Servicing the Public Debt: The Role of Expectations

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We study models in which debt repudiation — openly or through inflation — is possible. The government maximizes the utility of the representative individual, and we focus on no-precommitment equilibria of the Barro-Gordon type. We show cases in which the existence of government bonds generate multiple perfect-foresight equilibria. However, price indexation and/or interest-rate ceilings are shown to be possible solutions of the equilibrium multiplicity problem.

As one looks around the world one is impressed by the wide variety of languages, cultures, and even perceptions of the same phenomena. However, the topic of the government budget is one of the few where the range of issues and degree of emotions appears to find a common denominator. The government budget lies at the basis of political campaigns and plays a significant role in the evaluation of government policy.

Recently, economic theory has begun to catch up with political reality by not only studying the optimality of fiscal and monetary policy in a context where explicit account is taken of the government's budget constraint—the cradle of political disputes —but it has gone a step further by examining the time consistency of optimal policy, that is, the issue of whether it is optimal to keep promises that were optimal to make in the past (for example, during the electoral campaign). The latter lies at the heart of the "credibility" dilemma faced by any serious politician.

Time inconsistency of optimal policy has received a lot of attention in the literature (see, for example, Finn Kydland and Edward Prescott, 1977; Guillermo Calvo, 1978a, b; Robert Barro and David Gordon, 1983a, b; Robert Lucas, Jr., and Nancy Stokey, 1983; Mats Persson, Torsten Persson, and Lars Svensson, 1987, among others). Most of the attention, however, was directed to (a) the identification of the factors responsible for time inconsistency, (b) mechanisms to ensure time consistency, and (c) the characterization of optimal policy when present planners take into account their inability to make future commitments—which leads to the equilibrium concept first examined in macroeconomics by Edmund Phelps and Robert Pollak, 1968). The latter has become the dominant equilibrium concept in this literature (see Kenneth Rogoff, 1987).

Phelps-Polak equilibria were studied and developed much further in several influential articles (for example, Kydland and Prescott, 1977; Barro and Gordon, 1983a, b); most of the examples, however, exhibit unique equilibrium solutions. Non-uniqueness was mentioned in Phelps (1975) and Calvo (1978b), and is one of the central subjects of recent game-theoretic models with an infinite horizon (see Rogoff, 1987), but, in all fairness, it must be said that non-uniqueness tends to be deemed more a theoretical possibility

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—reflecting, perhaps, the fact that the models are not entirely correct—rather than a characteristic of empirically relevant models.

The central topic of these notes is the discussion of a set of empirically motivated examples where non-uniqueness of Phelps-Pollak equilibria is clearly exhibited. The possible empirical relevance of the non-uniqueness issue becomes apparent when examining the role of interest-bearing government debt. The events of the 1970s and 1980s suggest very strongly that when governments become strapped for funds, they tend to rely more heavily on bonds' issuance. In fact, Barro (1979) suggests that this may even be optimal if governments are faced with a temporary income fall or a temporary increase in government expenditures. However, the consequent taxation postponement is not free from "credibility" problems: Will the additional debt be paid off in full, will the government find it optimal to resort to higher inflation or currency devaluation to diminish the burden of the debt, etc.? When these questions are posed in terms of more precise language they translate into the following: Is debt repudiation—directly, through interest rate taxes (say), or indirectly through inflation—a characteristic of Phelps-Pollak equilibria? Some answers can be found in Stanley Fischer, 1983; Henning Bohn, 1987; Herschel Grossman and John Van Huyck, 1987; and literature on sovereign countries' debt (see the recent survey by Jonathan Eaton, Mark Gersovitz, and Joseph Stiglitz, 1986). In this paper we pursue the discussion enquiring, specifically, whether the mere existence of government obligations may not be responsible for the existence of multiple Phelps-Pollak equilibria. This appears likely to happen because expected (partial) debt repudiation would tend to be reflected in the interest rate on government bonds (increasing it), while the higher the burden of the debt, the higher would be the temptation to repudiate it. Thus, it should be possible to generate an equilibrium with low interest and low repudiation, coexisting with a high-interest, high-repudiation equilibrium.

Our examples give a strong support to the above conjecture: Multiple solutions are possible even when our examples assume a finite horizon, and the existence of government bonds may be the triggering factor for non-uniqueness.

The implications for policy could be staggering; for, our results suggest that postponing taxes (i.e., falling into debt) may generate the seeds of indeterminacy; it may, in other words, generate a situation in which the effects of policy are at the mercy of people's expectations—gone would be the hopes of leading the economy along an "optimal" path.

For the sake of the exposition, we will first discuss these issues in terms of nonmonetary models with one representative individual. In Section I we will examine a two-period model, where the debt is contracted in the first period and repaid in the second; taxes are distorting and, hence, the government has an incentive to renege on the debt. In order to get an equilibrium with positive public debt, we assume that debt repudiation is costly, and that the cost is proportional to the amount being repudiated. In addition, we assume that individuals know the relevant model; hence, since there is no uncertainty, the equilibrium net rate of interest on public bonds (adjusted for debt repudiation) equals the rate of return on capital (the opportunity cost of private funds). The main result of the section is that if an equilibrium with a positive public debt exists, there are in general two equilibrium points: a "good" Pareto-efficient equilibrium in which there is no debt repudiation, and a "bad" Pareto-inefficient equilibrium where debt is partially repudiated.

