Smithian Growth through Creative Organization*

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Abstract

We consider an endogenous growth model in which appropriate organization fosters innovation, but because of contractibility problems, this benefit cannot be internalized. The organizational design element we focus on is the division of labor, which as Adam Smith argued, facilitates invention by observers of the production process. However, entrepreneurs choose its level only to facilitate monitoring their workers. Whether there is innovation and growth depends on the interaction of the markets for labor and for inventions. Because of a credit market imperfection, the relative scarcity of entrepreneurs and workers depends on the wealth distribution. A high level of specialization is chosen when the wage share is low, i.e. when there are few wealthy. But in this case there are also few entrepreneurs and a consequent small market for innovations, which discourages inventive activity. When there are many wealthy, the innovation market is large, but the rate of invention is low because there is little specialization. Sustained technological progress and economic growth therefore require only moderate levels of inequality. The model also suggests that the growth rate need not be monotonic in the "quality of institutions," such as the degree of credit market imperfection.

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1 Introduction

Is technological progress born of proper organization? There are good reasons to believe that it is. Like other human endeavours, invention depends on incentives. The organization of firms, nature of contracting arrangements and enforcement of property rights all influence the production of inventions, much as they do for other goods.[1] Apart from the question of what forms of organization best promote invention, which for a full answer must depend on fields outside economics (e.g. cognitive and social psychology, industrial engineering), this observation raises the more narrowly economic questions concerning the conditions under which such creative organizations actually get adopted.

Economists have recently begun to address these questions, particularly for the case in which invention happens to be the defining objective of a firm (e.g., research activity in R&D laboratories, joint ventures formed to create a new product). In this case, the organization is designed to maximize the value of the inventions produced at least cost, and the firm will choose an allocation of control rights over the research process, property rights over resulting inventions, and whether to integrate the research team into the firm or subcontract it out (Aghion-Tirole, 1994).

A broader look at the economic growth process suggests, however, that such firms account for only a fraction of historical technological progress. In fact, organized R&D was rare before the twentieth century, and even today is confined to only a relatively small fraction of firms. In this paper we shall be investigating the possibility of technological progress as an *unintended* consequence of organizational design.

The industrial revolution provides a useful backdrop for examining the issues we have in mind. A distinguishing feature of the period was the rise of the factory system, in which production was carried out by workers gathered under one roof, with strict supervision, rigid discipline, and most important perhaps, a division of labor. There seems to be little consensus among economic historians on just what role the factory system actually played in fostering the rapid technological advances and economic growth that also characterized the era. Some commentators, like Landes

[1] This perspective has its most forceful exponents among scholars of "new growth theory," e.g., Romer (1986, 1990) and Aghion-Howitt (1992), but there are antecedents in the industrial organization literature, e.g Kamien-Schwartz,(1976) and Loury (1979).

(1969), seem to argue that the factory was largely epiphenomenal, merely an optimal organizational response to exogenous technological change. Others, like Cohen (1981), Millward (1981), and North (1981), suggest that it was the enhanced efficiency of the factory system itself relative to earlier forms of organization that generated greater surpluses, though it is difficult to see how this by itself could plausibly translate into increased rates of innovation and growth.

A third view, attributable to Adam Smith, affirms a causal role for the factory system. It places the emphasis less on its static benefit of making better use of current inputs to produce current output, than on a *dynamic* one: the factory, and in particular the division of labor into elementary tasks, provides a superior environment to inspire invention and refinement of productive techniques – but primarily for persons, such as workmen or outside observers, other than the factory's owner. By focusing an individual's attention, it makes it easier for him to improve on old techniques. Alternatively, and complementarily, by providing a "model" in human form of elementary tasks, it facilitates the development of machines that can better perform those tasks.[2]

Now, there is little evidence that anyone during the industrial revolution ever built a factory because he expected it to help him innovate. Given the nonrival nature of ideas and the difficulties in excluding them for long, this should not be surprising. Patent protection was only just coming into force in a few countries the mid-eighteenth century. Even in Britain, the first industrial nation, the patent system was hardly optimally suited for foster invention: patents were expensive and the right went not to the inventor but simply to the first person to file (Khan and Sokoloff, 2004). The incentives to invest in a creative organization – whether a factory with specialized labor or an R&D laboratory – would have been weak, given the large number of potential witnesses to an idea (both within the firm and without) and the concomitant uncertainty of who would benefit. Contracts preventing employees from quitting firms and starting their own enterprises based on ideas learned while in their

[2]From the *Wealth of Nations*: "I shall only observe, therefore, that the invention of all those machines by which labor is so much facilitated and abridged seems to have been originally owing to the division of labor. Men are much more likely to discover easier and readier methods of attaining any object when the whole attention of their minds is directed towards that single object that when it is dissipated among great variety of things."

previous firms were (and still are) hard to enforce (how does one prove where another got an idea?) and in any case would not help against those who might observe the production process from outside.[3]

Thus the creative role of the division of labor could only be harnessed via some other economic mechanism that would have induced the widespread adoption of the factory and the concomitant surge of technical progress. Fortunately, the division of labor had other benefits, as Smith himself enumerated. Among them was the enhanced ability to monitor workers, as many writers have noticed: a worker assigned to only a small number of tasks will be less able to disguise shirking as downtime between tasks, or to find opportunities to embezzle either inputs or outputs undetected. Unlike the invention benefit, the monitoring benefit of divided labor is a private one, appropriable by the entrepreneur.

Our analysis focuses on the possibility of endogenous growth driven by these dual features of the division of labor. Starting from a standard dynamic occupational choice model (e.g. Banerjee-Newman, 1993), we construct a model of endogenous growth and technical progress borne of organizational choices and investments in innovation. The economy has overlapping generations; agents make investments when young and earn incomes in the labor market when old. Individuals can invest in an education that enables them to become entrepreneurs who run firms, but the credit market for education is imperfect, and this option is therefore limited to the wealthy; the rest of the population become workers who are employed by the entrepreneurs. As in the earlier occupational choice models, the distribution of wealth affects the competitive wage via the relative scarcity of entrepreneurs and workers.

Entrepreneurs choose the degree of labor division. A large number of complementary tasks are needed to produce the economy's consumption good; a worker may be assigned to perform any or all of them. A high degree of specialization simply means that each worker is responsible for only a small fraction of the total number of tasks.

Dividing labor is costly because it requires resources to coordinate and assemble

[3] Even if they could invoke such laws, entrepreneurs would have had a difficult time eliciting inventive effort from employees who, if not held up completely, could expect only a fraction of the surplus from their inventions. For this reason, as recent comparisons (Saxenian, 1996) of Silicon Valley in California and Route 128 in Massachusetts suggest, it is anything but clear that such "non-compete" clauses are socially desirable.

the components produced by each worker (see for instance, Becker and Murphy, 1992; or Acemoglu et al., 2005, which offers a somewhat different account of the costs of dividing labor). As we discussed, it confers two benefits that we emphasize in the model. First, it enhances the ability of the entrepreneur to monitor her workers. The fewer tasks the worker is responsible for, the more likely it is he can be detected if he is not doing what he's supposed to do. This monitoring capability directly benefits the entrepreneur, as it allows her to reduce the size of her wage bill while ensuring that her workers do not shirk.

Secondly, it increases the arrival rate of productivity-increasing ideas (inventions). This benefit is largely external to the particular entrepreneur who divides her labor force; indeed we model the invention process in the simplest possible way by supposing that *any* young individual may attempt to improve one of the tasks by engaging in costly "cogitation," i.e., studying the production process in the economy at large.[4] An individual will focus on a subset of tasks, corresponding to the scope of responsibility of a typical worker. The supposition is that the more finely divided the tasks are, the easier it is to discern a way to improve one of them.

The condition of the labor market determine an entrepreneur's organizational choice: when wage shares are high, she need not monitor intensively (workers have much to lose if caught, so only a small detection probability is needed), and labor division is fairly coarse.[5] When instead wage shares are low, labor is more finely divided. Since wage shares are high when there is a small fraction of poor, innovation will be difficult when the fraction of wealthy is large.

This result would seem to imply that poverty enhances growth. But there is a countervailing effect. The decision a youth makes to cogitate or vegetate depends on the expected income that accrues to an inventor. We make a "free enterprise"

[4]In fact during the Industrial Revolution inventors came from all walks of life. Many were unaffiliated with any firm, some (as Smith avers) were workmen in factories, and others drawn from the supervisory or managerial ranks. Few had rare technical expertise or extensive financial resources. For instance, for the U.S. in the period 1790-1846, Khan and Sokoloff (1990) show that more than 70% of them are discovered by individuals who were not professional inventors (i.e. that in their career discovered only one innovation).

