of factors and frameworks outlined in Section 3. Section 9 concludes.

2 Background and Literature Review

Concern about the downside to new technology dates at least to Ned Ludd's destruction of two stocking frames in 1779 near Leichester, England. Ludd, a weaver, was whipped for indolence before taking revenge on the machines. Popular myth has Ludd escaping to Sherwood Forest to organize secret raids on industrial machinery, albeit with no Maid Marian.

More than three decades later – in 1812, 150 armed workers – self-named Luddites – marched on a textile mill in Huddersfield, England to smash equipment. The British army promptly killed or executed 19 of their number. Later that year the British Parliament passed The Destruction of Stocking Frames, etc. Act, authorizing death for vandalizing machines. Nonetheless, Luddite rioting continued for several years, eventuating in 70 hangings.

Sixty-five years later, Marx (1867) echoed Ned Ludd's warning about machines replacing humans.

> *Within the capitalist system all methods for raising the social productivity of labour are put into effect at the cost of the individual worker; all means for the development of production undergo a dialectical inversion so that they become means of domination and exploitation*

of the producers; ... they alienate from him the intellectual potentialities of the labour process in the same proportion as science is incorporated in it as an independent power...

Keynes (1933) also discussed technology's potential for job destruction writing in the midst of the Great Depression that

> *We are being afflicted with a new disease of which some readers may not yet have heard the name, but of which they will hear a great deal in the years to come – namely, technological unemployment. This means unemployment due to our discovery of means of economizing the use of labor outrunning the pace at which we can find new uses for labor.*

But Keynes goes on to say that "this is only a temporary phase of maladjustment," predicting a future of leisure and plenty one hundred years hence. His contention that short-term pain permits long-term gain reinforced Schumpeter's 1942 encomium to "creative destruction".

In the fifties and sixties, with employment high and rapid real wage growth, Keynes' and Schumpeter's views held sway. Indeed, those raising concerns about technology were derided as Luddites.

Economic times have changed. Luddism is back in favor. Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011), and Autor and Dorn (2013) trace recent declines in employment and wages of middle skilled workers to outsourcing by smart machines. Margo (2013) points to similar *labor polarization* during the early stages of America's industrial revolution. Goos, Manning, and Salomons (2010) offer additional supporting evidence for Europe. However, Mishel, Shierholz, and Schmitt (2013) argue that 'robots' can't be 'blamed' for post-1970's U.S. job polarization given the observed timing of changes in relative wages and employment. A literature inspired by Nelson and Phelps (1966) hypothesizes that inequality may be driven by skilled workers more easily adapting to technological change, but generally predicts only transitory increases in inequality.

Our model supports some of the empirical findings and complements some of the theoretical frameworks in this literature. Its simple elements produce dynamic changes in labor market conditions, the nature and timing of which are highly sensitive to parameterization. But the model consistently features tech booms possibly followed by tech busts, evidence for which is provided in Gordon (2012) and Brynjolfsson and McAfee (2011).

A second prediction of our model is a decline, over time, in labor's share of national income. U.S. national accounts record a stable percent share of national income going to labor during the 1980's and 1990's. But starting in the 2000's labor's share has dropped significantly. Frey and Osborne (2013) try to quantify prospective human redundancy arguing that over 47 percent of current jobs will likely be automated in the next two decades. They also identify the priesthood, psychotherapy and coaching (parts of our service sector) as among the least subject to automation.

While our paper is about smart machines, it's also about endogenous technological change. Schumpeter is clearly the father of this literature. But other classic contributions include Arrow (1962), Lucas (1988), Romer (1990), Zeira (1998), and Acemoglu (1998). These later two papers endogenize the choice of technology. Zeira shows that countries with relatively high total factor productivity levels will adopt more capital-intensive techniques in producing intermediate inputs, leading to cross-country dispersion in per capita income. But this adoption of new technology benefits workers since the two inputs are perfect complements in production. Acemoglu also views technology as helping workers. In his model technology can be altered to make particular skill groups more productive. Hence, a temporary glut of one type of worker can initiate innovations culminating in higher productivity of such workers. Rourke, et. al. (2013) build on Acemoglu (1998), but they endogenize workers' decisions to become skilled and examine how these decisions influence the development of labor-complementing technology.

This literature's generally rather sanguine view of technology, namely as complimenting human effort, differs from that presented here. Rather than technology permanently assisting humans, it ultimately largely replaces them. Hemous and Olson (2014) depart somewhat by calibrating a model in which capital can substitute for low-skilled labor while complimenting high-skilled labor to explain trends in the labor share of income and inequality.

3 A Modeling Framework for Understanding Economic Impacts of Robots

The first ingredient of any model of robot competition is, of course, one or more production processes that can produce particular goods or services with little or no input from humans. The second ingredient is one or more human-based production processes of specific goods and services that do not admit the easy substitution of non-human for human input. The third ingredient is dynamics, since technological change generally doesn't happen over night and since it takes time for new technologies to fully impact the economy. The fourth ingredient is agents that are differentially susceptible to replacement by robots. The fifth and final ingredient is a description of the manner in which robotic technology evolves. This includes the inclination and ability of humans to produce technology that puts themselves out of work.

The first ingredient permits production of particular goods to become less human dependent as robots become more abundant and capable. This process may involve the termination of particular human-intensive production processes. The second ingredient insures that humans have somewhere to go when they are put out of work or out of good work by robots. Taken together the first two ingredients help us consider a basic question surrounding robotic competition: Will the reduction in the cost of goods produced by more advanced robots compensate workers for the lower wages? The third ingredient – dynamics – is essential for determining how physical capital – economic brawn – is impacted through time by robot competition. After all, the counterpart of investment is saving and saving is done by households, not robots. The fourth ingredient, agents that are differentially outmoded by robots, is key for assessing the impact of robots on inequality. And the fifth ingredient, endogenous development of robots, is the driving force of interest.

Our model has each of these ingredients, but not all varieties of them. We don't, for example, include an alternative goods-production technology strictly utilizing labor and capital. Were we to do so, the economy would discretely switch, at some point, from non-robotic to robotic good production. Nor do we assume that goods production requires any direct human input. Adding this feature would not materially alter the qualitative nature of our findings. Dynamics, the third ingredient, play a central role in our model and admit our central finding that better supply can, over time, mean worse demand. The fourth element – different skill groups – is covered by our inclusion of low-tech as well as high-tech workers. The presence of low-tech workers lets us consider whether technological change can flip the income distribution between people of different skill sets. Finally, our assumption that new software code is purchased provides a realistic means for endogenizing the development of robots.

4 Our Model

Agents consume the product of both sectors, goods and services. Goods, which can be consumed or invested, are produced using capital and code via a CES production function. The combination of capital and code that makes goods can be viewed as a smart machine or robot. Services, which are consumed when produced, are also created via CES production. New code is written by high-tech workers, and the stock of code is the sum of new and existing code. Old code requires maintenance, retention, and updating. This requirement is modeled as a form of depreciation. High and low-tech workers both live and consume for two periods, but work only when young.

Supply

Time t production of goods, Y_t, and services, S_t, follow (1) and (2),

$$Y_t = D_Y[\alpha(K_t)^{\frac{\varepsilon_y-1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y-1}{\varepsilon_y}}]^{\frac{\varepsilon_y}{\varepsilon_y-1}}, \tag{1}$$

$$S_t = D_S[\gamma(H_{S,t})^{\frac{\varepsilon_s-1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s-1}{\varepsilon_s}}]^{\frac{\varepsilon_s}{\varepsilon_s-1}}, \tag{2}$$

where $H_{S,t}$ is the amount of high-tech workers in the service sector, and G_t references low-tech workers. D_S and D_Y are total factor productivity terms, γ and α are CES parameters related to factor intensity, and ε_y and ε_s are CES elasticities. The stock of code A_t grows according to,

$$A_t = \delta A_{t-1} + zH_{A,t}, \tag{3}$$

where the "depreciation" factor is $\delta \in [0, 1)$. Higher δ means that legacy code is useful longer. $H_{A,t}$ is the amount of high-tech labor hired by good firms, and z is the productivity of high-tech workers writing code.

The good sector's demands for code, high-tech workers, and capital satisfy[3]

$$\max_{K_t, A_t} Y_t(A_t, K_t) - m_t A_t - r_t K_t, \tag{4}$$

where the price of a unit of goods is one, m_t is the rental rate for code, and r_t is the interest rate. Factor demands for services reflect,

$$\max_{H_{S,t}, G_t} q_t S_t(H_{S,t}, G_t) - w_t^G G_t - w_t^H H_{S,t}, \tag{5}$$

where q_t is the price of services, w_t^H is a high-tech worker's wage in the service sector, and w_t^G is a low-tech worker's wage.

Households save in the form of capital and code. Capital accumulation obeys

$$K_{t+1} = \phi I_t - p_t \delta A_t, \tag{6}$$

where I_t is the total resources of those born in t, ϕ is the saving propensity of the young, and $p_t \delta A_t$ is the value of code retained from the current period.

Factor prices satisfy

$$w_t^H = q_t D_S [\gamma (H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}} [\gamma (H_{S,t})^{-\frac{1}{\varepsilon_s}}], \tag{7}$$

$$w_t^G = q_t D_S [\gamma (H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}} [(1-\gamma)(G_t)^{-\frac{1}{\varepsilon_s}}], \tag{8}$$

$$r_t = D_Y [\alpha (K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}} [\alpha (K_t)^{-\frac{1}{\varepsilon_y}}], \tag{9}$$

and

$$m_t = D_Y [\alpha (K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}} [(1-\alpha)(A_t)^{-\frac{1}{\varepsilon_y}}]. \tag{10}$$

Households

Whether high-tech or low-tech, households maximize

$$u = (1-\phi)[(1-\kappa) log c_{y,t} + \kappa log s_{y,t}] + \phi[(1-\kappa) log c_{o,t+1} + \kappa log s_{o,t+1}], \tag{11}$$

[3] To understand this production function consider a firm which provides the service of 'making good chess moves'. Better chess playing smart machines are, in part, distinguished by how many game trees they can investigate and the level of sophistication with which they evaluate board positions and determine which sequences of moves to spend more computational time considering. Therefore, our firm can improve the quality of its output (the chess move it chooses) by increasing either of its inputs. It can either increase the quality of its chess program (increasing its efficiency units of code) or devote more computing time to investigating possible moves and counter-moves (rent more capital). While the logic of decreasing marginal returns to an input seems to hold for production of this type, this does not imply any specific structure on overall returns to scale. Here we restrict our attention to constant returns to scale production.

where $c_{y,t}$, $c_{o,t}$, $s_{y,t}$, $s_{o,t}$, are consumption of goods and services by the young and old, respectively.

Households maximize utility subject to,

$$c_{y,t} + q_t s_{y,t} + \frac{c_{o,t+1} + q_{t+1} s_{o,t+1}}{1 + r_{t+1}} = i_{j,t}, \tag{12}$$

where $i_{j,t}$ is total resources of group j. For low-tech workers,

$$i_{G,t} = w_t^G. \tag{13}$$

For high-tech workers laboring in the service sector,

$$i_{(H,S),t} = w_t^H, \tag{14}$$

and for high-tech workers writing code,

$$i_{(H,A),t} = z(m_t + \delta p_t), \tag{15}$$

where zm_t is revenue from renting out newly produced code and $z\delta p_t$ is revenue from the sale of the intellectual property. Note that like any asset price, p_t is a present value. The second component of the compensation of the code-writing high-tech workers reflect their sale of future rights to their newly written code or their retention and use of this code in their own firms.

