Entrepreneurial Finance and Non-diversifiable Risk[*]

Hui Chen[†]   Jianjun Miao[‡]   Neng Wang[§]

March 25, 2009

Abstract

Entrepreneurs face significant non-diversifiable business risks. We build a dynamic incomplete-markets model of entrepreneurial finance to demonstrate the important implications of non-diversifiable risks for entrepreneurs' interdependent consumption, portfolio allocation, financing, investment, and business exit decisions. The optimal capital structure is determined by a generalized tradeoff model where leverage via risky non-recourse debt provides significant diversification benefits. More risk-averse entrepreneurs default earlier, but also choose higher leverage, even though leverage makes his equity more risky. Non-diversified entrepreneurs demand both systematic and idiosyncratic risk premium. Cash-out option and external equity further improve diversification and raise the entrepreneur's valuation of the firm. Finally, entrepreneurial risk aversion can overturn the risk-shifting incentives induced by risky debt.

Keywords: Default, diversification benefits, entrepreneurial risk aversion, incomplete markets, private equity premium, hedging, capital structure, cash-out option, precautionary saving

JEL Classification: G11, G31, E2

[†]MIT Sloan School of Management, 50 Memorial Drive, Cambridge, MA 02142. Email: huichen@mit.edu. Tel.: 617-324-3896.

[‡]Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

[§]Columbia Business School and National Bureau of Economic Research. Address: Columbia Business School, 3022 Broadway, Uris Hall 812, New York, NY 10027. Email: neng.wang@columbia.edu. Tel.: 212-854-3869.

1 Introduction

Entrepreneurship plays an important role in fostering innovation and economic growth (Schumpeter (1934)). For reasons such as incentive alignment and informational asymmetry between the entrepreneur and financiers, entrepreneurs typically hold a significant non-diversified equity position in their businesses, and thus bear non-diversifiable entrepreneurial business risk.[1] For example, using data from the survey of consumer finance, Gentry and Hubbard (2004) report that active businesses on average account for 41.5 percent of entrepreneurs' total assets. Moskowitz and Vissing-Jorgensen (2002) document that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth.

Entrepreneurs are both producers and consumers. Like firms, they need to make investment/capital budgeting, financing, and business exit decisions. Like consumers, they also manage household finance and have preferences for intertemporal consumption smoothing. The non-diversifiable idiosyncratic risk that the entrepreneurs bear from their businesses makes the business decisions and household decisions interdependent. We integrate intertemporal household finance (consumption-smoothing, portfolio choice) with corporate finance, and provide a utility-maximizing framework to analyze the entrepreneurial firm's capital budgeting/investment, capital structure, and business exit decisions.

We model an infinitely-lived risk-averse entrepreneur who has access to an illiquid non-tradable investment project. The project requires a lump-sum investment to start up, and generates stochastic cash flows that bear both systematic and idiosyncratic risks. If he chooses to take on the project, the entrepreneur sets up a firm with limited liability.

Our baseline model analyzes the tradeoff between inside equity and external risky debt. Our focus on this tradeoff is motivated by the following empirical evidence. Heaton and Lucas (2004) document the high concentration of equity ownership in entrepreneurial firms and the importance of debt as a source of outside funding using data from the Survey of Small Business Finances (SSBF). For example, they report that the principal owner holds on average 81 percent of the firm's equity, and the median owner wholly owns the firm. These findings are consistent with results from earlier Fed surveys of small businesses, as reported in Berger and Udell (1998), Cole and Wolken (1996), and Petersen and Rajan (1994). Using a large data set of public and private

---

[1]Bitler, Moskowitz and Vissing-Jorgensen (2005) provide evidence that agency considerations play a key role in explaining why entrepreneurs on average hold large ownership shares.

firms in the United Kingdom, Brav (2009) finds that private firms rely almost exclusively on debt financing, have higher leverage ratios, and tend to avoid external capital markets, compared to their public counterparts. Furthermore, Leland and Pyle (1977) show that debt often dominates equity in settings with asymmetric information because debt is less information-sensitive.

After the firm is set up, the entrepreneur decides when to default on his debt (if the business does sufficiently poorly), and when to cash out by selling the firm (if the business is doing sufficiently well). In addition to the business exit decisions, he also chooses consumption and allocates his liquid wealth between a riskless asset and a diversified market portfolio (as in Merton (1971)).

While the entrepreneur can hedge the systematic component of his business risks using the market portfolio, he cannot diversify the idiosyncratic risks. Moreover, both default and cashing out options are costly. Therefore, the entrepreneur faces incomplete markets, and the idiosyncratic risk exposure has important effects on the entrepreneur's interdependent consumption, investment, financing, and business exit decisions.

While the entrepreneur can hedge the systematic component of his business risks using the market portfolio, he cannot diversify the idiosyncratic risks. Moreover, exit via default or cashing out is costly. Therefore, the entrepreneur faces incomplete markets, and the idiosyncratic risk exposure will have important effects on the entrepreneur's interdependent consumption, investment, financing, and business exit decisions.

Our main results are as follows. The default option embedded in non-recourse debt allows the entrepreneur to limit his liability when walking away from his business. Intuitively, the entrepreneur is long a default (put) option (i.e., a state-contingent sale of the firm to the financier), which is an insurance to the entrepreneur against the firm's potential poor performance in the future. Because the entrepreneur is exposed to the idiosyncratic risk from his concentrated ownership position in the firm, the diversification benefits of external risky debt are important when markets are incomplete.

The diversification benefits of debt are large. Even without the tax benefits of debt, the entrepreneurial firm still issues a significant amount of debt. The diversification benefits also lead to a seemingly counterintuitive prediction: more risk-averse entrepreneurs choose higher leverage. On the one hand, higher leverage increases the risk for the entrepreneur's equity stake. On the other hand, higher leverage/debt implies less equity exposure to the entrepreneurial project, making the entrepreneur's overall portfolio (including both his private equity in the firm and his liquid financial wealth) less risky. This overall portfolio composition effect dominates the high leverage effect

2

within the firm. The more risk-averse the entrepreneur, the stronger the need to reduce his firm risk exposure, therefore the higher the leverage.

The non-diversifiable risk and concentrated wealth in the business make the entrepreneur value his equity less than do diversified investors. The entrepreneur demands an extra premium for bearing the idiosyncratic risks of the firm. Thus, compared to a well-diversified firm, the entrepreneur tends to default earlier on his debt. While equity is a call option on firm assets, and hence is convex in firm cash flows, in our model, the private value of equity is not necessarily globally convex. When risk aversion and/or idiosyncratic volatility are sufficiently high, the entrepreneur's precautionary saving demand can make his private value of equity concave in cash flow.

This finding has important implications for risk shifting, an agency problem induced by risky debt. Jensen and Meckling (1976) point out that managers of public firms have incentives to invest in excessively risky projects after debt is in place because of the convexity feature of equity. In our model, when the degree of risk aversion is high enough, the entrepreneur's private value of equity (locally) decreases with the idiosyncratic volatility of the project. Thus, he may prefer to invest in a low idiosyncratic volatility project with debt in place, overturning the asset substitution result of Jensen and Meckling (1976). We find that, when the firm is not in distress, very low risk aversion is enough to make the entrepreneur prefer safer projects. Our model thus provides a potential explanation for the lack of empirical and survey evidence on asset substitution and risk-shifting incentives.

The standard option valuation analysis is no longer applicable to the default and cash-out options in our setting because these options are non-tradable and their underlying assets are illiquid. Under the assumption of constant absolute risk aversion (CARA) utility, we provide an analytically tractable framework to value these options. We also derive an analytical formula for the idiosyncratic risk premium demanded by the entrepreneur. The key determinants of the idiosyncratic risk premium are risk aversion, idiosyncratic volatility, and the sensitivity of entrepreneurial value of equity with respect to cash flow. The dynamic properties of the idiosyncratic risk premium are quite different from the systematic risk premium, especially when the firm is close to default or cash-out. Moreover, ignoring the idiosyncratic risk premium can lead to substantial downward bias in the estimates of the leverage ratio of entrepreneurial firms.

Finally, we extend our model to allow the entrepreneur to issue costly external equity. The more external equity issued, the smaller the entrepreneur's idiosyncratic risk exposure, but it also creates

an incentive problem for the entrepreneur, which lowers the expected growth rate of revenue. We show that the entrepreneur's dependence on external debt for diversification decreases when he has access to external equity. Intuitively, external equity is more effective than debt in transferring idiosyncratic risks to outside investors. The entrepreneur maximizes his *ex ante* private value of the firm by trading off the diversification benefits of equity against the costs of incentive problems.

This paper builds on the insight of Heaton and Lucas (2004), who are the first to model the diversification benefits of risky debt in a static model with asymmetric information (as in Leland and Pyle (1977)). We study the entrepreneur's consumption, portfolio choice, (debt/equity) financing, and exit (default/cash-out) decisions in a dynamic trade-off model. Our model is tractable and allows for analytical characterization of the dynamics of debt, equity, and the systematic and idiosyncratic risk premium demanded by the entrepreneur. We also incorporate a range of realistic features for entrepreneurial financing, including taxes, cash-out options, and external equity.

We integrate incomplete markets and diversification benefits into the tradeoff model of Leland (1994), thus bringing a new dimension to the structural credit risk/capital structure models.[2] Our generalized tradeoff model not only applies to entrepreneurs, but also to public firms with under-diversified managers. Our model is related to the incomplete-markets consumption smoothing/precautionary saving literature.[3] For analytical tractability reasons, we adopt the CARA utility specification as in Merton (1971), Caballero (1991), Kimball and Mankiw (1989), and Wang (2006). Our model contributes to this literature by extending the CARA-utility-based precautionary saving problem to allow the entrepreneur to reduce his idiosyncratic risk exposure via exit strategies such as cash-out and default.

This paper is also related to the real options literature.[4] The closest paper is Miao and Wang (2007), who analyze the impact of the entrepreneur's non-diversifiable idiosyncratic risks on his growth option exercising decision. The present paper analyzes the entrepreneurial firm's investment and financing (internal versus external, debt versus equity), and endogenous default and cash-out decisions.

---

[2]See Leland (1994), Goldstein, Ju, and Leland (2001), Strebulaev (2007), Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2008), Chen (2008), and earlier work of Black and Cox (1976), and Fischer, Heinkel, and Zechner (1989).

[3]Hall (1978) initiated the Euler equation approach to study intertemporal consumption behavior. See Deaton (1992) and Attanasio (1999) for surveys.

[4]See Brennan and Schwartz (1986), McDonald and Siegel (1986), Abel and Eberly (1994), and Dixit and Pindyck (1994).

2 Model setup

Investment opportunities An infinitely-lived risk-averse agent has a take-it-or-leave-it project at time 0, which requires a one-time investment I. The project generates a stochastic revenue process $\{y_t : t \geq 0\}$ that follows a geometric Brownian motion (GBM):

$$dy_t = \mu y_t dt + \omega y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given}, \tag{1}$$

where μ is the expected growth rate of the revenue, B_t and Z_t are independent standard Brownian motions, which are the sources of market (systematic) and idiosyncratic risks of the private business, respectively. The parameters ω and ϵ are the systematic and idiosyncratic volatility parameters of the revenue growth. The total volatility of revenue growth is

$$\sigma = \sqrt{\omega^2 + \epsilon^2}. \tag{2}$$

As we will show, these different volatility parameters ω, ϵ, and σ have different effects on the entrepreneur's decision making.

In addition, the agent has access to standard financial investment opportunities as in Merton (1971).[5] The agent allocates his liquid financial wealth between a riskfree asset which pays a constant rate of interest r and a diversified market portfolio (the risky asset) with returns R_t satisfying:

$$dR_t = \mu_p dt + \sigma_p dB_t, \tag{3}$$

where μ_p and σ_p are the expected return and volatility of the risky asset, respectively, and B_t is the standard Brownian motion introduced earlier. Let

$$\eta = \frac{\mu_p - r}{\sigma_p} \tag{4}$$

denote the after-tax Sharpe ratio of the market portfolio, and let $\{x_t : t \geq 0\}$ denote the entrepreneur's liquid (financial) wealth process. The entrepreneur invests the amount ϕ_t in the market portfolio (the risky asset) and the remaining amount $x_t - \phi_t$ in the riskfree asset.