[5]This relationship between wage and monitoring levels has also appeared in the efficiency wage literature, e.g. Acemoglu-Newman (2002).

assumption: anyone who comes up with an idea has property rights on it, and is therefore able to sell it to all takers. All ideas that are generated in one period are sold on an innovation market to the entrepreneurs at the beginning of the next period.[6] The investment decision for a would-be inventor will depend on the extent of the innovation market: the larger it is, the more revenue is available (since ideas are nonrival but temporarily excludable, the inventor's revenue is proportional to the number of entrepreneurs).

The fraction of poor therefore affects the innovation market as well as the labor market through a *market size effect*: when there are many poor, the number of buyers of inventions is small, and though it may be easier to invent, the revenue generated will be too small to justify the effort; the innovation market shuts down.[7] At the other extreme, a large demand for innovations, which comes when there are many rich agents, might be undone by the difficulty of inventing when workers are correspondingly not specialized because of high wages.

So the wealth distribution will determine the level of innovative activity: too many wealthy, and invention is too difficult; too few, and invention is not worth the trouble.[8]

[6] An idea takes time to diffuse. In the meantime it is excludable (this could be due to patent protection or because it takes time for the inventor to demonstrate to entrepreneurs how the idea is implemented).

[7] Something like this size effect can be gleaned by comparing industries in eightheenth century Britain. Watchmaking had a fine division of labor going back at least a century earlier, but it served only a small (luxury) market, and thus never experienced the high levels of innovation that affected other industries such as cotton and steel (Mathias, 1983).

[8] There is a second aspect to the revenue accruing to an inventor in the innovation market. Because of the complementarity of tasks, the willingness of an entrepreneur to pay for an invention depends on how many ideas are being sold in the market: the more there are in total, the more valuable each idea is (at any time there is a large number of entrepreneurs and inventors, so no one exercises power in either the product or labor markets). This introduces a strategic complementarity that leads to a potential coordination problem among the inventors: even if the number of entrepreneurs is high enough to admit an equilibrium in which everyone cogitates, there may also be an equilibrium where everyone vegetates.

Since we are interested in the possibility rather than the inevitability of growth, we take the optimistic tack of assuming that this coordination problem is always solved. It is clear though that in this model there is also a role to play by expectations: for instance an ethos of optimism and individual initiative can be self–fulfilling. The role of beliefs in coordinating growth is an already much-addressed issue in the literature (e.g. Murphy et al., 1989; Grief, 1994; Benabou-Tirole, Only at moderate levels of inequality can high rates of innovation be sustained.

Thinking dynamically, under a set of assumptions about parameter values that ensure that the incomes of entrepreneurs, inventors, and high-wage workers are large enough to finance their children's schooling while those of low paid workers are not, we can demonstrate the existence and local stability of steady states in which this relation between the distribution of wealth the rate of innovation and growth is maintained in the long run. An economy initially with many poor will tend toward collapse. One with many rich will make technological change slow or nonexistent, though it may appear statically affluent, with a high wage share and few resources lost to coordinating divided labor. Moderate levels of inequality, however, keep wage shares low so that specialization is high and ideas arrive easily, and at the same time provides enough of a market for them to induce people to invent. In short the model predicts an inverted U-shaped relation between the degree of inequality (more precisely, the population fraction that can afford to become entrepreneurs) and the rate of technological progress and economic growth.[9]

This result is suggestive of a possible explanation for the venerable economic historians' conundrum of why Britain among European nations was first to industrialize. Compared with some of its continental counterparts (notably France), in the late eighteenth century both had similar levels of technology, some form of patent system, free labor markets, and (as in our model) only rudimentary and imperfect credit markets. Yet France remained a nation of family farms and small enterprise for several decades, while Britain rapidly became a nation of factories and the seat of the Industrial Revolution (Deane, 1965; Shapiro, 1967; O'Brien and Keyder, 1978; Crafts, 1985; Crouzet, 1990; Mokyr, 1990). One difference was the distribution of wealth, which following the Revolution in France and the earlier enclosure movement in Eng-

forthcoming), which is why we suppress it here.

[9] Other "inverted U" relationships involving the rate of technical progress have been discussed in the literature, though not with respect to the level of inequality so far as we are aware. For instance, Aghion et al. (2005) finds such a relationship between innovation and the degree of competition (see also Mookherjee-Ray (1991) and Lai (1998), which consider market size and diffusion rates as well as degree of competition). These studies have been partial equilibrium in nature, with innovation endogenous and competitiveness or market size exogenous. In our case, inequality not only governs the innovation rate, but also is influenced by it, since it is determined in part by the incomes accruing to the inventors.

land was rather more unequal in England (Clapham, 1936; Grantham, 1975; Soltow, 1980). Meanwhile, many of the poorer (and more unequal) countries of Europe were even slower to industrialize.

The model has other implications for growth theory more generally besides the link it draws between inequality and technological progress. Raising the wage share "artificially," say via trade unions or minimum wage policy will tend to operate much like an increase in the fraction of wealthy. Though not typically Pareto improving, it can enhance static efficiency measured in terms of GDP because it reduces the coordination costs of the dividing labor. But for this very reason it will tend to slow the rate of technical progress and growth, so that after a while the static gains are swamped by the dynamic losses.[10]

Second, "institutional" improvements, such as the provision of public education (which reduces its private marginal cost) or the increased efficiency of credit markets need not have monotonic effects on the rate of technological progress. Both of these changes are akin to increase in the fraction wealthy; thus starting from very high cost education or very poorly functioning markets, both static and dynamic efficiency are likely to improve as output increases and the demand for inventions increases enough to activate the innovation market. But further improvements to these institutions will eventually reduce the division of labor and therefore the rate of technical progress and economic growth. An economy with moderately well functioning credit markets that has been rapidly growing a while will have higher productivity than one with perfect credit markets that has been growing slowly or not at all. It follows that total factor productivity need not be monotonic in the "quality of institutions," either over time or in cross section.[11]

[10] This conflict between static and dynamic effciency distinguishes ours from some more aggregative models of endogenous growth, such as the AK model.

[11] The model thereby also offers a mechanism for "reversal of fortune" phenomena that have been documented in historical cross-country comparisons of economic prosperity (e.g. Acemoglu et al., 2002).

2 The model

Consider a closed, one-consumption good, overlapping generation economy with a single perishable consumption good. Time is discrete, indexed by $t = 0, 1, 2, ...$ Agents live for two periods, youth and old age. At each date there is a continuum of agents, with a unit measure each of youth and old. Every individual is endowed with a unit of productive effort, a unit of cognitive effort, and some wealth in the form of the consumption good.

All individuals born at time t have preferences characterized by the utility

$$U^t(c_t^t, c_{t+1}^t, b_{t+1}^t) = c_t^t + \gamma(c_{t+1}^t)^{1-\beta}(b_{t+1}^t)^{\beta},$$

where c_t^t is the consumption in youth, c_{t+1}^t is the consumption in old age, b_{t+1}^t is a bequest passed on to the individual's child, $1 > \beta > 0$, and $\gamma = \beta^{-\beta}(1-\beta)^{\beta-1}$. Indirect income utility is therefore $y_t + y_{t+1}$, where y_t is income in period t.

These preferences imply that individuals contribute a fraction β of that income to their offspring (this becomes the offspring's inherited wealth), and have no strict incentive to borrow for consumption. Perishability implies there is no instrument of saving either, apart from education loans.[12] To simplify matters, we will make assumptions below that rule this instrument out as well. Thus all inherited wealth and first period income (net of schooling expenditures) is consumed in youth, so that all agents begin the second period of life with a zero stock of the consumption good. This structure simplifies the dynamic analysis below because it will involve consideration only of current income and not also of accumulated wealth.

2.1 Production

2.1.1 Technology

Production of the consumption good involves a unit measure of tasks or jobs indexed by $j \in [0, 1]$.The labor productivity at time t for job j is $a_t(j)$ and output is

[12]A positive demand for consumption loans requires that the interest rate be no greater than unity. Lending or saving requires that the interest rate be no less than unity. Any imperfection that drives a wedge between the borrower's and the lender's rate will prevent any borrowing from occurring in equilibrium. Obviously, nothing changes if we assume other values for γ.

$\exp(\int_0^1 \log[a_t(j)l_t(j)]dj)$, where $l_t(j)$ is the labor allocated to job j at t. Given this technology, labor is uniformly allocated over all jobs independently of $a_t(\cdot)$. Output per unit labor is therefore $A_t = \exp(\int_0^1 \log a_t(j)dj)$.

Technical progress follows a "quality ladder" (Grossman-Helpman, 1991). An invention improves the productivity of a single task by the multiplicative factor $(1+\gamma)$, transforming $a(j)$ to $(1+\gamma)a(j)$; there is no limit to number of times j can be improved over time. For simplicity, we suppose that j might get multiple improvements in a single period t; call the number of improvements $m_t(j)$; then define $a'_t(j) = (1+\gamma)^{m_t(j)}a(j)$.[13]

At the beginning of period t, the publicly available technology is

$$A_t = \exp(\int_0^1 \log a_t(j)dj),$$

and the state of the art, which will be employed only by entrepreneurs, is $A'_t = \exp(\int_0^1 \log a'_t(j)dj)$. The state of the art technology diffuses completely after production to become the public technology next period. If we let $m_t = \int_0^1 m_t(j)dj$, productivity growth is $A'_t/A_t = A_{t+1}/A_t = (1+\gamma)^{m_t}$.