In Section II we focus on a monetary economy with non-indexed bonds. Obviously, in this context inflation is equivalent to some form of repudiation, but *negative* repudiation has to be allowed for since price deflation cannot be ruled out. In order to cast the example in terms of a standard monetary model, we follow Barro and Gordon (1983a, b) and modify the previous model by assuming that price changes are costly. This is enough to give results that are qualitatively similar to the ones in the nonmonetary model. Typical of a monetary economy, however, the two equilibria are Pareto inefficient. At the (relatively) good equilibrium, inflation and the nominal interest rate on government bonds are lower than at the bad equilibrium. In addition, we show that if instead the nominal return on government bonds were fully indexed to the price level, then the first-best solution with precommitment can be attained, which suggests that, if inflation is the only means of debt repudiation, indexation of the public debt could lead the economy to a Pareto-superior outcome.

In Section III the analysis is extended to an open economy where foreigners hold a positive share of total public debt. The main purpose of this short section is to establish a link with the relatively well-developed theory of sovereign borrowing. We show that multiple equilibria still can occur in this new context, and that the effect on the bond interest rate of increasing foreigners' participation in that market depends on whether the economy settles down to the good or to the bad equilibrium.

Finally, Section IV summarizes some of the central findings, and discusses some extensions and suggestions for further research.

I. Debt Repudiation

There are two periods: period 0 and period 1, and two types of agents: identical competitive consumers (or individuals) and the government. In period 0 the government borrows b per capita units of output with a (gross) interest factor[1] R_b, that is, in period 1 consumers will receive R_b units of output per unit of bond they hold *and which has not being repudiated*. Therefore, if consumers expect that a proportion θ of total bonds will be repudiated, $0 \le \theta \le 1$, the *net* interest factor would be[2]

$$(1-\theta)R_b.$$

We assume consumers can accumulate physical capital with a constant net interest factor equal to $R>1$; hence, in a perfect foresight equilibrium with positive stocks of capital and government bonds, consumers should be indifferent between these two types of assets, and, consequently

$$(1)\qquad (1-\theta)R_b = R.$$

The budget constraint of the government in period 1 is

$$(2)\qquad x=(1-\theta)bR_b+g+\alpha\theta bR_b,$$

where b, x, and g are the per capita stock of bonds, taxes, and government expenditure, respectively, and α stands for the per capita cost per unit of repudiated debt[3] $(0\le\alpha<1)$. Thus, in other words, taxes are required to finance debt repayment, $(1-\theta)bR_b$, government expenditure, g, and the costs of debt repudiation, $\alpha\theta bR_b$. Furthermore, we assume that g is exogenous.[4]

By (2),

$$(3)\qquad \theta bR_b=\frac{bR_b+g-x}{1-\alpha}.$$

In period 1 individuals consume all of their wealth; thus, assuming that government expenditure does not directly affect private consumption in period 1, c, we have

$$(4)\quad c=y-z(x)+kR+(1-\theta)bR_b-x,$$

where y is endowment income, $z(x)$ is a function representing the "deadweight" cost

[1]Let r be an interest rate; we define the corresponding interest factor as $(1+r)$.

[2]Repudiation is a catchall word in this paper that includes anything from open repudiation to a tax on interest.

[3]α could be thought of as transactions costs associated with debt repudiation (legal fees, etc., when repudiation is open). In Section IV other types of costs are discussed. It should be stressed, however, that in this paper we are interested in examining the "mechanics" associated with these types of costs as a prelude to a more substantive analysis where those costs will be subject to a closer scrutiny.

[4]In a more realistic model, the expected net return on capital will also be a function of expected government policy (through expected capital levies, for example). The relationship between the latter and "capital flight" has recently been explored by Eaton (1987).

of taxation,[5] and k is per capita physical capital.[6] We assume

$$z(0)=z'(0)=0, \tag{5a}$$

$$z''(x)>0 \quad \text{for all } x, \tag{5b}$$

$$\lim_{x\to\infty} z'(x)=\infty=-\lim_{x\to-\infty} z'(x). \tag{5c}$$

The last Inada-type condition will simplify the presentation of our results.

Employing (3) in (4), we get

$$c=y-z(x)+kR+bR_b - (bR_b+g-x)/(1-\alpha)-x. \tag{6}$$

Consequently, a "benevolent" government that tries to maximize c in period 1 subject to its budget constraint, (2), and taking R_b as given (recall that, by assumption, R_b is negotiated at time 0), will choose x so as to minimize

$$z(x)-\frac{\alpha}{1-\alpha}x, \tag{7}$$

subject to (2), and $0\le\theta\le 1$. In other words, the problem is to minimize (7) by choosing x such that the associated θ in budget constraint (2) is in the interval $[0,1]$. This condition is equivalent to restricting x in the following manner,

$$g+\alpha bR_b\le x\le g+bR_b. \tag{8}$$

Notice that the left-most expression in (8) corresponds to total debt repudiation, $\theta=1$, in which case taxes are equal to government consumption, g, plus the cost of total repudiation, αbR_b. On the other hand, the right-most expression corresponds to no repudiation, $\theta=0$; thus taxes have to be equal to g plus debt repayment.

For the sake of definiteness, we will assume $g>0$ and $b>0$, unless otherwise stated; thus, by (5), if $R_b>0$ (the only relevant case for the model of this section), the minimization of (7) subject to (8) is attained at some unique $x>0$. Notice that, due to (8), the solution will depend on g, and, most importantly for our discussion here, it will also depend on R_b, the interest factor on bonds (which, as argued above, is a predetermined variable in period 1).