Entrepreneurial firm If the entrepreneur decides to start the project, he runs it by setting up a limited-liability entity, such as a limited liability company (LLC) or an S corporation. The LLC

---

[5]It is straightforward to consider entering the labor market as an alternative to running entrepreneurial business, which provides an endogenous opportunity cost of taking on the entrepreneurial project. Such an extension does not change key economics of our paper in any significant way.

or S corporation allows the entrepreneur to face single-layer taxation for his business income and makes the debt non-recourse. We may extend the model to allow for personal guarantee of debt to varying degrees. This feature effectively makes debt recourse to varying degrees. The entrepreneur finances the initial one-time lump-sum cost I via his own funds (internal financing) and external financing. In the benchmark case, we assume that the only source of external financing is debt. See Petersen and Rajan (2002), Heaton and Lucas (2004), and Brav (2009) for evidence that debt is the primary source of financing for most entrepreneurial firms.[6] One interpretation of the external debt is bank loans. The entrepreneur uses the firm's assets as collateral to borrow, so that the debt is secured.

We assume that debt is issued at par and is interest-only (consol) for tractability reasons as in Leland (1994) and Duffie and Lando (2001). Let b denote the coupon payment of debt and F_0 denote the par value of debt. Debt is priced competitively in that the lender breaks even on the risk-adjusted basis. We further assume that debt is only issued at time 0 and remains unchanged until the entrepreneur exits. Allowing for dynamic capital structure before exit will not change the key economic tradeoff that we focus on: the impact of idiosyncratic risk on entrepreneurial financing decisions.

After debt is in place, at any time $t > 0$, the entrepreneur has three choices: (1) continuing his business; (2) defaulting on the outstanding debt, which leads to the liquidation of his firm; (3) cashing out by selling the firm to a diversified buyer.

While running the business, the entrepreneur receives income from the firm in the form of cash payments (operating profit net of coupon payments). Negative cash payments are interpreted as cash injections by the entrepreneur into the firm. Notice that trading riskless bonds and the diversified market portfolio alone does not help the entrepreneur diversify the idiosyncratic business risk. He can sell the firm and cash out, which requires a fixed transaction cost K. The default timing T_d and cash-out timing T_u are not contractible at time 0. Instead, the entrepreneur chooses the default/cash-out policy to maximize his own utility after he chooses the time-0 debt level. Thus, there is an inevitable conflict of interest between financiers and the entrepreneur. The choices of default and cash-out resemble American-style put and call options on the underlying non-tradeable entrepreneurial firm. Since markets are incomplete for the entrepreneur, we cannot price the entrepreneur's options using the standard dynamic replication argument (Black-Scholes-Merton).

---

[6]In Section VII, we introduce external equity as an additional source of financing.

At bankruptcy, the outside lender takes control and liquidates/sells the firm. Bankruptcy *ex post* is costly as in standard tradeoff models of capital structure. We assume that the liquidation/sale value of the firm is equal to a fraction α of the value of an all equity (unlevered) public firm, $A(y)$. The remaining fraction $(1 - \alpha)$ is lost due to bankruptcy costs. We also assume that absolute priority is enforced, and abstract away from any *ex post* renegotiation between the lender and the entrepreneur.

Before the entrepreneur can sell the firm, he needs to retire the firm's debt obligation at par F_0. We make the standard assumption that the buyer is well diversified. He will optimally relever the firm as in the complete-markets model of Leland (1994). The value of the firm after sale is the value of an optimally levered public firm, $V^*(y)$.

After the entrepreneur exits from his business (through default or cash-out), he "retires" and lives on his financial income. He then faces a standard complete-markets consumption and portfolio choice problem.[7]

Taxes   We consider a simple tax environment. The entrepreneurial firm pays taxes on his business profits at rate τ_e. When $\tau_e > 0$, issuing debt has the benefit of shielding part of the entrepreneur's business profits from taxes. For a public firm, the effective marginal tax rate is τ_m. Unlike the entrepreneurial firm, the public firm is subject to double taxation (at the corporate and individual levels), and τ_m captures the net tax rate (following Miller (1977)). Finally, τ_g denotes the tax rate on the capital gains upon sale. Naturally, higher capital gain taxes will delay the timing of cash-out.

Entrepreneur's objective   The entrepreneur derives utility from consumption $\{c_t : t \geq 0\}$ according to the following time-additive utility function:

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t} u(c_t)\, dt\right], \tag{5}$$

where $\delta > 0$ is the entrepreneur's subjective discount rate and $u(\cdot)$ is an increasing and concave function. The entrepreneur's objective is to maximize his lifetime utility by optimally choosing consumption (c_t), financial portfolio (ϕ_t), and whether to start his business. If he starts his business, he also chooses the financing structure of the firm (coupon b), and the subsequent timing decisions of default and cash-out (T_d, T_u).

---

[7]Extending our model to allow for sequential rounds of entrepreneurial activities will complicate our analysis. We leave this extension for future research.

In general, incomplete markets imply that the entrepreneur cannot fully diversify his business risk and hence cannot fully separate his investment from consumption decisions. Indeed, provided that $u'(c)$ is convex, the entrepreneur's precautionary motive will determine his intertemporal consumption smoothing.[8]

3 Model solution

First, in Section III.A, we report the complete-markets solution for firm value and financing decisions when the firm is owned by diversified investors. Then, we analyze the entrepreneur's interdependent consumption/saving, portfolio choice, default, and initial investment and financing decisions. The complete-markets solution of Section III.A serves as a natural benchmark for us to analyze the impact of non-diversifiable idiosyncratic risk on entrepreneurial investment, financing and valuation.

3.1 Complete-markets firm valuation and financing policy

Consider a public firm owned by diversified investors. Because equityholders internalize the benefits and costs of debt issuance, they will choose the firm's debt policy to maximize *ex ante* firm value by trading off the tax benefits of debt against bankruptcy and agency costs. The results in this case are well-known.[9] In Appendix A, we provide the after-tax value of an unlevered public firm $A(y)$ in equation (A.19), and the after-tax value of a public levered firm $V^*(y)$ in equation (A.21).

Next, we turn to analyze the entrepreneur's decision problem before he exits from his business.

3.2 Entrepreneur's problem

The significant lack of diversification invalidates the standard finance textbook valuation analysis for firms owned by diversified investors. As a result, the standard two-step complete-markets (Arrow-Debreu) analysis[10] (i.e., first value maximization and then optimal consumption allocation) no longer applies. This non-separability between value maximization and consumption smoothing has important implications for real economic activities (e.g., investment and capital budgeting) and the valuation of claims that an entrepreneur issues to finance his investment activities.

---

[8]Leland (1968) is among the earliest studies on precautionary saving models. Kimball (1990) links the degree of precautionary saving to the convexity of the marginal utility function $u'(c)$.

[9]For example, see Leland (1994), Goldstein, Ju, and Leland (2001), and Miao (2005).

[10]Cox and Huang (1989) apply this insight to separate intertemporal portfolio choices from consumption in continuous-time diffusion settings.

8

We solve the entrepreneur's problem by backward induction. First, we summarize the entrepreneur's consumption/saving and portfolio choice problem after he retires from his business via either cashing out or defaulting on debt. This "retirement-stage" optimization problem is the same as in Merton (1971), a dynamic complete-markets consumption/portfolio choice problem. Second, we solve the entrepreneur's joint consumption/saving, portfolio choice, and default decisions when the entrepreneur runs his private business. Third, we determine the entrepreneur's exit decisions (his cash-out and default boundaries) by comparing his value functions just before and after retirement. Finally, we solve the entrepreneur's initial (time-0) investment and financing decisions taking his future decisions into account.

Conceptually, our model setup applies to any utility function $u(c)$ under technical regularity conditions. For analytical tractability, we adopt the CARA utility throughout the remainder of the paper.[11] That is, let $u(c) = -e^{-\gamma c}/\gamma$, where $\gamma > 0$ is coefficient of absolute risk aversion, which also measures precautionary motive. We emphasize that the main results and insights of our paper (the effect of non-diversifiable idiosyncratic shocks on investment timing) do not rely on the choice of this utility function. As we show below, the driving force of our results is the precautionary savings effect, which is captured by utility functions with convex marginal utility such as CARA. While CARA utility does not capture wealth effects, it helps reduce the dimension of our double-barrier free-boundary problem, which makes the problem much more tractable compared to constant relative risk aversion (CRRA) utility.

Consumption/saving and portfolio choice after exit. After exiting from his business (via either default or cash-out), the entrepreneur no longer has any business income, and lives on his financial wealth. The entrepreneur's optimization problem becomes the standard complete-market consumption and portfolio choice problem (e.g., Merton (1971)). We summarize the results in Appendix B.

Entrepreneur's decision making while running the firm. We summarize the solution for consumption/saving, portfolio choice, default trigger y_d, and cash-out trigger y_u in the following theorem.

---

[11]The CARA utility specification proves tractable in incomplete-markets consumption-saving problems with labor income. Kimball and Mankiw (1989), Caballero (1991), Svensson and Werner (1993), and Wang (2006) have all adopted this utility specification in various precautionary saving models. Miao and Wang (2007) use this utility specification to analyze a real option exercising problem when the decision maker faces uninsurable idiosyncratic risk from his investment opportunity.

Theorem 1 *The entrepreneur exits from his business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever comes first. Prior to exit, for given liquid wealth x and revenue y, he chooses his consumption and portfolio rules as follows:*

$$\overline{c}(x, y) = r\left(x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{6}$$

$$\overline{\phi}(x, y) = \frac{\eta}{\gamma r \sigma_p} - \frac{\omega}{\sigma_p} y G'(y), \tag{7}$$

where $G(\cdot)$ and y_d solve the free boundary problem given by the differential equation:

$$rG(y) = (1 - \tau_e)(y - b - w) + (\mu - \omega\eta)yG'(y) + \frac{\sigma^2 y^2}{2}G''(y) - \frac{\gamma r \epsilon^2 y^2}{2}G'(y)^2, \tag{8}$$

subject to the following (free) boundary conditions:

$$G(y_d) = 0 \tag{9a}$$

$$G'(y_d) = 0 \tag{9b}$$

$$G(y_u) = V^*(y_u) - F_0 - K - \tau_g(V^*(y_u) - K - I) \tag{9c}$$

$$G'(y_u) = (1 - \tau_g)V^{*\prime}(y_u) \tag{9d}$$

where complete-markets firm value $V^*(y)$ is defined in (A.21), and the value of external debt $F_0 = F(y_0)$ is given in (C.6).

Equation (6) states that consumption is equal to the annuity value of the sum of financial wealth x, certainty equivalent wealth $G(y)$, and two constant terms capturing the effects of the expected excess returns and the wedge $\delta - r$ on consumption. The key is to note that $G(y)$ is the risk-adjusted subjective valuation of the entrepreneur's business project. Equation (7) gives the entrepreneur's portfolio holding, where the first term is the standard mean-variance term as in Merton (1971), and the second term gives the entrepreneur's hedging demand as he uses the market portfolio to dynamically hedge the entrepreneurial business risk.

The differential equation (8) provides a valuation equation for the certainty equivalent wealth $G(y)$ from the entrepreneur's perspective. In the standard CAPM model, only systematic risk demands a risk premium under the complete-markets setting. Since the systematic volatility of revenue growth is ω, the risk-adjusted expected growth rate of revenue in the CAPM model is

$$\nu = \mu - \omega\eta. \tag{10}$$

If we drop the last nonlinear term in (8), the differential equation becomes the standard pricing equation: setting the instantaneous expected return of an asset under the risk-neutral measure (RHS) equal to the riskfree rate (LHS). The last term in (8) captures the additional discount due to the non-diversifiable idiosyncratic risk. Intuitively, the higher the risk aversion parameter γ or the idiosyncratic volatility ϵy, the larger the discount on $G(y)$ due to idiosyncratic risk. The next section provides more detailed analysis on the impact of idiosyncratic risk on $G(y)$.

Equation (9a) comes from the value-matching condition for the entrepreneur's default decision. It states that the private value of equity $G(y)$ upon default is equal to zero. Equation (9b), often referred to as the smooth-pasting condition, can be interpreted as the optimality condition for the entrepreneur in choosing default.

Now we turn to the cash-out boundary. Because the entrepreneur pays the fixed cost K and triggers capital gains when cashing out, he naturally has incentive to wait before cashing out. However, waiting also reduces his diversification benefits *ceteris paribus*. The entrepreneur optimally trades off tax implications, diversification benefits, and transaction costs when choosing the timing of cashing out. The value-matching condition (9c) at the cash-out boundary states that the private value of equity upon the firm's cashing out is equal to the after-tax value of the public firm value after the entrepreneur pays the fixed costs K, retires outstanding debt at par F_0, and pays capital gains taxes. The smooth-pasting condition (9d) ensures that the entrepreneur optimally chooses his cash-out decision.