All variables – wages, profits and outside options – will be expressed as multipliers of A'

2.1.2 Occupations

At the beginning of an agent's youth, his parents pledge to him a bequest, and the youth goes to school. Schools come in two varieties: a basic school, where students learn the basic skills to become workers and "university," where students learn more advanced techniques that will give them a chance to be entrepreneurs.

While in university, students learn whether they have the talent to be entrepreneurs. Students are successful (i.e., have the requisite talent) with probability $\alpha \in (0,1)$ and unsuccessful otherwise. Thus, university is necessary but not sufficient

[13] Making the alternate assumption that only one improvement can occur per period, entails allowing for the possiblity of duplication by several inventors. In that contingency, their rents are reduced (possibly to zero). This complicates the computation of the return to cogitation, but changes relatively little in terms of conclusions.

for being an entrepreneur. Ability realizations are independent across individuals.[14]

While basic schooling is free, there is an extra cost hA' for going to university.[15] Because of financial market imperfections, schooling must be financed during the period and students cannot vouch their future earnings. Therefore only young agents whose parents have pledged at least hA' can elect to become university students. We shall make parametric assumptions that guarantee that *all* youths attend university who can afford to do so. We shall spell these out below.

All successful graduates of university are potential entrepreneurs, who hire a fixed number $n \geq 2$ of workers and manage them, where we assume that $\alpha(n+1) > 1$. Such a graduate might instead choose to operate the technology on his own as a small entrepreneur (or "artisan"). Before deciding whether to be an entrepreneur or artisan, he participates in the market for inventions and purchases m inventions at cost q leading to an innovated technology of $A(m)$; m, q and $A(m)$ will be derived below.

2.1.3 Contractibility Assumptions

We make the following contractibility assumptions, some of which we have already discussed.

- Worker effort is not perfectly observable.
- Cogitation is not observable.
- The source of ideas is not attributable (for instance, there are "glass factories"), hence entrepreneurs cannot claim ownership of them.
- There is a credit market imperfection: loans between periods do not exist, and school is affordable only if $\beta y \geq hA'$, where y is the parent's current income.

[14] This assumption guarantees some downward mobility and avoids the triviality that everybody converges to the same wealth level.

[15] The cost of education is proportional to A' so that the credit constraint has the potential to remain binding as the economy grows. This can be justified by supposing that there is no technical progress in education, and the teacher's opportunity cost is to be a worker or entrepreneur, in which case his earnings would be proportional to the index A'. See Banerjee-Newman (1994) or Mookherjee-Ray (2002) .

2.1.4 Division of Labor and Labor Contracting

On the labor market, entrepreneurs offer contracts (σ, w) consisting of a *degree of specialization* σ and a normalized wage w. The set of jobs can be subdivided into a number of (equal-size) subsets that we call *components.* Denoting the number of these components by σ, each component contains $1/\sigma$ jobs; hence $\sigma = 1$ corresponds to the early manufacturing days where artisans were put under the same roof but continued to do all the jobs involved in producing the good, while $\sigma \geq 2$ may correspond to an assembly line system. Workers are specialized in producing individual components; given the production technology, in order to produce a unit of the good, it is necessary to combine one unit of each of the σ components.

When $\sigma = 1$, a worker spreads his unit of labor time uniformly over all the jobs; hence a worker has to spend $1/\sigma$ of units of labor time to produce one unit of a component consisting of $1/\sigma$. Absent coordination problems, it does not make a difference in terms of total output whether each of n workers does all the jobs (is completely unspecialized), or is σ-specialized, with n/σ workers assigned to each component and producing σ components each: either way, output is nA'.

However, as in Becker and Murphy (1992), we assume that specialization generates coordination problems. For instance, in an assembly line, each worker has to spend time taking the component from the previous worker in line, assembling it with its own component and passing everything to the next component. In many firms producing complex products, seamless integration between components often requires a large number of meetings, reducing time available for production. When there are σ components, each worker specialized in one component will have to spent time coordinating with $\sigma - 1$ producers of the other components and the cost in time units is $c(\sigma - 1)$, with $c > 0$. Hence, total time available for production is now only $1 - c(\sigma - 1)$. It follows that a worker can produce $\sigma(1 - c(\sigma - 1))$ components. With n workers, n/σ workers are assigned to each component and total output is $n(1 - c(\sigma - 1))A'$.[16]

[16] Note that we assume that there is no *net* output gain from specialization (unlike e.g., Costinot, 2005); that is, we consider situations where the cost of coordination dominates the productivity gains from specialization. If we follow Smith and view specialization as first leading to large productivity gains but then leading to smaller productivity gains, thet net effect of specialization on productivity

While specialization generates a coordination cost it also yields a monitoring benefit. This is why specialization can arise: while inefficient from a total output point of view, it allows the entrepreneur to pay lower incentive wages to the workers, and therefore to get a larger payoff for himself. In our model, shirking takes the form of workers engaging in a *sideline activity* – having return $\mu A'$, $\mu \leq 1$. It is not possible to distinguish a worker doing job j for the firm or for himself. A worker assigned to a component consisting of $1/\sigma$ jobs will spend only $1/\sigma$ of his time on jobs that are part of that component if he shirks, whereas he spends all his time on the component if he works. Random monitoring will therefore detect shirking with probability

$$d(\sigma) = 1 - \frac{1}{\sigma}.$$

If the worker is detected shirking, it is optimal to punish him maximally: he loses both his wage and the "booty" $\mu A'$. Hence, shirking yields a benefit of $(1 - d(\sigma))(w + \mu) A'$ while working yields wA'. It follows that the worker will work when the following incentive compatibility condition is satisfied

$$w \geq \frac{\mu}{\sigma - 1}. \tag{1}$$

Incentive compatibility is independent of the degree of technology used within a firm; the set of feasible contracts available to an entrepreneur is the set of pairs (σ, w) satisfying (1).

From the perspective of the entrepreneur, the lower output brought by specialization is the price to pay for having a larger share. Statically, high degrees of specialization reduce aggregate output: a larger wage share would result in greater output, but smaller profits for entrepreneurs. However, from a dynamic perspective, specialization may enhance growth of aggregate output insofar as it facilitates innovation.

We assume that the parameters μ and c satisfy

$$\sqrt{c\mu} < 1/4, \tag{2}$$

is first positive and then negative as specialization increases as long as coordination costs do not decrease with specialization. Since the monitoring benefit increases with specialization, it follows that in the first region it is profit maximizing to increase specialization. $\sigma = 1$ in our model can then be reinterpreted as the maximum value of specialization for which there are net output gains from specialization.

which implies that entrepreneurship is preferable to artisanship over a range of wages and that the entrepreneur's profit is convex in the number of inventions he purchases, a fact that will be useful in our analysis of the market for innovations.

2.1.5 Innovation

While entrepreneurs will choose the degree of specialization in by optimizing over the tradeoff between coordination costs and monitoring benefits, specialization enables third parties to easily find ways to improve the productivity of tasks. A specialized worker, for instance, is focused on just a few tasks and has more time to think about each one and to learn its attributes and see where it might be improved than does a worker who has many things to do.

For modeling purposes, we think of inventions as coming not from workers (though it would be straightforward if somewhat more algebra-intensive matter to allow them to do this by substituting work in youth for regular school), but by outside observers. Someone studying production processes from the outside has a clearer signal of just what one job's features are when the workers who perform it are highly specialized: a worker who has just a few tasks becomes a human model for a machine that might be invented to replace him.

An individual can generate an idea on how to improve one job at most. While a youth, any agent, whether in university or not, may study and tinker with the existing technology in firms at a cost $\theta A'$ in current consumption (θ might be a fraction of time devoted to cogitation instead of consumption). He randomly selects a component for study and arrives at an idea for improvement to one of the tasks in the component with probability $p(\sigma)$, where $p(\cdot)$ is an increasing function of σ; this arrival probability is independent of managerial talent. If an invention is obtained, the agent becomes active on the invention market at the beginning of the next period, and can charge a license price for his invention, before taking his place on one side or the other of the labor market.[17]

If the total measure of innovations in a period is m, and an entrepreneur purchases

[17] It would not substantially alter our results to modify the model to assume heterogeneous arrival rates, dependent for instance on whether one is in university, or to allow youthful agents to become workers instead of students and to tinker with the technology available in their factory.

all of them (below we verify that this is the case), then the technology employed by that entrepreneur is

$$A(m) = A(1+\gamma)^m. \tag{3}$$

3 Markets and Prices

In each period, three markets operate: education, innovation, and labor. We have already described the functioning of the education market in section 2.1.2. We think of the innovation and labor markets as operating sequentially in that order. The demand side of the labor market consists of smart, educated old agents (entrepreneurs and artisans) and the supply side are uneducated or low ability old agents who become the workers (employed or unemployed). The innovation market consists of entrepreneurs and artisans on one side and inventors on the other. Inventors are the same age as entrepreneurs and will later be entering the labor market on one side or the other, depending on their educational background (this is a simplifying assumption: if selling inventions was an occupation distinct from being a worker or entrepreneur, the opportunity cost of innovating would need to be taken in to account; this would tend to reinforce the effects we identify.)