High-tech workers are mobile between sectors. Assuming, as we do, no specialization, high-tech workers work in both sectors and receive the same total compensation regardless of where they work.

$$w_t^H = z(m_t + \delta p_t). \tag{16}$$

Household demands satisfy,

$$s_{y,t} = \frac{\kappa(1-\phi) i_{j,t}}{q_t}, \tag{17}$$

$$c_{y,t} = (1-\kappa)(1-\phi) i_{j,t}, \tag{18}$$

$$s_{o,t+1} = \frac{1 + r_{t+1}}{q_{t+1}} [\kappa \phi i_{j,t}], \tag{19}$$

and

$$c_{o,t+1} = [1 + r_{t+1}][(1-\kappa)\phi i_{j,t}]. \tag{20}$$

Equilibrium

Equilibrium requires

$$Y_t = C_{y,t} + C_{o,t} + K_{t+1} - K_t, \tag{21}$$

and

$$S_t = S_{y,t} + S_{o,t}, \tag{22}$$

where C_y, C_o, S_y, S_o, are total consumption demand of goods and services by the young and old respectively.

Asset-market clearing entails equal investment returns on capital and code, i.e.,

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s+1-t} m_{s+1}, \tag{23}$$

where $R_{s,t}$ is the compound interest factor between t and s, i.e.,

$$R_{s,t} = \prod_{j=t}^{s} (1 + r_j). \tag{24}$$

Solving the Model

We calculate the economy's perfect foresight transition path following an immediate and permanent increase in the rate of code retention due, for example, to the development of the silicon chip. The solution is via Gauss-Seidel iteration (see Auerbach and Kotlikoff, 1987). First, we calculate the economy's initial and final steady states. This yields initial and final stocks of capital and code. These steady-state values provide, based on linear interpolation, our initial guesses for the time paths of the two input stocks. Next, we calculate associated guesses of the time paths of factor prices as well as the price paths of code and services. Step three uses these price paths and the model's demand, asset arbitrage, and labor market conditions to derive new paths of the supplies of capital and code. The new paths are weighted with the old paths to form the iteration's next guesses of capital and code paths. The convergence of this iteration, which occurs to a high degree of precision, implies market clearing in each period.

5 Simulating Transition Paths

The models' main novelty is the inclusion of the stock of code in the production of goods. When the code retention rate, δ equals zero, good sector production is conventional – based on contemporaneous amounts of capital and labor (code writers). But when δ rises, good production depends not just on capital and current labor, but, implicitly, on dead high-tech workers as well. We study the effects of this technological change by simulating an immediate and permanent increase in δ.

The increase in δ initially raises the compensation of code-writing high-tech workers. This draws more high-tech workers into code-writing, thereby raising high-tech worker compensation in both sectors. In most parameterizations, the concomitant reduction in service output raises the price of services. And, depending on the degree to which high-tech workers compliment low-tech workers in producing services, the wages of low-tech workers will rise or fall.

Things change over time. As more durable code comes on line, the marginal productivity of code falls, making new code writers increasingly redundant. Eventually the demand for code-writing high-tech workers is limited to those needed to cover the depreciation of legacy code, i.e., to retain, maintain, and update legacy code. The remaining high-tech workers find themselves working in the service sector. The upshot is that high-tech workers can end up potentially earning far less than in the initial steady state.

What about low-tech workers?

The price of services peaks and then declines thanks to the return of high-tech workers to the sector. This puts downward pressure on low-tech workers' wages and, depending on the complementarity of the two inputs in producing services, low-tech workers may also see their wages fall. In this case, the boom-bust in high-tech workers' compensation generates a boom-bust in low-tech compensation. In the extreme, if high and low-tech workers are perfect substitutes, their wages move in lock step.

The economy's dynamic reaction to the higher δ depends on the impact on capital formation. The initial rise in earnings of at least the high-tech workers can engender more aggregate saving and investment. The increased capital makes code and, thus, high-tech workers more productive. But if the compensation of high-tech and, potentially, low-tech workers falls, so too will the saving of the young and the economy's supply of capital. Less capital means lower marginal productivity of code and higher interest rates. This puts additional downward pressure on new code rental rates as well as on the price of future rights to the use of code.

We next consider four possible transition paths, labeled Immiserizing Growth, Felicitous Growth, The First Will be Last, and Better Tasting Goods. Each simulation features an immediate and permanent rise in the code-retention rate. But the dynamic impact of this technological breakthrough can be good for some and bad for others depending on the size of the shock and other parameters. After presenting these cases, we examine the sensitivity of long-run outcomes to parameter assumptions more systematically.

Immiserating Growth

Figure 1 shows that a positive tech shock (the code-preservation rate, δ, rises from 0 to .7) can have very negative long-term consequences. The simulation assumes Cobb-Douglas production of goods and linear production of services; i.e., both types of workers are perfect substitutes in producing services ($\varepsilon_S = \infty$).

As the top left panel indicates, national income quickly rises – by 13 percent. But it ultimately declines, ending up 28 percent below its initial steady-state value. Since preferences are logarithmic, expenditures on goods and services change by the same percentage. In the case of services, however, this occurs not only through changes in output levels, but also via changes in relative price.

The relative price of services first rises and then falls dramatically, while service output does the opposite. Good output moves pari passus with national income. Hence, in the long-run, both young and old agents end up consuming 28 percent less goods. And while their consumption of services is 27 percent larger, it's not worth very much at the margin. In fact, its price is 43 percent lower than before the technological breakthrough.

Both types of worker earn the same under this parameterization. Their compensation initially jumps 16 percent and then starts to fall dramatically. In the long run all workers end up earning 44 percent less than was originally the case!

What happens to the welfare of different agents through time? The initial elderly are essentially unaffected by the tech boom. The initial young experience a 14 percent rise in lifetime utility, measured as a compensating differential relative to their initial steady-state utility. But those born in the long run are 17 percent worse off.

The top right chart helps explain why good times presage bad times. The stock of code shoots up and stays high. But the stock of capital immediately starts falling. After six periods there is over 50 percent more code, but 65 percent less capital.

The huge long-run decline in the capital stock and associated rise in its marginal product (the interest rate) has two causes. First, as just stated, wages, which finance the acquisition of capital, are almost cut in half by the implicit competition with dead workers. Second, the advent of a new asset – durable code – crowds out asset accumulation in the form of capital. When δ rises, all workers immediately enjoy an increase in their compensation. This leads to more saving, but not more saving in the form of capital. Instead, their extra saving as well as some of the saving they originally intended to do is used to acquire claims to legacy code. Initially, when the stock of code is small, its price is high. And, later, when the stock of code is large, its price is low – some 56 percent below its initial value. However, the total value of code increases enough to significantly crowd out investment in capital along the entire transition path.

Another way to understand capital's crowding out is to view legacy code, which coders can sell or retain when the code retention rate rises, as a form of future labor income. This higher resource permits more consumption of goods by low-tech workers (and high-tech workers, since they are paid the same) when the shock hits. And this additional good consumption means less goods are saved and invested. But the knock-on effect of having less capital in the economy is lower labor compensation. This reduces the consumption through time of workers, but also their saving.

What happens to labor's share of national income? Initially it rises slightly.

But, in the long run, labor's share falls from 75 to 57 percent. This reflects the higher share of output paid to legacy code. The long-run decline in labor's share of national income arises in all our simulations except those in which preferences shift toward the consumption of goods at the same time as the code retention rate rises.

Felicitous Growth

As figure 2 shows, the tech boom need not auger long-term misery. A higher saving rate is the key. In the immiserating growth case above, we assumed a saving rate, ϕ, of .2. This generated a ratio of consumption when young to consumption when old of 1.5 in the initial steady state and .9 in the long run steady state. Here we assume a saving rate of .95 while holding fixed the model's other parameter values. The result is that good times can be good for good. But the road is rocky. Output ends up permanently higher, but only after an intervening depression. Output of both goods peaks in the period after the shock, with national income rising 52 percent. But in the long-run, it is only 20 percent higher – a major decline from its peak. The long-run expansion in output reflects less capital decumulation. In the prior simulation the capital stock immediately declined. Here the capital stock temporarily increases 14 percent above its initial value.

A less rapid decline in the capital stock and higher service prices boosts the common wage in the short term and leaves it above its initial value in the long run. After peaking 50 percent above its initial value, the wage falls, ending up only 2 percent higher. The stock of code ends up more than twice as high. But the capital stock, notwithstanding the high rate of saving, declines by 35 percent.

The respective increase and decrease in the stocks of code and capital produce a significant rise in the economy's interest rate – 74 percent in the long run. Although the labor compensation of high and low-tech workers ends up very close to where it started, this increase in the interest rate permits those living in the future to consume 20 percent more.

Why does a high enough saving rate keep the δ shock from reducing long-run welfare? The answer is that whatever happens to the stock of code, a higher saving rate entails a higher capital stock and, therefore, higher labor compensation payments to high-tech workers. In the two above examples, we've considered widely varying saving rates. If, instead, we consider an intermediate value of $\phi = .5$, long-run national income still decreases, but by less – only 10 percent compared with its initial steady state value. Figure 7 shows how long-run output varies with ϕ and δ.

The First Will Be Last

If high and low-tech workers are compliments in producing services, their wage and utility paths will diverge. Consider, for example, the model with table 2's parameters shown in figure 3. As is always the case, the initial effect for high-tech worker of the δ shock is positive. Indeed, immediately after the shock hits, high-tech workers make 43 percent more than in the previous period. But low-tech workers, who, in this case, need high-tech workers to be productive, see their wages rise only 10 percent as the share of high-tech workers working in services immediately falls from 50 percent to 38 percent.

However, as code accumulates and capital decumulates, high-tech workers start earning less in code-writing and move in great number back to the service sector. Ultimately, 68 percent of high-tech workers work in the service sector. And their return to that sector drives down their wage compared both its initial value and to the long-run wage of low-tech workers. Indeed, in the final steady state, high-tech workers earn 14 percent less than in the initial steady state. Low-tech workers, in contrast, earn 17 percent more. But, interestingly, in period 3 their wage peaks 41 percent above its original value. This rise and fall in the wages of low-tech workers reflects, in part, the rise and fall in the price of services.

Better Tasting Goods

We next consider how an increase in the preference for goods coincident with an increase in code retention changes the economy's transition path. Our assumption above that the share of each type of good in consumption is fixed is an important one. It is reasonable given that there is no strong evidence about whether technological innovations are shifting consumption demands towards or away from goods that are relatively labor intensive or about their true elasticities of substitution in household consumption.[4] Figure 4 displays the consequences of having κ fall from .5 to .25 at the same time δ rises. Other parameters are those in the 'First Will Be Last' case.

This additional shock has a dramatic impact on the path of national income. When the shock hits national income drops 2.5 percent. In the long run it drops 48 percent, which is far larger than the 17-percent long-run decline in the previous case.

What explains this result? Shouldn't a shift in preferences towards products that have become easier to produce be economically beneficial? As in previous cases, immiseration is caused by capital decumulation. Capital stocks in this case decrease 40 percent in the period after the shock, and 84 percent in the long-run. Capital decumulation is exacerbated by the κ shock in three ways. First, increased immediate consumption demand for goods (i.e., reduced demand for services) increases the share of high-tech workers working as coders. This translates, after one period, into more legacy code and lower labor com-

[4]It is also unclear how to choose reasonable initial values for κ, but this has little impact on the dynamic effect of code accumulation. Immiseration is still possible for high κ.

pensation, the source of saving and capital formation. Second, the increase in immediate good consumption reduces the amount of capital available to invest. Third, the shift in demand toward goods limits the rise in the price of services. This, too, has a negative impact on wages and capital formation. Figure 8 shows the sensitivity of the model to κ shocks of different sizes. Even without a δ shock, a shift in preferences towards goods is bad for long-run outcomes.