Initial financing and investment decisions. Next, we complete the model solution by endogenizing the entrepreneur's initial investment and financing decision. The entrepreneurial firm has two financial claimants: inside equity (entrepreneur) and outside creditors. The entrepreneur values his ownership at a certainty equivalent value $G(y)$. Diversified lenders price debt in competitive capital markets at $F(y)$, which does not contain the idiosyncratic risk premium because outside investors are fully diversified. Thus, the total *private value* of the entrepreneurial firm is

$$S(y) = G(y) + F(y). \tag{11}$$

We may interpret $S(y)$ as the total value that an investor needs to pay in order to acquire the entrepreneurial firm by buying out the entrepreneur and the debt investors.

At time 0, the entrepreneur thus chooses debt coupon b to maximize the private value of the

firm:

$$b^* = \operatorname*{argmax}_{b} \ S\left(y_0; b\right). \tag{12}$$

Intuitively, the entrepreneur internalizes the benefits and costs of debt financing, and markets competitively price the firm's debt. In Appendix B, we show that (12) indeed arises from the entrepreneur's utility maximization problem stated in (B.18). Note the conflicts of interest between the entrepreneur and external financiers. After debt is in place, the entrepreneur will no longer maximize the total value of the firm $S(y)$, but his private value of equity $G(y)$. Theorem 1 has already captured the conflict of interest between the entrepreneur and outside creditors.

The last step is to characterize whether the agent wants to undertake the project. He makes the investment and starts up the firm at time 0 if his life-time utility with the project is higher than that without the project. This is equivalent to the condition that $S\left(y_0\right) > I$.

We may interpret our model's implication on capital structure as a generalized tradeoff model of capital structure for the entrepreneurial firm, where the entrepreneur trades off the benefits of outside debt financing (diversification and potential tax implications) against the costs of debt financing (bankruptcy and agency conflicts between the entrepreneur and outside lenders). The natural measure of leverage from the entrepreneur's point of view is the ratio between the public value of debt $F(y)$ and the private value of firm $S(y)$,

$$L(y) = \frac{F(y)}{S(y)}. \tag{13}$$

We label $L(y)$ as private leverage to reflect the impact of idiosyncratic risk on the leverage choice. Note that the entrepreneur's preferences (e.g., risk aversion) influence the firm's capital structure. The standard argument that shareholders can diversify for themselves and hence diversification plays no role in the capital structure decisions of public firms is no longer valid for entrepreneurial firms.

So far, we have focused on the parameter regions where the entrepreneur first establishes his firm as a private business and finances its operation via an optimal mix of outside debt and inside equity. We now point out two special cases. First, when the cost of cashing out is sufficiently small, it is optimal for the entrepreneur to sell the firm immediately ($y_u = y_0$). The other special case is when asset recovery rate is sufficiently high, or the entrepreneur is sufficiently risk averse, so that he raises as much debt as possible and defaults immediately ($y_d = y_0$). Both cases lead to immediate exit. In our analysis below, we consider parameter values that rule out these cases.

4 Risky debt, endogenous default, and diversification

We now investigate a special case of the model in Theorem 1 which highlights the diversification benefits of risky debt. For this purpose, we shut down the cash-out option (by setting the cash-out cost K to infinity, making the cash-out option worthless).

We use the following (annualized) baseline parameter values: riskfree interest rate $r = 3\%$, expected growth rate of revenue $\mu = 4\%$, systematic volatility of growth rate $\omega = 10\%$, idiosyncratic volatility $\varepsilon = 20\%$, market price of risk $\eta = 0.4$, and asset recovery rate $\alpha = 0.6$. We set the effective marginal Miller tax rate τ_m to 11.29% as in Graham (2000) and Hackbarth, Hennessy, and Leland (2007).[12] In our baseline parametrization, we set $\tau_e = 0$, which reflects the fact that the entrepreneur can avoid taxes on his business income completely by deducting various expenses. Shutting down the tax benefits also allows us to highlight the diversification benefits of debt. Later, we consider the case where $\tau_e = \tau_m$, which can be directly compared with the complete-markets model. We set the entrepreneur's rate of time preference $\delta = 3\%$, and consider three values of the risk aversion parameter $\gamma \in \{0, 1, 2\}$. Finally, we set the initial level of revenue $y_0 = 1$.

Private value of equity $G(y)$ and default threshold. Figure 1 plots private value of equity $G(y)$ and its derivative $G'(y)$ as functions of y. The top and the bottom panels plot the results for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. When $\tau_e = 0$, the entrepreneur with very low risk aversion ($\gamma \to 0$, effectively complete-markets) issues no debt, because there are neither tax benefits ($\tau_e = 0$) nor diversification benefits ($\gamma \to 0$). Equity value is equal to the present discounted value of future cash flows (the straight dash line shown in the top-left panel). A risk-averse entrepreneur has incentive to issue debt in order to diversify idiosyncratic risks. The entrepreneur defaults when y falls to y_d, the point where $G(y_d) = G'(y_d) = 0$. When $\tau_e = \tau_m$, the entrepreneurial firm issues debt to take advantage of tax benefits in addition to diversification benefits. The bottom two panels of Figure 1 plot this case.

The derivative $G'(y)$ measures the sensitivity of private value of equity $G(y)$ with respect to revenue y. As expected, private value of equity $G(y)$ increases with revenue y, i.e., $G'(y) > 0$. Analogous to Black-Scholes-Merton's observation that firm equity is a call option on firm assets, the entrepreneur's private equity $G(y)$ also has a call option feature. For example, in the bottom

---

[12]We may interpret τ_m as the effective Miller tax rate which integrates the corporate income tax, individual's equity and interest income tax. Using the Miller's formula for the effective tax rate , and setting the interest income tax at 0.30, corporate income tax at 0.31, and the individual's long-term equity (distribution) tax at 0.10, we obtain an effective tax rate of 11.29%.
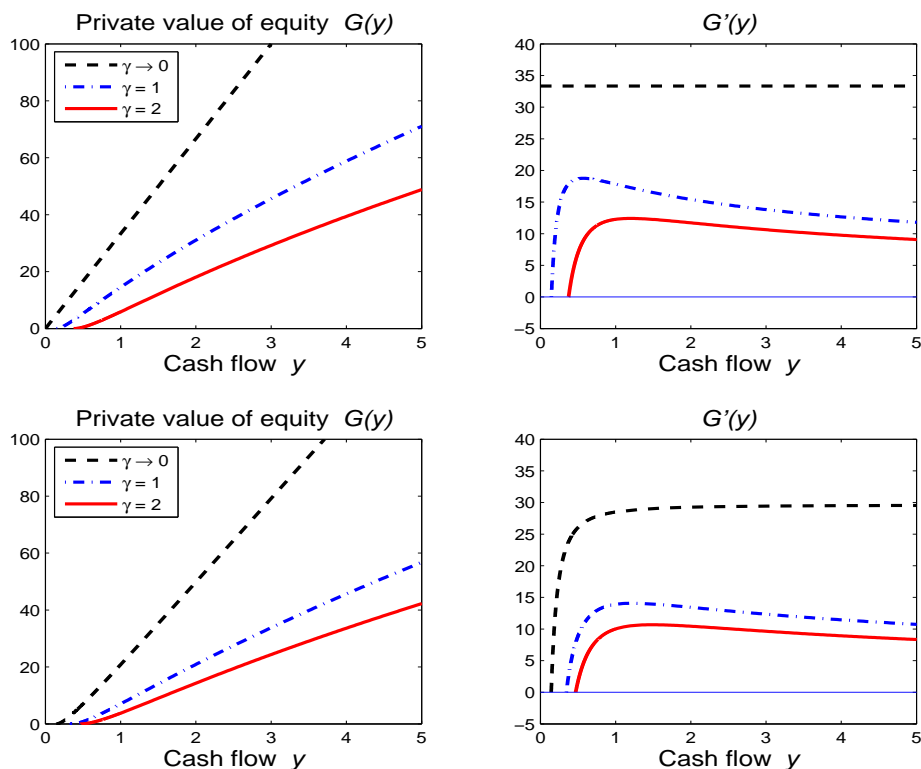
Figure 1: **Private value of equity** $G(y)$**: debt financing only.** The top and bottom panels plot $G(y)$ and its first derivative $G'(y)$ for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. We plot the results for two levels of risk aversion ($\gamma = 1, 2$) and the benchmark complete-market solution ($\gamma \to 0$).

panels of Figure 1 ($\tau_e = \tau_m$), when γ approaches 0 (complete markets case), equity value is convex in revenue y, reflecting its call option feature.

Unlike the standard Black-Scholes-Merton paradigm, neither the entrepreneurial equity nor the firm is tradable. When the risk-averse entrepreneur cannot fully diversify his project's idiosyncratic risks, the global convexity of $G(y)$ no longer holds, as shown in Figure 1 for cases where $\gamma > 0$. The entrepreneur now has precautionary saving demand to partially buffer against the project's non-diversifiable idiosyncratic shocks. This precautionary saving effect induces concavity in $G(y)$. When revenue y is large, the precautionary saving effect is large due to high idiosyncratic volatility ϵy, and the option (convexity) effect is small because the default option is further out of the money. Therefore, the precautionary saving effect dominates the option effect for sufficiently high y, making $G(y)$ concave in y for high y. The opposite is true for low y, where the convexity effect dominates.

Table 1: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: DEBT FINANCING ONLY.

This table reports the results for the setting where the entrepreneur only has access to debt financing and no option to cash out. The parameters are reported in Section IV. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0\%, 11.29\%$ (τ_m)) and three levels of risk aversion. The case "$\gamma \to 0$" corresponds to the complete-markets (Leland) model.

	coupon	public debt	private equity	private firm	private leverage (%)	credit spread (bp)	10-yr default probability (%)
	b	F_0	G_0	S_0	L_0	CS	$p_d(10)$
				$\tau_e = 0$			
$\gamma \to 0$	0.00	0.00	33.33	33.33	0.0	0	0.0
$\gamma = 1$	0.31	8.28	14.39	22.68	36.5	72	0.4
$\gamma = 2$	0.68	14.66	5.89	20.55	71.3	166	12.1
				$\tau_e = \tau_m$			
$\gamma \to 0$	0.35	9.29	20.83	30.12	30.9	75	0.3
$\gamma = 1$	0.68	14.85	7.02	21.86	67.9	159	9.5
$\gamma = 2$	0.85	16.50	3.77	20.27	81.4	213	22.3

The precautionary saving effect also causes a more risk-averse entrepreneur to discount cash flows at a higher rate. For a given level of coupon b, the entrepreneur values his inside equity lower (smaller $G(y)$), thus is more willing to default and walk away. Moreover, a more risk-averse entrepreneur also has a stronger incentive to diversify idiosyncratic risks by selling a bigger share of his firm, which implies a larger coupon b, a higher default threshold, and a higher debt value, *ceteris paribus*. The two effects reinforce each other. Figure 1 confirms that both $G(y)$ and the default threshold y_d increase with risk aversion γ.

Capital structure for entrepreneurial firms. First, we consider the special case where risky debt only offers diversification benefits for the entrepreneur and has no tax benefits ($\tau_e = 0$). Then, we incorporate the tax benefits of debt into our analysis.

The top panel in Table 1 provides results for the entrepreneurial firm's capital structure when $\tau_e = 0$. If the entrepreneur is very close to being risk neutral ($\gamma \to 0$), the model's prediction is essentially the same as the complete-market benchmark. In this case, the standard tradeoff theory of capital structure implies that the entrepreneurial firm will be entirely financed by equity (since debt provides no benefits). The risk-neutral entrepreneur values the firm at its market value 33.33.

For $\gamma = 1$, the entrepreneur borrows $F_0 = 8.28$ in market value with coupon $b = 0.31$, and values his non-tradable equity G_0 at 14.39, giving the private value of the firm $S_0 = 22.68$. The drop in S_0 is substantial (from 33.33 to 22.68, or about 32%) when increasing γ from zero to one. This drop in S_0 is mainly due to the risk-averse entrepreneur's discount of his non-tradable equity position for bearing non-diversifiable idiosyncratic business risks. The default risk of debt contributes little to the reduction of S_0 (the 10-year cumulative default probability rises from 0 to 0.4% only).

In Section III, we introduced the natural measure of leverage for entrepreneurial firms: private leverage L_0, given by the ratio of public debt value F_0 to private value of the firm S_0. Private leverage L_0 naturally arises from the entrepreneur's maximization problem and captures the entrepreneur's tradeoff between private value of equity and public value of debt in choosing debt coupon policy. For $\gamma = 1$, the private leverage ratio is about 36.5%.