To calculate the general equilibrium in these markets, it helpful to work backward.

3.1 Labor Contracts

Suppose that an entrepreneur has purchased m inventions and has technology $A(m)$ at the time the labor market opens. Since the profit is $n(1-c(\sigma-1)-w)A(m)$, a profit maximizing contract will bind the incentive compatibility condition (1) and the maximum possible profit is achieved at $(\overline{\sigma}, \underline{w}) = (1+\sqrt{\frac{\mu}{c}}, \sqrt{c\mu})$.Whether the entrepreneur achieves this profit depends on the condition of the labor market: if there is unemployment, he can always find would-be workers who are receiving zero and can therefore hire them and expect them to work for the wage $\underline{w}$. If there is full employment, however, every entrepreneur must guarantee his workers the equilibrium market payoff u^*, and the normalized wage must satisfy

$$wA(m) \geq u^*. \tag{4}$$

Hence the profit is

$$\pi\left(A\left(m\right),u^{*}\right)=\begin{cases} n\left(1-2\sqrt{c\mu}\right)A\left(m\right) & \text{if } \sqrt{c\mu}A\left(m\right)\geq u^{*} \\ n\left(1-c\frac{\mu A(m)}{u^{*}}-\frac{u^{*}}{A(m)}\right)A\left(m\right) & \text{if } \sqrt{c\mu}A\left(m\right)\leq u^{*}. \end{cases}$$

The agent will actively demand labor only if entrepreneurship is at least as attractive as artisanship, that is, only if $\pi\left(A\left(m\right),u^{*}\right)\geq A\left(m\right)$, which places an upper bound $\bar{w}$ on the equilibrium wage share and a corresponding lower bound $\underline{\sigma}$ on the degree of specialization.[18] Notice that (2) coupled with $n\geq 2$ implies $\bar{w}>\underline{w}$.

Thus a sufficient parametric condition that all agents who can afford university will want to go there is

$$\alpha(1-\bar{w})>h; \tag{5}$$

we impose this condition throughout.[19]

3.2 Revenue from Innovations

We now describe the market for innovations. Let e be the measure of potential entrepreneurs (equal with probability one to α times the previous period's enrollment of university students), A the index of the technology available at the beginning of the period, and u^{*} the labor market equilibrium worker payoff. There is a measure ι of inventors, each having one idea to license.

From the previous subsection, the return to a potential entrepreneur (i.e. a successful university graduate) who acquires m innovations when the labor market gives an outside option of u^{*} to the workers is

$$V\left(m,u^{*}\right)=\max\left\{\pi\left(A\left(m\right),u^{*}\right),A\left(m\right)\right\}. \tag{6}$$

If he uses the publicly available technology, his return is $V(0)$. It is straightforward to see that $V\left(m,u^{*}\right)$ is an increasing function of m, so that each entrepreneur would like to acquire all available innovations.

[18] Namely, $\bar{w}=\frac{1}{2}(\frac{n-1}{n}+\sqrt{(\frac{n-1}{n})^{2}-4c\mu})$ and $\underline{\sigma}=1+\frac{n-1-\sqrt{(n-1)^{2}-4n^{2}c\mu}}{2nc}$.

[19] If (5) fails, then the maximum equilibrium wage will be given by the condition that agents are indifferent between going to university or not; the subsequent analysis will be similar to what we carry out, but slightly more intricate.

We imagine that each inventor visits each entrepreneur once, and makes a take it or leave it offer, teaching him the idea in exchange for a price. Since every inventor has a monopoly on his particular idea, he extracts all of the surplus from each entrepreneur. An entrepreneur is willing to pay up to $V(\iota, u^*) - V(0, u^*)$ for the set of available inventions.

An entrepreneur's valuation of a single invention depends on how many other inventions she has already purchased. Thus, an inventor's position among the inventors lined up to sell to a particular entrepreneur will determine how much he can extract. If his position in line is uniformly and independently distributed across the population of entrepreneurs, the expected revenue accruing to an inventor will almost surely be

$$e\frac{(V(\iota, u^*) - V(0, u^*))}{\iota} \equiv eq(\iota, u^*). \tag{7}$$

In the Appendix we establish that under assumption (2), V is convex in ι and therefore that q is increasing in ι. It is straightforward to see that q is proportional to A (the current publicly available technology) and therefore bounded above given u^* and A.

3.3 Number of Innovators

Now ι is itself the outcome of cogitation decisions taken in the previous period. Because $q(\iota, u^*)$ is increasing, there will often be multiple equilibria in the game among would-be inventors: the more people cogitate, the more valuable it is to cogitate. If all cogitate, the measure of inventions is $p(\sigma)$; if none do it is of course 0.

Suppose $k \in [0,1]$ of the population choose to cogitate when the prevailing degree of specialization is σ. The expected number of inventions is then $kp(\sigma)$ and, since the probability of being successful in university is independent of being successful at inventing, the expected revenue accruing to one cogitator is almost surely $p(\sigma)\, eq(kp(\sigma), u^*)$. Cogitation is chosen only if it weakly exceeds the cost θA, where A is the current public technology A (the same as last period's state of the art: $A_t = A'_{t-1}$).

If $p(\sigma)\, eq(0, u^*) < \theta A$ but $p(\sigma)\, eq(p(\sigma), u^*) \geq \theta A$, it is an equilibrium to have all agents cogitate $(k = 1)$, and it is also an equilibrium to have all agents vegetate $(k = 0)$. The first Pareto dominates the second. As we discussed in the Introduction, we will focus on the cogitation equilibrium when it exists. (There will also be a third equilibrium in which agents mix between vegetating and cogitating, but because $q(\cdot, u^*)$ is strictly increasing, this has no counterpart in any finite approximation to the continuum economy, and in any case is also Pareto dominated, so we ignore it too.) Details of this derivation are in the Appendix.

If $p(\sigma) eq(0, u^*) > \theta A$, then of course the only equilibrium is for everyone to cogitate, while if $p(\sigma) eq(p(\sigma), u^*) < \theta A$, then everyone vegetates.

If e is small, the only equilibrium will be $k = 0$, while a necessary condition for $k = 1$ is that e be large enough. Thus, we have a market size effect for innovation: the larger the number of entrepreneurs, the more likely that there will be innovation, simply because a large market enhances the return to inventive activity.

To be more precise about this requires pinning down the profits made by entrepreneurs, which also affect the revenue from invention, and this in turn depends on the labor market. So full determination of the extent of innovation entails consideration of the general equilibrium of innovation and labor markets.

3.4 Labor and Invention Markets in Dynamic General Equilibrium

Let S_t be the fraction of young agents who attend university in period t. Then $e_t = \alpha S_{t-1}$. The labor market equilibrium at t is therefore determined by S_{t-1}: the supply of labor is $1-\alpha S_{t-1}$, while the demand is $n\alpha S_{t-1}$. Therefore today's wage is a function of yesterday's measure of university students.

S_{t-1} and S_{t-2} affect the innovation market at t. We have already noted that S_{t-1} determines the size of the demand for innovations at t. It also determines the wage and profit at t, and therefore the value of innovations. However while a high price for inventions is important for an active invention market, it is also crucial that the probability of becoming an inventor is not too small. This probability depends on the degree of specialization σ_{t-1}, which by our previous observation depends on S_{t-2}, the number of students in university at $t-2$. Once the wages, profits and innovation equilibrium are known at t, we have all the information needed to predict the future course of the economy, which is therefore fully described by a second-order dynamical system with S as state variable.

A first observation is that the labor market at t generically in a condition of "excess demand" ($n\alpha S_{t-1} > 1 - \alpha S_{t-1}$) or excess supply ($n\alpha S_{t-1} < 1 - \alpha S_{t-1}$). In the first case, $w_t = \bar{w}$, and some of the successful university graduates will become artisans; in the latter case, $w_t = \underline{w}$, and some agents (other than successful graduates) will be unemployed. Only if $n\alpha S_{t-1} = 1 - \alpha S_{t-1}$, i.e. $S_{t-1} = \frac{1}{\alpha(n+1)} \equiv \bar{S}$ could the wage assume an interior value. (Note from our assumptions in Section 2.1.2 that $\bar{S} < 1$.) From the dynamics section it will be clear that $S_t = \bar{S}$ is nongeneric, and we therefore ignore this possibility in what follows.