The Large Range of Potential Outcomes

As just demonstrated, the model's reaction to the δ shock is highly sensitive to parameter values. We now consider this sensitivity in more detail. Figure 5 jointly displays our previous results. Table 3 shows additional results for several different parameter combinations. The table's baseline simulation (row one) assumes intermediate parameter values. Subsequent rows show the impact of sequentially modifying one parameter. Figure 6 plots the path of national income for each row of the table.

These simulations teach several new things. First, high-tech workers benefit from substitutability in the goods sector. In the perfect substitutability case the productivity of high-tech workers is independent of supplies of code and capital.

Second, as one can show analytically, with both Cobb-Douglas production and preferences, the path of the capital-to-code ratio is independent of the relative supplies of the two typtes of workers. Since the compensation of high-tech workers is pegged to the capital-to-code ratio, reducing the number of high-tech workers, holding fixed the number of low-tech workers, leaves the compensation of high-tech workers unchanged. In contrast, holding fixed the number of high-tech workers and reducing the number of low-tech workers raises the compensation of low-tech workers. In the former case of fewer high-tech workers, the number of coders is fully supplemented by movement of high-tech workers from services into coding. In the latter case of fewer low-tech workers, the number of high-tech workers coding and working in the service sector stays the same along the transition path. Furthermore, high-tech workers earn the same in services because the rise in the price of services exactly offsets their lower marginal productivity resulting from having fewer low-tech workers with whom to work.[5]

Third, a positive δ shock always produces a tech boom with increases in both the price of code and the wage of high-tech workers.[6] In most simulations, the

[5] To get some intuition for these results, consider the case that the δ shock arises in the context of a smaller number of low-tech workers. If the price of services rises by the same percentage that the number of low-tech workers falls, there will be no change in the value of service output, measured in goods. Nor will there be any change in the other component of national income – good output. Since national saving is a fixed share of national income (given the Cobb-Douglas preferences), the path of the capital stock as well as the path of the code stock will not change from what would otherwise have been the case. So high-tech workers, whose numbers are unchanged, earn the same amount in total and per person. In contrast, low-tech workers experience a rise in their wages.

[6] This can be shown analytically.

boom is short lived, auguring a major tech and saving bust. Fourth, in most simulations capital becomes relatively scarce compared to code leading to a rise in interest rates. Finally, the δ shock generally raises labor share in the short run and lowers it in the long run.

Figure 7 presents a contour graph of long-run national income. Its top half considers combinations of shocks to δ and the saving rate ϕ assuming table 1's values of the other parameters. Redder areas denote higher long-run national income relative to the initial steady state. Bluer areas denote the opposite. Long-run national income increases most when δ is large and the saving rate is high. It decreases the most when the δ shock is high and the saving rate is low.

Figure 7's bottom half considers joint shocks to the saving rate and code-writing productivity (z). Higher values of each reinforces their individual positive impacts on long-run national income. As opposed to δ shocks, shocks to code-writing productivity (z) enhance all agents' welfare. The reason is simple – this shock makes living, but not dead high-tech workers more productive.

The top half of figure 8 examines joint shocks to δ and κ – services' preference share. As discussed in the Better Tasting Goods case, δ shocks do more long-run damage when they are accompanied by a rise in the preference for goods. This is not surprising. In the short run, higher good demand elicits more code production, which eventuates in a larger long-run stock of code. Yes, there is a larger long-run demand for coders to maintain and retain the larger code stock. But this permanently larger code stock entails perpetually greater competition of new high-tech workers with dead high-tech workers and means permanently lower incomes to high-tech workers as well as low-tech workers. Stated differently, if human automation is accompanied by increased demand for goods that can be automated, the long-run economic fallout is worse and, potentially, far worse than would arise were non-automatable goods to become relatively more desirable.

The bottom panel in figure 8 considers combinations of the saving rate, ϕ, and the good sector's elasticity of substitution, ε_y. It shows the aforementioned sensitivity of long-run output to the substitutability of code for capital. It also indicates that this sensitivity is greater for low than for high saving rates. Higher substitutability moderates the negative effects of capital's crowding out that occurs with low savings.

6 The Role of Property Rights and Rivalry

To this point we've assumed that code is private and rival. Specifically, we've assumed that when one firm uses code it is unavailable for rent or use by other firms. But unlike capital, code represents stored information that may be non-rival in its use. Non-rivalry does not however necessarily imply non-excludability. Patents, copyrights, trade secrets, and other means can be used to limit code's unlicensed distribution. On the other hand, the government can turn code into a public good by mandating it be open source.

This section explores two new scenarios. The first is that code is non rival and non excludable in its use, i.e., it is a public good. The second is that code is non rival, but excludable. To accommodate these possibilities we modify our model in two ways. We assume that each firm faces a fixed cost of entry. And we assume that each firm is endowed with a limited supply of public code. These assumptions ensure a finite number of firms operating with non-trivial quantities of capital. To compare these two new settings with what came above – the case of private (rival and excludable) code, we rewrite our baseline model with the two new assumptions.

Rival, Excludable (Private) Code

With a fixed public code endowment and fixed entry costs, profit maximization satisfies:

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t} - m_t a_{j,t}, \tag{25}$$

where $\pi_{j,t}$ are profits for firm j at time t, $F(\bullet)$ is the same CES production function as in the baseline model, $k_{j,t}$ is the amount of capital rented by the firm, $a_{j,t}$ is the amount of code rented by the firm, $H_{j,t}$ is the amount of high-tech labor hired by the firm, $\overline{A}$ is the exogenously set amount of free code in the economy, and C is the cost of creating a new firm. This cost must be paid each period. In equilibrium all firms have zero profits.

$$0 = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_t - m_t a_{j,t}. \tag{26}$$

Market clearing conditions are,

$$\sum a_{j,t} = \delta A_{t-1}, \tag{27}$$

$$\sum k_{j,t} = K_t, \tag{28}$$

$$\sum H_{j,t} = H_{A,t}, \tag{29}$$

$$Y = c_{o,t} + c_{y,t} - K_t + K_{t+1} + NC, \tag{30}$$

where N is the number of firms. Since all firms are identical, (26) provides an expression for N, the number of firms.

$$0 = NF(\frac{K_t}{N}, zH_t + \frac{1}{N}\delta A_{t-1} + \overline{A}) - NC - r_t K_t - m_t \delta A_{t-1} \tag{31}$$

Firms enter up to the point that the value of the public code they obtain for free, namely $\overline{A}$, equals their fixed cost of production. Thus,

$$\overline{A} F_{a,t} = C. \tag{32}$$

This fixes the marginal product of code at $\frac{C}{\overline{A}}$ in every period. Intuitively, new firms can acquire a perfect substitute for new code, and, thus, new coders at a fixed cost by setting up shop and gaining access to $\overline{A}$ in free code. Given that good production obeys constant returns to scale, fixing code's marginal product means fixing the ratio of capital to code. This, in turn, fixes the interest rate. Hence, the rental rates of coders and capital are invariant to the increase in δ.

Although the increase in δ doesn't raise the current productivity of coders, it does raise the present value of their labor compensation. The reason is that coders can now sell property rights to the future use of their invention. Hence, unlike our initial model, this variant with fixed costs and a free endowment of code does not admit immiserating growth absent some additional assumptions.[7]

Were the number of firms to remain fixed, the jump in δ would entail more code per firm with no higher capital per firm. This would mean a lower marginal productivity of code, which (32) precludes. It would also mean a negative payoff to setting up a new firm. Hence, the number of firms must shrink in order to raise the level of capital per firm as needed to satisfy (32).

To solve the model an additional step is added to the iteration procedure. Given a guess of prices and stocks in a period, (31) is used to calculate N. This guess of N in each period is included in the next iteration to calculate new prices.[8]

Figure 9 shows transition paths for key variables for this excludable, non-rival model based on Table 4's parameter values. Note that high-tech workers earn 14 percent more in the long run and enjoy commensurately higher utility. Low-tech workers are also better off. There is also a modest increase in the economy's capital stock.

Non-Rival, Non-Excludable (Public) Code

Consider next the case that code, in the period after it is produced, is a pure public good used simultaneously by every firm. This possibility could arise by government edict, the wholesale pirating of code, or reverse engineering.

Profits are now

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t}, \tag{33}$$

as firms no longer need to rent their stock of code ($a_{j,t}$), where

$$a_{j,t} = \delta A_{t-1} \forall j \tag{34}$$

As before, firm entry and exit imply zero profits,

$$0 = NF(\frac{K_t}{N}, zH_t + \delta A_{t-1} + \overline{A}) - NC - r_t K_t. \tag{35}$$

and

$$\delta A_{t-1} + \overline{A} F_{a,t} = C. \tag{36}$$

[7]If the number of firms is fixed due to oligopilization of the industry, equation (32) would not hold, in which case the marginal productivity of code would again decrease as it accumulates.

[8]In what follows, we consider only equilibria in which high-tech workers work in both sectors. If the public endowment is large enough in a period, goods firms will require no new code.

Finally, with investment in code no longer crowding out investment in capital,

$$K_{t+1} = \phi I_t. \tag{37}$$

Figure 10 shows results for this case again with Table 4's parameter values. The initial steady state is the same as in the prior case of excludable rival code. However, the response to the jump in δ are dramatically different. The jump in δ has no immediate impact on the economy because high-tech workers no longer hold copyright to their code.

In the period after the shock, the economy begins to react. The stock of free public code, which now includes both $\overline{A}$ plus all of the economy's legacy code, is larger. This induces more firm entry. Indeed, the number of firms more than doubles. As indicated in equation (36), with more free code available, new entrants can cover the fixed costs of entry with a lower value per unit of free code, i.e., with a lower marginal product of code. The lower marginal product of code and, thus, of coders leads to an exodus of high-tech workers from coding into services. In the long run, the number of high-tech workers hired for their coding skills falls by 30 percent and their wage falls by 25 percent. National income peaks at 5 percent above its initial level in this period. The interest rate rises by 35 percent and the wage of low-tech workers decreases by 10 percent.

The economy's transition is characterized by a series of damped oscillations as periods of relatively high coder hiring is followed by periods of plentiful free code and relatively low coder hiring. Most importantly, the long-run impact of this change is a net immiseration with long-run national income 8 percent below its initial steady state level.

As in the baseline model, the main mechanism for immiseration is the reduction of the high-tech wage leading to lower capital accumulation. A secondary reason is the inefficiency introduced due to high-tech workers no longer being able to internalize the full value of their creation of new code.

Non-Rival, Excludable (Private) Code

A third possibility is that code is excludable, but non-rival in its use, permitting high-tech workers to license all their code to all firms. The equations for the rival, excludable model hold with the following exceptions. First, profits are given by

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + \delta A_{t-1} + \overline{A}) - C - r_t k_{j,t} - m_t \delta A_{t-1} \tag{38}$$

Second, the price of code reflects its use by all firms.

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s+1-t} m_{s+1} N_{s+1}. \tag{39}$$

As shown in figure 11, the δ shock produces a felicitous transition path, indeed far better than the rival excludable case. As in the rival, excludable case, firms entry satisfies equation (32). Hence, the marginal product of new code is fixed. So is the marginal product of capital, i.e., the interest rate.

7 Endogenous Production Technology

So far we've assumed a single means of producing goods. Here we permit good producers to switch between more and less code-intensive production techniques. To keep matters simple we assume the good sector's production function is Cobb-Douglas and that good producers can choose the parameter on A (and thus on K) such that $\alpha \in [\alpha_1, \alpha_2]$. In the initial steady state, $\alpha_1 = \alpha_2$, but when δ is shocked, the range of possible technologies is expanded as well.

