Classical monetary economics

1. Quantity theory of money defined
2. The German hyperinflation episode studied by Cagan
3. Lucas’s two illustrations: money and inflation, inflation and interest rates
4. Linear difference equations
5. Rational expectations: Cagan Money Demand Example
1. Quantity theory of money

• Developed along separate links in US (Irving Fisher), UK (Marshall and Keynes) and likely other places in terms of models in 19th century and early 20th century
• All models: private individuals use some particular asset or small set of related assets as media of exchange (one side of most non-barter exchanges) leading to a demand for money
• Many models: government has some extent of control on supply of money
• Examples: gold and silver coins, US dollars, bank deposits
• Model \( M \cdot v = P \cdot t \) (Fisherian form: M is stock of money, v is velocity of money, P is price level and t is volume of real transactions
Key proposition

• Increases in M, holding velocity and real transactions fixed, should proportionately increase P.
• Example: Fisher’s work on the effect of gold discoveries during the 19th century on the price level
• Cagan’s dissertation study is part of Studies in the Quantity Theory of Money, an influential book including contributions by Friedman and his students that aimed at restoring quantity-theoretic reasoning to macroeconomics in the 1950s after it had fallen into disrepute during the Keynesian revolution
Often associated with

- Monetary neutrality: idea that changes in the level of the money stock do not affect real activity, but only the price level, at least in the “long run”
- Superneutrality: idea that changes in money growth do not affect real activity, but only the inflation rate, at least in the “long run”
- Fisherian analysis of interest rate, which is that expected inflation raises the nominal interest rate one-for-one at least in the “long-run”
- Monetarist view of inflation: “inflation is everywhere and always a monetary phenomenon”.

Cagan

• The strategy of Cagan was to look at a series of historical episodes – hyperinflation episodes, which he defines as involving inflation at more than 50% per month -- in which high rates of money growth made it plausible that all other factors would be not too important

• But, in this setting, a simple QT approach with $v$ and $t$ constant was not sufficient. High expected inflation caused substitution away from money holding
2. One of Cagan’s hyperinflations: Germany 1920-1923

- Weimar republic issued a large number of paper marks
- Inability or unwillingness to directly tax the economy in the aftermath of WWI led to use of “inflation tax”.
- Historical period is nicely summarized in Wikipedia entry (although one must be careful with this source more generally): http://en.wikipedia.org/wiki/Inflation_in_the_Weimar_Republic
Depreciation of paper mark against gold

Classic picture of mark value

Table of logarithms

<table>
<thead>
<tr>
<th>Log(10^n)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(10)</td>
<td>2.3</td>
</tr>
<tr>
<td>Log(100)</td>
<td>4.6</td>
</tr>
<tr>
<td>Log(1000)</td>
<td>6.9</td>
</tr>
<tr>
<td>Log(10^n)</td>
<td>n \log(10) = n \times 2.3</td>
</tr>
<tr>
<td>Log(10^{12})</td>
<td>27.6</td>
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</table>
Cagan’s data on money and price level
4.6 means that price level was 100 times greater than in prior month: Cagan defines **hyperinflation** as greater than 50% per month ($\log(1.5)=.41$)
Demand for money

• Cagan presumed real money demand depends negatively on the expected rate of inflation over the holding period (here a month), as a cost of money holding.

• He posited a semi-log form that has become a classic example:

\[ \log(M_t/P_t) = k + \alpha e_t \]
Modeling expected inflation

• Cagan assumed an **adaptive expectations** model of the form

\[ e_t - e_{t-1} = \theta \star (\pi_t - e_{t-1}) \]

\[ \pi_t = \log(P_t / P_{t-1}) = \log(P_t) - \log(P_{t-1}) \]

• This form meant that expected inflation (\( e \)) increases (decreases) whenever actual inflation exceeds (falls below) previously expected inflation
Two views of Cagan’s
German regression

1. Visualize links between two series over June 1920 through end of 1922

2. Estimate OLS specification (we do it with fixed theta (θ), while Cagan estimated k,α for each fixed θ and then chose best fit)
Eyeball regression

June 1920-December 1922

Real balances and expected inflation through end of 1922
OLS regression (fixed $\theta$)

$\alpha = -6.11$
Puzzle #1: End of Episode
Revenue from Money Creation

\[
\left( \frac{M_t - M_{t-1}}{P_t} \right) = \left( \frac{M_t}{P_t} \right) - \left( \frac{P_{t-1}}{P_t} \right) \left( \frac{M_{t-1}}{P_{t-1}} \right)
\]

\[
= m_t - \frac{1}{\pi_t} m_{t-1}
\]
Revenue from money creation

• Inflation tax: if $P_t = 10P_{t-1}$ then 90% of pre-existing cash balances have been taxed away ($\pi=10$, $1/\pi = 1/10$)