The maximum problem for the government in period 1 is depicted in Figure 1.[7] In the figure we portray the two functions associated with constraint (8). Given R_b, the government is thus constrained to choose x between the two straight lines stemming from g on the vertical axis. If x^* is located as in Figure 1, the government will be able to attain the unconstrained maximum x^*, if R_b lies between $\underline{R}$ and $\overline{R}$ (see Figure 1). On the other hand, if $R_b<\underline{R}$, the maximum will be attained on the upper bound (where there is no repudiation, $\theta=0$), while if $R_b>\overline{R}$, the maximum lie on the lower bound (where repudiation is total, $\theta=1$). Consequently, the set of best responses or "reaction function" for the government in period 1, given R_b, is depicted by the heavy line of broken segments in Figure 1. Notice that when $R_b \le \underline{R}$ the government will be induced to repudiate none of its debt (i.e., set $\theta=0$), whereas if $R\ge\overline{R}$ repudiation will be total (i.e., $\theta=1$). In between $\underline{R}$ and $\overline{R}$, repudiation will be partial, and—by (2), and recalling that in this region $x=x^*$—the repudiation share, θ, will increase with R_b; thus, in other words, *the optimal repudiation share in period 1 is an increasing function of the interest rate contracted in period 0.*

We will be interested in an equilibrium concept in which individuals at time 0 are able to predict the optimal government policy in period 1 (i.e., the optimal government

[5]Alternatively, one could follow Barro (1979) and assume $z(x)$ measures tax collection costs; under this interpretation, $z(x)$ should be included in the RHS of (2).

[6]There is no real need to keep track of the nonnegativity of c, because in this model one can always ensure that by setting endowment income, y, sufficiently large.

[7]I am very grateful to Elhanan Helpman for suggesting Figures 1 and 2, which greatly improved the intuitive appeal of the presentation.

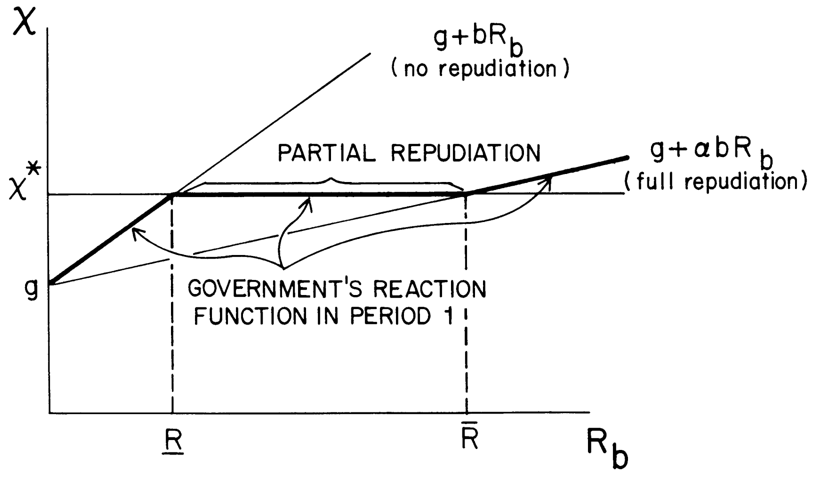

FIGURE 1. DETERMINATION OF GOVERNMENT'S REACTION FUNCTION IN PERIOD 1

decision about x and θ at time 1 as discussed above) with perfect certainty. Under these circumstances (1) holds and, hence, by (2),

$$x = g + (1-\alpha)bR + \alpha bR_b. \tag{9}$$

The latter is a "consistency condition" that must be satisfied by the government's budget constraint at equilibrium, where, by definition, the public can predict exactly the repudiation share, θ, and it is indifferent between holding bonds or capital (i.e., equation (1) is satisfied). Notice, incidentally, that since θ is nonnegative, (1) implies that equation (9) is only relevant over the range where $R_b \geq R$, that is, where the interest rate on public bonds exceeds or equals that of physical capital.

The consistency condition and the government's reaction function are depicted in Figure 2 for the case in which $x^* > g + bR$. Equilibria are found at points of intersection; in the present case these are E^0 and E^1. At E^0 the interest factor on bonds, R_b, is equal to that on capital, R, meaning that the public expects no repudiation; on the other hand, the government's optimal response in period 1 is to set $x = g + bR$, which, as indicated in Figures 1 and 2, lies on the no-repudiation section of its reaction function. Expectations are, thus, fulfilled. Notice that at this equilibrium solution the government would wish to increase x above $g + bR$ and toward x^*, but that is impossible because it would call for setting $\theta < 0$ (negative repudiation), which has been ruled out by assumption.[8]

In the other equilibrium, E^1, repudiation is partial and, thus, the associated interest factor, denoted R_b^1 in Figure 2, is larger than that on capital, R. At this equilibrium, the government is able to attain the unconstrained optimum value of x, x^*.

Our previous discussion covered the case in which $x^* > g + bR$. If, contrariwise, $x^* < g + bR$, the curve depicting the consistency condition would stem from a point like A in Figure 2; it is easy to see that the latter and the government's reaction function will never

[8]This type of constraint will be relaxed in Section II. However, one simple way to extend the present example to allow for $\theta < 0$ would be to assume that negative repudiation is also costly. It is easy to see that this would remove any incentive from the government at time 1 to set $\theta < 0$, because the latter would involve bigger tax distortions and repudiation costs than if $\theta = 0$.