This is simulated via an additional step in the iteration process. After a guess of the path of code and capital is made, an $\alpha \in [\alpha_1, \alpha_2]$ is selected in every period to maximize good output. Subsequently, prices are calculated from marginal products and a new guess of the path of inputs is made.

Given the inputs, and the prevailing stocks of code and capital, output is convex in α. Hence, firms will produce using either the lowest or highest value.[9] This results in the economy flipping back and forth repeatedly, although not necessarily every period, from the most to the least capital-intensive technology. Since our solution method relies on the economy reaching a stable steady state, we set α to a fixed value, namely α_2, far enough in the future such that the transition path for the initial several hundred periods is unaffected.

Figure 12 presents results based on table 5's parameter values. Unlike the previous figures, the absolute amounts of capital and code stocks reflect the dependency of the choice of technology on the ratio of the two stocks. In the initial steady state, the code stock consists just of newly produced code and, naturally, is low. The economy is in a capital-intensive steady state. After the δ shock, code begins to accumulate. In the fourth period, sufficient code is accumulated to lead producers to switch technologies toward more code-intensive production. But the switch to code-intensive production raises wages and, thus, workers' saving. Due to our assumed high saving rate ($\phi = .9$), the increase in saving more than offsets the increase in the value of claims to code and the capital stock increases. If the saving rate were lower, capital stocks would not rise, and the economy would remain permanently in a code-intensive equilibrium. In this case, however, the increase in saving is large enough to drive producers to adopt a capital-intensive technology in the next period. This leads to lower wages, which, over time, means a lower capital-code ratio and a subsequent switch back to code-intensive production.

This ongoing cycle has important welfare implications. High-tech workers who are young when the code-intensive technology is used will earn a high wage when young and high interest rates when old. Those unfortunate enough to be young in a period when a high alpha is chosen will earn low wages while young and low interest rates when old.

Because a period in our model corresponds to roughly 30 years, this cycle of technologically driven booms and busts bears a striking resemblance to the 'long-wave' theories of early economists such as Schumpeter and Kondratieff.

[9] $Y''(\alpha) = B\frac{A}{K}^{\alpha}(log(A) - log(K))^2$ must always be positive for K and A greater than zero

While evidence for the existence of such cycles is limited (Mansfield 1983), this model's long-wave cycles reflect a different mechanism from those in Rosenberg and Frischtak (1983).

8 Conclusion

Will smart machines, which are rapidly replacing workers in a wide range of jobs, produce economic misery or prosperity? Our two-period, OLG model admits both outcomes. But it does firmly predict three things - a long-run decline in labor share of income (which appears underway in OECD members), tech-booms followed by tech-busts, and a growing dependency of current output on past software investment.

The obvious policy for producing a win-win from higher code retention is taxing those workers who benefit from this technological breakthrough and saving the proceeds. This will keep the capital stock from falling and provide a fund to pay workers a basic stipend as their wages decline through time. Other policies for managing the rise of smart machines may backfire. For example, restricting labor supply may reduce total labor income. While this may temporarily raise wages, it will also reduce investment and the long-term capital formation on which long-term wages strongly depend. Another example is mandating that all code be open source. This policy removes one mechanism by which capital is crowded out, but it leads firms to free ride on public code rather than hire new coders. This reduces wages, saving, and, in time, the capital stock.

Our simple model illustrates the range of things that smart machines can do for us and to us. Its central message is disturbing. Absent appropriate fiscal policy that redistributes from winners to losers, smart machines can mean long-term misery for all.

References

1. Acemoglu, D. 1998. Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113 No. 4 (November): 1055-1089.

2. Acemoglu, D. & Autor, D. 2011. Skills, tasks and technologies: Implications for employment and earnings. *Handbook of Labor Economics* 4: 1043-1171.

3. Acemoglu, D., Autor, D. H., Dorn, D., Hanson, G. H., & Price, B. 2014. Return of the solow paradox? IT, productivity, and employment in U.S. manufacturing. *American Economic Review American Economic Association* 104 No. 5 (May): 394-399.

4. Arrow, K. A. 1962. The economic implications of learning by doing. *The Review of Economic Studies* 29 No. 3 (June): 155-173

5. Autor, D. H., Levy, F., & Murnane, R. J. 2003. The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics* 118 No.4 (Novemeber): 1279-1333.

6. Autor, D. H., & Dorn, D. 2013. The growth of low-skill service job and the polarization of the US labor market. *American Economic Review* 103 No. 5 (August): 1553-1597.

7. Brynjolfsson, E., & McAfee., A. 2011. *Race Against the Machine.* Lexington Digital Frontier Press.

8. Frey, C. B., & Osborne, M. A. 2013. The future of employment: how susceptible are jobs to computerisation? Oxford University (September).

9. Goos, M., Manning, A., & Salomons, A. 2010. Explaining job polarization in Europe: The roles of technology, globalization and institutions. *Centre for Economic Performance Discussion Papers.* No. 1026. (November).

10. Hémous, D., & Olsen, M. 2013. The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality. Horizontal Innovation and Income Inequality

11. Katz, L. F., & Margo, R. A. 2013. Technical change and the relative demand for skilled labor: The united states in historical perspective. *National Bureau of Economic Research* No. w18752.

12. Kelly, K. *The Three Breakthroughs That Have Finally Unleashed AI on the World.Wired Online Edition*, October 27, 2014. Website: http://www.wired.com/2014/10/future-of-artificial-intelligence/

13. Keynes, J. M. 1933. Economic possibilities for our grandchildren 1930. *Essays in persuasion.* New York: W.W.Norton & Co., 1963, pp. 358-373.

14. Kotlikoff, L. J. & Sachs, J. D. 2012. Smart machines and long-term misery.(Working Paper No. 18629). Retrieved from Nationl Bureau of Economic Research website: http://www.nber.org/papers/w18629

15. Kondratieff, N.D., & Stolper, W. F. 1935 The long waves in economic life. *The Review of Economics and Statistics* 17 No. 6 (November): 105-115.

16. Lucas, R. 1988. On the mechanics of economic development. *Journal of Monetary Economics* 22 (February): 3-42.

17. Madrigal, A. *Inside Google's secret drone delivery program.The Atlantic Online Edition*, August 28, 2014. Website: http://www.theatlantic.com/technology/archive/2014/08/inside-googles-secret-drone-delivery-program/379306/

18. Mansfield, E. 1983. Long waves and technological innovation. *The American Economic Review* 73 No. 2 (May): 141-145.

19. Marx, K. 1867. Capital: A Critique of Political Economy. Penguin Classics Vol 1 (1992).

20. Mishel, L., Schmitt, J., & Shierholz, H. 2013. Don't blame the robots. Assessing the job polarization explanation of growing wage inequality. *CEPR Working Paper* November 19, 2013. National Archives United Kingdom Government.

21. Luddites: The Growth of Political Rights in Britain in the 19th Century. Website of the National Archives. Retrieved January 20, 2015, from http://www.nationalarchives.gov.uk/education/politics/g3/

22. May 2013 OES National Industry-Specific Occupational Employment and Wage Estimates. (n.d.). Retrieved January 20, 2015, from http://www.bls.gov/oes/current/oessrci.htm

23. Nelson, R. & Phelps, E. 1966. Investment in humans, technological diffusion, and economic growth. *The American Economic Review*, 56(1/2):pp. 69–75

24. Romer, P. 1990. Endogenous Technological Change. *Journal of Political Economy* 98, no. 5 (September): 71-102

25. Rosenberg, N., & Frischtak, C. R. 1983. Long waves and economic growth: a critical appraisal. *American Economic Review* 73 No. 2 (May): 146-151.

26. Rourke, K. H., Rahman, A. S., & Taylor, A. M. 2013. Luddites, the industrial revolution, and the demographic transition. *The Journal of Economic Growth* 18 No. 4 (December): 373-409.

27. Schumpeter, J. A. 1939. *Business cycles* (Vol. 1, pp. 161-74). New York: McGraw-Hill.

28. Zeira, J. 1998. Workers, Machines, and Economic Growth. *The Quarterly Journal of Economics* 113 No. 4 (November): 1091-1117.

Table 1
Parameters for Immiserating Growth

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	∞
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code Retention Rate	0 shocked to 0.7
ϕ	Saving Rate	0.2
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Table 1: This table gives parameter values for the first illustration of the effects of a one-time, permanent increase in the depreciation rate, δ, from zero to .7. We take the intermediate value of .5 for κ, α, and γ. The productivity terms z, D_Y, and D_S, are set to one. σ takes its CD value of zero, ρ, the CES substitution parameter, takes on the perfect-substitute value of 1. ϕ is the saving rate. In this and all subsequent simulations invoking an elasticity of 1 (except for the endogenous technology extension) the true elasticity is actually 1.0001

Figure 1
Immiserating Growth

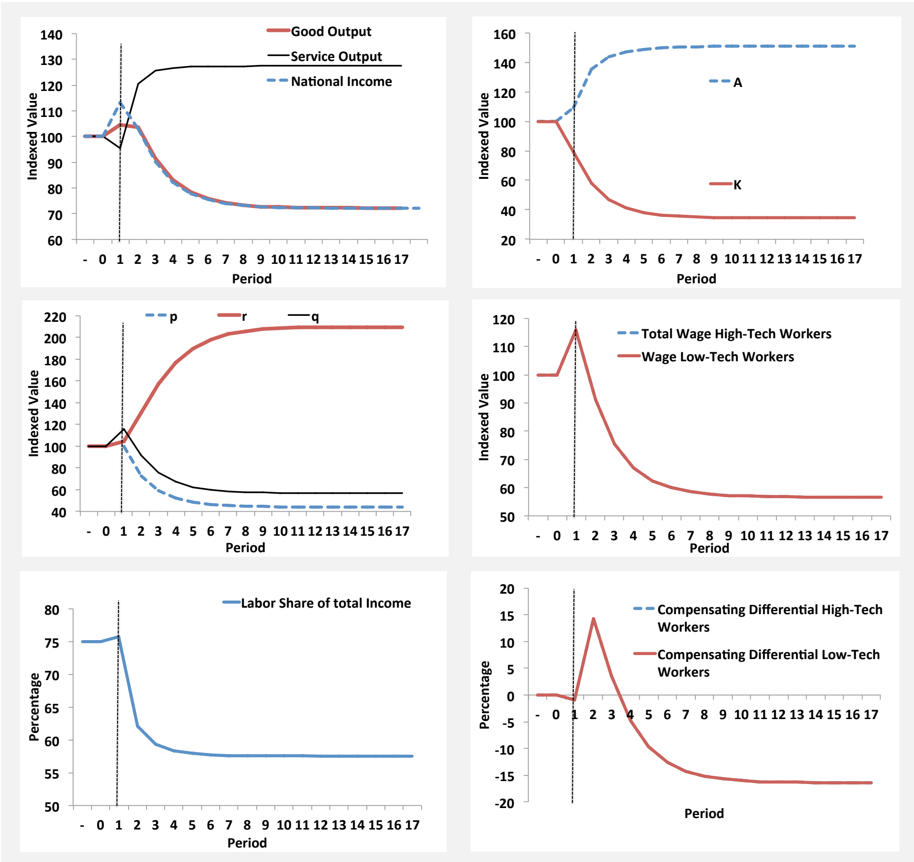


Figure 1: Transition paths based on table 1. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices are $r = 1.737$, $q = .349$, and $p = .043$.