With a higher risk aversion level $\gamma = 2$, the entrepreneur borrows more ($F_0 = 14.66$) with a higher coupon ($b = 0.68$). He values his remaining non-tradable equity at $G_0 = 5.89$, and the implied private leverage ratio $L_0 = 71.3\%$ is much higher than 36.5%, the value for $\gamma = 1$. The more risk-averse entrepreneur takes on more leverage, because he has stronger incentive to sell more of the firm to achieve greater diversification benefits. With greater risk aversion, default is more likely (the 10-year cumulative default probability is 12.1%), and the credit spread is higher (166 basis points over the riskfree rate).

Next, we incorporate the effect of tax benefits for the entrepreneur into our generalized tradeoff model of capital structure for entrepreneurial firms. To compare with the complete-markets benchmark, we set $\tau_e = \tau_m = 11.29\%$. Therefore, the only difference between an entrepreneurial firm and a public firm is that the entrepreneur faces non-diversifiable idiosyncratic risks.

The first row of the lower panel of Table 1 gives the results for the complete-markets benchmark. Facing positive corporate tax rates, the public firm wants to issue debt, but is also concerned with bankruptcy costs. The optimal tradeoff for the public firm is to issue debt at the competitive market value $F_0 = 9.29$ with coupon $b = 0.35$. The implied initial leverage is 30.9% and the 10-year cumulative default probability is tiny (0.3%).

Similar to the case with $\tau_e = 0$, an entrepreneur facing non-diversifiable idiosyncratic risks wants to issue more risky debt to diversify these risks. The second panel of Table 1 shows that the entrepreneur with $\gamma = 1$ borrows 14.85 (with the coupon rate $b = 0.68$), higher than the level for the public firm. The private leverage more than doubles to 67.9%. Not surprisingly, the entrepreneur

Table 2: DECOMPOSITION OF PRIVATE LEVERAGE FOR ENTREPRENEURIAL FIRMS

This table compares a private firm owned by a risk-averse entrepreneur with a public firm. There is no option to cash out. We assume $\tau_e = \tau_m$, while the rest of the parameters are reported in Section IV. All the results are for initial revenue $y_0 = 1$.

	10-yr default probability (%)	public debt	equity value	firm value	financial leverage (%)	credit spread (bp)
	$p_d(10)$	F_0	G_0	S_0	L_0	CS
$\gamma = 2$ $(b = 0.85, y_d = 0.47)$	22.3	16.50	3.77	20.27	81.4	213
Public $(b = 0.85, y_d = 0.47)$	22.3	16.50	11.10	27.60	59.8	213
Public $(b = 0.85, y_d = 0.35)$	9.8	17.71	11.56	29.26	60.5	178
Public $(b = 0.35, y_d = 0.14)$	0.3	9.29	20.82	30.11	30.9	75

faces a higher default probability and the credit spread of his debt is also higher. With $\gamma = 2$, debt issuance increases to 16.50, and private leverage increases to 81.4%.

Determinants of capital structure decisions. To further demonstrate the important role of idiosyncratic risks in determining the capital structure of entrepreneurial firms, we now turn to two comparisons.

First, consider an econometrician who has correctly identified the entrepreneurial firm's debt coupon $b = 0.85$ and default threshold $y_d = 0.47$, but does not realize that the entrepreneur's subjective valuation $G(y; b, y_d)$ is lower than the corresponding public equity value $E(y; b, y_d)$ due to non-diversifiable idiosyncratic risk. Indeed, he assigns the entrepreneur's equity with a value at $E_0 = 11.10$ instead of the subjective valuation $G_0 = 3.77$, thus obtaining a leverage ratio of 59.8%, substantially lower than the entrepreneur's private leverage $L_0 = 81.4\%$. The large difference between the private and market leverage ratios highlights the economic significance of taking idiosyncratic risks into account. Simply put, standard corporate finance methodology potentially underestimates the leverage of entrepreneurial firms.

Second, we highlight the impact of the entrepreneur's endogenous default decision. The public and the entrepreneurial firms have significantly different leverage decisions because both debt issuance and default decisions on debt (given the same level of debt coupon outstanding) are different. To see the quantitative effects of endogenous default decisions on leverage, we hold the coupon rate on outstanding debt fixed. That is, consider a public firm that has the same technology/environment parameters as the entrepreneurial firm. Moreover, the two firms have the same

17

debt coupons ($b = 0.85$).

Facing the same coupon $b = 0.85$, the public firm defaults when revenue reaches the default threshold $y_d = 0.35$, which is lower than the threshold $y_d = 0.47$ for the entrepreneurial firm. Intuitively, facing the same coupon b, the entrepreneurial firm defaults earlier than the public firm because of the entrepreneur's aversion to non-diversifiable idiosyncratic risk. The implied shorter distance-to-default for the entrepreneurial firm translates into a higher 10-year default probability (22% for the entrepreneurial firm versus 10% for the public firm) and a higher credit spread (213 basis points for the entrepreneurial firm versus 178 basis points for the public firm). Defaulting optimally for the public firm raises its value from $S_0 = 27.60$ to $S_0 = 29.26$.

The preceding two comparisons help explain the differences in leverage ratios between the entrepreneurial firm and the public firm. First, fixing both the coupon and the default threshold, the entrepreneur's subjective valuation (due to non-diversifiable risks) has significant impact on the implied leverage ratio. Ignoring subjective valuation substantially underestimates the entrepreneurial firm's leverage. Second, facing the same coupon, the entrepreneurial firm defaults earlier than the public firm, which reduces the value of debt and lowers the leverage ratio, but the quantitative effect seems small. Third, diversification motives make the entrepreneur issue more debt than the public firm, which further raises the leverage ratio of the entrepreneurial firm. While the numerical results are parameter specific, the analysis provides support for our intuition that the entrepreneur's need for diversification and subjective valuation discount for bearing non-diversifiable idiosyncratic risks are key determinants of the private leverage for an entrepreneurial firm.

5 Cash-out option as an alternative channel of diversification

We now turn to a richer and more realistic setting where the entrepreneur can diversify idiosyncratic risks through both the default and cash-out options. The entrepreneur avoids the downside risk by defaulting if the firm's stochastic revenue falls to a sufficiently low level. In addition, when the firm does well enough, the entrepreneur may want to capitalize on the upside by selling the firm to diversified investors.

In addition to the baseline parameter values from Section IV, we set the effective capital gains tax rate from selling the business $\tau_g = 10\%$, reflecting the tax deferral advantage of the tax timing option.[13] We set the initial investment cost for the project $I = 10$, which is 1/3 of the market

---

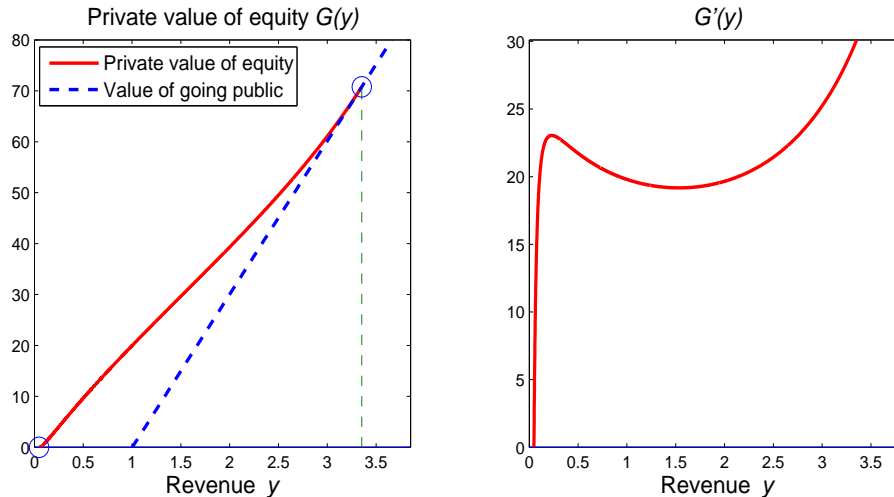[13]In Appendix D.1, we investigate the effects of different capital gain taxes.

Figure 2: **Private value of equity $G(y)$: debt financing and cash-out option.** We plot the results with the following parameters: $\gamma = 1$, $\tau_e = 0$, $\tau_g = 10\%$, $I = 10$, and $K = 27$. The remaining parameters are from the benchmark case reported in Section IV.

value of project cash-flows. We choose the cash-out cost $K = 27$ to generate a 10-year cash-out probability of about 20% (with $\gamma = 2$), consistent with the success rates of venture capital firms (Hall and Woodward (2008)).

Cash-out option: Crowding out debt. Figure 2 plots the private value of equity $G(y)$ and its first derivative $G'(y)$ for an entrepreneur with risk aversion $\gamma = 1$ when he has the option to cash out. The function $G(y)$ smoothly touches the horizontal axis on the left and the dash line denoting the value of cashing out on the right. The two tangent points give the default and cash-out thresholds, respectively. For sufficiently low values of revenue y, the private value of equity $G(y)$ is increasing and convex because the default option is deep in the money. For sufficiently high values of y, $G(y)$ is also increasing and convex because the cash-out option is deep in the money. For revenue y in the intermediate range, neither default nor cash-out option is deep in the money. In this range, the precautionary saving motive may be large enough to induce concavity. As shown in the right panel of Figure 1, $G'(y)$ first increases for low values of y, then decreases for intermediate values of y, and finally increases for high values of y.

Table 3 provides the capital structure information of an entrepreneurial firm with both cash-out and default options. For brevity, we only report the results for the case $\tau_e = \tau_m$. When the market is complete, the firm's cash-out option is essentially an option to adjust the firm's capital

19

Table 3: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: DEBT FINANCING AND CASH-OUT OPTION

This table reports the results for the setting where the entrepreneur has access to both public debt financing and cash-out option to exit from his project. The parameters are: $I = 10$, $K = 27$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, and $y_0 = 1$. The rest of the parameters are from the benchmark case in Section IV.

	coupon	public debt	private equity	private firm	private leverage (%)	10-yr default prob (%)	10-yr cash-out prob (%)
	b	F_0	G_0	S_0	L_0	$p_d(10)$	$p_u(10)$
$\gamma \to 0$	0.35	9.29	20.83	30.12	30.9	0.3	0.0
$\gamma = 1$	0.55	12.45	9.57	22.02	56.5	4.2	12.3
$\gamma = 2$	0.66	13.68	6.24	19.92	68.7	10.1	23.3

structure (recall that there is no diversification benefit for public firms). In this case, given our calibrated fixed cost K, the 10-year cash-out probability is essentially zero and hence this option value is close to zero for the public firm. Therefore, we expect that the bulk of the cash-out option value for entrepreneurial firms comes from the diversification benefits, not from the option value of readjusting leverage.

For a risk-averse entrepreneur, the prospect of cashing out lowers the firm's incentive to issue debt. When $\gamma = 1$, debt coupon is $b = 0.55$, lower than the level $b = 0.68$ for the firm which only has the default option. The private leverage ratio L_0 at issuance is 56.5%, with a credit spread at 138 basis points, compared to the private leverage ratio $L_0 = 67.9\%$ and credit spread of 159 basis points when the firm only has the default option. The 10-year default probability is close to zero, but the 10-year cash-out probability is 12.3%, which is economically significant (recall that the 10-year cash-out probability for a public firm is zero). For more risk averse entrepreneurs (e.g., $\gamma = 2$), the private leverage ratio is 68.7%, smaller than 81.4% for the setting without the cash-out option. Given the opportunity to sell his business to public investors, the entrepreneur substitutes away from risky debt and relies more on the future potential of cashing out to diversify his idiosyncratic risks.