In equilibrium, all entrepreneurs operate with the state-of-the-art technology A'. If $w = \bar{w}$, then an entrepreneur with the state of the art technology is indifferent between operating at large and small scale. Anyone deviating to the older technology (i.e. forgoing the innovation market) would then strictly prefer to operate at small scale, since the wage share he would pay would have to exceed $\bar{w}$ in order to induce workers to participate (if $\bar{w} = u^*/A'$, $w_0 = u^*/A > \bar{w}$). Thus, if u^* is such that $w = \bar{w}$, $V(m, u^*) = A(1+\gamma)^m$ and $V(0, u^*) = A$. On the other hand, if $w = \underline{w}$, then even an entrepreneur with the old technology can always find workers who are willing to work and be incentive compatible at $\underline{w}$. In this case, $V(m, u^*) = A(1+\gamma)^m n\left(1 - 2\sqrt{c\mu}\right)$ and $V(0, u^*) = An\left(1 - 2\sqrt{c\mu}\right)$.

Thus, the condition as to whether the innovation market will be active at t can be written

$$\alpha S_{t-1}[(1+\gamma)^{p(\sigma_{t-1})} - 1]\pi \geq \theta,$$

where $\pi = n\left(1 - 2\sqrt{c\mu}\right)$ or 1 depending on whether there is excess supply or excess demand in the labor market.

We can now summarize how S_{t-1} and σ_{t-1} affect equilibrium at t in the following

Proposition 1 *Let S_{t-1} be the fraction of university students and σ_{t-1} the degree of specialization at $t-1$. Then in period t:.*

(i) If $S_{t-1} < \frac{1}{\alpha(n+1)}$, the equilibrium contract is $(\overline{\sigma}, \underline{w})$, and the market for inventions is active only if

$$\alpha S_{t-1}[(1+\gamma)^{p(\sigma_{t-1})} - 1]n\left(1 - 2\sqrt{c\mu}\right) \geq \theta \tag{8}$$

(ii) If $S_{t-1} > \frac{1}{\alpha(n+1)}$, the equilibrium contract is $(\underline{\sigma}, \overline{w})$, and the market for inventions is active only if $\alpha S_{t-1}[(1+\gamma)^{p(\sigma_{t-1})} - 1] \geq \theta$.

It will be helpful to define two further values for S. Suppose the previous period's degree of specialization was $\bar{\sigma}$; let S_0 be the minimum number of university students that allows a currently active invention market:

$$S_0 = \frac{\theta}{\alpha(1-2\sqrt{c\mu})n((1+\gamma)^{p(\bar{\sigma})}-1)}.$$

Similarly, let S_1 be the corresponding level of S when current *and* previous wages are $\bar{w}$:

$$S_1 = \frac{\theta}{\alpha((1+\gamma)^{p(\underline{\sigma})}-1)}$$

Obviously, $S_0 < S_1$, but the relationships between S_0 or S_1 and $\bar{S}$ are ambiguous.

4 Rudimentary Dynamics

A full characterization of the dynamics of the model is beyond the scope of this paper. However, a number of interesting conclusions can be derived by limiting attention to some special parametric cases.

We therefore make the following simplifying assumptions.

h1 $p(\underline{\sigma}) < \frac{\log(1+\theta/\alpha)}{\log(1+\gamma)}$, i.e., $S_1 > 1$: it follows that the invention market is not operative if the previous period's specialization is $\underline{\sigma}$, that is if $S_{t-1} > \overline{S}$.

h2 $p(\overline{\sigma}) > \frac{\log\left(1+\frac{\theta(n+1)}{n\left(1-2\sqrt{c\mu}\right)}\right)}{\log(1+\gamma)}$, i.e., $S_0 < \bar{S}$: the invention market can be operative if specialization at $t-1$ is $\overline{\sigma}$, that is if $S_{t-1} < \overline{S}$, as long as $S_{t-1} \geq S_0$. If $S_{t-1} < S_0$, the small number of entrepreneurs makes the expected return on inventions too small to encourage cogitation.

h3 $\beta\underline{w} < h$: children of low wage workers who did not get a return from invention cannot go to university.

h4 $\min\left\{\beta\overline{w}, \frac{\beta\theta}{p(\overline{\sigma})n\left(1-2\sqrt{c\mu}\right)}\right\} > h$: children of high wage workers and children of inventors can go to university.[20]

[20] Under (h1) and (h2), the minimum revenue that an inventor can generate occurs when $S_{t-1} = S_0$ and $S_t > \bar{S}$. From Proposition 1 (ii) and the formula for S_0, this is just $\frac{\theta}{p(\bar{\sigma})n\left(1-2\sqrt{c\mu}\right)}$.

The state variable is S and as we have already emphasized, today's state is a function of the two last period states. However, our assumptions enable use to reduce the recursion to one period only. First, note that the assumptions imply the following.

Lemma 2 *Assume h1-h4.*
(i) If $S_{t-1} < S_0$*, then* $S_t = \alpha S_{t-1}$
(ii) If $S_0 \leq S_{t-1} < \bar{S}$*, then* $S_t = \alpha S_{t-1} + p(\sigma_{t-1})(1 - \alpha S_{t-1})$
(iii) If $S_{t-1} > \bar{S}$*,* $S_t = 1$.

Proof. (i) There is excess supply but the condition for an operative invention market is not satisfied; workers get a low wage $\underline{w}$ and their children cannot go to university. The only children who can go to university are those of the αS entrepreneurs: since their profits exceed $\bar{w}$, by (h4), they can afford the tuition.
(ii) There is excess supply, hence the wage is$\underline{w}$, but now the invention market is operative. As before children of entrepreneurs go to university school, and there are αS of them. Also by (h4), children of the workers who invent can also go to university, and there are $\alpha p(\sigma)(1 - S)$ of them.
(iii) There is excess demand, hence high wages, and all children can go to university.
■

As a consequence of Lemma 2, the dynamics are simplified to a first order system. Indeed, given S_{t-1}, the only other information required is σ_{t-1}, and then only if $S_{t-1} < \bar{S}$. Now if $S_{t-1} > \bar{S}$; then by (iii), $S_t = 1$ and therefore if there is excess demand in one period, there will be excess demand forever after. It follows that when $S_{t-1} < \bar{S}$, $S_{t-2} < \bar{S}$ as well, so that $\sigma_{t-1} = \overline{\sigma}$. In other words, S_{t-2} enters only through its determination of σ_{t-1}, and knowing S_{t-1} is enough to conclude what the value of σ_{t-1} must be.

We can therefore write the dynamics in the following simplified form:

$$S_t = \begin{cases} \alpha S_{t-1}, \text{ if } S_{t-1} < S_0 \\ \alpha S_{t-1} + p(\bar{\sigma})(1 - \alpha S_{t-1}), \text{ if } S_0 \leq S_{t-1} < \bar{S} \\ 1, \text{ if } S_{t-1} \geq \bar{S} \end{cases}$$

There are two or three steady states, depending on whether the fixpoint S^* of $\alpha S + p(\overline{\sigma})(1 - \alpha S)$ lies in $[S_0, \bar{S})$. In either case, all steady states are locally stable.

If $S^* = \frac{p(\overline{\sigma})}{[1-\alpha+\alpha p(\overline{\sigma})]} < S_0$, we are in the "dismal case": the two steady states are $S = 0$ and $S = 1$: both cases are incompatible with growth. When $S = 0$, the economy has collapsed; $S = 1$, by contrast is a statically prosperous economy in which everyone attends university and become a small scale entrepreneur. By (h1) this implies that there is stagnation: given the low degree of specialization, the arrival rate of ideas is too low to induce anyone to attempt to invent. Collapse eventually occurs if the economy starts below $\bar{S}$; stagnant prosperity results if it begins above $\bar{S}$. There might however, be a short period of innovation in case the economy happens to start in $[S_0, \bar{S}]$, but collapse soon follows. See Figure 1.