Figure 2
Felicitous Growth
(higher saving rate, $\phi = .95$)

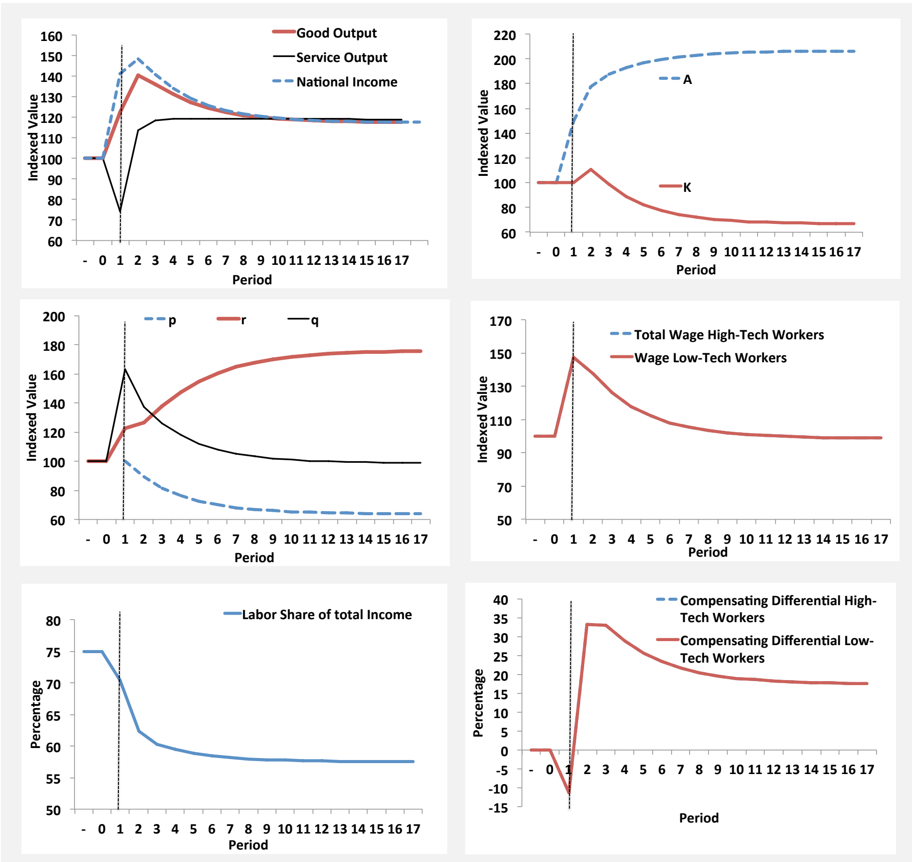


Figure 2: Transition paths based on table 1, with the exception of a higher saving rate ($\phi = .95$). "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices are $r = .454$, $q = 2.204$, and $p = .631$.

Figure 3
The First Will Be Last

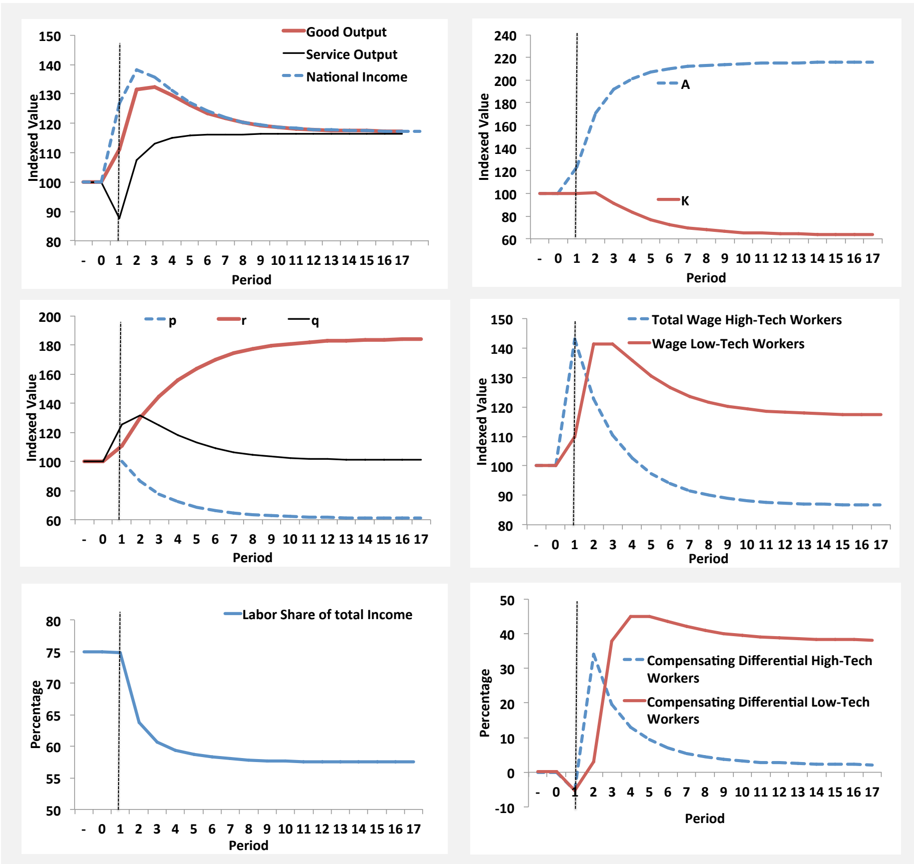


Figure 3: Transition paths based on table 2. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices are $r = .529$, $q = 1.317$, and $p = .398$.

Figure 4
Better Tasting Goods

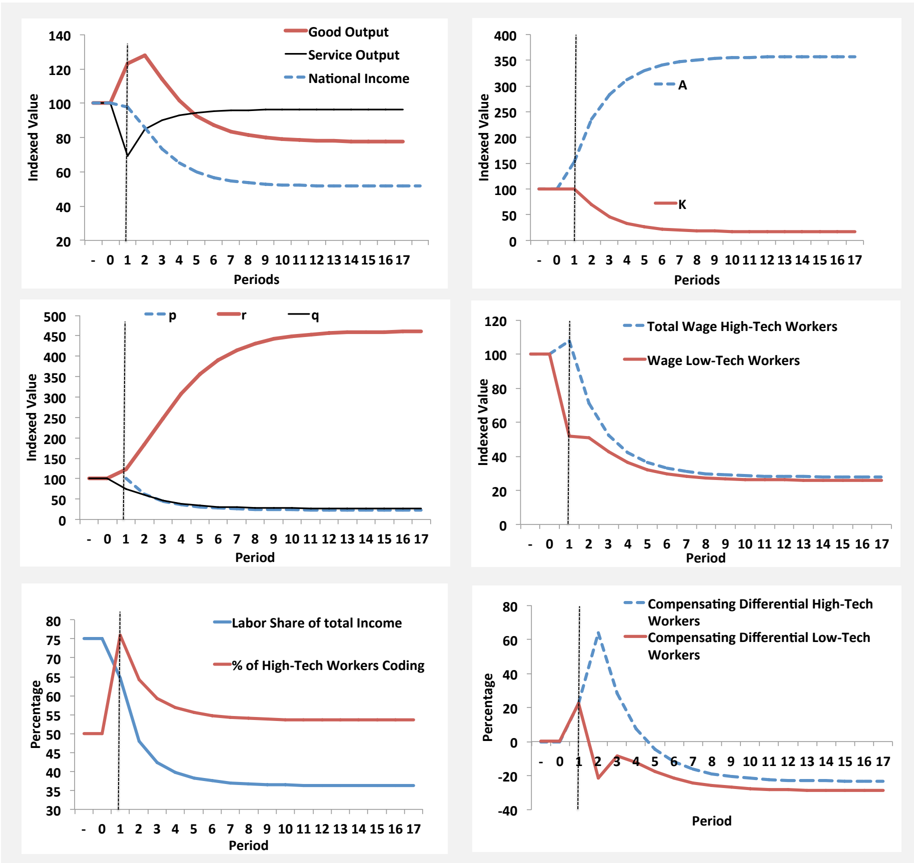


Figure 4: Transtion paths based on table 2, except in addition to the δ shock, κ is simultaneously shocked from .5 to .25. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = .587$, $q = .784$, and $p = .200$.

Table 2
Parameters for The First Will Be Last

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	1
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code-Retention Rate	0 shocked to 0.7
ϕ	Saving Rate	0.7
H	High-Tech Worker Quantity	2
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Table 3: Sensitivity Analysis

Baseline

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	**0.0**	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	121	117.5	106.4	100.0	112.6	100.0	116.3	106.1	54.4	124.4	66.8	56.3	20.3	-47.4
2	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	118	116.0	109.1	96.6	143.0	89.6	116.2	121.7	61.9	109.2	65.6	43.4	8.9	-38.3
3	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	114	113.2	109.6	88.3	152.4	83.6	110.6	131.4	60.3	101.3	65.6	40.5	4.5	-37.7
4	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	112	111.0	109.7	82.1	156.0	80.0	106.0	137.8	58.1	96.7	64.8	39.9	2.3	-38.5
10	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	107.0	109.6	71.9	159.8	74.2	98.0	149.1	53.7	89.5	62.7	40.0	-1.0	-40.5
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	106.7	109.5	71.1	160.0	73.8	97.4	150.0	53.4	88.9	62.5	40.0	-1.2	-40.7

Service Taste Shock κ Jumps from 0.5 to 0.75

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.5**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.8
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	203	117.2	120.6	100.0	87.1	100.0	178.9	93.4	95.0	168.4	58.4	43.6	20.3	-47.3
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	231	129.2	123.6	140.0	98.2	116.7	230.1	83.7	138.8	190.8	67.2	27.3	8.9	-38.2
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	251	136.7	124.4	173.0	96.4	130.9	264.4	74.7	163.3	214.0	72.0	23.7	4.5	-37.6
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	266	142.0	124.8	200.0	93.4	142.3	290.4	68.4	180.6	233.3	74.6	22.6	2.3	-38.4
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	305	154.1	125.6	275.8	85.2	172.2	358.3	55.6	225.0	285.3	80.2	21.2	-1.0	-40.4
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	313	156.5	125.8	293.4	83.5	178.7	373.3	53.4	234.8	296.7	81.3	20.9	-1.2	-40.6

Good Taste Shock κ Jumps from 0.5 to 0.25

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.5**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	78	117.2	81.6	100.0	149.3	100.0	69.2	122.2	24.6	97.1	70.5	74.7	22.3	-68.9
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	67	102.4	84.1	66.0	208.1	67.6	54.4	177.6	22.2	66.7	56.4	66.8	-4.5	-68.2
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	60	91.9	84.8	44.9	233.3	52.4	43.7	227.9	18.4	51.9	52.3	64.6	-18.2	-71.0
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	56	85.4	85.0	34.4	244.7	44.8	37.7	266.5	16.0	44.4	49.0	64.1	-25.7	-73.2
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	51	76.7	84.9	23.1	255.6	36.1	30.2	332.7	12.8	35.6	44.0	64.0	-34.9	-76.5
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	51	76.3	84.9	22.7	255.9	35.7	30.0	335.6	12.7	35.3	43.8	64.0	-35.2	-76.6

Low Saving Rate ϕ = 0.1

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	0.0	**0.1**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	49.9	100.0	75.1	50.1	0.0	-50.0
1	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	100	100.8	110.5	100.0	102.3	100.0	105.6	101.2	52.2	106.8	77.5	51.2	6.8	-47.9
2	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	92	93.2	113.1	78.8	129.1	78.5	93.3	128.1	51.5	84.4	70.0	39.0	-9.3	-44.6
3	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	88	88.1	113.8	63.7	136.6	68.6	83.4	146.6	47.2	73.8	67.4	36.1	-15.9	-46.2
4	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	85	85.0	113.9	55.8	138.9	63.6	77.9	158.0	44.2	68.5	65.3	35.4	-19.0	-47.6
10	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	82	81.6	114.0	47.8	140.3	57.9	71.6	172.3	40.8	62.8	62.6	35.1	-22.3	-49.6
Steady State	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	81	81.4	114.0	46.9	140.4	57.9	71.5	172.6	40.7	62.6	62.5	35.1	-22.4	-49.6

High Saving Rate ϕ = 0.9

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	0.0	**0.9**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	131	126.9	103.2	100.0	121.5	100.0	117.7	110.2	52.2	132.9	60.9	60.8	25.0	-50.9
2	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	132	129.4	106.3	104.4	154.4	95.0	126.1	121.6	65.1	122.3	62.3	46.8	16.4	-38.0
3	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	130	128.9	107.0	102.0	164.1	92.1	125.2	126.8	66.6	117.7	63.7	43.5	13.7	-35.7
4	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	129	128.0	107.2	99.2	167.5	90.3	123.2	129.9	66.0	115.1	63.6	42.7	12.4	-35.5
5	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	128	127.3	107.1	97.1	168.9	89.1	121.5	131.9	65.1	113.4	63.4	42.6	11.7	-35.8
10	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	126	125.7	107.0	92.7	170.9	86.8	118.0	135.8	63.1	110.3	62.7	42.8	10.5	-36.7
Steady State	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	125	125.4	106.9	91.8	171.3	86.3	117.2	136.5	62.7	109.6	62.5	42.8	10.3	-36.9

In all examples, the δ is shocked from 0 in the initial steady state to .5. Simulations subsequent to the baseline have one (highlighted) parameter changed. Endogenous variables are indexed. Service output is in market value terms.