Idiosyncratic risk, leverage, and risk premium. We now turn to the impact of idiosyncratic volatility on leverage and risk premium. Figure 3 shows its effect on leverage. In complete-markets models, an increase in (idiosyncratic) volatility ϵ raises default risk, hence the market leverage
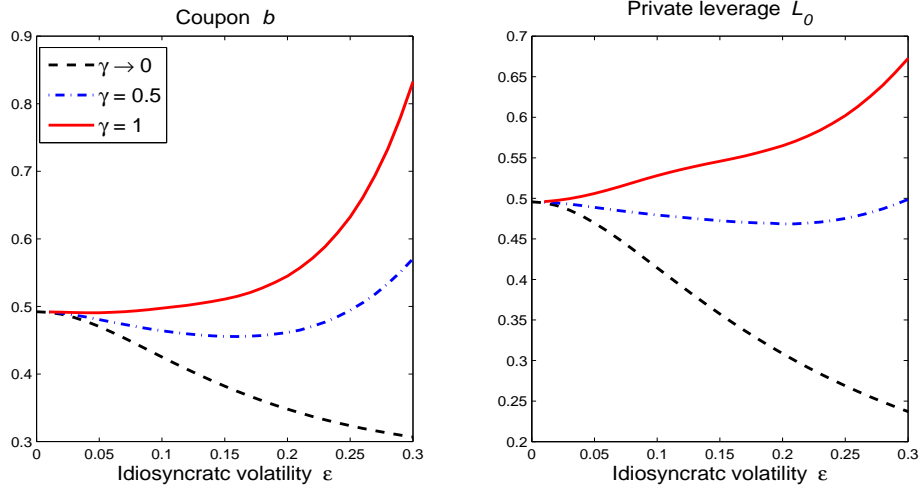
Figure 3: **Comparative statics – optimal coupon and private leverage with respect to idiosyncratic volatilities ϵ: the case of debt and cash-out option.** The two panels plot the optimal coupon b and the corresponding optimal private leverage L_0 at $y_0 = 1$. In each case, we plot the results for two levels of risk aversion ($\gamma = 1, 2$) alongside the benchmark complete-market solution ($\gamma \to 0$). The remaining parameters are the same as in Table 3.

ratio and the coupon rate for the public firm decrease with idiosyncratic volatility. By contrast, risk-averse entrepreneurs take on more debt to diversify their idiosyncratic risks when ϵ is higher. For $\gamma = 1$, both coupon and leverage become monotonically increasing in ϵ. This result implies that the private leverage ratio for entrepreneurial firms increases with idiosyncratic volatility even for mild risk aversion.

For public firms, risk premium is determined by the firm's non-diversifiable systematic risk. For entrepreneurial firm, both systematic and idiosyncratic risks matter for the firm's investment decisions. Without loss of generality, we decompose the entrepreneur's risk premium into two components: the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$. Rearranging the valuation equation (8) gives:

$$\pi^s(y) = \eta\omega\frac{G'(y)}{G(y)}y = \eta\omega\frac{d\ln G(y)}{d\ln y}, \tag{14}$$

$$\pi^i(y) = \frac{\gamma r}{2}\frac{(\epsilon y G'(y))^2}{G(y)}. \tag{15}$$

The systematic risk premium $\pi^s(y)$ defined in (14) takes the same form as in standard asset pricing models. It is the product of the (market) Sharpe ratio η, systematic volatility ω, and the elasticity of $G(y)$ with respect to y, where the elasticity captures the impact of optionality on the
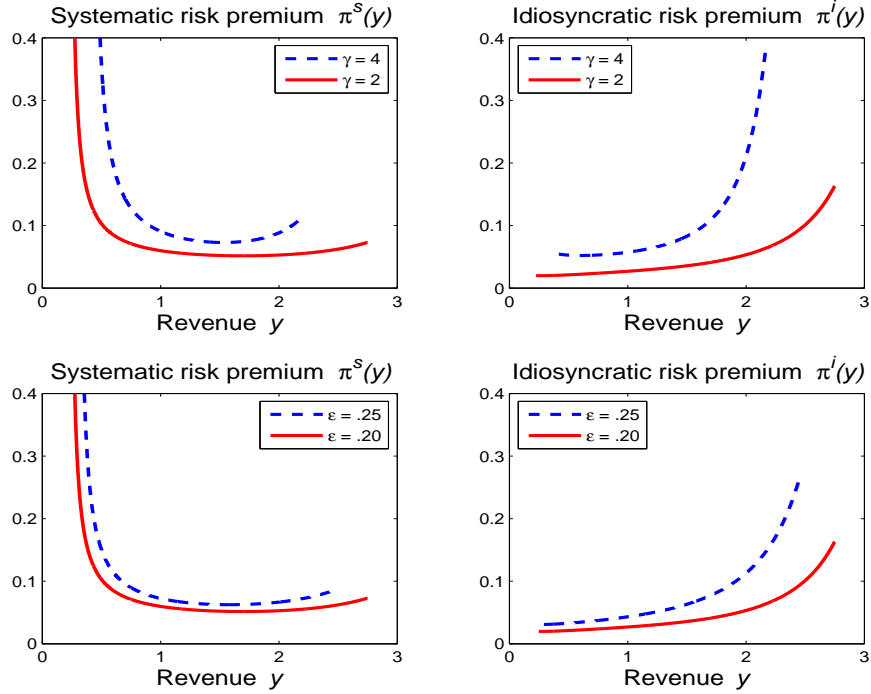
21

Figure 4: **Systematic and idiosyncratic risk premium.** This figure plots the systematic and idiosyncratic risk premium for entrepreneurial firms. The top panels plot the results for two levels of risk aversion ($\gamma = 2, 4$). The bottom panels plot the results for two levels of idiosyncratic volatility ($\epsilon = 0.20, 0.25$). We assume $\gamma = 2$ (when changing ϵ), $\epsilon = 0.2$ (when changing γ). The remaining parameters are the same as in Table 3.

risk premium.[14]

Unlike $\pi^s(y)$, the idiosyncratic risk premium $\pi^i(y)$ defined in (15) *directly* depends on risk aversion γ and $(\epsilon y G'(y))^2$, the conditional (idiosyncratic) variance of the entrepreneur's equity $G(y)$. The conditional (idiosyncratic) variance term reflects the fact that the idiosyncratic risk premium $\pi^i(y)$ is determined by the entrepreneur's precautionary saving demand, which depends on the conditional variance of idiosyncratic risks (Caballero (1991) and Wang (2006)).

We examine the behavior of these risk premia in Figure 4. The entrepreneur's equity is a levered position in the firm. When the firm approaches default, the systematic component of the risk premium $\pi^s(y)$ behaves similarly to the standard valuation model. That is, the significant leverage effect around the default boundary implies that the systematic risk premium diverges to

---

[14]Despite this standard interpretation for the systematic risk premium, it is worth pointing out that $\pi^s(y)$ also indirectly reflects the non-diversifiable idiosyncratic risks that the entrepreneur bears, and risk aversion γ indirectly affects $\pi^s(y)$ through its impact on $G(y)$.

infinity when y approaches y_d. When the firm approaches the cash-out threshold, the cash-out option makes the firm value more sensitive to cash flow shocks, which also tends to raise the systematic risk premium.

The idiosyncratic risk premium $\pi^i(y)$ behaves quite differently. Figure 4 indicates that the idiosyncratic risk premium is small when the firm is close to default, and it increases with y for most values of y. The intuition is as follows. The numerator in (15) reflects the agent's precautionary saving demand, which depends on the conditional idiosyncratic variance of the changes in the certainty equivalent value of equity $G(y)$ and risk aversion γ. Both the conditional idiosyncratic variance and $G(y)$ increase with y. When y is large, the conditional idiosyncratic variance rises fast relative to $G(y)$, generating a large idiosyncratic risk premium.

6 Project choice: asset substitution versus risk sharing

Jensen and Meckling (1976) point out that there is an incentive problem associated with risky debt: After debt is in place, managers have incentive to take on riskier projects to take advantage of the option-type of payoff structure of equity. However, there is little empirical evidence in support of such risk shifting behaviors.[15] One possible explanation is that managerial risk aversion can potentially dominate the risk shifting incentives. Our model provides a natural setting to investigate these two competing effects quantitatively.

We consider the following project choice problem. Suppose the risk-averse entrepreneur can choose among a continuum of mutually exclusive projects with different idiosyncratic volatilities ϵ in the interval $[\epsilon_{\min}, \epsilon_{\max}]$ *after debt is in place*. Let F_0 be the market value of existing debt with the coupon payment b. The entrepreneur then chooses idiosyncratic volatility $\epsilon^+ \in [\epsilon_{\min}, \epsilon_{\max}]$ to maximize his own utility. As shown in Section III, the entrepreneur effectively chooses ϵ^+ to maximize his private value of equity $G(y_0)$, taking the debt contract (b, F_0) as given. Let this maximized value be $G^+(y_0)$.

In a rational expectations equilibrium, the lender anticipates the entrepreneur's *ex post* incentive of choosing the level of idiosyncratic volatility ϵ^+ to maximize $G(y_0)$, and prices the initial debt contract accordingly in competitive capital markets. Therefore, the entrepreneur *ex ante* maximizes the private value of the firm, $S(y_0) = G^+(y_0) + F_0$, taking the competitive market debt pricing into account. We solve this joint investment and financing (fixed-point) problem.

---

[15]See Andrade and Kaplan (1998), Graham and Harvey (2001), and Rauh (2008), among others.
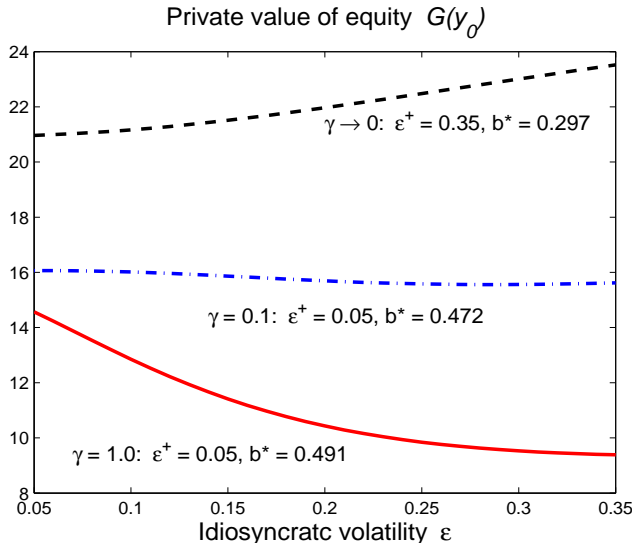
Figure 5: **Private equity value as function of idiosyncratic volatility after optimal debt is in place.** This figure plots the private value of equity for different choices of idiosyncratic volatility ϵ after debt issuance. The coupon is fixed at the optimal value corresponding to given risk aversion. We assume $\epsilon_{\min} = 0.05$, $\epsilon_{\max} = 0.35$. The remaining parameters are the same as in Table 3.

Figure 5 illustrates the solution of this optimization problem. We set $\epsilon_{\min} = 0.05, \epsilon_{\max} = 0.35$. When $\gamma \to 0$, the entrepreneur chooses the highest idiosyncratic volatility project with $\epsilon_{\max} = 0.35$. The optimal coupon payment is 0.297. In this case, the entrepreneur effectively faces complete markets. The Jensen and Meckling (1976) argument applies because the market value of equity is convex and the *ex post risk shifting* problem arises. When the entrepreneur is risk averse, he demands a premium for bearing the non-diversifiable idiosyncratic risks, which tends to lower his private value of equity $G(y_0)$. When this effect dominates, the entrepreneur prefers projects with lower idiosyncratic volatility. For example, for $\gamma = 1$, the entrepreneur chooses the project with $\epsilon_{\min} = 0.05$, with the corresponding optimal coupon payment 0.491. Even when the degree of risk aversion is low (e.g., $\gamma = 0.1$, which implies an idiosyncratic risk premium of 2 basis points for $\epsilon = 0.05$, or 20 basis points for $\epsilon = 0.20$), we still find that the risk aversion effect dominates the risk shifting incentive.

From this numerical example, we find that in our model, even with low risk aversion, the precautionary saving incentive tends to dominate the asset substitution incentive in normal times. Note that our argument applies to public firms as well, provided that: (i) managerial compensation is tied to firm performance; (ii) managers are not fully diversified, behave in their own interests,

24

and are entrenched. Thus, the lack of empirical evidence supporting asset substitution may be simply due to the non-diversifiable idiosyncratic risks faced by risk-averse decision makers.

7 External equity

While debt is the primary source of financing for most entrepreneurial (small-business) firms, high-tech startups are often financed by venture capital, which often use external equity in various forms as the primary source of financing. This financing choice particularly makes sense when the liquidation value of firm's assets is low (e.g., computer software firms), and incentive alignments are more important. Hall and Woodward (2008) provide a quantitative analysis for the lack of diversification of venture-capital-backed entrepreneurial firms. In this section, we extend the baseline model of Section II by allowing the entrepreneur to issue external equity at $t = 0$, and study the effect of external equity on the diversification benefits of risky debt.[16]

If it is costless to issue external equity, a risk-averse entrepreneur will want to sell the entire firm to the VC right away. We motivate the costs of external equity through the agency problems of Jensen and Meckling (1976). Intuitively, the more concentrated the entrepreneur's ownership is, the better incentive alignment (Berle and Means (1931) and Jensen and Meckling (1976)). Let ψ denote the fraction of equity that the entrepreneur retains and hence $1 - \psi$ denote the fraction of external equity. Consider the expected growth rate of revenue μ in equation (1). We capture the incentive problem of ownership in reduced form by making μ an increasing and concave function of the entrepreneur's ownership ψ (i.e., $\mu'(\psi) > 0$ and $\mu''(\psi) < 0$). Intuitively, the concavity relation suggests that the incremental value from incentive alignment becomes lower as ownership concentration rises, *ceteris paribus*.