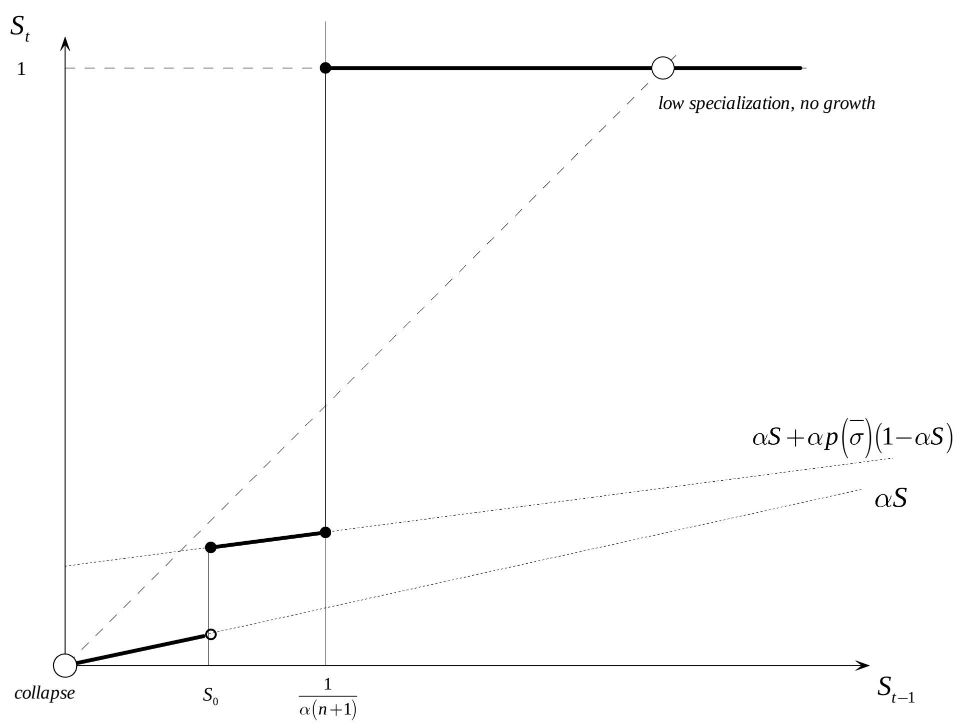

Figure 1: The dismal case: no long term growth is possible.

A slightly less dismal case occurs if $S^* > \bar{S}$; in this case $p(\bar{\sigma})$ is relatively large, and the $\alpha S + p(\overline{\sigma})(1 - \alpha S)$ branch lies above the 45°-line. Permanent high growth is not possible, though again the economy may experience growth for a few periods. The basin of attraction for collapse is smaller than in the dismal case, consisting only of the interval $[0, S_0)$.

The case of greatest interest is the "hopeful" one in which $S_0 \leq S^* < \bar{S}$, in which there is another locally stable steady state at S^*. Any economy beginning in the interval $[S_0, \bar{S})$ converges to S^*. Here the wage share is low and there is a high degree of division of labor, and a technological growth rate of $(1+\gamma)^{p(\overline{\sigma})}$ (see Figure 2).

Assumption (h1) could be relaxed to admit the possibility of a cogitation equilibrium for levels of S exceeding $\bar{S}$ – so that for instance $\bar{S} < S_1 < 1$ – but growth, equal to $(1+\gamma)^{p(\underline{\sigma})}$ will be slower than at S^*, since $p(\sigma)$ is increasing.

If we relax assumption (h2), so that $S_0 > \bar{S}$, then we are in a truly dismal case where the innovation market is *never* operative and the economy proceeds either to collapse or to prosperous stagnation.

Note except in the nongeneric case that $S^* = \bar{S}$, the economy cannot spend any time at $\bar{S}$ unless it happens to start there, so that we are justified in ignoring the cases of intermediate wages.

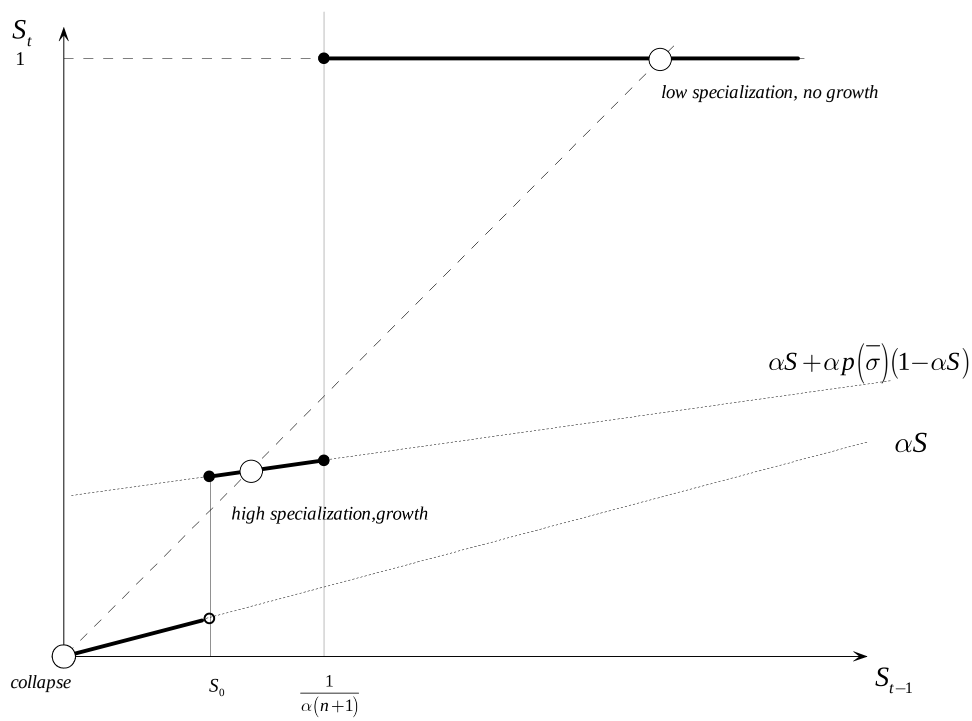

Figure 2: The possibility of long-term technological progress.

The hopeful case depicted in Figure 2 is the one that suggests the possibility of an inverse U relation between the degree of inequality (measured by $1/S$) : economies with *either* high or low degrees of inequality (low or high S) grow slowly or not at all, while those with middling levels are the ones that generate sustained technical progress.

4.1 Comparative Dynamics

Here we consider three types of changes: reductions in the cost of education, improvements in managerial scope, and increases in technological complexity. The first is similar to the effects of an improvement in credit market institutions, though to model that, the analysis is slightly more intricate and is omitted. The latter two are suggestive of some of the challenges that an economy that relies on the Smithian growth mechanism faces when confronted by "meta" technological changes or exposure to more developed economies.

4.1.1 "Institutional" Improvements (h)

Model this as a reduction in h. The leading interpretation is a reduction in the (private) cost of education either through technological improvement or more plausibly through state subsidy. An improvement in the functioning of the credit market will work very similarly: given a wealth distribution, either changes increases the fraction of the population that can afford the higher education.

For a formal treatment, it is helpful to assume that in addition to the bequest out of parent's earnings, individuals receive a random endowment ω of the consumption good. This has distribution $\Omega(\omega)$ and the extra wealth realization is independent across individuals and from the parent's occupation. The set of individuals who attend school at t will now be those for whom $\omega \geq h - \beta y_t$, where y_t is the parent's income. Notice that this modification doesn't change the value of S_0 and $\bar{S} = \frac{1}{\alpha(n+1)}$.

But it does of course change the dynamics of S. Assume we are in the case where there are three steady states. Now we have (letting p denote $p(\bar{\sigma})$)

$$S_t = \begin{cases} \alpha S_{t-1} + n\alpha S_{t-1}(1-\Omega(h-\beta\sqrt{c\mu})) + (1-\alpha(n+1)S_{t-1})(1-\Omega(h)), \\ \qquad \text{if } S_{t-1} < S_0 \\ \alpha S_{t-1} + (1-\alpha S_{t-1})p + (1-\alpha S_{t-1})(1-p)[1-\frac{n\alpha S_{t-1}}{1-\alpha S_{t-1}}\Omega(h-\beta\sqrt{c\mu}) - (1-\frac{n\alpha S_{t-1}}{1-\alpha S_{t-1}})\Omega(h)], \\ \qquad \text{if } S_0 \leq S_{t-1} < \bar{S} \\ 1, \qquad \text{if } S_{t-1} > \bar{S} \end{cases}$$

Starting with high levels of h for which $\Omega(h-\beta\sqrt{c\mu}) = 1$, little changes substantially until $\Omega(h) < 1$. If the economy is below S_0 to start, it will now converge to a level of S that is higher than zero. Further reductions in h will allow for convergence to a level of S that exceeds S_0, .which then implies a "takeoff" into industrialization and high growth. Depending on parameters, the economy may eventually leave the interval $[S_0, \bar{S})$, and the growth process slow down, or it may converge to a steady state in that interval in which case it will enjoy permanent high growth (and a more educated workforce than the high growth steady state before the reduction in h). Finally, if h falls further still, everyone goes to the upper level school, and growth slows. This accords with the contrasting experiences of Germany (especially Prussia), which was relatively late to industrialize despite early widespread public education, and Britain, the first industrial nation, which nevertheless was rather slow to educate the masses (West, 1975).

Thus, "improvements in institutions" may have ambiguous effects, depending on where the economy is to begin with. An economy that has very poorly functioning credit markets or costly education will generally be helped by improvements in these institutions. But economies in which these institutions are functioning moderately well may actually be hurt.