• Amount of revenue depends on:
  – Change in nominal money divided by price level
  – New real balances issued less value of prior period real balances
Historical revenue

![Graph showing historical revenue from 1920.5 to 1924. The x-axis represents time from 1920.5 to 1924, and the y-axis represents actual real revenue in units of $10^{-3}. The graph shows fluctuations in revenue over time.](image-url)
Qualifications

• Germany had a mixed money system
  – Pure paper money
  – Bank deposits

• Deposits can be interest bearing

• Depreciation of deposits shifts revenue to banks not to government (but to government if banks required to hold low interest government bonds)
Revenue from alternative constant inflation rates

![Graph showing estimated real revenue vs. inflation (actual=expected)].
Comments

• “Laffer curve” for revenue from inflation
• Can find top analytically

\[
\max_E R = \exp(k + \alpha e)[1 - \exp(-e)]
\]

FOC: \[
\exp(k + \alpha e)\{\alpha[1 - \exp(-e)] + \exp(-e)\} = 0
\]

\[
\exp(-E) = \frac{-\alpha}{1 - \alpha} \approx \frac{6}{7} \Rightarrow e = -\log(6 / 7) = .154
\]

(analogy to monopoly pricing)

• Puzzle #2: inflation tax exceeds optimal level (about 15% per month)
Cagan on two puzzles and changing expectations

• Puzzle 1: Raises possibility of expected currency reform (page 55)

• Puzzle 2: Raises possibility that government’s immediate need for revenue dominated over longer term considerations (page 83)

• Changes in expectation coefficient (for Cagan, this is b, for us it is theta: these are linked by theta=1-exp(-b) which is approximately equal to b for small b. (page 58 and below)
3. Lucas’s illustrations

• Two key ideas related to QTM
  – Higher money growth leads to higher inflation when it is sustained (Friedman: “inflation is always and everywhere a monetary phenomenon”)
  – Higher money growth leads to higher expected inflation when it is sustained and thus to higher nominal interest rate (Fisher and Friedman)

• Not every model delivers these, but some important reference ones do, and Lucas seeks to see if the data are consistent with such models
Cross-country evidence

Sample Averages from Sixteen Latin American Countries, 1950-69

Annual Rate of MI Growth

Annual Rate of CPI Inflation
Short-term associations

Figure 2

Figure 3
Lucas’s low frequency components

Figure 10
Annual Rate of M1 Growth
Smoothed Data for 2nd Quarters, 1955-75

Figure 11
Annual Rate of Treasury Bill Rate
Smoothed Data for 2nd Quarters, 1955-75
Lucas’s procedure for smoothing data

\[ s_t^x = \alpha \sum_{i=-\infty}^{\infty} \beta^{|i|} x_{t+i} \]

with \( \alpha = \left[ 1 + 2 \frac{\beta}{1 - \beta} \right]^{-1} = \frac{1 - \beta}{1 + \beta} \)

so that \( 1 = \alpha \sum_{i=-\infty}^{\infty} \beta^{|i|} \)
Why manipulate data using such a procedure?

- Highlighting “low frequency” or “trend” component because theory is partial (e.g., does not include short-run departures from neutrality or changes in real interest rate)
- Other procedures have been developed for this purpose (see laboratory work with Eviews and self-guided work with MATLAB)
Other related approaches

• Elimination of seasonals and other “high frequency” parts of series via smoothing
• Year-over-year changes to highlight recent developments
• Detrending: \( (x - s) \) in Lucas’s setting
• Detrending plus smoothing to focus on “business cycle components”
4. Difference Equations

• A common element of the Cagan and Lucas studies is that they are concerned with dynamic elements.
• In discrete time, this involves analysis of difference equations.
• Systems of linear difference equations are used a lot because they are easy to solve and manipulate.
• These are also used in analysis of rational expectations models, for the same reason.
Example of difference equation

• Cagan’s adaptive expectations model

\[ e_t - e_{t-1} = \theta \cdot [\pi_t - e_{t-1}] \]

• Can be solved \textit{recursively}, given a series on inflation and a value for \( \theta \) as follows

\[ e_0 : \text{ given} \]
\[ e_1 = \theta \pi_1 + (1 - \theta)e_0 \]
\[ e_2 = \theta \pi_2 + (1 - \theta)e_1 \]

and so on
A difference equation solution for E
Uses of difference equations in class

- Forecasting (difference equations with shocks are called autoregressions and if they involve a vector of variables they are called vector autoregressions)
- Solutions of linear rational expectations models
- Linear approximation solutions of nonlinear RE models
- Other approximate solutions of RE models
Coverage

• Not really core material of this class (you should have picked up this up in macro class EC502)
• But we’ll try to help you with aspects of this material, so that you can effectively access course elements and so that you can benefit from lab sessions
• Some self-guided study materials will be provided, which are associated with particular lectures. This lecture’s material is called difeq.pdf
4. Rational expectations