More specifically, we model the growth rate μ as a quadratic function of the entrepreneur's ownership ψ, $\mu(\psi) = -0.02\psi^2 + 0.04\psi + 0.03$, with $\psi \in [0, 1]$. This functional form is chosen such that the maximum expected growth rate is 5%, when the entrepreneur owns the entire firm ($\psi = 1$), while the lowest growth rate is 3%, when the entire firm is sold ($\psi = 0$). The parameters for $\mu(\psi)$ are chosen to keep the agency costs of external equity modest so as to highlight the substitution effect of external equity.

After external debt (with coupon b) and equity (with share $1 - \psi$ of the firm ownership) are

---

[16]Initial equity issuance together with cashing out can be viewed as a two-step procedure to unload the entrepreneur's private holdings. Gradual sales of ownership, e.g., as DeMarzo and Urosevic (2006) consider for large shareholders, is less applicable for private business owners due to the lack of liquidity, which we capture with the fixed cost K.

issued at $t = 0$, the entrepreneur's optimal policies, including consumption/portfolio rule and default/cash-out policies, are summarized in the following theorem.

Theorem 2 *The entrepreneur exits from his business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever occurs first. When the entrepreneur runs his firm, he chooses his consumption and portfolio rules as follows:*

$$\overline{c}(x,y) = r\left(x + \psi G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{16}$$

$$\overline{\phi}(x,y) = \frac{\eta}{\gamma r \sigma_p} - \frac{\psi \omega}{\sigma_p} y G'(y), \tag{17}$$

where $(G(\cdot), y_d, y_u)$ solves the free boundary problem given by the differential equation:

$$rG(y) = (1 - \tau_e)(y - b - w) + \nu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) - \frac{\psi \gamma r \epsilon^2 y^2}{2} G'(y)^2, \tag{18}$$

subject to the following (free) boundary conditions:

$$G(y_d) = 0 \tag{19a}$$

$$G'(y_d) = 0 \tag{19b}$$

$$\psi G(y_u) = \psi V^*(y_u) - F_0 - K - \tau_g (\psi V^*(y_u) - K - (I - (1 - \psi) E_0)) \tag{19c}$$

$$\psi G'(y_u) = (1 - \tau_g)\psi V^{*\prime}(y_u) \tag{19d}$$

where complete-markets firm value $V^*(y)$ is defined in (A.21), the value of external debt $F_0 = F(y_0)$ is given in (C.6), and the value of external equity $E_0 = E(y_0)$ is given by (C.10).

Equation (18) shows how the partial ownership ψ affects the entrepreneur's private value of equity. A more concentrated inside equity position (higher ψ) raises the last nonlinear term, which raises the idiosyncratic risk premium that the entrepreneur demands. The ownership ψ also affects the boundary conditions at cash-out. The value-matching condition (19c) at the cash-out boundary states that, upon cashing out, the entrepreneur's ownership is worth fraction ψ of the after-tax value of the public firm value net of (1) the amount required to retire outstanding debt at par F_0, (2) fixed costs K, and (3) capital gains taxes. The smooth-pasting condition (19d) also reflects the effects of partial ownership.

Finally, at time $t = 0$, the entrepreneur chooses debt coupon b and initial ownership ψ to maximize the private value of the firm $S(y)$, which now has three parts: inside equity (entrepreneur's ownership), diversified outside equity, and outside debt:

$$S(y) = \psi G(y) + (1 - \psi)E_0(y) + F(y). \tag{20}$$

26

Table 4: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: EXTERNAL DEBT/EQUITY AND CASH-OUT OPTION

This table reports the results for the setting where the entrepreneur has access to both public debt and equity financing, and cash-out option to exit from his project. We assume $\tau_e = \tau_m$, $\mu(\psi) = -0.02\psi^2 + 0.04\psi + 0.03$, while the rest of the parameters are reported in Table 3. All the results are for initial revenue $y_0 = 1$.

	ownership	public debt	public equity	private equity	private firm	private leverage (%)	10-yr default prob (%)	10-yr cash-out prob (%)
	ψ	F_0	$(1-\psi)E_0$	ψG_0	S_0	L_0	$p_d(10)$	$p_u(10)$
$\gamma \to 0$	1.00	15.23	0.00	30.07	45.30	33.6	0.4	0.0
$\gamma = 1$	0.69	16.00	8.50	8.26	32.76	48.8	3.8	11.3
$\gamma = 2$	0.65	15.93	9.50	5.67	31.10	51.2	6.0	15.4

The results are reported in Table 4. If the entrepreneur is risk-neutral, he will clearly prefer to keep 100% ownership. In this case, all the equity in the firm is privately held, the private leverage is 33.6%, and the 10-year probabilities of default and cash-out are both close to zero. An entrepreneur with $\gamma = 1$ lowers his ownership to 69%, which reduces the growth rate to 4.8% (only a 0.2% drop). However, the coupon rises from 0.55 to 0.66, and private leverage rises from 33.6% to 48.8%. The increase in demand for debt due to diversification is economically sizeable, especially considering that the increase is partially offset by the reduced tax benefit of debt due to lower expected growth rates. The 10-year default and cash-out probabilities rise to 3.8% and 11.3% respectively. When $\gamma = 2$, the ownership drops to 65%, while private leverage rises further to 51.2%. The 10-year cash-out probability also rises to 15.4%.

These results demonstrate that entrepreneurial firms have sizeable demand for risky debt and cash-out options for diversification purpose, even when the agency costs of external equity are small.

8   Concluding remarks

Entrepreneurial investment opportunities are often illiquid and non-tradable. Entrepreneurs cannot completely diversify away project-specific risks for reasons such as incentives and informational asymmetry. Therefore, the standard law-of-one-price-based valuation/capital structure paradigm in corporate finance cannot be directly applied to entrepreneurial finance. An entrepreneur acts

both as a producer making dynamic investment, financing, and exit decisions for his business project, and as a household making consumption/saving and portfolio decisions. The dual roles of the entrepreneur motivate us to develop a dynamic incomplete-markets model of entrepreneurial finance that centers around the non-diversification feature of the entrepreneurial business.

We show that more risk-averse entrepreneurs use higher leverage for greater diversification benefits and default earlier. This prediction holds not only in the baseline setting where the source of external financing is risky debt, but is robust when we introduce additional channels of diversification such as cash-out option and external equity. In addition to compensation for systematic risks, the entrepreneur also demands sizable premium for bearing idiosyncratic risks, which increase with his risk aversion, his equilibrium inside ownership, and the project's idiosyncratic variance. Ignoring this idiosyncratic risk premium can lead to a large upward bias when computing the private value of entrepreneurial equity. Finally, even for low to moderate levels of risk aversion, the idiosyncratic risk premium significantly weakens the risk shifting incentives (Jensen and Meckling (1976)) for non-diversified managers.

Our paper also makes methodological contributions to financial valuation and decision making. This paper extends the standard complete-markets Black-Scholes-Merton option pricing and real options methodology to settings where investment opportunities are illiquid and not marked-to-market, and decision makers are not diversified. Our framework can also be used to value the stock options of non-diversified executives or to analyze how these executives make capital structure and investment decisions. See Carpenter, Stanton, and Wallace (2008) for a recent study on the optimal exercise policy for an executive stock option and implications for firm costs.

We have taken a standard optimization framework where the entrepreneur's utility only depends on his consumption. A significant fraction of entrepreneurs view the non-pecuniary benefits of being their own bosses as a large component of rewards. It has also been documented that less risk-averse (see Gentry and Hubbard (2004), De Nardi, Doctor, and Krane (2007)) and more confident/optimistic individuals are more likely to self-select into entrepreneurship. We also do not model the fundamental frictions causing markets to be incomplete and entrepreneurs to be non-diversified. We view endogenous incomplete markets as a complementary perspective, which can have fundamental implications such as promotion of entrepreneurship and contract design. We leave these extensions for future research.

Appendix

A Market valuation and capital structure of a public firm

Well-diversified owners of a public firm face complete markets. Given the Sharpe ratio η of the market portfolio and the riskfree rate r, there exists a unique stochastic discount factor (SDF) $(\xi_t : t \geq 0)$ satisfying (see Duffie (2001)):

$$d\xi_t = -r\xi_t dt - \eta \xi_t dB_t, \quad \xi_0 = 1. \tag{A.1}$$

Using this SDF, we can derive the market value of the unlevered firm, $A(y)$, the market value of equity, $E(y)$, and the market value of debt $D(y)$. The market value of the firm is equal to the sum of equity value and debt value:

$$V(y) = E(y) + D(y). \tag{A.2}$$

Under the risk-neutral probability measure Q, we can rewrite the dynamics of the revenue (1) as follows:

$$dy_t = \nu y_t dt + \omega y_t dB_t^Q + \epsilon y_t dZ_t, \tag{A.3}$$

where ν is the risk-adjusted drift defined by $\nu \equiv \mu - \omega\eta$, and B_t^Q is a standard Brownian motion under Q satisfying $dB_t^Q = dB_t + \eta dt$.

A.1 Valuation of an unlevered public firm

Throughout the appendix, we derive our results assuming that there is a flow operating cost w for running the project. The operating cost w generates operating leverage and hence the option to abandon the firm has positive value. The results reported in this paper are for the case $w = 0$. Appendix D.2 provides results for the case $w > 0$.

We start with the after-tax unlevered firm value $A(y)$, which satisfies the following differential equation:

$$rA(y) = (1 - \tau_m)(y - w) + \nu y A'(y) + \frac{1}{2}\sigma^2 y^2 A''(y). \tag{A.4}$$

This is a second-order ordinary differential equation (ODE). We need two boundary conditions to obtain a solution. One boundary condition describes the behavior of $A(y)$ when $y \to \infty$. This condition rules out speculative bubbles. To ensure that $A(y)$ is finite, we assume $r > \nu$ throughout the paper. The other boundary condition is related to abandonment. As in the standard option

exercise models, the firm is abandoned whenever the cash flow process hits a threshold value y_a for the first time. At the threshold y_a, the following value-matching condition is satisfied

$$A(y_a) = 0, \tag{A.5}$$

because we normalize the outside value to zero. For the abandonment threshold y_a to be optimal, the following smooth-pasting condition must also be satisfied:

$$A'(y_a) = 0. \tag{A.6}$$

Solving equation (A.4) and using the no-bubble condition and boundary conditions (A.5)-(A.6), we obtain

$$A(y) = (1 - \tau_m) \left[\left(\frac{y}{r - \nu} - \frac{w}{r} \right) - \left(\frac{y_a}{r - \nu} - \frac{w}{r} \right) \left(\frac{y}{y_a} \right)^{\theta_1} \right], \tag{A.7}$$

where the abandonment threshold y_a is given in

$$y_a = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} w, \tag{A.8}$$

where

$$\theta_1 = -\sigma^{-2} \left(\nu - \sigma^2/2 \right) - \sqrt{\sigma^{-4} \left(\nu - \sigma^2/2 \right)^2 + 2r\sigma^{-2}} < 0. \tag{A.9}$$

A.2 Valuation of a levered public firm

First, consider the market value of equity. Let y_d be the corresponding default threshold. After default, equity is worthless, in that $E(y) = 0$ for $y \leq y_d$. This gives us the value matching condition $E(y_d) = 0$. Before default, equity value $E(y)$ satisfies the following differential equation:

$$rE(y) = (1 - \tau_m)(y - w - b) + \nu y E'(y) + \frac{1}{2}\sigma^2 y^2 E''(y), \; y \geq y_d. \tag{A.10}$$

When $y \to \infty$, $E(y)$ also satisfies a no-bubble condition. Solving this ODE and using the boundary conditions, we obtain

$$E(y; y_d) = (1 - \tau_m) \left[\left(\frac{y}{r - \nu} - \frac{w + b}{r} \right) - \left(\frac{y_d}{r - \nu} - \frac{w + b}{r} \right) \left(\frac{y}{y_d} \right)^{\theta_1} \right]. \tag{A.11}$$

Equation (A.11) shows that equity value is equal to the after-tax present value of profit flows minus the present value of the perpetual coupon payments plus an option value to default. The term $(y/y_d)^{\theta_1}$ may be interpreted as the price of an Arrow-Debreu security contingent on the event of default. The optimal default threshold satisfies the smooth-pasting condition,

$$\left. \frac{\partial E(y)}{\partial y} \right|_{y=y_d} = 0, \tag{A.12}$$

30

which gives

$$y_d^* = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} (b + w). \tag{A.13}$$

After debt is in place, there is a conflict between equityholders and debtholders. Equityholders choose the default threshold y_d to maximize equity value $E(y; y_d)$.