If one were to measure the rate of TFP growth across economies with different qualities of institutions, one may therefore find that growth rates are not monotonic in the quality measure. Similarly, since the levels TFP will depend on the history of their growth, neither should there be any expectation of finding a monotonic pattern in a cross-country regression of TFP levels on institutional quality.

4.1.2 Managerial Scope (n)

Consider figures 1 and 2. Increasing n makes excess supply less likely since $1/\left(\alpha(n+1)\right)$ decreases, but also makes S_0 smaller. Hence starting from a situation like in Figure 1, a larger managerial scope will eventually render the recursion diagram similar to that of Figure 2. Starting from a situation as in Figure 2, increasing n will first preserve the steady state with high specialization and growth, but then for n large enough, growth will stop and the economy will either collapse or go to the low specialization, no growth, steady state. On the other hand an economy on the path to collapse might be "rescued" by an increase in managerial scope.

4.1.3 Technological Complexity

Complexity of the technology can be represented by the task to population ratio; call it χ (thus far, it has been assumed equal to one). The production technology becomes $\exp(\int_0^{\chi} \log[a_t(j)l_t(j)]dj)$. The growth rate is now $(1+\gamma)^{p(\sigma)/\chi}$, and the threshold value S_0 of educated that sustains growth is $\frac{\theta}{\alpha[(1+\gamma)^{p(\sigma)/\chi}-1]n(1-2\sqrt{c\mu})}$. With simple technologies (χ small), growth rates sustained by the Smithian mechanism are high, and the economy is more likely be able to sustain growth because S_0 is small and $\bar{S}$ is unchanged.[21] An exogenous increase in the complexity of technology will lower growth rates and increase the likelihood that technological progress comes to a halt.

If one interprets this complexity as originating in a world technological frontier, one that may be expanding through "internalized" R&D, an implication of this observation is that Smithian growth may be a particularly inadequate mechanism by which a developing country might catch up to the rest of the world. Although an increase in χ instantaneously increases output (assuming the $a(j)$ of the "new" steps are equal on average to the rest), over the longer run, this benefit would be overshadowed by the slowdown in growth, and a Smithian country will fall further and further behind the R&D-driven frontier.

[21] It may be reasonable to suppose that increased complexity would increase the coordination cost for a given choice of σ, since there are now more tasks to coordinate. For instance the coordination parameter cost might be written as $c\chi$. All else the same, the division of labor would be reduced with increased complexity and the effects on growth exacerbated.

5 Conclusions

We have analyzed a causal link between the organization of firms and economic growth. Starting from the "Smithian" idea that there is an increased likelihood of innovations in the production process when labor is more specialized, we show that entrepreneurs may be induced to choose innovation-enhancing organizations even though intractable contractual incompleteness and incentive problems prevent them from appropriating the returns to innovations. The conditions that do this in a laissez-faire market equilibrium depend on a constellation of factors: a free enterprise legal environment that allows an individual with an idea to sell it on a "monopolistically competitive" market; an imperfect credit market that restricts entry into entrepreneurship; and a distribution of wealth that is neither too equal nor too unequal. An economy that satisfies the first two conditions may violate the third and never grow. An economy that does grow initially may eventually violate the distributional condition, or may, for reasons having to do with changed expectations, improvements in credit markets or subsidized education, switch to a no-growth equilibrium, with firms that are too unspecialized to foster further Smithian innovation: "trickle down" effects may eventually limit growth.

The chart in Figure 3 plots average growth rates of per capita GDP against inequality for several European countries for the period 1820-1870. Growth rates are from Maddison (2001), and inequality is measured as the ratio of skilled to unskilled wages, taken from Allen (2005).[22] The inverted U pattern is clearly displayed, with growth rates for the lowest skill premium countries (Netherlands and France) slightly higher than those for the highest skill premium country (Spain). (The contrast between the Netherlands, for which the wage data are from (Protestant) Amsterdam, and Belgium, for which wages are from (Catholic) Antwerp, is also striking.) Obviously evidence of this sort is at best indicative (for instance, institutions are not identical across countries, and we would rather have TFP growth data than GDP growth), but it does accord broadly with the basic predictions of our model.

[22] Since Allen's wage data are for Milan, "Italy" is actually Lombardy 1836-1857, with the growth the rate from Pichler (2001) calibrated to Maddison's other estimates using the two authors' estimates for Austria (Maddison's estimate for all of Italy is somewhat lower than our imputed estimate for Lombardy; this haslittle effect on the basic pattern).

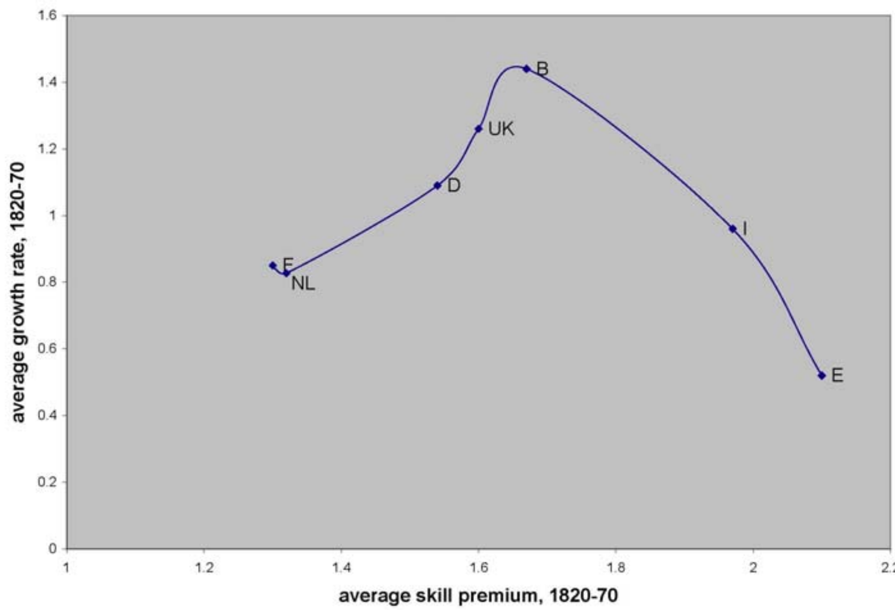

Figure 3: Inequality vs. growth. Sources: Allen (2005), Pichler (2001), and Maddison (2001, p.186). Countries in sample: Netherlands, France, Germany, United Kingdom, Belgium, Italy, Spain

The analysis also offers a specific interpretation to the view, expressed by some historians and economists (e.g., Crafts, 1985; Acemoglu and Zilibotti, 1997), that the Industrial Revolution occurred because of "luck." Many factors, partly institutional, partly technological , and partly distributional, must fall into place in order for a period of sustained technological growth to emerge. As our model suggests, the path to sustained prosperity is a narrow one, difficult to find, and easily lost.

6 Appendix

The expressions for technology, returns and growth rates are derived in the standard way, namely by thinking of the continuum as the limit of "large-square" (here population and tasks) finite economies.

Specifically, the production technology is $\prod_{j=1}^{J}[a(j)l(j)]^{1/J}$, the population is N, and we take limits as N and J go to infinity while N/J remains constant (for simplicity set the ratio to unity).

Let E be the number of potential entrepreneurs. There are I inventors, each having one idea to license; inventors line up in random order at each entrepreneur and make a take it or leave it offer for licensing their innovation, then move on to the next entrepreneur.[23]

The counterparts of expressions (3) and (6) for M inventions purchased are

$$A(M/J) = A(1+\gamma)^{M/J},$$
$$V(M/J,u) = \max\{\pi(A(M/J),u), A(M/J)\}. \tag{9}$$

An entrepreneur who has already purchased $M-1$ inventions is willing to pay $V(M/J,u) - V((M-1)/J,u)$ for an additional innovation, which is positive for any value of M. Therefore all inventors in line will sell their innovations; the Mth in line offers a price $V(M) - V(M-1)$. It follows that that the entrepreneur pays for the I inventions

$$\begin{aligned}
&\sum_{M=1}^{I}[V(M/J,u) - V((M-1)/J,u)] \\
&= V(I/J,u) - V(0,u) \\
&= \max\{\pi(A(I/J),u), A(I/J)\} - \max\{\pi(A,u), A\}.
\end{aligned}$$

Hence, the inventors are able to extract all the surplus from the potential entrepreneur.