Table 3 Continued

Perfect Substitutability in Service Output $\varepsilon_s = \infty$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	∞	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	100.0	100.0	75.0	66.7	0.0	0.0
1	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	120	116.1	111.0	100.0	119.5	100.0	122.9	109.3	122.8	122.8	69.1	79.7	17.6	17.6
2	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	113	111.3	112.9	97.9	137.7	89.0	112.2	118.6	112.2	112.2	68.0	52.0	14.1	14.1
3	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	107.3	112.9	86.6	143.0	82.2	103.6	128.5	103.6	103.6	66.7	49.5	10.4	10.4
4	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	105	104.7	112.8	79.0	145.7	78.0	98.1	135.8	98.1	98.1	65.3	49.5	7.8	7.8
10	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	100	100.4	112.5	67.5	149.8	71.4	89.6	149.0	89.6	89.6	62.7	50.0	3.7	3.7
Steady State	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	100	100.1	112.5	66.7	150.1	71.0	89.0	150.0	88.9	88.9	62.5	50.0	3.4	3.4

Strong Complementarity in Service Output $\varepsilon_s = 0.5$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	**0.5**	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	31.5	100.0	75.0	43.9	0.0	-68.4
1	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	121	117.7	106.0	100.0	112.3	100.0	116.0	106.0	32.1	124.4	66.7	49.2	20.4	-68.9
2	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	118	116.7	108.6	96.5	143.6	89.5	116.7	122.0	41.4	108.9	65.3	38.4	8.3	-58.8
3	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	115	114.1	109.1	88.5	153.8	83.6	111.5	131.8	41.4	101.0	65.4	35.9	3.8	-57.4
4	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	113	112.0	109.2	82.6	157.6	80.1	107.2	138.1	40.3	96.5	64.6	35.4	1.5	-57.6
10	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	108.1	109.1	72.6	161.4	74.5	99.4	149.1	37.3	89.4	62.7	35.4	-1.7	-59.0
Steady State	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	107.8	109.1	71.9	161.6	74.1	98.8	150.0	37.1	88.9	62.5	35.4	-1.9	-59.1

Strong Complementarity in Good Output $\varepsilon_y = 0.8$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	**0.8**	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	52.2	100.0	73.0	47.9	0.0	-47.8
1	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	116	113.1	108.2	100.0	111.3	100.0	113.4	106.3	56.0	119.8	66.7	53.3	18.4	-44.6
2	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	111	109.1	111.1	96.8	137.1	87.4	110.5	121.5	62.4	102.1	65.2	39.0	5.3	-35.7
3	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	105	104.4	111.7	87.9	143.1	79.7	102.9	131.1	59.6	92.7	65.0	35.7	0.0	-35.7
4	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	101	100.7	111.8	80.8	144.6	74.6	96.8	137.8	56.4	86.6	63.9	35.0	-3.1	-36.9
10	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	92	92.3	111.9	65.8	145.0	64.1	83.3	153.6	48.6	74.5	60.8	34.7	-9.3	-40.7
Steady State	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	91	90.8	111.9	63.5	145.0	62.4	81.1	156.5	47.3	72.5	60.2	34.7	-10.3	-41.4

Perfect Substitutability in Good Output $\varepsilon_y = \infty$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	∞	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	45.5	100.0	80.0	54.6	0.0	-54.5
1	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	146	136.0	97.5	100.0	119.0	100.0	131.8	100.0	52.6	150.0	64.1	65.0	27.4	-55.3
2	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	149	144.1	99.5	94.6	163.6	100.0	146.3	100.0	64.9	150.0	66.7	56.8	23.5	-46.6
3	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	150	147.3	99.9	86.4	182.7	100.0	149.2	100.0	67.5	150.0	69.0	55.0	22.7	-44.7
4	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	150	148.7	100.0	81.0	191.5	100.0	149.8	100.0	68.1	150.0	69.6	54.7	22.6	-44.4
10	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	150	150.0	100.0	75.1	199.9	100.0	150.0	100.0	68.2	150.0	70.0	54.6	22.5	-44.2
Steady State	1.0	∞	0.5	0.5	1.0	1.0	0.5	0.5	1.0	150	150.0	100.0	75.0	200.0	100.0	150.0	100.0	68.2	150.0	70.0	54.6	22.5	-44.2

Code Writing Productivity Shock z Jumps from 1 to 1.5

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Good Output	Service Output	K	A	p	q	r	Wage High-Tech Workers	Wage Low-Tech Workers	Labor Share	% High-Tech Workers Coding	Compensating Differential High-Tech Workers	Compensating Differential Low-Tech Workers
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	0.5	0.5	**1.0**	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	160	157.8	105.3	100.0	176.2	100.0	136.0	132.7	61.8	149.7	59.5	58.8	35.2	-44.2
2	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	163	162.3	108.6	112.3	221.7	96.3	149.7	140.5	78.8	142.1	61.2	44.6	25.9	-30.1
3	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	163	162.4	109.4	112.5	234.0	94.3	150.5	144.2	81.7	138.6	62.9	41.1	23.3	-27.3
4	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	162	161.8	109.5	110.9	237.6	93.0	149.5	146.4	81.7	136.7	63.0	40.2	22.3	-26.9
10	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	160	160.2	109.5	107.0	239.9	91.1	146.4	149.7	80.2	133.6	62.6	40.0	21.0	-27.3
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	160	160.1	109.5	106.7	240.1	90.9	146.2	150.0	80.1	133.4	62.5	40.0	20.9	-27.4

In all examples δ is shocked from 0 in the initial steady state to .5. Simulations subsequent to the baseline have one (highlighted) parameter changed. Endogenous variables are indexed. Service output is in market value terms.

Figure 5
Comparing Four Case Studies

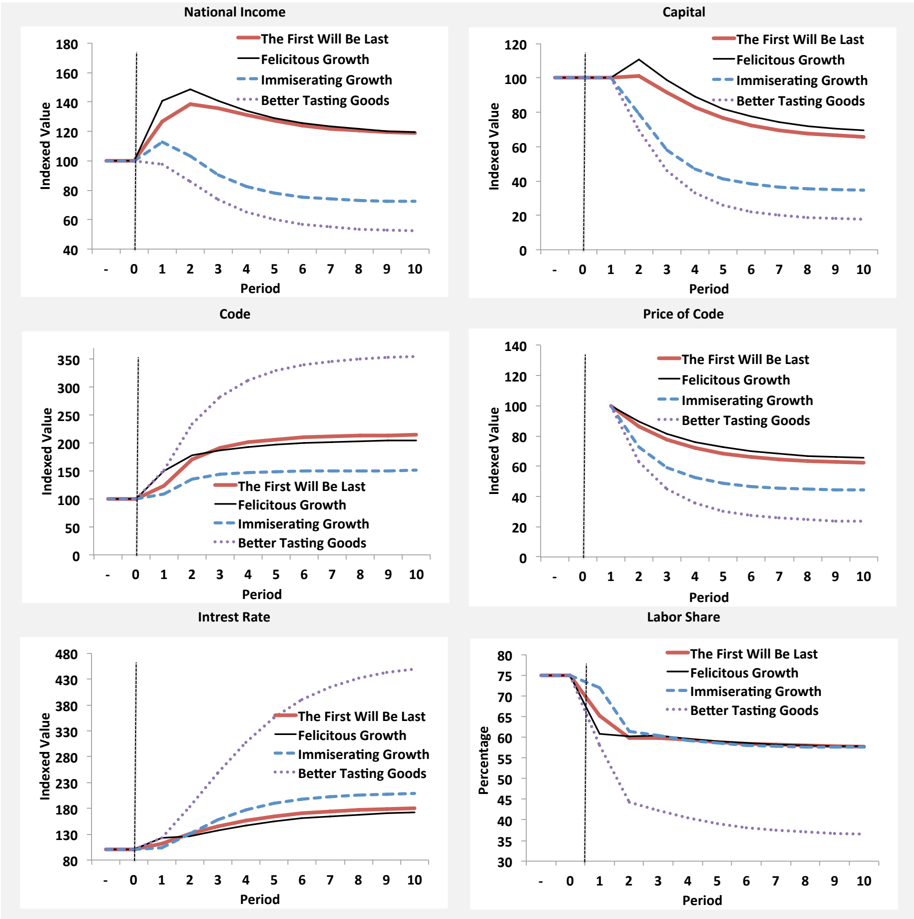


Figure 5: Transition paths from the first 4 cases presented (immiserating growth, etc.) superimposed.

Figure 6
Comparing National Incomes In Sensitivity Analysis

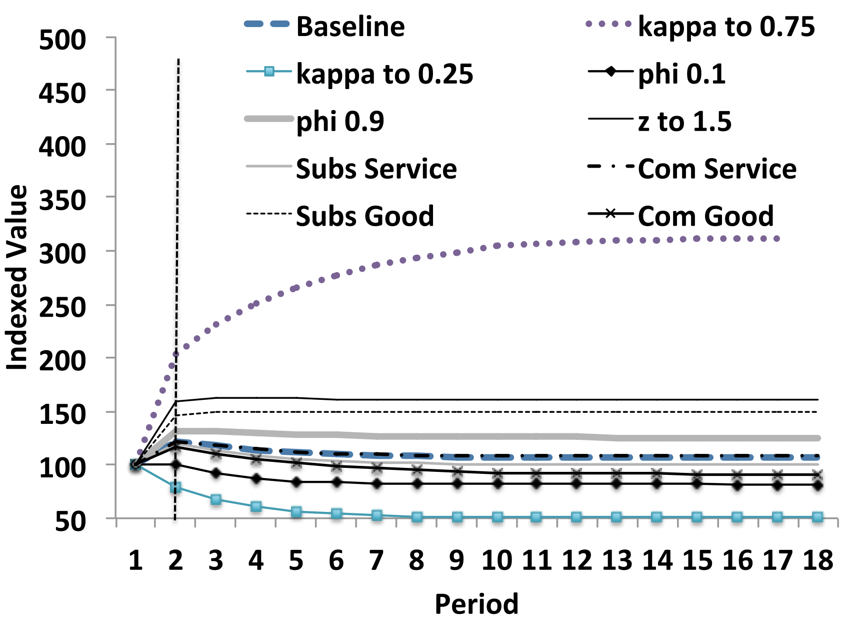


Figure 6: Illustration of the 10 sensitivity analysis cases superimposed. 'Subs' refer to cases in which the production technology of a sector is more substitutable. 'Com' refer to cases in which the production technology is more complimentary.