The market value of debt before default satisfies the following differential equation:

$$rD(y) = b + \nu y D'(y) + \frac{1}{2}\sigma^2 y^2 D''(y), \ y \geq y_d. \tag{A.14}$$

The value-matching condition is given by:

$$D(y_d) = \alpha A(y_d). \tag{A.15}$$

We also imposes a no bubble condition when $y \to \infty$. Solving the valuation equation, we have

$$D(y) = \frac{b}{r} - \left[\frac{b}{r} - \alpha A(y_d)\right] \left(\frac{y}{y_d}\right)^{\theta_1}, \tag{A.16}$$

For a given coupon rate b and default threshold y_d, using equation (A.2), we may write the market value of the levered firm value $V(y; y_d)$ as follows:

$$V(y; y_d) = A(y) + \frac{\tau_m b}{r}\left[1 - \left(\frac{y}{y_d}\right)^{\theta_1}\right] - (1 - \alpha) A(y_d)\left(\frac{y}{y_d}\right)^{\theta_1}, \tag{A.17}$$

Equation (A.17) shows that the levered market value of the firm is equal to the after-tax unlevered firm value plus the present value of tax shields minus bankruptcy costs.

While y_d^* is chosen to maximize $E(y)$, coupon b is chosen to maximize *ex ante* firm value $V(y)$. Substituting (A.13) into (A.17) and using the following first-order condition:

$$\frac{\partial V(y_0)}{\partial b} = 0, \tag{A.18}$$

we obtain the optimal coupon rate b^* as a function of y_0. We also verify that the second order condition is satisfied.

Now consider the special case without operating cost $(w = 0)$. First, from (A.7), the value of an unlevered public firm becomes

$$A(y) = (1 - \tau_m)\left[\frac{y}{r - \nu} - \frac{y_a}{r - \nu}\left(\frac{y}{y_a}\right)^{\theta_1}\right], \tag{A.19}$$

For a levered public firm, we have an explicit expression for the optimal coupon:

$$b^* = y_0 \frac{r}{r - \nu} \frac{\theta_1 - 1}{\theta_1}\left(1 - \theta_1 - \frac{(1 - \alpha)(1 - \tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}. \tag{A.20}$$

Substituting (A.13) and (A.20) into (A.17), we obtain the following expression for $V^*(y)$, the firm value when debt coupon is optimally chosen:

$$V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{(1-\alpha)(1-\tau_m)\theta_1}{\tau_m} \right)^{1/\theta_1} \right] \frac{y}{r-\nu}. \tag{A.21}$$

Note that this firm value formula only applies at the moment of debt issuance and will equal to firm value when the entrepreneur cashes out.

B   Proof of Theorem 1 and 2

Theorem 1 is a special case of Theorem 2. Thus, we only prove the results in the general case where the entrepreneur has partial ownership ψ of the firm.

After exit (via default or cashing out), the entrepreneur solves the standard complete-markets consumption/portfolio choice problem (Merton (1971)). His wealth follows

$$dx_t = (r(x_t - \phi_t) - c_t)dt + \phi_t(\mu_p dt + \sigma_p dB_t). \tag{B.1}$$

The entrepreneur's value function $J^e(x)$ is given by the following explicit form:

$$J^e(x) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta-r}{\gamma r^2} \right) \right]. \tag{B.2}$$

The consumption and portfolio rules[17] are given by

$$\bar{c}(x) = r \left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta-r}{\gamma r^2} \right), \tag{B.3}$$

$$\bar{\phi}(x) = \frac{\eta}{\gamma r \sigma_p}. \tag{B.4}$$

Before exit, the entrepreneur's financial wealth evolves as follows:

$$dx_t = (r(x_t - \phi_t) + \psi(1-\tau_e)(y - b - w) - c_t)dt + \phi_t(\mu_p dt + \sigma_p dB_t), \quad 0 < t < \min(T_d, T_u). \tag{B.5}$$

---

[17] An undesirable feature of CARA-utility models is that consumption and wealth could potentially turn negative. Cox and Huang (1989) provide analytical formulae for consumption under complete markets for CARA utility with non-negativity constraints. In our incomplete-markets setting, imposing non-negativity constraints substantially complicates the analysis. Intuitively, requiring consumption to be positive increases the entrepreneur's demand for precautionary saving because he will increase his saving today to avoid hitting the constraints in the future. The induced stronger precautionary saving demand will likely strengthen our results (such as diversification benefits of outside risky debt).

Using the principle of optimality, we claim that the entrepreneur's value function $J^s(x,y)$ satisfies the following HJB equation:

$$\delta J^s(x,y) = \max_{c,\phi} u(c) + (rx + \phi(\mu_p - r) - c + \psi(1 - \tau_e)(y - b - w)) J_x^s(x,y)$$

$$+ \mu y J_y^s(x,y) + \frac{(\sigma_p \phi)^2}{2} J_{xx}^s(x,y) + \frac{\sigma^2 y^2}{2} J_{yy}^s(x,y) + \phi \sigma_p \omega y J_{xy}^s(x,y). \quad \text{(B.6)}$$

The first-order conditions (FOC) for consumption c and portfolio allocation ϕ are as follows:

$$u'(c) = J_x^s(x,y) \quad \text{(B.7)}$$

$$\phi = \frac{-J_x^s(x,y)}{J_{xx}^s(x,y)} \left(\frac{\mu_p - r}{\sigma_p^2}\right) + \frac{-J_{xy}^s(x,y)}{J_{xx}^s(x,y)} \frac{\omega y}{\sigma_p}, \quad \text{(B.8)}$$

We conjecture that $J^s(x,y)$ takes the following exponential form:

$$J^s(x,y) = -\frac{1}{\gamma r} \exp\left[-\gamma r\left(x + \psi G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \quad \text{(B.9)}$$

As shown in Miao and Wang (2007), $G(y)$ is the entrepreneur's certainty equivalent wealth per unit of the entrepreneur's inside equity of the firm. Under this conjectured value function, it is easy to show that the optimal consumption rule and the portfolio rule are given by (16) and (17), respectively. Substituting these expressions back into the HJB equation (B.6) gives the differential equation (18) for $G(y)$.

We now turn to the boundary conditions. First, consider the lower default boundary. Since equity is worthless at default, the entrepreneur's financial wealth x does not change immediately after default. In addition, the entrepreneur's value function should remain unchanged at the moment of default. That is, the following value-matching condition holds at the default boundary $y_d(x)$:

$$J^s(x, y_d(x)) = J^e(x). \quad \text{(B.10)}$$

In general, the default boundary depends on the entrepreneur's wealth level. Because the default boundary is optimally chosen, the following smooth-pasting conditions at $y = y_d(x)$ must be satisfied:[18]

$$\left.\frac{\partial J^s(x,y)}{\partial x}\right|_{y=y_d(x)} = \left.\frac{\partial J^e(x)}{\partial x}\right|_{y=y_d(x)} \quad \text{(B.11)}$$

$$\left.\frac{\partial J^s(x,y)}{\partial y}\right|_{y=y_d(x)} = \left.\frac{\partial J^e(x)}{\partial y}\right|_{y=y_d(x)} \quad \text{(B.12)}$$

---

[18]See Krylov (1980), Dumas (1991), and Dixit and Pindyck (1994) for details on the smooth-pasting conditions.

These two conditions equate the marginal value of wealth and the marginal value of revenue before and after default.

At the instant of cashing out, the entrepreneur retires debt at par, pays fixed cost K, and sells his firm for $V^*(y)$ given in (A.17). We assume that the shares owned by existing equity-holders are converted $1:1$ into the shares of the new firm. Then the entrepreneur pays capital gains taxes on the sale. His wealth x_{T_u} immediately after cashing out satisfies

$$x_{T_u} = x_{T_u-} + \psi V^*(y_{T_u}) - F_0 - K - \tau_g \left(\psi V^*(y_{T_u}) - K - (I - (1-\psi)E_0) \right). \tag{B.13}$$

The entrepreneur's value function at the payout boundary $y_u(x)$ satisfies the following value-matching condition:

$$J^s(x, y_u(x)) = J^e(x + \psi V^*(y_u(x)) - F_0 - K - \tau_g \left(\psi V^*(y_u(x)) - K - (I - (1-\psi)E_0) \right)). \tag{B.14}$$

The entrepreneur's optimality implies the following smooth-pasting conditions at $y = y_u(x)$:

$$\left. \frac{\partial J^s(x,y)}{\partial x} \right|_{y=y_u(x)} = \left. \frac{\partial J^e(x + \psi V^*(y) - F_0 - K - \tau_g(\psi V^*(y) - K - (I-(1-\psi)E_0)))}{\partial x} \right|_{y=y_u(x)} \tag{B.15}$$

$$\left. \frac{\partial J^s(x,y)}{\partial y} \right|_{y=y_u(x)} = \left. \frac{\partial J^e(x + \psi V^*(y) - F_0 - K - \tau_g(\psi V^*(y) - K - (I-(1-\psi)E_0)))}{\partial y} \right|_{y=y_u(x)} \tag{B.16}$$

Using the conjectured value function (B.9), we show that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use y_d and y_u to denote the default and cash-out thresholds, respectively. Using the value matching and smooth-pasting conditions (B.10)-(B.12) at y_d, we obtain (19a) and (19b). Similarly, using the value-matching and smooth-pasting conditions (B.14)-(B.16) at y_u, we have (19c) and (19d).

Finally, we characterize the entrepreneur's investment and financing decision at $t = 0$. Let $\underline{x}$ denote the entrepreneur's endowment of financial wealth. If the entrepreneur chooses to start his business, his time-0 financial wealth x_0 immediately after financing is

$$x_0 = \underline{x} - (I - F_0 - (1-\psi)E_0). \tag{B.17}$$

At time zero, the entrepreneur chooses a coupon rate b and equity share ψ to solve the following problem:

$$\max_{b,\psi} J^s \left(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0 \right), \tag{B.18}$$

34

subject to the requirement that outside debt and equity are competitively priced, i.e., $F_0 = F(y_0)$, and $E_0 = E_0(y_0)$. In Appendix C, we provide an explicit formulae for $F(y)$ and $E_0(y)$.

The entrepreneur will decide to launch the project if his value function from the project (under the optimal capital structure) is higher than the value function without the project,

$$\max_b J^s(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0) > J^e(\underline{x}). \tag{B.19}$$

C Market values of the entrepreneurial firm's outside debt and equity

When the entrepreneur neither defaults nor cashes out, the market value of his debt $F(y)$ satisfies the following ODE:

$$rF(y) = b + \nu y F'(y) + \frac{1}{2}\sigma^2 y^2 F''(y), \ y_d \leq y \leq y_u. \tag{C.1}$$

At the default trigger y_d, debt recovers the fraction α of after-tax unlevered firm value, in that $F(y_d) = \alpha A(y_d)$. At the cash-out trigger y_u, debt is retired and recovers its face value, in that $F(y_u) = F_0$. Solving (C.1) subject to these boundary conditions gives

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right)\overline{q}(y) + \left[\alpha A(y_d) - \frac{b}{r}\right]\underline{q}(y), \tag{C.2}$$

where

$$\overline{q}(y) = \frac{y^{\theta_1}y_d^{\theta_2} - y^{\theta_2}y_d^{\theta_1}}{y_u^{\theta_1}y_d^{\theta_2} - y_u^{\theta_2}y_d^{\theta_1}}, \tag{C.3}$$

$$\underline{q}(y) = \frac{y^{\theta_2}y_u^{\theta_1} - y^{\theta_1}y_u^{\theta_2}}{y_u^{\theta_1}y_d^{\theta_2} - y_u^{\theta_2}y_d^{\theta_1}}. \tag{C.4}$$

Here, θ_1 is given by (A.9) and

$$\theta_2 = -\sigma^{-2}(\nu - \sigma^2/2) + \sqrt{\sigma^{-4}(\nu - \sigma^2/2)^2 + 2r\sigma^{-2}} > 1. \tag{C.5}$$

Equation (C.2) admits an intuitive interpretation. It states that debt value is equal to the present value of coupon payments plus the changes in value when default occurs and when cash-out occurs. Note that $\overline{q}(y_0)$ can be interpreted as the present value of a dollar if cash-out occurs before default, and $\underline{q}(y_0)$ can be interpreted as the present value of a dollar if the entrepreneur goes bankrupt before cash-out. Using $F_0 = F(y_0)$, we have that the initial debt issuance is given by

$$F_0 = \frac{b}{r} - \left(\frac{b}{r} - \alpha A(y_d)\right)\frac{\underline{q}(y_0)}{1 - \overline{q}(y_0)}. \tag{C.6}$$

Similarly, for the outside equity claim, we have the following valuation equation:

$$rE_0(y) = (1 - \tau_e)(y - w - b) + \nu y F'(y) + \frac{1}{2}\sigma^2 y^2 F''(y), \ y_d \le y \le y_u, \tag{C.7}$$

subject to the following boundary conditions:

$$E_0(y_u) = V^*(y_u), \tag{C.8}$$

$$E_0(y_d) = 0. \tag{C.9}$$

Solving the above valuation equation, we have that the value of outside equity $E_0(y)$ is given by

$$
\begin{aligned}
E_0(y) &= (1 - \tau_e)\left(\frac{y}{r - \nu} - \frac{w + b}{r}\right) + \left[V^*(y_u) - (1 - \tau_e)\left(\frac{y_u}{r - \nu} - \frac{w + b}{r}\right)\right]\overline{q}(y) \\
&\quad - (1 - \tau_e)\left(\frac{y_d}{r - \nu} - \frac{w + b}{r}\right)\underline{q}(y).
\end{aligned} \tag{C.10}
$$

The initial outside equity issuance E_0 is then given by $E_0 = E_0(y_0)$.