• How should we think about people’s beliefs in reality? within a model?
• Expected inflation seems really central in hyperinflation.
• It might be revealed to us in asset prices but, in a sense, this only pushes the question back one step
• One view is that people work hard to forecast the future, using information that has proved valuable previously and doing so efficiently
• Another view is that some people have general perspectives – they are bulls or bears, optimists or pessimists

• Some suggests that there are general swings in optimism or pessimism relative to “reasonable” beliefs (Hurricane hysteria)

• To assess these views, a necessary benchmark is that people face uncertainties but that they have “rational expectations”

• Most models in this class are constructed under this assumption, but we will also look at some alternatives at times.
RE in the Cagan model

• We will replace $e_t$ with $E\pi_{t+1} | I_t$, where $I_t$ means “information used in forming expected inflation at date t”

• The most basic way of solving a RE model is recursive forward substitution, much like the procedure that we just used for the linear difference equation.

• We are also going to use the fact that the inflation rate is the log difference of the price level
RE solution of Cagan model

\[ m_t - p_t = \alpha^* [E_t(p_{t+1} - p_t)] \]

\[ p_t = \frac{1}{1 - \alpha} m_t + \frac{-\alpha}{1 - \alpha} E_t p_{t+1} \quad \text{just algebra if } p \text{ is known} \]

\[ p_t = (1 - \beta)m_t + \beta E_t p_{t+1} \quad \text{makes further algebra easier} \]

\[ E_t p_{t+1} = E_t \{(1 - \beta)m_{t+1} + \beta E_t p_{t+2}\} \quad \text{rational expectations} \]

\[ E_t p_{t+1} = \{(1 - \beta)E_t m_{t+1} + \beta E_t p_{t+2}\} \quad \text{law of iterated expectations} \]

\[ E_t p_{t+k} = \{(1 - \beta)E_t m_{t+k} + \beta E_t p_{t+k+1}\} \quad \text{repetition (k step ahead)} \]

\[ p_t = (1 - \beta)\{m_t + \beta E_t m_{t+1} + \beta^2 E_t m_{t+2} + ... \} \]

\[ p_t = (1 - \beta)\{\sum_{k=0}^{\infty} \beta^k E_t m_{t+k}\} \]
Discussion

• Price level today depends on future path of money stock.

• Work example with expected shift in level of money at one future date.
  – Use intuition
  – Use algebra

• Discuss other examples: stock prices and earnings, etc.
solution with money “process”

\[ m_t - m = \rho (m_{t-1} - m) + s_t \quad \text{with } \rho < 1 \]

\[ m_{t+1} - m = \rho (m_t - m) + s_{t+1} \]

\[ \Rightarrow E_t (m_{t+1} - m) = \rho (m_t - m) \]

\[ m_{t+k} - m = \rho^k (m_t - m) + s_{t+k} + \rho s_{t+k-1} + \ldots + \rho^{k-1} s_{t+1} \]

\[ \Rightarrow E_t (m_{t+k} - m) = \rho^k (m_t - m) \]

\[ p_t = (1 - \beta) \sum_{k=0}^{\infty} \beta^k [m + \rho^k (m_t - m)] \]

\[ = m + \frac{(1 - \beta)}{1 - \beta \rho} (m_t - m) \]
Economics

• Price level moves one-for-one with permanent level of money \((m)\)

• Price level moves less than one-for-one with other variations in money (since \(\rho<1\), coefficient on \((m_t-m)\) is \(<1\)).

• Why is this? Keynes and Cagan stressed expectations as source of variations in price level.
Implication for expected inflation: it is negative when money is high

\[ e_t = E_t p_{t+1} - p_t \]

\[ = E_t \left[ m + \frac{(1 - \beta)}{1 - \beta \rho} (m_{t+1} - m) \right] - \left[ m + \frac{(1 - \beta)}{1 - \beta \rho} (m_t - m) \right] \]

\[ = \frac{(1 - \beta)}{1 - \beta \rho} \left\{ E_t (m_{t+1} - m) - (m_t - m) \right\} \]

\[ = \frac{(1 - \beta)}{1 - \beta \rho} \left\{ (\rho - 1)(m_t - m) \right\} \]
Chart 1: Logarithm of the UK annual price level, 1661-2003, and UK annual inflation, 1662-2003

Elisabeth Schumpeter index for prices of consumer goods

EC541: Session 1
Link to Lucas

• Suppose that money has two parts: permanent and transitory variations

\[ m_t = \tau_t + x_t \]

\[ \tau_t = \tau_{t-1} + s_{\tau t} \quad [\text{trend or permanent component}] \]

\[ x_t = \rho x_{t-1} + s_{x t} \quad [\text{serially correlated but transitory}] \]

• Then, must smooth out transitory parts to find classical link between money and price level (Lucas does for money growth and inflation, but idea is same) or hold expected inflation fixed