Table 4
Parameters for Institutional Simulations

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	1
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code Retention Rate	0 shocked to 0.25
ϕ	Saving Rate	0.5
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Good Sector	1
D_s	TFP in Service Sector	1
C	Firm Setup cost	.055
$\overline{A}$	Exogenous Free Code	.25

Table 4: This table gives parameter values for illustrations of the effects of a one-time, permanent increase in the depreciation rate, δ, from zero to .25 given different institutional settings.

Figure 7
Long-Run National Income Impact of Alternative Saving and Code-Retention and Productivity Shocks

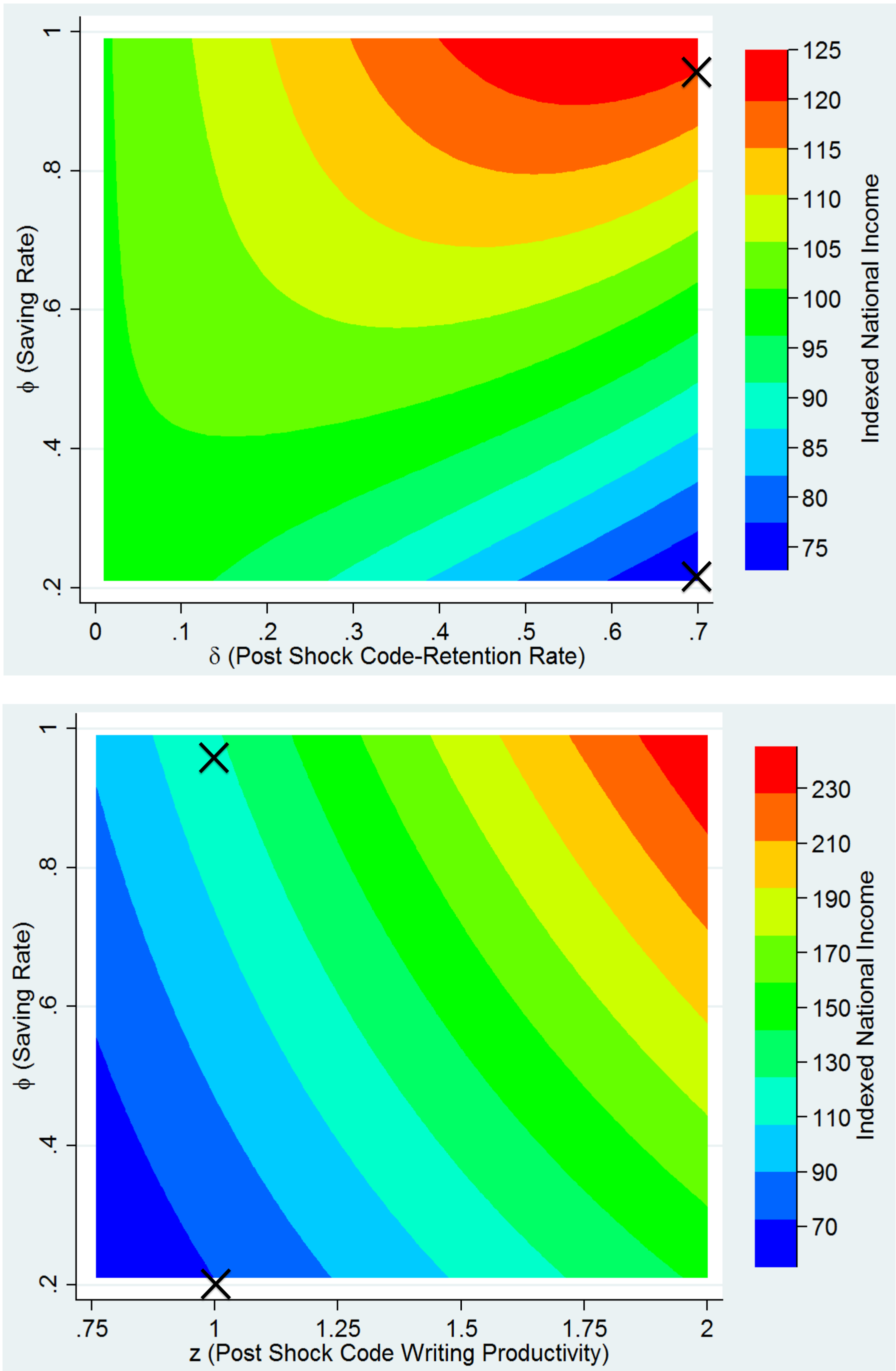
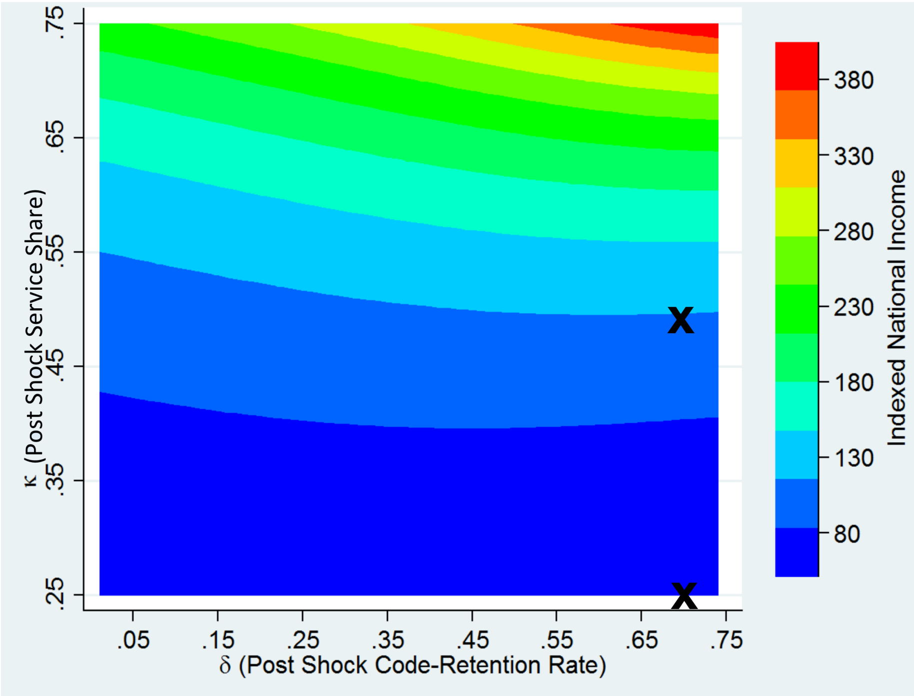


Figure 7: Long-run national income (NI) indexed by pre-shock steady state NI. Parameters not on axes are given in table 1. X's denote parameter combinations with transition paths discussed in the text.

Figure 8
Long-Run National Income Impact of Alternative Saving, Elasticity of Substitution, and Taste Shocks



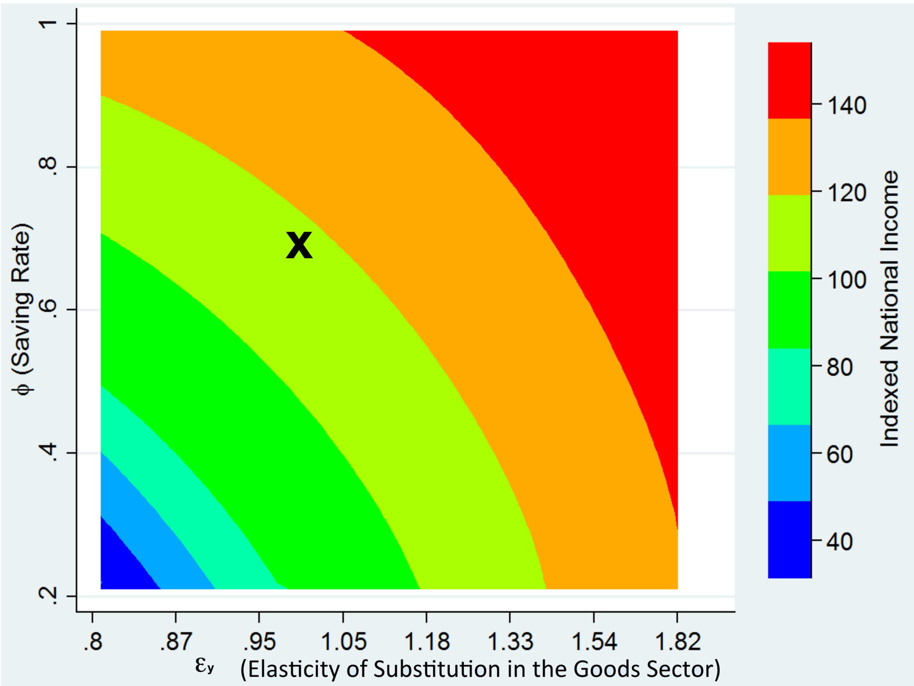


Figure 8: Long-run national income (NI) indexed by pre-shock steady state NI. Parameters not on axes are given in table 2. X's denote parameter combinations with transition paths discussed in the text.

Figure 9
Rival, Excludable (Private) Code

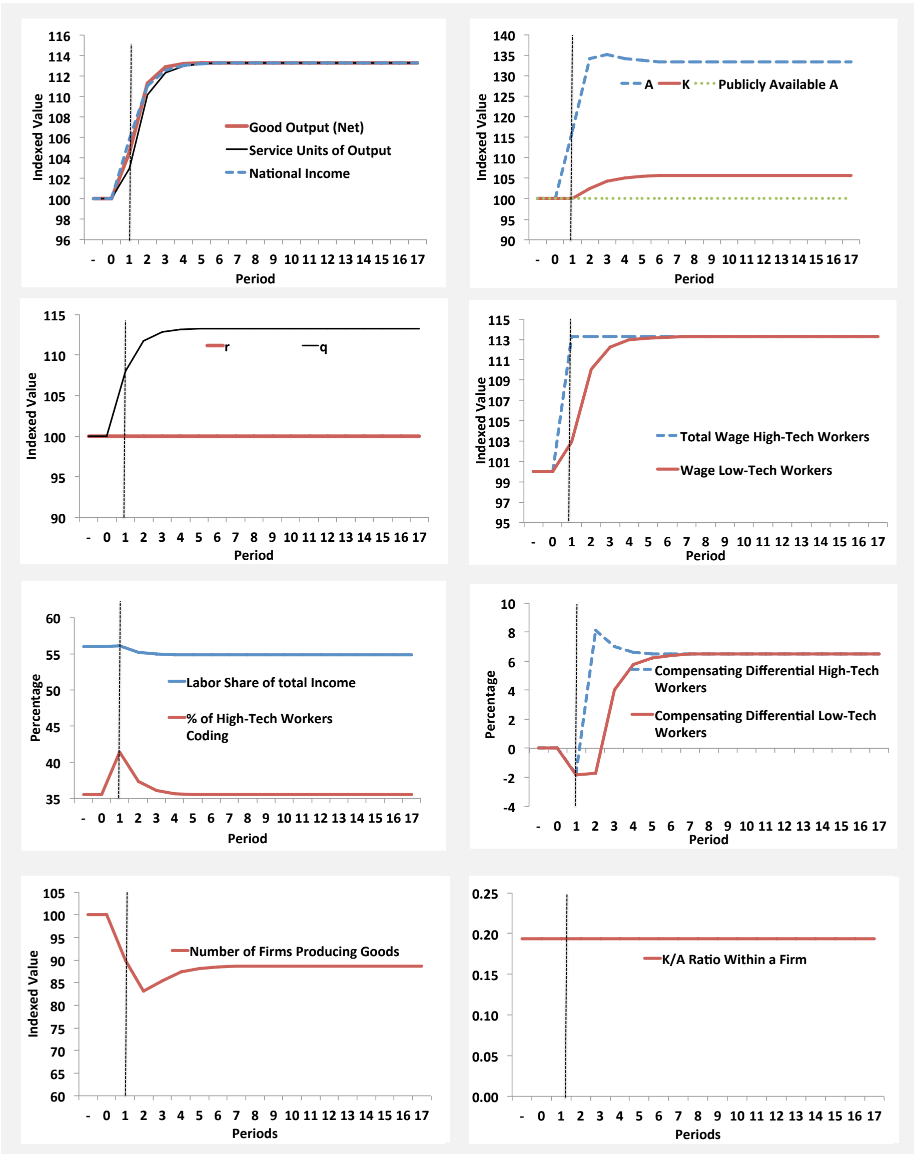


Figure 9: Transition paths based on Table 4. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .382$, and $p = .117$.