Since the position in a line is random and independent of the position in other lines, if there are E potential entrepreneurs, the expected revenue from an invention

[23] The construction below leads in fact to the same outcome as the Shapley value – if one assumes that the entrepreneurs are "trading posts" and do not enter also randomly in the bargaining process.

when there are I inventions is

$$q\left(I/J,u\right)E=\frac{E}{I}\left[V\left(I/J,u\right)-V\left(0,u\right)\right]. \tag{10}$$

We now show that the price for and innovation is increasing in the number I of inventors by showing that if $\sqrt{c\mu}<1/4$, then (i) $V\left(M/J,u\right)$ is convex in M and therefore that (ii) $q\left(I/J,u\right)$ is increasing in I. Recall that

$$\pi\left(A\left(M/J\right),u\right)=\begin{cases}\bar{\pi}\left(A\left(M/J\right),u\right)\equiv n\left(1-2\sqrt{c\mu}\right)A\left(M/J\right), \text{ if } \sqrt{c\mu}A\left(M/J\right)\geq u\\ \hat{\pi}\left(A\left(M/J\right),u\right)\equiv n\left(1-c\frac{\mu A(M/J)}{u}-\frac{u}{A(M/J)}\right)A\left(M/J\right), \text{ if } \sqrt{c\mu}A\left(M/J\right)\leq u\end{cases}$$

(i) $A\left(M/J\right)$ is increasing. Indeed, treating M as a continuous variable, we have $\frac{\partial A(M/J)}{\partial M}=\frac{\log(1+\gamma)}{J}A\left(M/J\right)>0$. So there is at most one M^{*} s.t. $\sqrt{c\mu}A(M^{*}/J)=u$.

$A(M/J)$ is convex because $\frac{\partial^{2}A(M/J)}{\partial M^{2}}=\left(\frac{\log(1+\gamma)}{J}\right)^{2}A\left(M/J\right)>0$.

If $M\geq M^{*}$, $\pi\left(A\left(M/J\right),u\right)=\bar{\pi}\left(A\left(M/J\right),u\right)$, which is convex since $A(M/J)$ is and $1-2\sqrt{c\mu}>0$.

If $M<M^{*}$, $\pi\left(A\left(M/J\right),u\right)=\hat{\pi}\left(A\left(M/J\right),u\right)$. Hence,

$$\frac{1}{n}\frac{\partial\pi\left(A\left(M/J\right),u\right)}{\partial M}=\frac{\log\left(1+\gamma\right)}{J}A\left(M/J\right)-2c\frac{\log\left(1+\gamma\right)}{J}\frac{\mu A\left(M/J\right)^{2}}{u}$$

and

$$\begin{aligned}\frac{1}{n}\frac{\partial^{2}\pi\left(A\left(M/J\right),u\right)}{\partial M^{2}} &= \left(\frac{\log\left(1+\gamma\right)}{J}\right)^{2}A\left(M/J\right)-4c\left(\frac{\log\left(1+\gamma\right)}{J}\right)^{2}\frac{\mu A\left(M/J\right)^{2}}{u}\\ &\propto A\left(M/J\right)\left(1-4c\frac{\mu A\left(M/J\right)}{u}\right)\end{aligned}$$

which is nonnegative if and only if $1-4c\frac{\mu A(M/J)}{u}\geq 0$, or when $u\geq 4c\mu A\left(M/J\right)$. Since $M<M^{*}$, $\sqrt{c\mu}A(M/J)<u$, whence $1-4c\frac{\mu A(M/J)}{u}>1-4\sqrt{c\mu}$. Thus it is enough to assume that $\sqrt{c\mu}\leq\frac{1}{4}$ for convexity of $\pi\left(A\left(M/J\right),u\right)$ in this region.
By definition, $\bar{\pi}\left(A\left(M^{*}/J\right),u\right)=\hat{\pi}\left(A\left(M^{*}/J\right),u\right)$. We have $\frac{\partial\bar{\pi}(A(M/J),u)}{\partial M}|_{M=M^{*}}=n\frac{\log(1+\gamma)}{J}A(M^{*}/J)(1-2\sqrt{c\mu})$. Meanwhile,

$$\frac{\partial\hat{\pi}\left(A\left(M/J\right),u\right)}{\partial M}|_{M=M^{*}}=n\frac{\log(1+\gamma)}{J}A(M^{*}/J)(1-2\sqrt{c\mu}).$$

Hence, at M^{*}, the left and right derivatives of $\pi\left(A\left(M/J\right),u\right)$ are well defined and equal, and so $\pi\left(A\left(M/J\right),u\right)$ convex. Since the maximum of convex functions is convex, $V(M/J,u)$ is convex.

(ii) $q(M/J,u) = [V(M/J,u) - V(0,u)]/M$. Since $V(M/J,u)$ is convex in M, so is $V(M/J,u) - V(0,u)$, and the latter is equal to 0 at $M = 0$. Thus the average $[V(M/J,u) - V(0,u)]/M$ is increasing in M.

Let S be the fraction of the population attending university and K the number who cogitate. Then E and I are independent binomial random variables, with $E \sim \mathrm{B}(SN,\alpha)$ and $I \sim \mathrm{B}(K, p(\sigma))$. If K agents cogitate, the expected revenue for an inventor is $\mathrm{E}\left(\frac{E}{I}[V(I/J,u) - V(0,u)]\right) = \mathrm{E}(E)\mathrm{E}\left(\frac{[V(I/J,u)-V(0,u)]}{I}\right) = \mathrm{E}\left(\frac{E}{N}\right)\mathrm{E}\left(\frac{[V(I/N,u)-V(0,u)]}{I/N}\right) = \alpha S q(I/N,u)$, where we have taken $J = N$.

Therefore, if there are already $K-1$ agents who cogitate, an agent who cogitates has an expected revenue from cogitating of

$$L(K,p(\sigma),u) = \alpha S p(\sigma) \sum_{I=0}^{K} \Pr(I|K,p(\sigma))\, q(I/N,u), \tag{11}$$

where $\Pr(I|K,p(\sigma))$ is the binomial probability $\binom{K}{I} p(\sigma)^I (1-p(\sigma))^{K-I}$.

Note that the distribution $\Pr(I|K-1,p(\sigma))$ is dominated in the first order by the distribution $\Pr(I|K,p(\sigma))$. To verify this, it is enough to show that $\frac{\sum_{I=0}^{M}\binom{K-1}{I}p^I(1-p)^{K-1-I}}{\sum_{I=0}^{M}\binom{K}{I}p^I(1-p)^{K-I}}$ is nonincreasing in $M \le K-1$. That is,

$$\frac{\sum_{I=0}^{M}\binom{K-1}{I}p^I(1-p)^{K-1-I}}{(1-p)K\sum_{I=0}^{M}\frac{1}{K-I}\binom{K-1}{I}p^I(1-p)^{K-1-I}} \ge \frac{\sum_{I=0}^{M+1}\binom{K-1}{I}p^I(1-p)^{K-1-I}}{(1-p)K\sum_{I=0}^{M+1}\frac{1}{K-I}\binom{K-1}{I}p^I(1-p)^{K-1-I}}$$

$$= \frac{\sum_{I=0}^{M}\binom{K-1}{I}p^I(1-p)^{K-1-I} + \binom{K-1}{M+1}p^{M+1}(1-p)^{K-M-2}}{(1-p)K\sum_{I=0}^{M}\frac{1}{K-I}\binom{K-1}{I}p^I(1-p)^{K-1-I} + \frac{(1-p)K}{K-M-1}\binom{K-1}{M+1}p^{M+1}(1-p)^{K-M-2}},$$

or

$$\sum_{I=0}^{M}\binom{K-1}{I}p^I(1-p)^{K-1-I} \ge$$

$$\frac{\binom{K-1}{M+1}p^{M+1}(1-p)^{K-M-2}}{\frac{(1-p)K}{K-M-1}\binom{K-1}{M+1}p^{M+1}(1-p)^{K-M-2}}(1-p)K\sum_{I=0}^{M}\frac{1}{K-I}\binom{K-1}{I}p^I(1-p)^{K-1-I}$$

$$\Longleftrightarrow \sum_{I=0}^{M}\binom{K-1}{I}p^I(1-p)^{K-1-I} \ge \sum_{I=0}^{M}\frac{K-M-1}{K-I}\binom{K-1}{I}p^I(1-p)^{K-1-I},$$

which is clearly satisfied since $I \le M+1$.

Since $q(I/N,u)$ is increasing in I, $L(K,p(\sigma),u) > L(K-1,p(\sigma),u)$. It follows that either $K=N$, so that all agents cogitate or that $K=0$, so none do. For if $L(K-1,p(\sigma),u) \geq \theta A$, any vegetating agent strictly benefits by cogitating instead.

Supposing that all agents cogitate, the number of innovators is the binomial random variable I with mean $Np(\sigma)$ and variance $Np(\sigma)(1-p(\sigma))$. The growth rate is $A'/A = (1+\gamma)^{I/N}$ and the expected revenue per inventor is $\alpha Sq(I/N,u)$. Taking limits as $N \to \infty$, by Bernstein's Theorem (e.g. Billingsley, 1995, p. 87) , the growth rate is almost surely $(1+\gamma)^{p(\sigma)}$ and the expected revenue almost surely $p(\sigma)\alpha Sq(p(\sigma),u) = \alpha S[V(p(\sigma),u) - V(0,u)]$, which are the expressions given in the text.

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