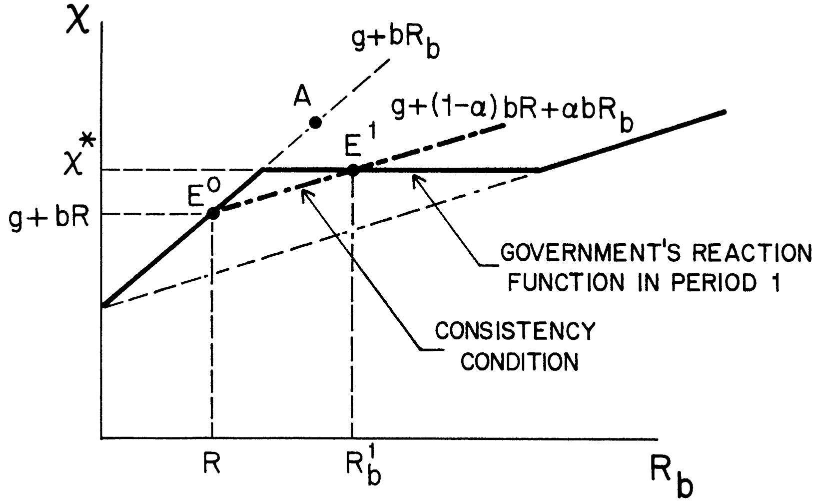

FIGURE 2. DETERMINATION OF EQUILIBRIUM

cross, because the slope of the consistency-condition curve is equal to that of the right-most section of the reaction function. Consequently, there is no equilibrium with a positive stock of bonds.

Finally, in the borderline case where $x^* = g + bR$, equilibrium is unique and $R_b^1 = R = \underline{R}$. The following proposition summarizes our findings:

PROPOSITION 1: *If* $x^* > g + bR$, *then there exist two equilibrium solutions; one of the solutions exhibits no repudiation, while in the other the public debt is partially repudiated. On the other hand, if* $x^* < g + bR$, *there is no equilibrium, and in the borderline case that* $x^* = g + bR$, *equilibrium is unique and there is no repudiation.*

Our model has several interesting implications. In the first place, notice that, by (7), x^* is an increasing function of the cost of repudiation, α; hence, given the initial stock of debt, there is some critical value for the cost of repudiation below which no equilibrium with a positive amount of bonds would exist. The possibility of nonexistence of equilibrium is intuitively clear for the case $\alpha = 0$; for, a benevolent government would repudiate 100 percent of the debt given that there are no direct repudiation costs, and that setting $\theta = 1$ eliminates the deadweight cost of taxes required to service the debt. But, of course, this cannot be an equilibrium with a positive stock of bonds. In addition to confirming this basic intuition, our example shows that the nonexistence problem could arise with a positive α, and that the critical value for the latter depends positively on the value of the outstanding debt.

The most important implication of the analysis, however, is that problems do not necessarily go away when repudiation costs are set above the critical level; for, in such a case the economy will normally exhibit two equilibria: a nonrepudiation equilibrium in which taxes equal $g + bR$ (recall Figure 2), and another equilibrium in which the debt is partially repudiated and taxes are set at $x^* > g + bR$. Notice that, by (1) and (4), in equilibrium we have

$$(10) \qquad c = y - z(x) + (k + b)R - x.$$

Therefore, these two solutions can be Pareto-ranked, and the nonrepudiation solution is the dominant one. Unfortunately, however, without further restrictions the economy could end up at any one of these equilibria. The interest factor in the "good" equilibrium is always R; however, the value of R_b and θ at the "bad" equilibrium (E^1 in Figure 2) are sensitive to changes in b. Somewhat paradoxically, as it can easily be established employing Figure 2, at the bad equilibrium *the bond interest factor is smaller the larger is the total outstanding government debt*. It should be noted, however, that the consumption cost of being at the bad equilibrium is independent of total debt[9] (because $x = x^*$, and the latter does not depend on b). But, on the other hand, x^* depends positively on α; therefore, our analysis reveals that while repudiation costs are important for ensuring the existence of equilibria with a positive stock of bonds, by (5), (7), and (10), the larger are the repudiation costs, the larger will also be the consumption cost associated with the bad equilibrium. Thus, *although increasing the costs of repudiation enhances the "credibility" of the government in the capital markets, the latter may be bought at the cost of lower consumption if the economy ends up at the bad equilibrium.*

The way the above "game" has been structured is consistent with situations in which the government at time 0 auctions off b, and lets the market free to determine the associated interest factor, R_b. An alternative[10] would be for the government at time 0 to refuse to sell bonds at interest factors which are equal to or exceed R_b^1 (recall Figure 2). Thus, under our assumptions it follows that the only possible equilibrium would be $R_b = R$, the first best. This is an important observation which suggests that free auctioning of the public debt may be dominated by a system in which the government stops selling bonds after the implied interest rates go beyond certain bounds. This observation will take up further relevance when we discuss it in the context of the next section.

[9]This invariance does not hold in the model discussed in Section IV.

[10]This fine point was suggested to me by Assaf Razin.