D Capital gain taxes and operating leverage

First, we analyze the case where the capital gains tax is zero. Then, we extend the baseline model of Section II to allow for operating leverage.

D.1 Effects of capital gain taxes

In the presence of capital gains taxes with $\tau_g = 0.10$, the benefit from cash-out falls. Table 5 shows that the 10-year cash-out probability decreases, and the entrepreneur takes on more debt in order to diversify idiosyncratic risks. However, the quantitative effects are small in our numerical example. We may understand the intuition from the value-matching condition (9c). At the cash-out threshold y_u, when $\psi = 1$, the entrepreneur obtains less value $(1 - \tau_g)V^*(y_u)$, but enjoys tax rebate $\tau_g(K + I)$. Thus, these two effects partially offset each other, making the effect of capital gains taxes small. Clearly, if the cash-out value is sufficiently large relative to the cash-out and investment costs, then the effect of the capital gains tax should be large.

D.2 Effects of operating leverage

How does operating leverage affect an entrepreneurial firm's financial leverage? Intuitively, operating leverage increases financial distress risk, and thus should limit debt financing. Panel 3 of Table 6 confirms this intuition for the complete-markets case (the limiting case with $\gamma \to 0$). As the

Table 5: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: CAPITAL GAIN TAXES

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0, \tau_m(11.29\%)$), two capital gain tax rates ($\tau_g = 0, 10\%$), and two levels of risk aversion ($\gamma = 1, 2$). The case "$\gamma \to 0$" corresponds to the complete-market model, where the "cash-out" option effectively allows the firm to adjust leverage once.

	coupon	public debt	private equity	private firm	private leverage (%)	credit spread (bp)	10-yr default probability (%)	10-yr cash-out probability (%)
	b	F_0	G_0	S_0	L_0	CS	$p_d(10)$	$p_u(10)$
				$\tau_e = 0$, $\tau_g = 0$				
$\gamma = 1$	0.11	3.20	19.95	23.14	13.8	32	0.0	12.3
$\gamma = 2$	0.42	10.11	10.36	20.47	49.4	115	1.9	24.1
				$\tau_e = \tau_m$, $\tau_g = 0$				
$\gamma = 1$	0.54	12.29	9.92	22.22	55.3	138	3.9	15.5
$\gamma = 2$	0.66	13.57	6.47	20.04	67.7	186	9.8	26.8

operating cost w increases from 0.2 to 0.4, the 10-year default probability rises from 2.2% to 6.2%, and the firm issues less debt. On the other hand, equity value also decreases because operating costs lower the operating profits. As a result, the effect on financial leverage ratio is ambiguous. In our numerical examples, this ratio increases with operating costs.

Our analysis above shows that risky debt has important diversification benefits for entrepreneurial firms. This effect may dominate the preceding "crowding-out" effect of operating leverage. Table 6 confirms this intuition. As w increases from 0.2 to 0.4, an entrepreneur with $\gamma = 1$ raises debt with increased coupon payments from 0.59 to 0.62. However, the market value of debt decreases because both the 10-year default probability and the cash-out probability increase with w. The private equity value also decreases with w and this effect dominates the decrease in debt. Thus, the private leverage ratio rises with operating costs. This result also holds true for a more risk-averse entrepreneur with $\gamma = 2$. Note that the more risk-averse entrepreneur relies more on risky debt to diversify risk. As a result, the 10-year default probability increases substantially from 26.9% to 50.6% for $\gamma = 2$. But the 10-year cash-out probability decreases from 23.7% to 22.3%.

Table 6: The Effects of Operating Leverage: The case of debt financing and cash-out option

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau_e = \tau_m$, $\tau_g = 10\%$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two levels of risk aversion ($\gamma = 1, 2$) alongside the complete-market solution ($\gamma \to 0$).

	coupon	public debt	private equity	private firm	private leverage (%)	credit spread (bp)	10-yr default probability (%)	10-yr cash-out probability (%)
	b	F_0	G_0	S_0	L_0	CS	$p_d(10)$	$p_u(10)$
					$\gamma \to 0$			
$w = 0.2$	0.35	8.03	16.73	24.76	32.4	132	2.2	0.0
$w = 0.4$	0.33	6.72	13.40	20.12	33.4	194	6.2	0.0
					$\gamma = 1$			
$w = 0.2$	0.59	10.94	6.34	17.28	63.3	237	14.1	13.1
$w = 0.4$	0.62	9.41	3.98	13.39	70.3	356	28.4	13.5
					$\gamma = 2$			
$w = 0.2$	0.73	11.95	3.57	15.53	77.0	315	26.9	23.7
$w = 0.4$	0.84	10.48	1.57	12.05	86.9	503	50.6	22.3

References

Abel, A. B., and J. C. Eberly, 1994, "A unified model of investment under uncertainty," *American Economic Review*, 84, 1369–1384.

Andrade, G., and S. N. Kaplan, 1998, "How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Became Distressed," *Journal of Finance*, 53, 1443–1493.

Attanasio, O. P., 1999, "Consumption," *In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1B. North-Holland, Amsterdam.*

Berger, A. N., and G. F. Udell, 1998, "The economics of small business finance: The roles of private equity and debt markets in the financial growth cycle," *Journal of Banking & Finance*, 22, 613–673.

Berle, A. A., and G. C. Means, 1932, *The Modern Corporation and Private Property*, Harcourt, Brace & World, New York.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev, 2008, "The Levered Equity Risk Premium and Credit Spreads: A Unified Framework," *Working paper.*

Bitler, M. P., T. J. Moskowitz, and A. Vissing-Jrgensen, 2005, "Testing Agency Theory with Entrepreneur Effort and Wealth," *Journal of Finance*, 60, 539–576.

Black, F., and J. Cox, 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31, 351–367.

Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637–659.

Brav, O., 2009, "Access to Capital, Capital Structure, and the Funding of the Firm," *Journal of Finance*, 64, 263–308.

Brennan, M., and E. S. Schwartz, 1985, "Evaluating natural resource investments," *Journal of Business*, 58, 135–157.

Caballero, R. J., 1991, "Earnings uncertainty and aggregate wealth accumulation," *American Economic Review*, 81, 859–871.

Carpenter, J. N., R. Stanton, and N. Wallace, 2008, "Optimal Exercise of Executive Stock Options and Implications for Firm Cost," Working paper.

Chen, H., 2008, "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," Working Paper.

Cole, R. A., and J. D. Wolken, 1996, "Bank and Nonbank Competition for Small Business Credit: Evidence from the 1987 and 1993 National Surveys of Small Business Finances," *Federal Reserve Bulletin*, pp. 983–995.

Cox, J. C., and C. Huang, 1989, "Optimum Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process," *Journal of Economic Theory*, 49, 33–83.

De Nardi, M., P. Doctor, and S. D. Krane, 2007, "Evidence on entrepreneurs in the United States: data from the 19892004 survey of consumer finances," *Economic Perspectives*, pp. 18–36.

Deaton, A., 1992, *Understanding Consumption*, Oxford University Press, Oxford.

DeMarzo, P. M., and B. Urosevic, 2006, "Ownership Dynamics and Asset Pricing with a Large Shareholder," *Journal of Political Economy*, 114, 774–815.

Dixit, A. K., and R. S. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press, Princeton, N.J.

Duffie, D., 2001, *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton, 3rd edn.

Duffie, D., and D. Lando, 2001, "Term Structures of Credit Spreads with Incomplete Accounting Information," *Econometrica*, 69, 633–664.

Dumas, B., 1991, "Super contact and related optimality conditions," *Journal of Economic Dynamics and Control*, 4, 675–685.

Fischer, E. O., R. Heinkel, and J. Zechner, 1989, "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance*, 44, 19–40.

Gentry, W. M., and R. G. Hubbard, 2004, "Entrepreneurship and Household Saving," *Advances in Economic Analysis & Policy*, 4, 1053–1053.

Goldstein, R., N. Ju, and H. Leland, 2001, "An EBIT-Based Model of Dynamic Capital Structure," *Journal of Business*, 74, 483–512.

Graham, J. R., and C. Harvey, 2001, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics*, 60, 187–243.

Hackbarth, D., C. A. Hennessy, and H. E. Leland, 2007, "Can the Trade-off Theory Explain Debt Structure?," *Review of Financial Studies*, 20, 1389–1428.

Hackbarth, D., J. Miao, and E. Morellec, 2006, "Capital Structure, Credit Risk, and Macroeconomic Conditions," *Journal of Financial Economics*, 82, 519–550.

Hall, R. E., 1978, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, 971–987.

Hall, R. E., and S. E. Woodward, 2008, "The Burden of the Nondiversifiable Risk of Entrepreneurship," Working Paper, Stanford University.

Heaton, J. C., and D. J. Lucas, 2004, "Capital Structure, Hurdle Rates, and Portfolio Choice Interactions in an Entrepreneurial Firm," Working Paper, Northwestern University.

Jensen, M. C., and W. H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305–360.

Kimball, M., and N. Mankiw, 1989, "Precautionary saving and the timing of taxes," *Journal of Political Economy*, 97, 863–879.

Kimball, M. S., 1990, "Precautionary Saving in the Small and in the Large," *Econometrica*, 58, 53–73.

Krylov, N., 1980, *Controlled Diffusion Processes*, Springer, Berlin.

Leland, H., 1994, "Corporate Debt Value, bond covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.

Leland, H. E., 1968, "Saving and uncertainty: the precautionary demand for savings," *Quarterly Journal of Economics*, 82, 465–473.

Leland, H. E., and D. H. Pyle, 1977, "Informational Asymmetrics, Financial Structure, and Financial Intermediation," *Journal of Finance*, 32, 371–387.

McDonald, R., and D. Siegel, 1986, "The value of waiting to invest," *Quarterly Journal of Economics*, 101, 707–728.

Merton, R. C., 1971, "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373–413.

Merton, R. C., 1973a, "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867–887.

Merton, R. C., 1973b, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4, 141–183.

Miao, J., 2005, "Optimal Capital Structure and Industry Dynamics," *Journal of Finance*, 60, 2621–2659.

Miao, J., and N. Wang, 2007, "Investment, Consumption and Hedging under Incomplete Markets," *Journal of Financial Economics*, 86, 608–642.

Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32, 261–275.

Moskowitz, T. J., and A. Vissing-Jorgensen, 2002, "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?," *American Economic Review*, 92, 745–778.

Myers, S. C., and N. S. Majluf, 1984, "Corporate Financing and Investment Decisions when Firms Have Information That Investors do not Have," *Journal of Financial Economics*, 13, 187–221.

Petersen, M. A., and R. G. Rajan, 1994, "The Benefits of Lending Relationships: Evidence from Small Business Data," *Journal of Finance*, 49, 3–37.

Rauh, J. D., 2008, "Risk Shifting versus Risk Management: Investment Policy in Corporate Pension Plans," *Forthcoming, Review of Financial Studies*.

Schumpeter, J., 1934, *The Theory of Economic Development: An Inquiry into Profits Capital Credit Interest and the Business Cycle*, Cambridge, MA.

Strebulaev, I. A., 2007, "Do Tests of Capital Structure Theory Mean What They Say?," *Journal of Finance*, 62, 1747–1787.

Svensson, L., and I. Werner, 1993, "Nontradeal assets in incomplete markets: pricing and portfolio choice," *European Economic Review*, 37, 1149–1168.

Wang, N., 2006, "Generalizing the Permanent-Income Hypothesis: Revisiting Friedman's Conjecture on Consumption," *Journal of Monetary Economics*, 53, 737–752.