Figure 10
Non-Rival, Non-Excludable (Public) Code

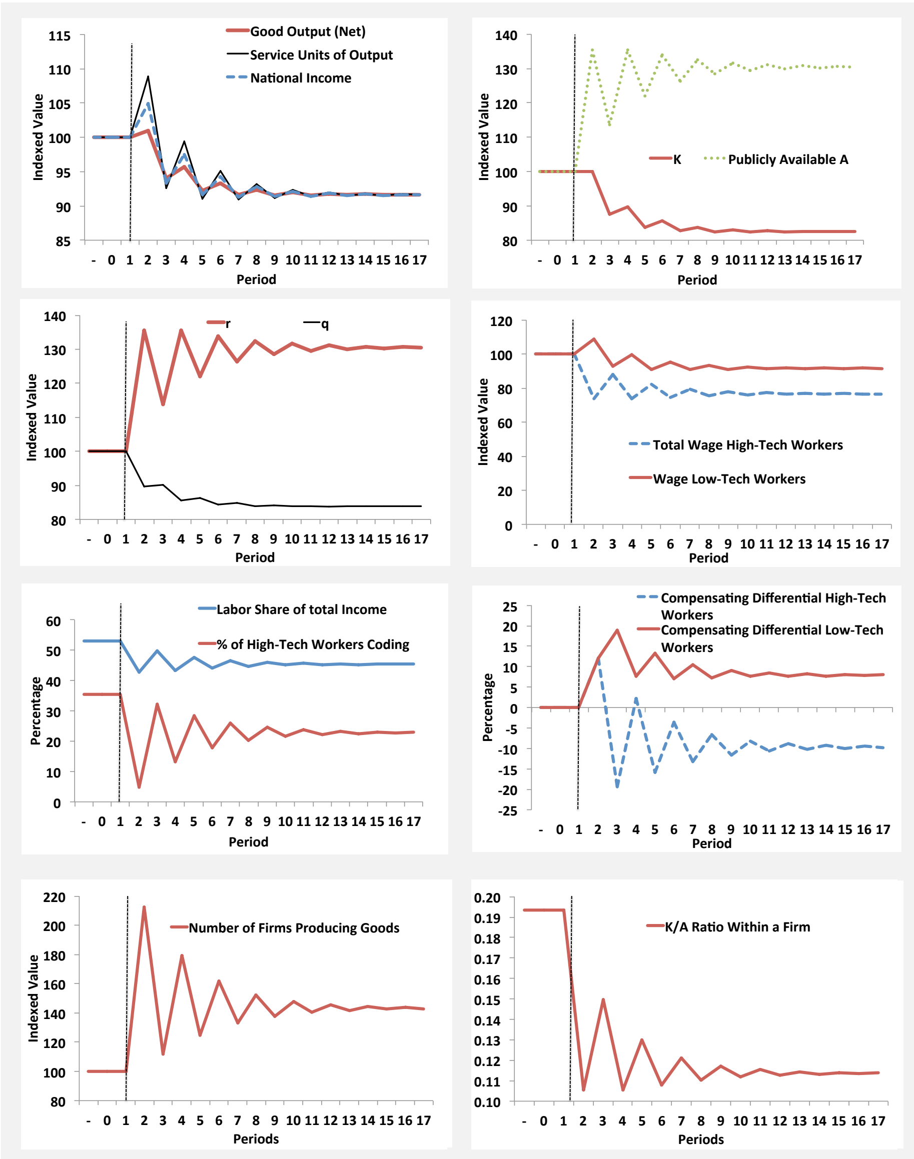


Figure 10: Transition paths based on Table 4. All parameters are identical to Figure 10 except equations are modified as detailed in the text. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .353$, and $p = NA$.

Figure 11
Non-Rival, Excludable (Private) Code

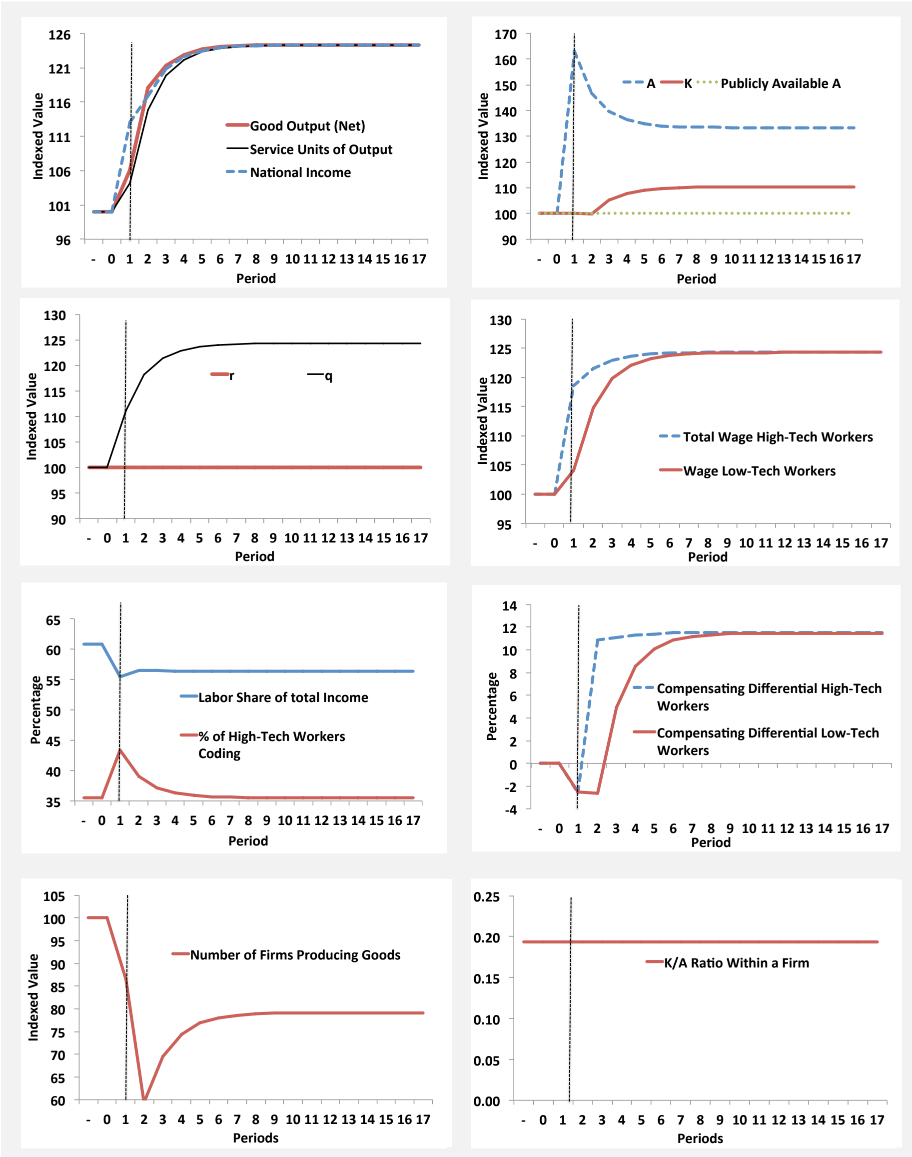


Figure 11: Transition paths based on Table 4. All parameters are identical to Figure 10 except equations are modified as detailed in the text. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .393$, and $p = .164$.

Table 5
Parameters for the Endogenous Technology Extension

Model Parameter	Role	Value
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	[0.3, 0.5]
δ	Code-Retention Rate	0 shocked to 0.6
ϕ	Saving Rate	0.9
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	10
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Figure 12
Endogenous α

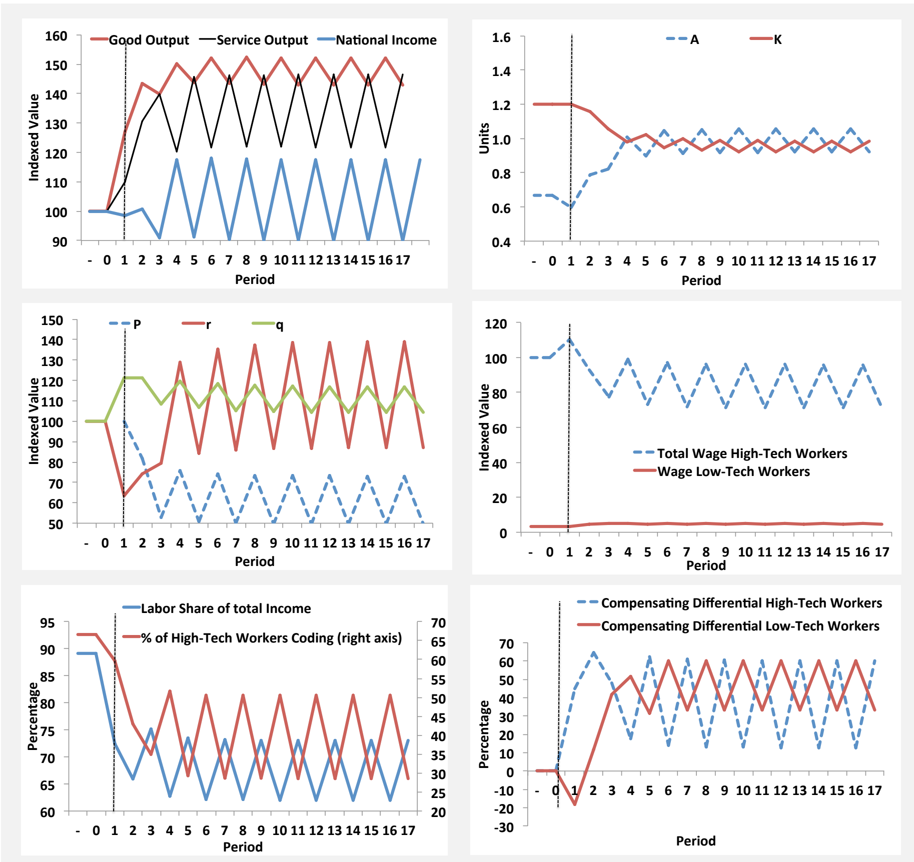


Figure 12: Transition paths based on table 3. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Wage of low-tech workers is indexed to the initial steady state wage of high-tech workers.