II. Money and Non-Indexed Debt

The concept of partial repudiation takes a more familiar form in the context of a monetary model, because when variables are expressed in real terms, changes in the rate of inflation imply changes in the real value of assets which are not indexed to the price level. Two important assets of this kind are (high-powered) money and nominal government debt. The literature has paid a great deal of attention to the former (for example, Barro and Gordon, 1983a,b); Lucas and Stokey, 1983; Calvo (1978a,b), but there is growing awareness about the importance of understanding the role of nominal debt (for example, Henning Bohn 1987; Grossman and Van Huyck, 1987; Persson, Persson, and Svensson, 1987). As far as I know, however, none of the available papers addresses the issue of non-uniqueness of equilibria, which is the main focus of the following discussion.

To minimize the use of new notation, we redefine R_b as the nominal interest factor on nominal non-indexed bonds (i.e., $R_b = 1 + i$, where i is the nominal interest rate from periods 0 to 1). Thus, if we let P_t stand for the price level in period $t = 0,1$, then the *real* interest factor on period-0 bonds would be

$$R_b P_0 / P_1. \tag{11}$$

We can, therefore, think of the ratio P_0/P_1 as the share of the debt which is not repudiated in period 1; hence, using the notation of previous section, we will write

$$P_0/P_1 = (1-\theta). \tag{12}$$

We define the rate of inflation between periods 0 and 1, π, as follows,

$$\pi = \frac{P_1 - P_0}{P_0}. \tag{13}$$

Thus, by (12) and (13)

$$\theta = \frac{\pi}{1+\pi}. \tag{14}$$

Consequently, each rate of inflation, π, $-1 < \pi$, is associated with a unique rate of repudiation, θ. We are thus entitled to carry out our discussion in terms of θ instead of π. Notice, however, that contrary to the previous section, θ could be negative because it is only constrained to satisfy

$$(15) \qquad -\infty < \theta \leq 1.$$

In line with our previous results, it is easy to convince oneself that if the costs of inflation/repudiation are nil, then a social welfare maximizer will repudiate 100 percent of the debt, that is, he will set $\theta = 1$—or, equivalently, $\pi = \infty$. Therefore, the existence of a well-defined interior equilibrium solution requires, once again, the introduction of (social) repudiation costs. As a matter of fact, one could translate the model of Section II in monetary terms using (14), and interpret those costs as being related to, for example, the Olivera-Tanzi effect. However, I believe it will be more useful to develop the analysis in terms of a more conventional monetary model in which inflation costs directly affect the consumers' utility functions (see Barro and Gordon, 1983a, b), and where explicit account is taken of the inflation tax on high-powered money.

Let the supply of high-powered money in period t, $t = 0, 1$, be denoted by M_t. For the sake of simplicity, we assume that the monetary authorities can directly determine the price level P, by, for example, setting the exchange rate. Furthermore, we assume that the demand for money satisfies[11]

$$(16) \qquad M/P = \kappa, \qquad \kappa > 0$$

The revenue from inflation is conventionally defined as the amount of real resources that the government can obtain by the associated sale of high-powered money; thus, in period 1 it would be given by

$$(17) \qquad \frac{M_1 - M_0}{P_1} = (M_1/P_1) - (M_0/P_0)(P_0/P_1) = \kappa\theta,$$

where the last equality follows from (12) and (16).

The government budget constraint in period 1 is given by

$$(18) \qquad x = (1-\theta)bR_b + g - \kappa\theta.$$

This is similar to (2) above, except that inflation is assumed not to be costly for the government, and the inflation tax is subtracted from total government expenditure (inclusive of debt service) in order to calculate the amount of required conventional taxes.

Effective consumption, c, is defined as follows,

$$(19) \qquad c = y - z(x) + kR + (1-\theta)bR_b - x - \kappa\theta - \Re(\theta),$$

where $\Re(\cdot)$ is the inflation-cost function. Again, this is very similar to (4) above, except that we have quite naturally subtracted the inflation tax, and, as discussed at the outset, we assume that inflation is reflected in a direct welfare cost for consumers.

We assume

$$(20a) \qquad \Re(0) = \Re'(0) = 0,$$

$$(20b) \qquad \Re''(\theta) > 0 \quad \text{for all } \theta.$$

We will first study the optimal problem with precommitment, that is, the problem of maximizing c with respect to θ *in period* 0. Since we maintain the assumption of perfect foresight, equation (1)—the Fisher equation—holds. Therefore, the above-mentioned problem can be formally stated as follows:

$$\max_{\theta \leq 1} c,$$

[11]The following simple form is sufficient for conveying the central insights of this section. Extensions are possible; however, see fn. 14.

subject to (1), (18), and (19).[12] The latter is equivalent to

$$(21)\quad \underset{\theta \leq 1}{\text{Min}}\left[z(g+bR-\kappa\theta)+\Re(\theta)\right],$$

which, by (5) and (20) has a unique solution, denoted θ^{FB} (for "first best"). Since the function in (21) is strictly convex, one can readily prove the following useful proposition:

PROPOSITION 2: *Assume that* (1) *holds, and let c^i be the effective consumption associated with θ^i, $i=1,2$; then, if $\theta^{\text{FB}} \leq \theta^1 < \theta^2$, we have that $c^2 < c^1$.*

In other words, the above proposition states that if inflation is higher than its first-best level, then social welfare—as measured by effective consumption— is a decreasing function of inflation.

We now turn to the second-best situation which is the focus of our analysis. The government is not able to precommit the inflation level, and, therefore, the government maximizes social welfare in period 1, taking the interest rate factor R_b and period-0 inflationary expectations as given. Recalling (19), this problem is equivalent to

$$(22)\quad \underset{\theta \leq 1}{\text{Min}}\, F(\theta; R_b)$$

$$\equiv \underset{\theta \leq 1}{\text{Min}}\left[z(g+b(1-\theta)R_b-\kappa\theta)+\Re(\theta)\right].$$

Notice that in the first-best problem (21) the government takes (1) into account because it is maximizing from the perspective of period 0 and consumers would not be in an (interior) equilibrium if the Fisher equation does not hold. In (22), however, the government controls θ taking R_b as given, because it maximizes effective consumption in period 1 in an environment in which R_b has been negotiated in the previous period (i.e., period 0), and, by assumption, R_b is not contingent on future policy or events.

By (22), at an interior[13] (second-best) optimum we have

$$(23)\quad F_\theta \equiv -z'(x)(bR_b+\kappa)+\Re'(\theta)=0,$$

implying, by (5) and (20) that optimal $\theta > 0$. The second-order condition is always satisfied because F is strictly convex with respect to θ. Also, notice that, by (5) and (20), the minimization problem (22) always has a solution, and by strict convexity, the solution is unique; we denote it θ^{SB} (for "second best"). By (23), there exists some function $\varphi(\cdot,\cdot)$, such that

$$(24)\qquad \theta^{\text{SB}}=\varphi\left(\overset{+}{bR_b},\overset{+}{g}\right),$$

where signs over the variables indicate those of the corresponding partial derivatives at an interior solution. Consequently, the (second-best) rate of inflation is an increasing function of nominal debt service, bR_b, and government expenditure, g.

An important implication of the above is that if there exists a positive stock of government bonds, b, then θ^{SB} is an increasing function of the nominal interest rate factor, R_b; but, by Fisher equation (1), equilibrium θ is also an increasing function of R_b and thus—like in Figure 1—the two schedules (i.e., the government's reaction function φ and the Fisher equilibrium relationship) are upward sloping, which, once again, opens the door for multiple equilibria.

It is important to notice that in the absence of government debt (i.e., $b=0$), equilibrium will be unique because θ^{SB} would be independent of R_b. Thus, in the present simple context, the familiar inflation tax on high-powered money plays no significant role in causing non-uniqueness.[14]

[12]This is like the optimization problem examined in Phelps (1973). Its solution differs from Milton Friedman's Optimum Quantity of Money ($\theta = 0$ in the present context) because, contrary to Friedman, we assume taxes are distorting.

[13]In what follows we will concentrate on interior solutions (i.e., $\theta < 1$), corresponding to cases in which the inflation rate is less than infinity (recall (14)).

[14]This result could be extended to the case in which the demand for money is sensitive to the rate of interest

In order to complete the non-uniqueness argument, the next step will be to show a simple example of non-uniqueness when $b > 0$. Consider the case in which

$$(25) \qquad \Re(\theta) = \frac{\beta}{2}\theta^2.$$

Therefore, by (1) and (23), we have

$$(26) \qquad z'(x) = \beta \frac{\theta}{bR/(1-\theta)+\kappa},$$

and, by (1) and (18), we get

$$(27) \qquad x = bR + g - \kappa\theta.$$

Equilibrium relationships (26) and (27) are depicted in Figure 3 for the limit case in which the demand for money is zero, that is, $\kappa = 0$. By continuity, it is quite clear that multiplicity of solutions would continue to hold for κ in some neighborhood of 0. It is, however, very interesting to note that in the present example uniqueness would prevail if κ is sufficiently large.[15] This is perfectly consistent with our previous observation that the existence of high-powered money is conducive to uniqueness. The result is also highly suggestive; since the value of κ may have a lot to do with the inflationary history of a country, the above result indicates that *multiple solutions may be a bigger problem for countries which have recently suffered from high inflation*.[16]

By (21), the first-order condition for a first-best optimum is

$$(28) \qquad -z'(g + bR - \kappa\theta)\kappa + \Re'(\theta) = 0.$$

Thus, recalling that in Section II we assumed $g > 0$ and $b > 0$, we have $\theta^{\mathrm{FB}} > 0$. On the other hand, recalling (18), the first-order condition for a second-best optimum is at equilibrium—that is, when (1) holds—

$$(29) \qquad -z'(g + bR - \kappa\theta)(bR + \kappa) + \Re'(\theta) = 0.$$

Notice that, by (5), (20), (28), and (29), the first-and second-best optima satisfy $\kappa\theta < g + bR$, implying that in neither one of these optima will the inflation tax be enough to finance total government expenditures. Consequently, by (5), (20), (28), and (29), we have, at equilibrium,

$$(30) \qquad \theta^{SB} > \theta^{FB}.$$

In other words, we have shown that optimal inflation with precommitment is lower than in any of the equilibrium solutions obtained under no precommitment. Consequently, recalling Proposition 2, we deduce that equilibria without precommitment can be ranked according to their associated inflation rates: given any two of these equilibria, the one exhibiting the largest rate of inflation yields the smallest social welfare (as measured by effective consumption, see (19)). In terms of the example associated with Figure 3, therefore, we are entitled to call θ^0 the "good," and θ^1 the "bad" equilibrium.[17]

Price indexation of government debt has important effects on this economy since it removes the inflation incentives associated with the debt burden. With full-price indexa-

if the elasticity of $z'(x)$ with respect to x is less than unity, and the inflation tax does not exceed regular taxes, the normal case. Our proof, however, relies on the existence of more than two periods, and will, therefore, not be presented here. For an infinite-horizon model of non-uniqueness along the present lines, but without government bonds, see Maurice Obsfeld (1988).

[15] The exact condition is $\kappa > bR + g$, that is, real monetary balances must exceed total government expenditure, including the inflation-adjusted service on the public debt.

[16] This point emerged during a useful conversation with Sweder van Wijnbergen.

[17] Multiple equilibria is by no means a necessary feature of these models. Uniqueness would, in fact, prevail if, for example, $\Re(\theta) = \pi^2$; the latter is related to the work of Barro and Gordon (1983a,b) and Bohn (1987). For a somewhat detailed discussion of these issues, see Calvo (1987).

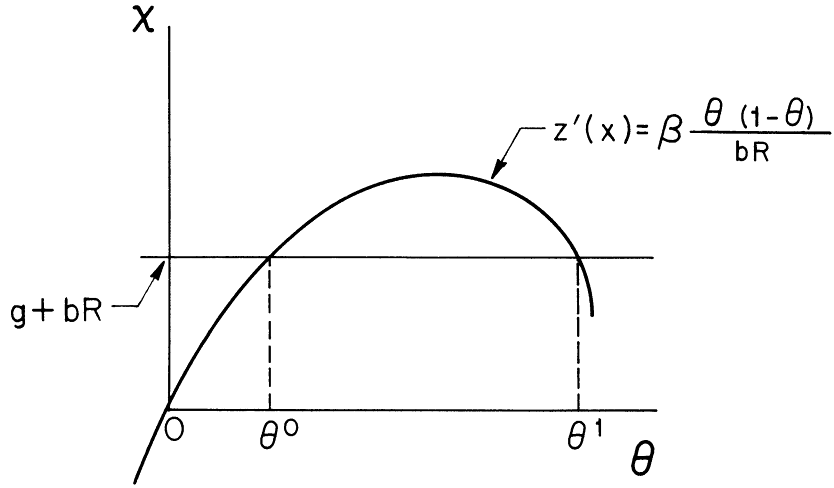

FIGURE 3. MONETARY EXAMPLE

tion and an interest-insensitive demand for money (i.e., constant κ, as in the present model), it is easy to see that the no precommitment equilibrium coincides with the first best.[18]

An alternative to indexation—which was discussed at the end of Section I—is to put bounds on interest rates; this could ensure that the solution settles at θ^0 in Figure 3 rather than at θ^1. However, one can easily show that such a solution is inferior to bond indexation, because the latter results in lower inflation[19] (recall Proposition 2 and (30)).[20]

As the example depicted in Figure 3 shows, an increase in the cost of inflation (i.e., an increase in β, recall (25)), has ambiguous results on welfare. A rise in β causes the curve in Figure 3 to shift upward, raising inflation in the bad equilibrium, and lowering it in the good one. Clearly, net consumption, c, will decrease in the bad equilibrium (because β and inflation go up). In the good equilibrium (i.e., θ^0), on the other hand, the picture is less clear, because inflation goes down, which tends to be welfare improving. However, if $\kappa = 0$, a simple manipulation using (26) and (27) shows that welfare increases with β. Thus, as in Section I, the effects of increasing the costs of inflation/repudiation are ambiguous: they could be welfare-improving in the good equilibrium, but they definitely lower welfare in the bad equilibrium.

Our example sheds some light on the issue studied by Bennett McCallum (1984) of whether monetary policy could be separated from fiscal considerations. His discussion has to do with the long-run (literally when $t \to \infty$) feasibility of policies that keep money supply at a constant level while running a fiscal deficit. In our model the relationship between the rate of inflation and fiscal policy is very direct for the case of non-indexed bonds (still the dominant kind of govern-

[18] Indexation would not suffice for attaining the first-best solution if the demand for money is sensitive to the rate of interest. I conjecture, however, that there are relevant examples where one could still show that debt indexation increases social welfare.

[19] This is not intended to be a thorough analysis of the welfare economics of bond indexation since there are some important aspects of the issue which have not been taken into account. For example, as pointed out to me by Nissan Liviatan, it has been argued that indexation may reduce the base of the inflation tax because in such a case bonds become better substitutes for domestic money. This aspect is not covered by our discussion because κ was assumed to be independent of the degree of indexation.

[20] Bounds on interest rates may be particularly difficult to implement when there exists an active private capital market. See Calvo (1987) for some discussion on this issue.

ment bonds in the United States and several other countries), and confirms and complements McCallum's insight that there are no grounds for expecting fiscal policy to have no monetary implications even though, for instance, the monetary authority promises to keep a constant price level (or a constant money supply in a setup where the latter is explicitly modeled). In our model this happens because the government may not commit itself *credibly* to zero inflation in the presence of non-indexed debt, since the level of the latter determines the set of equilibrium inflation rates that could be sustained *ex post*.[21] This relationship between the stock of government bonds and inflation, incidentally, will hold true even when the equilibrium is unique (see Bohn, 1987).

Our results add a pessimistic note to the view that price controls might help leading the economy to the "good" solution,[22] which appears to have played some role in the design of the recent "heterodox" stabilization plans in Argentina, Brazil, and Israel. This is so, because in our example the government has direct control on the price level, but, once again, non-uniqueness arises due to the fact that its *ex post* behavior is not immune to *ex ante* expectations. As a matter of fact, our previous discussion suggests that interest-rate pegging of some sort may be as much a crucial ingredient of a credible stabilization effort as price controls are, and that interest rate pegging could in some occasions be all that is needed for credibility.

III. International Debt. A Short Detour

Borrowing and lending among sovereign states has recently received considerable attention (see, for instance, the recent survey by Eaton, Gersovitz, and Stiglitz, 1986), so a few words establishing the connection between the present paper and that type of literature may be in order.

A common characteristic of our examples is that the "size of the penalty" (i.e., the cost of repudiation) is a function of the degree of repudiation; in the example of Section II the cost was proportional to the value of the debt, whereas in that of Section III the cost was simply a function of the fraction that was repudiated. In the international debt literature, on the other hand, it is typically assumed that the cost of repudiation bears no relationship to the size of the debt. This explains why it was relatively so easy for us to generate multiple equilibria while in that literature equilibrium is typically unique.[23]

The presence of foreign debt adds an interesting twist to our previous models. Thus, for example, we could assume that only a proportion γ, $0 \le \gamma \le 1$, of the total public debt is owned by domestic residents, while $(1-\gamma)$ is owned by foreigners whose welfare is of no concern to the local authorities. Thus equation (4) now becomes

$$(4') \qquad c = y - z(x) + kR + \gamma(1-\theta)bR_b - x,$$

and (19) is similarly modified. One can now examine the relationship between γ and the equilibrium outcomes. This is a straightforward exercise which, however, shows that the results depend very strongly on the model. In the model of Section I, an increase in foreign ownership of the public debt (i.e., a fall in γ) induces a fall in the bond interest rate at the bad equilibrium, but no change at the good one. On the other hand, in the monetary model of Section II (with $\kappa = 0$), interest rates in both equilibria are affected, rising in the bad equilibrium and falling in the good one. There does not seem, therefore, to be a clear-cut relationship between the participation of foreigners in the official

[21] Notice that our arguments establish a link between the stock of non-indexed debt and inflation, in contrast with McCallum (1984) who emphasized the relation between fiscal deficits and inflation.

[22] For recent models that could be employed to support that point of view, see Michael Bruno and Fischer (1985) and Miguel Kiguel (1986).

[23] An exception is the recent work of Eaton (1987), where, however, the issue of debt repudiation is not central to the analysis.

bonds market, and the degree of repudiation or inflation.

IV. Concluding Remarks

The central message that comes across this paper is that expectations may play a crucial role in the determination of equilibrium when the government debt is auctioned off to the public, and there is no attempt to manage expectations or to peg interest rates on the government debt. In our examples, we saw that the nominal interest rate is not simply a passive reflection of people's inflationary expectations, but rather that the nominal interest rate is actually one of the main determinants of inflation. Consequently, a credible anti-inflationary policy would have to implement rules to prevent nominal interest rates to become unduly high. The two simple mechanisms that were suggested in our discussion were: (a) price indexation of the public debt, and (b) refraining from issuing new government bonds when their interest rate exceeds some well-defined bounds. In more general terms, however, our discussion pointed out to the advisability of governments taking a more active stance on nominal interest rates. In my opinion, this implication of the formal analysis is likely to be particularly relevant in situations where the stabilization program is launched in the midst of very high inflation.

The analysis has been conducted in terms of models with one representative consumer because so much of the related literature has been couched in this form (prominently Barro, 1979; and Lucas and Stokey 1983). However, I suspect that one of the important reasons for debt repudiation to be costly (other than due to "reputation" costs) is the fact that not all individuals are alike and, therefore, the incidence of repudiation is not uniform.[24] Consider, for example, a model of the type discussed by Thomas Sargent and Neil Wallace (1982) where only a subset of total population holds non-interest bearing public debt (cash, say), while the rest of society holds high-denomination non-indexed bonds. For the sake of the argument, suppose, in addition, that everybody pays the same amount of taxes. Clearly, if the planner adheres to utilitarian principles, for example, debt repudiation could be costly (or beneficial), because the latter would be an instrument of wealth redistribution. In that context, the planner may end up repudiating less than 100 percent of the debt (held by the "rich," say) because of the loss it would cause cash-holders (the "poor"). I think the analysis of these types of models should provide us with a deeper understanding of these issues. It is my feeling, however, that the central insights of this paper will carry over to those models.

Another interesting extension would be to introduce uncertainty. Notice, first, that the above two-period examples would not exhibit random equilibria, because once the interest rate is determined in period 0, there exists only one optimum response for the government in period 1. An easy extension, however, would be to make the repudiation cost, α, random. Thus, if individuals are risk neutral, for example, R_b *times* the expected repayment share would in equilibrium equal the interest factor on capital R; the repudiation shares for each state of nature, on the other hand, would depend on R_b (as in our previous analysis). Therefore, the equilibrium values of R_b will be deeply intertwined with the probability distribution of α. Clearly, since R_b is just one number, we will observe situations where *ex post* the *actual* net return on bonds exceeds (or falls short of) that of physical capital. Consequently, these models could help explain phenomena like the high *ex post* real interest rates observed at the beginning of anti-inflationary programs (see, for instance, Rudiger Dornbusch, 1985, and Jeffrey Sachs, 1986).

Finally, an important extension of the analysis would be to allow for more than two periods,[25] and the simultaneous existence of explicit repudiation (for example, interest rate taxes) and inflation.

[24]See Carol Rogers (1986) and Calvo and Obstfeld (1988) for models which highlight the link between time inconsistency and wealth distribution.

[25]For some progress on this front, see Calvo (1987).

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